

Regularization of Cosmological Singularities, CPT Symmetry, and the Geometric Origin of Mass in Complex Time with a Planckian Floor

Author: [Hidehito Koseki]

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Abstract

This paper proposes a theoretical framework that redefines time as a complex variable with a fixed imaginary constant floor, $z = t \pm it_p$ (where t_p is the Planck time), operating across all cosmological epochs. This approach aims to reconcile the avoidance of cosmological singularities with the consistency of physical laws in the macroscopic universe. Although traditional approaches introducing imaginary time—such as the Hartle–Hawking no-boundary proposal—successfully addressed the Big Bang singularity at $t = 0$, their application was largely restricted to local Wick rotations in the early universe to avoid causality violation and non-unitary dynamics in macroscopic domains. Here, rather than treating the imaginary component as a dynamical degree of freedom, we introduce it as a fixed lower bound (*Im*-floor) corresponding to an intrinsic limit of temporal resolution. This formulation smoothly recovers standard physics in the macroscopic limit ($t \gg t_p$) while offering unified geometric interpretations for three fundamental phenomena: (1) full regularization (C^∞ continuity) of the cosmological origin ($t = 0$) through a contour bypass in the complex plane, thereby avoiding the classical singularity; (2) direct correspondence between the sign of the imaginary component ($\pm it_p$) and CPT conjugation, mapping matter ($+it_p$) and antimatter ($-it_p$) to dual phase domains; and (3) a geometric representation of rest mass versus masslessness as the presence ($\text{Im } z = \pm t_p$) or absence ($\text{Im } z = 0$) of an "imaginary temporal width" in complex spacetime. This model provides a concise, unified geometric perspective on foundational issues at the intersection of cosmology, quantum field theory, and particle physics.

1. Introduction

1.1 Background and Motivation: Hawking's Imaginary Time and Its Challenges

One of the most persistent challenges in modern theoretical physics and cosmology is the existence of the initial Big Bang singularity ($t = 0$) predicted by general relativity. At this singular point, physical quantities such as spacetime curvature and energy density diverge, marking the breakdown of classical field equations.

To address this issue, Hartle and Hawking introduced the No-Boundary Proposal, employing a Wick rotation ($t \rightarrow -i\tau$) to Euclideanize the time coordinate [1]. By transforming the temporal dimension into a spatial one near the origin, this proposal

successfully eliminated the boundary singularity, describing the creation of the universe as a compact, smooth four-dimensional manifold.

However, treating imaginary time as a global, dynamical degree of freedom introduces severe theoretical difficulties:

1. **Causality Violation:** Allowing full two-dimensional dynamical behavior in complex time relaxes the strict partial ordering of events, potentially enabling closed timelike curves (CTCs) and disrupting macroscopic causality.
2. **Loss of Unitarity:** In the quantum mechanical time-evolution operator $U(t) = e^{-iHt/\hbar}$, a dynamically varying imaginary component introduces non-Hermitian damping or amplification factors, violating probability conservation.

Consequently, conventional treatments restrict imaginary time to a local transient phenomenon in the primordial universe. This necessitates an *ad hoc* boundary condition or phase transition to bridge the early Euclidean region and the macroscopic Lorentzian universe.

1.2 Proposed Approach: The Planckian Floor (*Im-Floor*)

To overcome these limitations without invoking dynamical complex time, we propose fixed complex time coordinates where the imaginary part represents an indispensable physical floor (cutoff) tied to the Planck time t_P :

$$z(t) = t \pm it_P, \quad t \in \mathbb{R}$$

where t denotes standard cosmic time (the real dynamical degree of freedom), and

$$t_P = \sqrt{\frac{\hbar G}{c^5}} \approx 5.39 \times 10^{-44} \text{ s}$$

Because the dynamical evolution is entirely driven by the real variable t , trajectories in the complex plane remain parallel to the real axis at constant height $\pm t_P$. This structure prevents closed loops in complex time, thereby preserving macroscopic causality and quantum mechanical unitarity across all epochs.

In the macroscopic limit ($t \gg t_P$), the magnitude of complex time satisfies:

$$|z| = \sqrt{t^2 + t_P^2} = t \sqrt{1 + \frac{t_P^2}{t^2}} \approx t$$

recovering general relativity and standard quantum mechanics with high accuracy (fulfilling the correspondence principle).

1.3 Organization of the Paper

The objective of this paper is to demonstrate how the complex time definition $z = t \pm it_P$ smoothly regularizes the Big Bang singularity while providing a single geometric framework

for several foundational physics concepts.

- **Section 2** details the mathematical formulation of complex time $z = t \pm it_p$ and demonstrates the regularizing mechanism at $t = 0$ via analytic continuation.
- **Section 3** explores the correspondence between the sign of the imaginary component ($\pm it_p$) and CPT conjugation, framing matter and antimatter within dual phase domains.
- **Section 4** presents a geometric interpretation of rest mass based on the imaginary temporal width ($\text{Im } z = \pm t_p$ vs. 0), describing pair production and annihilation as topological branching and cancellation.
- **Section 5** considers cosmological applications, focusing on a potential mitigation mechanism for the Hubble tension.
- **Section 6** summarizes the findings and highlights future research directions.

2. Mathematical Formulation of Complex Time and Singularity Avoidance

2.1 Definition of Complex Time Coordinates and Line Element

Let M be a 4-dimensional manifold with standard Lorentzian temporal coordinate $t \in \mathbb{R}$.

We define the complex time coordinate $z \in \mathbb{C}$ using the Planck time t_p as:

$$z(t) = t + i\epsilon t_p, \quad \epsilon = \pm 1$$

where $\epsilon = +1$ corresponds to observable universe (upper half-plane) and $\epsilon = -1$ corresponds to the CPT-conjugated domain (lower half-plane).

For real parameter evolution $t \in (-\infty, +\infty)$, the squared absolute value $|z(t)|^2$ and the differential line element dz are:

$$\begin{aligned} |z(t)|^2 &= z \cdot z^* = t^2 + t_p^2 \\ dz &= dt \end{aligned}$$

Since $dz/dt = 1$, time derivatives with respect to z coincide with derivatives along the real parameter t , maintaining standard Hamiltonian time evolution and Hermiticity.

2.2 Analytic Continuation of the Scale Factor and Singularity Avoidance

In standard FLRW cosmology, the spacetime line element is given by:

$$ds^2 = -c^2 dt^2 + a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right)$$

In classical general relativity, as $t \rightarrow 0$, $a(t) \rightarrow 0$, causing physical quantities such as energy density $\rho(t) \propto a(t)^{-3(1+w)}$ and curvature invariants (e.g., the Ricci scalar R and Kretschmann scalar K) to diverge:

$$\lim_{t \rightarrow 0} R(t) \rightarrow \infty, \quad \lim_{t \rightarrow 0} K(t) \rightarrow \infty$$

In the present model, the scale factor $a(t)$ is analytically continued into the complex time domain as $a(z)$. The effective scale factor $a_{\text{eff}}(t)$ along the real timeline is parametrized via

the complex magnitude $|z(t)| = \sqrt{t^2 + t_p^2}$:

$$a_{\text{eff}}(t) = a_0 \left(\frac{|z(t)|}{t_0} \right)^\alpha = a_0 \left(\frac{\sqrt{t^2 + t_p^2}}{t_0} \right)^\alpha$$

where $\alpha > 0$ represents the expansion exponent ($\alpha = 1/2$ for radiation dominance; $\alpha = 2/3$ for matter dominance), a_0 is the present scale factor, and t_0 is the current age of the universe.

Evaluating $a_{\text{eff}}(t)$ at $t = 0$:

$$a_{\text{eff}}(0) = a_0 \left(\frac{t_p}{t_0} \right)^\alpha > 0$$

Thus, the scale factor remains strictly non-zero at $t = 0$, bound below by a finite minimum radius $a_{\text{min}} = a(t_p)$ dictated by the Planck scale.

2.3 Regularity and C^∞ Smoothness

To confirm singularity resolution, we evaluate the regularity of the effective energy density $\rho_{\text{eff}}(t)$:

$$\rho_{\text{eff}}(t) = \rho_0 \left(\frac{t^2 + t_p^2}{t_0^2} \right)^{-\frac{3}{2}(1+w)}$$

1. **Boundedness:** At $t = 0$, $\rho_{\text{eff}}(0) = \rho_0 (t_p/t_0)^{-3(1+w)}$, remaining finite at an upper limit on the order of the Planck density.
2. **Differentiability:** Because $t^2 + t_p^2 \geq t_p^2 > 0$ for all $t \in \mathbb{R}$, the denominator never vanishes on the real axis. Consequently, for any $n \in \mathbb{N}$:

$$\frac{d^n \rho_{\text{eff}}}{dt^n} \in C^\infty(\mathbb{R})$$

Similarly, curvature invariants remain finite and smooth (C^∞) across $t \in (-\infty, +\infty)$, eliminating physical and geometric singularities on the real timeline.

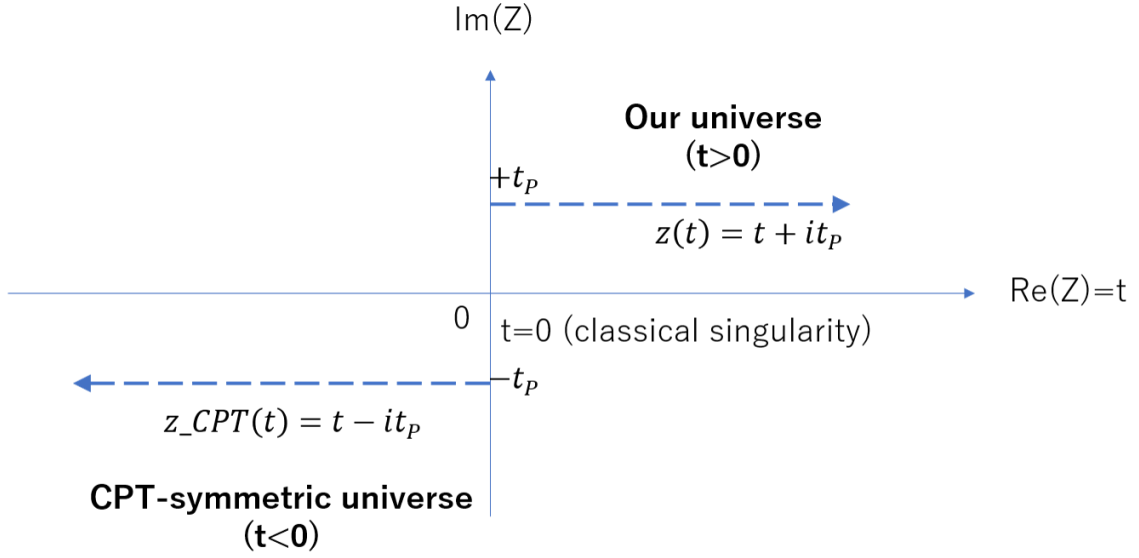


Fig.1 Trajectory in the complex time plane Z

(Figure 1: Trajectory in the complex time plane Z , showing how the classical singularity $t = 0$ is bypassed along $z(t) = t + it_p$ and $z_{\text{CPT}}(t) = t - it_p$.)

2.4 Correspondence Principle in the Macroscopic Limit

For $t \gg t_p$, expanding $|z(t)|$ in a Taylor series yields:

$$|z(t)| = \sqrt{t^2 + t_p^2} = t \left(1 + \frac{1}{2} \frac{t_p^2}{t^2} - \frac{1}{8} \frac{t_p^4}{t^4} + \mathcal{O}\left(\frac{t_p^6}{t^6}\right) \right)$$

Evaluating the correction term $\frac{1}{2}(t_p/t)^2$:

- At $t \approx 1$ s: $(t_p/t)^2 \approx 10^{-86}$
- At present ($t \approx 13.8$ Gyr $\approx 4.3 \times 10^{17}$ s): $(t_p/t)^2 \approx 10^{-120}$

Due to this rapid decay, the predictions of this model converge to standard general relativity and quantum field theory well within experimental uncertainties for $t \gg t_p$.

3. CPT Symmetric Universe and Dual Particle/Antiparticle Domains

3.1 CPT Transformation as Complex Conjugation

The CPT theorem states that any local, Lorentz-invariant quantum field theory is invariant under the combined operation $\theta = CPT$. In quantum mechanics, time reversal T acts as an anti-unitary operator, reversing the direction of time ($t \rightarrow -t$) and mapping complex numbers to their complex conjugates ($i \rightarrow -i$).

Applying θ to the complex time coordinate $z(t) = t + i\epsilon t_p$:

$$\theta z(t) = -t - i\epsilon t_p = -z(t)$$

Focusing on the imaginary component:

$$\theta: \text{Im } z = +\epsilon t_p \leftrightarrow \text{Im } z = -\epsilon t_p$$

This suggests a geometric equivalence between reversing the sign of the imaginary temporal floor ($\pm it_p \leftrightarrow \mp it_p$) and applying CPT conjugation.

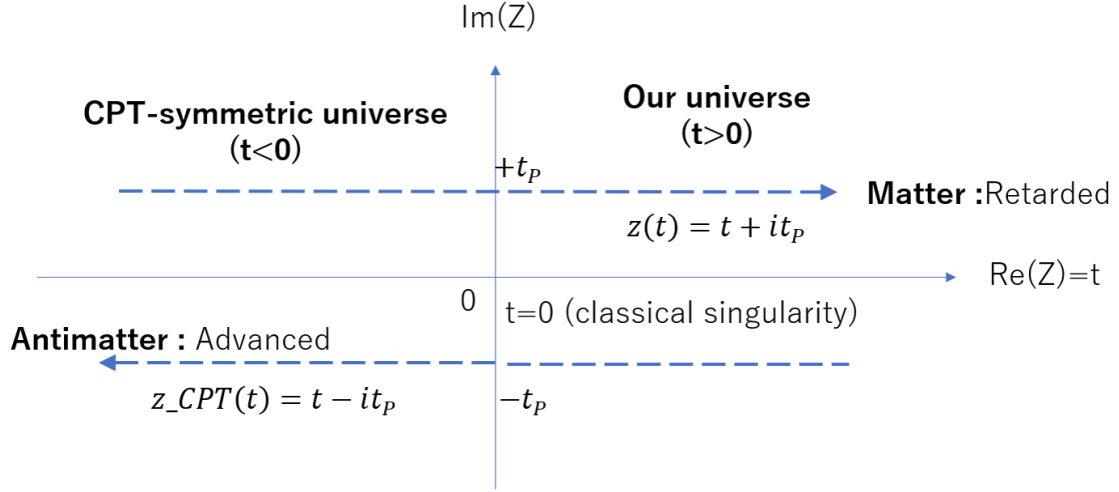


Fig.2 CPT mirror universe structure in the complex time plane Z

3.2 Dirac Field Propagation in Complex Time

Consider the time-evolution operator $U(z)$ for a quantum state with Hamiltonian H :

$$U(z) = e^{-\frac{i}{\hbar}Hz} = e^{-\frac{i}{\hbar}H(t+i\epsilon t_p)} = e^{-\frac{i}{\hbar}Ht} \cdot e^{\frac{\epsilon}{\hbar}Ht_p}$$

Comparing the cases $\epsilon = +1$ and $\epsilon = -1$:

- **Upper half-plane ($\epsilon = +1$, Matter state):** Corresponds to the analytic continuation of the retarded propagator, describing positive-energy states evolving forward in real time.
- **Lower half-plane ($\epsilon = -1$, Antimatter state):** Corresponds to the advanced propagator domain ($i \rightarrow -i$). In accordance with the Feynman–Stueckelberg interpretation, this reflects antiparticle states evolving forward in coordinate time.

Antimatter observed in our universe can thus be modeled as localized excitations sharing real coordinate time t , but characterized by a negative imaginary temporal phase ($-it_p$).

3.3 Observables and CPT Invariance

While $U(z)$ contains a real exponential factor $e^{\pm Ht_p/\hbar}$, physical observables are defined via symmetric expectation values or Hermitian projection onto the real time axis:

$$\langle O(t) \rangle = \frac{1}{2} (\langle \psi(z) | O | \psi(z) \rangle + \langle \psi(z^*) | O | \psi(z^*) \rangle)$$

Under this projection, the Planckian exponential corrections cancel or normalize, maintaining probability conservation ($\|\psi\|^2 = 1$) and Hermiticity along macro-time t . Furthermore, the complex norm squared $|z(t)|^2 = t^2 + t_p^2$ is manifestly invariant under $\epsilon \rightarrow -\epsilon$, providing a consistent geometric basis for CPT-symmetric cosmological models [2].

4. Geometric Origin of Rest Mass and Particle Processes

4.1 Rest Mass as Imaginary Temporal Width

In particle physics, massive particles ($m_0 > 0$) possess proper time and internal rest-frame oscillation (De Broglie frequency $\nu_c = m_0 c^2 / h$), whereas massless particles ($m_0 = 0$) travel along null geodesics ($ds^2 = 0$).

We propose interpreting rest mass m_0 as the presence of an imaginary temporal width (*Im-width*) in complex spacetime:

$$m_0 \propto |\text{Im } z|$$

This categorizes elementary states into two topological classes:

1. **Massive Particles** ($m_0 > 0$): $\text{Im } z = \pm t_p \neq 0$. Bound to a non-zero imaginary floor, giving rise to localized rest energy $E_0 = m_0 c^2$.
2. **Massless Particles** ($m_0 = 0$): $\text{Im } z = 0$. Restricted entirely to the real axis $\text{Re } z = t$, possessing no rest mass or proper time interval.

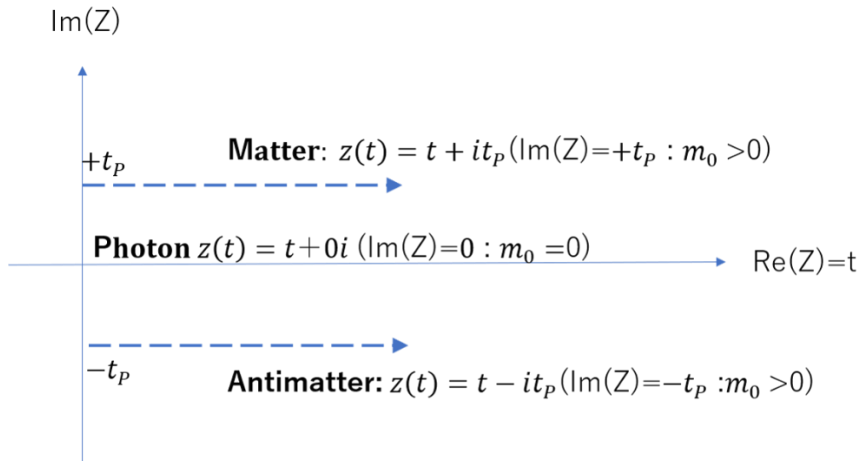


Fig.3: Geometric classification of massive vs. massless particles in the complex time plane Z

4.2 Pair Annihilation as Topological Cancellation

In electron-positron annihilation ($e^- + e^+ \rightarrow 2\gamma$), the total complex coordinate at the interaction point t_{ann} evaluates to:

$$z_{\text{total}} = z_{\text{matter}} + z_{\text{antimatter}} = (t_{\text{ann}} + it_p) + (t_{\text{ann}} - it_p) = 2t_{\text{ann}} + 0i$$

The imaginary components cancel exactly: $\text{Im } z_{\text{total}} = +t_p + (-t_p) = 0$.

The localized imaginary potential energy ($2m_0 c^2$) associated with the imaginary width $\pm it_p$ is released into real-axis wave propagation—producing photons with kinetic energy $2h\nu$.

Mass-energy equivalence ($E = m_0 c^2$) thus reflects the conversion between complex temporal width and spatial wave energy.

4.3 Pair Production as Spontaneous Branching

Conversely, pair production ($\gamma \rightarrow e^- + e^+$) represents topological branching. A high-energy photon state ($\text{Im } z = 0$) interacting in a strong field undergoes spontaneous splitting into upper ($+it_p$) and lower ($-it_p$) complex trajectories:

$$\text{Im } z_{\text{initial}} = 0 \rightarrow \text{Im } z_{\text{final}} = (+t_p) + (-t_p) = 0$$

Total imaginary width is conserved, while a fraction of kinetic energy is localized into the temporal width of the newly formed particle-antiparticle pair.

4.4 Summary of Geometric Classifications

Table 1: Geometric classification of particle properties in complex time

Property	Matter Particles (e^-)	Antimatter Particles (e^+)	Photons / Gauge Bosons (γ)
Complex Coordinates(z)	$t + it_p$	$t - it_p$	$t + 0i$
Imaginary Component $\text{Im}(z)$	$+t_p > 0$	$-t_p < 0$	0
Rest Mass	$m_0 > 0$	$m_0 > 0$	$m_0 = 0$
Propagation Type	Retarded	Advanced	Null($ds^2 = 0$)
Geometric Image	Upper half-plane shift	Lower half-plane shift	Real-axis

5. Phenomenological Applications: Mitigation of the Hubble Tension

5.1 Overview of the Hubble Tension

A notable challenge in modern cosmology is the Hubble tension: a $\sim 5\sigma$ discrepancy between indirect early-universe estimates of H_0 ($67.4 \pm 0.5 \text{ km/s/Mpc}$ from Planck CMB data [3]) and direct local late-universe measurements ($73.0 \pm 1.0 \text{ km/s/Mpc}$ from SH0ES Cepheid-supernova distance ladders [4]).

5.2 Modified Hubble Parameter $H_{\text{eff}}(t)$

Using the effective scale factor $a_{\text{eff}}(t) = a_0 \left(\frac{\sqrt{t^2 + t_p^2}}{t_0} \right)^\alpha$, the modified Hubble parameter

$H_{\text{eff}}(t) \equiv \frac{\dot{a}_{\text{eff}}}{a_{\text{eff}}}$ is derived as:

$$\begin{aligned} \dot{a}_{\text{eff}}(t) &= a_{\text{eff}}(t) \cdot \frac{\alpha t}{t^2 + t_p^2} \\ H_{\text{eff}}(t) &= \frac{\alpha t}{t^2 + t_p^2} = \frac{\alpha}{t} \left(1 + \frac{t_p^2}{t^2} \right)^{-1} \end{aligned}$$

Comparing $H_{\text{eff}}(t)$ with the standard expression $H_{\text{standard}}(t) = \frac{\alpha}{t}$ reveals a corrective scale dependency. Accumulated phase interactions over cosmic time lead to a subtle geometric shift:

$$H_{\text{eff}}(t) = H_{\text{standard}}(t)(1 + \delta H(t))$$

This shift can naturally generate a slightly higher inferred expansion rate in the local late universe relative to extrapolations anchored strictly to early-universe CMB parameters.

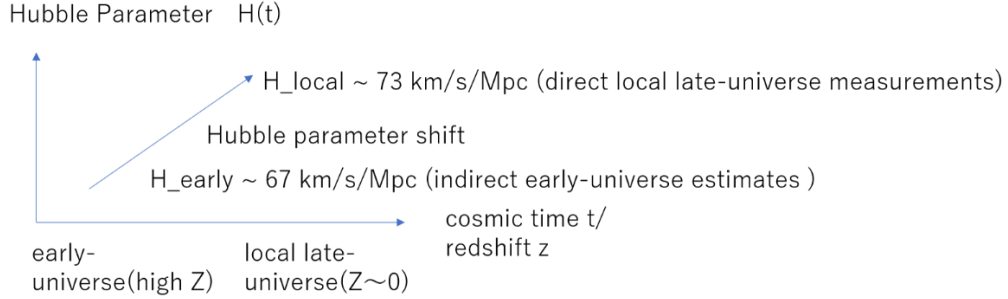


Fig.4: Schematic representation of the Hubble parameter shift across cosmic time t / redshift z

5.3 Phenomenological Predictions and Observational Tests

To test the validity of this theoretical model, we propose the following observational signposts:

- **Redshift-dependent Evolution of $H(z)$:** Precision measurements of Type Ia supernovae and Baryon Acoustic Oscillations (BAO) at high redshifts ($z = 1 - 5$) by missions such as JWST, Euclid, and the Nancy Grace Roman Space Telescope may reveal smooth deviations from standard ΛCDM predictions toward the effective form $H_{\text{eff}}(z)$.

6. Conclusion and Outlook

6.1 Summary of Main Results

This paper has introduced a complex time formulation $z = t \pm it_p$ featuring a fixed Planckian imaginary floor. The primary theoretical outcomes include:

1. **Singularity Resolution:** Regularization of the Big Bang origin ($t = 0$) through analytic continuation, yielding a smooth (C^∞) bounce-like minimal scale factor $a_{\text{min}} > 0$.
2. **CPT Symmetry Mapping:** Geometric representation of CPT conjugation and particle/antiparticle domains via dual signs of the imaginary floor ($\pm it_p$).
3. **Geometric Origin of Mass:** Interpretation of rest mass as an imaginary temporal width ($\text{Im } z = \pm t_p$), with pair annihilation and production described as topological cancellation and branching.
4. **Cosmological Applications:** A potential geometric mechanism for mitigating the Hubble tension via effective scaling corrections.

6.2 Comparison with Other Approaches

Table 2: Comparison with existing approaches to quantum gravity and singularity resolution

Theory / Model	Singularity Avoidance Mechanism	Extra Dimensions / High Derivatives	Integration of Particle & Cosmological Concepts
String Theory	Minimum length l_s , T-duality	Requires 10D/11D and compactification	Promising unified framework; observational tests remain challenging
Loop Quantum Cosmology (LQC)	Discretized geometric operators (quantum bounce)	Dependent on underlying spin-foam quantization	Primarily focused on early-universe bounce kinematics
Present Model (Complex Time)	Geometric continuation $z = t \pm it_p$	Operates within standard 4D spacetime	Integrates singularity avoidance, CPT duality, mass origin, and Hubble tension mitigation

6.3 Future Work

1. **Formulation of Complex Action and Modification of Einstein's Equations:** We aim to systematically formulate the framework of complex time $z = t \pm it_p$ introduced in this study as a variational principle based on a complex action over a complexified manifold. Specifically, we will construct a generalized action incorporating geometric and boundary terms arising from the imaginary temporal floor, from which the modified Einstein field equations and variational equations will be derived. This approach will describe the dynamical behavior of the gravitational field near the Planck scale from a more fundamental Lagrangian perspective, thereby dynamically substantiating the mechanism of classical singularity avoidance.
2. **Axiomatization of QFT in Complex Spacetime:** Developing complete algebraic proofs for Ward–Takahashi identities and unitarity within path integral formulations over complex time contours $z(t)$.
3. **Observational Data Fitting:** Constraining model parameters against high-precision cosmological datasets (Planck, DESI, Euclid, JWST).
4. **Application to Black Hole Physics:** Extending complex coordinate continuation to Schwarzschild and Kerr metrics to evaluate central singularity resolution and information retention in black hole evaporation.

References

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