

VECTOR DIVISION AND GEOMETRIC INTERPRETATION OF COMPLEX EXPRESSIONS FOR FUNDAMENTAL THEOREMS IN ELEMENTARY GEOMETRY

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ABSTRACT. Coordinates or complex numbers may be regarded as quotients of a vector divided by a basis vector. This paper employs vector quotients to interpret the geometric meaning of complex numbers and transformed complex expressions for fundamental theorems in trigonometry. It demonstrates that different algebraic transformations admit distinct geometric interpretations, and the resulting conclusions are coordinate-independent.

Keywords: vector quotient, complex numbers, elementary geometry, coordinate-free geometry.

1. INTRODUCTION

The establishment of analytic geometry built a bridge between geometry and algebra, marking a major milestone in the history of mathematics. Nevertheless, in practical geometric reasoning, Li Hongbo [1] observed that classical automated deduction methods such as Wu's method and Gröbner basis algorithms often suffer from prohibitive computational complexity. Even when the final geometric conclusion takes a concise form, the intermediate expressions tend to be extremely cumbersome, rendering the extraction of geometric meaning difficult or even infeasible.

Leibniz once envisioned an algebraic system that could directly represent geometric primitives including points and directed segments, without recourse to coordinate systems. In such a system, addition, subtraction, multiplication and division would each carry transparent geometric interpretations.

Within Euclidean geometry, many problems tractable via analytic geometry admit simpler treatments using complex numbers [2]. Still, the historical development of complex numbers was tortuous. The vector interpretation of complex numbers plays an important role in establishing their status in mathematics. In 1797, Norwegian surveyor Caspar Wessel proposed that complex numbers may be interpreted as planar points or directed line segments, thereby establishing the geometric representation of complex numbers together with their operational rules [3]. Gauss further refined this idea, systematically relating complex numbers to plane vectors and clarifying their bijective correspondence.

This natural correspondence between complex numbers and plane vectors underpins the geometric interpretation of complex identities.

It should be emphasized that the notion of a vector in synthetic geometry differs substantially from the vector concept in linear algebra. While vector division is undefined in standard linear algebra, Ramírez JEH and Roca OE [12] introduced

definitions of vector division for 2D and 3D spaces; however, no practical application cases for these definitions have yet been reported. Geometrically, a vector is intuitively a directed segment endowed with magnitude and direction.

In 2023, Peng Xicheng, Zhang Jingzhong, Chen Mao and Liu Sannyuya [4] noted that vectors can be interconverted via scaling and rotation, yet they did not elaborate how such transformations are formally implemented. Several fundamental questions remain unresolved: Can we define multiplication and division operations between complex numbers and vectors? Why do complex numbers admit division while ordinary vectors do not? Such questions have generated wide-ranging discussion without reaching consensus.

A critical historical puzzle arises: why have the multiplication and division operations for directed line segments proposed by Wessel [3] never been incorporated into standard mathematics textbooks? A further question arises: are the geometric results formulated with directed segments in works by Liang Shaohong [5] and Johnson [6], such as the Menelaus theorem, genuinely vector statements?

Ambiguities of this kind have led to problematic claims. Zhang Jianyue, chief editor of China's standard high-school mathematics textbooks, asserted in a pedagogical paper that "complex numbers are vectors and vectors are complex numbers." This claim has sparked substantial controversy and criticism, and he has not responded to these criticisms to date.

Textbooks typically explain the geometric interpretation of the product of two complex numbers by treating one complex number as a vector and the other as a scaling-rotation operator. Li Hongbo [1], however, argued that although complex multiplication can encode vector scaling and rotation, it lacks intrinsic geometric meaning because its definition relies on the choice of the real axis. This position stands in tension with the conventional view. Zhang Bo [8] contended that an unclear definition of vector multiplication does not obstruct the proof of geometric theorems.

A deeper foundational question emerges: does the geometric meaning of relations expressed via coordinates or complex numbers depend on the coordinate system? Traditional expositions of complex arithmetic generally answer in the affirmative.

In 2023, Peng Xicheng and Zhang Jingzhong et al. [4] employed the concept of vector division to unify geometric conditions and conclusions within a single equation, though they did not supply a rigorous definition of vector division itself. In Peng Xicheng's monograph published in 2025 [9], a conjecture built upon vector division was proposed: every equational theorem in elementary geometry can be expressed as a simple, intuitive identity. Notably, the vector-division expressions appearing in their earlier paper [4] were omitted from this book, possibly to avoid conflict with the standard linear-algebraic fact that vector division is undefined.

In algebra, division is understood as the inverse operation of multiplication. Zhou Ning and Lin Xinjian [10] defined multiplication and division operations for plane vectors using the trigonometric form of complex numbers. However, they argued that such definitions are limited exclusively to two-dimensional cases and cannot be extended to higher-dimensional spaces, thus lacking universal value and theoretical significance.

Li Hongbo [1] discussed complex multiplication and questioned its intrinsic geometric meaning, but did not address division. Zhang Bo [8] explicitly maintained

that plane vectors can support division, yet also failed to provide an explicit rigorous definition.

These constructions have not entered standard textbooks, indicating that they have not received widespread adoption within the mathematical community.

Chinese senior high school textbooks introduce the historical development of complex numbers and vectors, but omit discussion of the operational relationships between these two mathematical objects [11]. At one time, an extended-thinking question “Do vectors have division?” appeared immediately after the section on vector dot products. This question triggered extensive public discussion, though this question was later removed from current textbook editions with no official explanation.

The present paper further clarifies how the concept of planar vector division accounts for the geometric interpretation of complex identities.

Like analytic geometry, the complex-number method for proving geometric theorems relies fundamentally on the line equation. Evan Chen [2] gave an extensive treatment of line equations but did not unpack their underlying coordinate-free geometric meaning.

The author classifies geometric meaning into two categories:

- Coordinate geometric meaning: interpretations dependent on the unit points of coordinate axes.
- Non-coordinate geometric meaning: interpretations independent of axis unit points, relying only on an origin or fully coordinate-free.

When converting complex polynomial equations into vector equations, it is challenging to define a vector multiplication operation corresponding to complex multiplication. Only after introducing vector division can we fully illuminate the geometric content encoded in vector equations.

Throughout this paper, unless otherwise specified, uppercase letters $A, B, C, \dots$ denote geometric points or position vectors on the plane, while lowercase letters $a, b, c, \dots$ denote the corresponding complex coordinates in a given coordinate system.

The remainder of this paper is organized as follows. Section 2 reviews basic concepts of plane vectors and complex numbers. Section 3 introduces the definition and algebraic properties of planar vector division. Section 4 demonstrates how vector division can interpret complex identities and prove elementary geometric theorems. Section 5 summarizes the main results and discusses future research directions.

1.1. Definitions and Theorems.

Definition 1 (Vector Quotient, Length Quotient and Direction Quotient). *Let A, B, C, D be four points on a Euclidean plane, and let θ denote the directed angle from vector $\overrightarrow{CD}$ to vector $\overrightarrow{AB}$. Let $|\overrightarrow{AB}|$ stand for the modulus of vector $\overrightarrow{AB}$.*

We define the length quotient of $\frac{\overrightarrow{AB}}{\overrightarrow{CD}}$ as

$$\lambda = \frac{|\overrightarrow{AB}|}{|\overrightarrow{CD}|},$$

which represents the ratio of the moduli of the two vectors.

We further define the direction quotient of $\frac{\overrightarrow{AB}}{\overrightarrow{CD}}$ as

$$v = e^{i\theta},$$

which characterizes the rotational relationship from the reference vector $\overrightarrow{CD}$ to the target vector $\overrightarrow{AB}$.

Based on the above two quantities, the vector quotient of $\overrightarrow{AB}$ with respect to $\overrightarrow{CD}$ is defined as the product of the length quotient and the direction quotient, which is a complex number denoted by $\frac{\overrightarrow{AB}}{\overrightarrow{CD}}$ and satisfies:

$$\frac{\overrightarrow{AB}}{\overrightarrow{CD}} = \lambda v = \frac{|\overrightarrow{AB}|}{|\overrightarrow{CD}|} e^{i\theta} \in \mathbb{C}.$$

Definition 2 (Vector-Complex Multiplication). Let $\mathbf{v}$ be an arbitrary plane vector, and let $z = re^{i\theta} \in \mathbb{C}$ be a complex number with $r \geq 0$ (modulus) and $\theta \in \mathbb{R}$ (argument). The product of z and $\mathbf{v}$, denoted by $z\mathbf{v}$, is a new plane vector satisfying the following rules:

- i) **Modulus Rule:** The modulus of the product vector is the product of the complex modulus and the vector modulus, i.e., $|z\mathbf{v}| = r|\mathbf{v}|$;
- ii) **Argument Rule:** The argument of the product vector is the sum of the argument of z and the argument of $\mathbf{v}$;
- iii) **Distributive Laws:** For any complex numbers $z_1, z_2 \in \mathbb{C}$ and any plane vectors $\mathbf{v}_1, \mathbf{v}_2$, the multiplication satisfies

$$z(\mathbf{v}_1 + \mathbf{v}_2) = z\mathbf{v}_1 + z\mathbf{v}_2, \quad (z_1 + z_2)\mathbf{v} = z_1\mathbf{v} + z_2\mathbf{v}.$$

Geometrically, multiplying a vector by a complex number $z = re^{i\theta}$ is equivalent to a **similar transformation** of the vector: scaling the vector by a factor of r and rotating it counterclockwise by an angle of θ around its starting point.

Definition 3 (Similar Vector). Based on the definitions of vector quotient and vector-complex multiplication, the concept of similar vectors can be naturally derived, which describes the vector obtained by applying the inherent similar transformation of two vectors to a third vector. Let $\mathbf{a}, \mathbf{b}, \mathbf{d}$ be arbitrary plane vectors, and let the vector quotient $z = \frac{\mathbf{a}}{\mathbf{b}} \in \mathbb{C}$. The **similar vector** of $\mathbf{d}$ determined by $\mathbf{a}$ and $\mathbf{b}$, denoted by $\mathbf{c}$, is defined as:

$$\mathbf{c} = z\mathbf{d} = \frac{\mathbf{a}}{\mathbf{b}}\mathbf{d}.$$

If the vector quotient equation $\frac{\mathbf{a}}{\mathbf{b}} = \frac{\mathbf{c}}{\mathbf{d}}$ holds, then $\mathbf{c}$ is the similar vector of $\mathbf{d}$ with respect to the reference vector pair $(\mathbf{a}, \mathbf{b})$.

Geometric Interpretation: The vector quotient $\frac{\mathbf{a}}{\mathbf{b}}$ uniquely determines a similar transformation (scaling ratio + rotation angle) that maps $\mathbf{b}$ to $\mathbf{a}$. The similar vector $\mathbf{c}$ is the result of applying this identical transformation to the vector $\mathbf{d}$.

Geometric Meaning of Complex Addition: Let u, v be two complex numbers, and let $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ be four planar vectors satisfying the vector quotient relations $u = \frac{\mathbf{B}}{\mathbf{A}}$ and $v = \frac{\mathbf{D}}{\mathbf{C}}$. Denote the sum of the two complex numbers as $w = u + v$, and let O be the origin of the plane Cartesian coordinate system. For an arbitrary planar

vector $\mathbf{X}$, the product of complex numbers and the vector satisfies the distributive law, leading to:

$$w\mathbf{X} = u\mathbf{X} + v\mathbf{X} = \frac{B}{A}\mathbf{X} + \frac{D}{C}\mathbf{X}.$$

Define the similar vectors derived from the vector quotient as $\frac{B}{A}\mathbf{X} = \mathbf{Y}$ and $\frac{D}{C}\mathbf{X} = \mathbf{Z}$, respectively. The geometric meaning of the complex addition $w = u + v$ can be interpreted by the following constructive steps: first, construct the similar vectors $\mathbf{Y}$ and $\mathbf{Z}$ corresponding to the vector triples $(B, A, \mathbf{X})$ and $(D, C, \mathbf{X})$ respectively based on the definition of similar vectors; then, denote the vector sum of $\mathbf{Y}$ and $\mathbf{Z}$ as $\mathbf{W}$, i.e., $\mathbf{W} = \mathbf{Y} + \mathbf{Z}$, which can be geometrically constructed by applying the **Parallelogram Law** to vectors $\mathbf{Y}$ and $\mathbf{Z}$.

From the above construction, the fundamental vector quotient identity for complex addition is derived:

$$\frac{W}{X} = \frac{B}{A} + \frac{D}{C}.$$

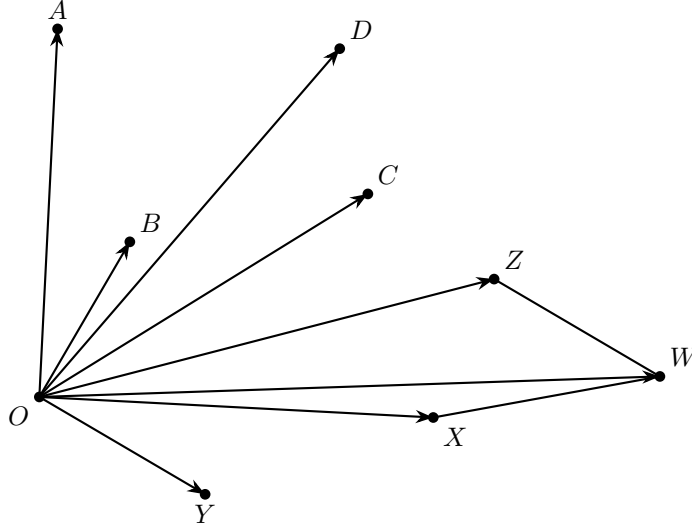


FIGURE 1. Geometric interpretation of complex addition

Let the directed angles be denoted as $\angle XOW = \gamma$, $\angle AOB = \alpha$ and $\angle COD = \beta$. Performing the **separation of real and imaginary parts** on the above vector quotient identity, we obtain the quantitative geometric relations between the moduli and directed angles of the vectors:

$$\begin{aligned} \frac{|W|}{|X|} \cos \gamma &= \frac{|B|}{|A|} \cos \alpha + \frac{|D|}{|C|} \cos \beta, \\ \frac{|W|}{|X|} \sin \gamma &= \frac{|B|}{|A|} \sin \alpha + \frac{|D|}{|C|} \sin \beta. \end{aligned}$$

Subtraction, multiplication and division of complex numbers can be explained similarly.

Special Case. If the vectors $\mathbf{A}$, $\mathbf{C}$ and $\mathbf{X}$ coincide (i.e., $\mathbf{A} = \mathbf{C} = \mathbf{X}$), this reduces to the **classical geometric interpretation of complex addition** presented in standard textbooks. In this case, $\mathbf{X}$ can be regarded as the **unit point** of the plane, and the position vector $\overrightarrow{OX}$ is aligned with the **real axis** of the complex plane. This classical interpretation is thus a special instance of the general vector quotient-based complex addition proposed above.

2. VECTOR, COORDINATE AND COMPLEX NUMBER

2.1. Corollary: Complex Representation of Vector Linear Combination.

Let O be the origin of the plane Cartesian coordinate system, and let $\overrightarrow{OX}, \overrightarrow{OY}$ be two non-collinear plane vectors with a common starting point O . For any point $Z \in \Pi$ (Euclidean plane), the position vector $\overrightarrow{OZ}$ can be expressed as the linear combination:

$$\overrightarrow{OZ} = \lambda \overrightarrow{OX} + \eta \overrightarrow{OY}, \quad \lambda, \eta \in \mathbb{R}.$$

Take $\overrightarrow{OX}$ as the basis vector of the plane Π (with its complex representation $x = 1$, i.e., X is the unit point). Let the vector quotient $u = \frac{\overrightarrow{OY}}{\overrightarrow{OX}}$, then from the definition of vector-complex multiplication, we have $\overrightarrow{OY} = u \overrightarrow{OX}$. Substituting into the linear combination yields:

$$\overrightarrow{OZ} = \lambda(u \overrightarrow{OX}) + \eta \overrightarrow{OX} = (\lambda u + \eta) \overrightarrow{OX}.$$

By the definition of vector quotient, the complex number corresponding to the position vector $\overrightarrow{OZ}$ with respect to the basis vector $\overrightarrow{OX}$ is:

$$z = \frac{\overrightarrow{OZ}}{\overrightarrow{OX}} = \lambda u + \eta.$$

This establishes the one-to-one correspondence between the linear combination of plane vectors and the algebraic operation of complex numbers.

For any two non-zero vectors $\overrightarrow{AB}, \overrightarrow{CD} \in \Pi$ (where $A, B, C, D \in \Pi$, and $a, b, c, d \in \mathbb{C}$ are the complex numbers corresponding to A, B, C, D respectively), the ratio of $\overrightarrow{AB}$ to $\overrightarrow{CD}$ (in the sense of complex numbers) satisfies:

$$\frac{\overrightarrow{AB}}{\overrightarrow{CD}} = \frac{b - a}{d - c},$$

and this ratio is independent of the choice of coordinate system or basis of Π .

Proof.

$$\frac{\overrightarrow{AB}}{\overrightarrow{CD}} = \frac{B - A}{D - C} \frac{X}{X} = \frac{\frac{B}{X} - \frac{A}{X}}{\frac{D}{X} - \frac{C}{X}} = \frac{b - a}{d - c}.$$

(where $X = 1$ is the complex number corresponding to the basis vector $\overrightarrow{OX}$) $\square$

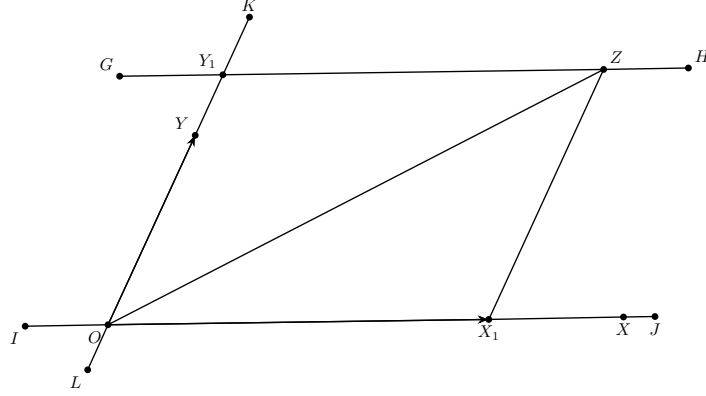


FIGURE 2. Schematic diagram of vectors and coordinates

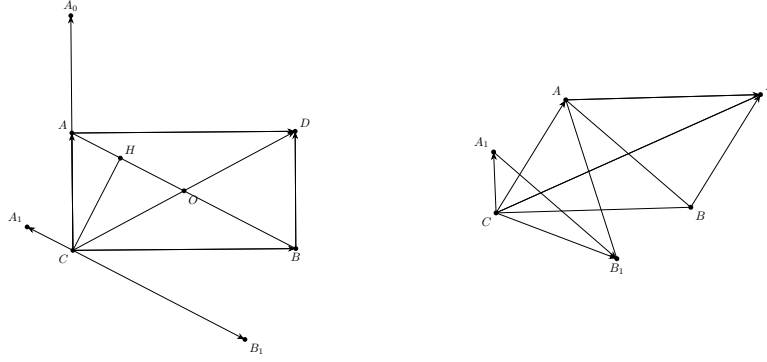


FIGURE 3. Geometric constructions of vector quotient

3. GEOMETRIC INTERPRETATIONS OF COMPLEX EXPRESSIONS FOR THREE FUNDAMENTAL THEOREMS AND RELATED IDENTITIES

3.1. Geometric Interpretations of Complex and Vector Difference-of-Squares Identities for the Pythagorean Theorem. Suppose C is the right-angle vertex and take C as the origin. From the Pythagorean theorem,

$$(a - b)(\overline{a - b}) = a\bar{a} + b\bar{b}.$$

Rearranging the identity:

$$a - b = \frac{a\bar{a}}{a - b} + \frac{b\bar{b}}{a - b}.$$

In vector form:

$$\overrightarrow{BA} = \overrightarrow{CA} \left(\frac{\overrightarrow{CA}}{\overrightarrow{BA}} \right) + \overrightarrow{CB} \left(\frac{\overrightarrow{CB}}{\overrightarrow{BA}} \right).$$

Geometric interpretation: Extend segment CA to A_0 with A as the midpoint of CA_0 . By the projection theorem, $\overrightarrow{AH} = \overrightarrow{AA_0} \frac{\overrightarrow{CA}}{\overrightarrow{BA}}$, and $\overrightarrow{HB}$ is obtained similarly. Clearly,

$$\overrightarrow{AB} = \overrightarrow{AH} + \overrightarrow{HB}.$$

Assume A lies on the imaginary axis and B on the real axis. Then $\bar{a} = -a, \bar{b} = b$. The Pythagorean theorem becomes

$$b^2 - a^2 = (b + a)(b - a),$$

which can be rewritten in vector form:

$$B \frac{B}{A+B} - A \frac{A}{A+B} = B - A.$$

Construct points $B_1 = B \frac{B}{A+B}$, $A_1 = A \frac{A}{A+B}$. Then $\overrightarrow{B_1 A_1} = \overrightarrow{B A}$. This interpretation also applies to parallelograms.

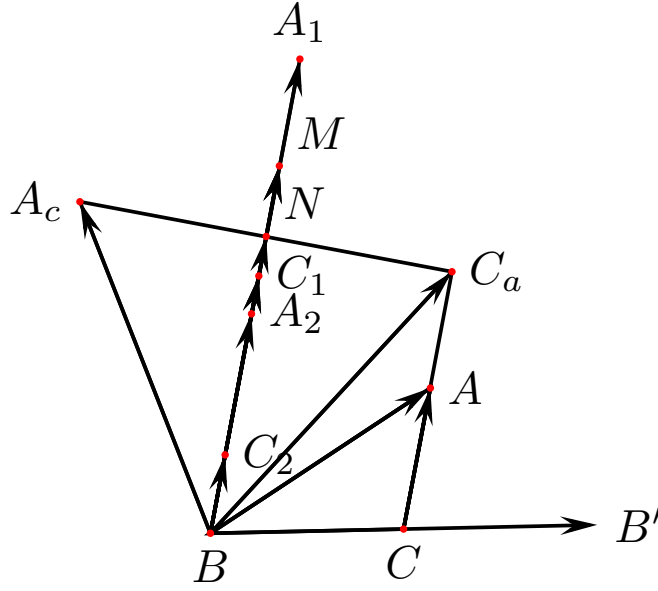


FIGURE 4. Geometric construction of vector quotient for the law of cosines and distance between two points

3.2. Law of Cosines and Distance Formula between Two Points. Suppose B is placed at the origin. The complex form of the law of cosines reads

$$(a - c)(\overline{a - c}) = a\bar{a} + c\bar{c} - (a\bar{c} + \bar{a}c).$$

Let $\angle ACB = C_0$. Using Euler's formula and the definition of vector quotient, we obtain

$$a\bar{c} + \bar{a}c = 2|a||c|\cos C_0.$$

Define vectors:

$$A_1 = A \overline{\left(\frac{A}{A-C}\right)}, \quad C_1 = C \overline{\left(\frac{C}{A-C}\right)}, \quad A_c = A \overline{\left(\frac{C}{A-C}\right)}, \quad C_a = C \overline{\left(\frac{A}{A-C}\right)}.$$

Let M, N be the midpoints of segments $A_1 C_1$ and $A_c C_a$, respectively. The vector relation

$$2\overrightarrow{MN} = \overrightarrow{C A}$$

holds.

Without expanding into the law of cosines, we may reinterpret the distance formula between two points by rearranging the original expression:

$$a - c = a \frac{\bar{a}}{\bar{a} - \bar{c}} - c \frac{\bar{a}}{\bar{a} - \bar{c}}.$$

Construct vectors

$$A_2 = A \left(\frac{A}{\bar{A} - \bar{C}} \right), \quad C_2 = A \left(\frac{A}{\bar{A} - \bar{C}} \right),$$

then

$$\overrightarrow{A_2 C_2} = \overrightarrow{AC}.$$

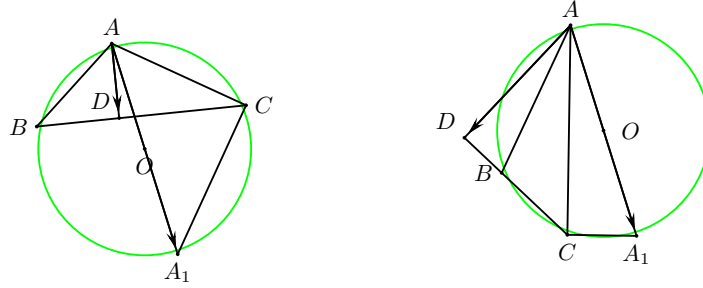


FIGURE 5. Two cases for the law of sines

3.3. Law of Sines and Similar Triangles. Take the circumcenter of $\triangle ABC$ as the origin. Let AD be the altitude to side BC , and let d denote the complex coordinate of point D .

$$d = a + b + c - \frac{bc}{a}.$$

From the geometric condition we have the complex identity

$$\frac{a - d}{a - b} = \frac{a - c}{2a}.$$

Taking moduli and rearranging yields the law of sines

$$\frac{|AC|}{\sin B} = 2R,$$

where B is the internal angle opposite side AC and R is the circumradius.

The geometric meaning of the complex identity

$$\frac{a - d}{a - b} = \frac{a - c}{2a}$$

is as follows: extend AO to meet the circumcircle at A_1 , so $A_1 = -a$. Rewritten as a vector equation:

$$\frac{\overrightarrow{AD}}{\overrightarrow{AB}} = \frac{\overrightarrow{AC}}{\overrightarrow{AA_1}}.$$

In the figure, $\triangle ABD \sim \triangle AA_1C$.

4. CONCLUSION AND FUTURE WORK

This paper systematically establishes a complete theoretical framework for planar vector quotients, clarifying the rigorous definitions of length quotient, direction quotient, and general vector quotient. It formulates the operational rules between complex numbers and plane vectors and constructs the concept of similar vectors, thereby providing a unified, coordinate-free geometric interpretation for complex algebraic operations. The present work fundamentally reveals the intrinsic logical relationships among plane vectors, coordinate systems, and complex numbers, resolving long-standing conceptual ambiguities in elementary geometric theory.

Compared with the abstract conformal geometric framework proposed by Li [1], which exceeds the cognitive level of ordinary high-school students, the vector quotient method developed in this paper is intuitive, concise, and highly adaptable to basic mathematics education. Students can independently derive corollaries and extended conclusions of classical geometric theorems, including the Pythagorean theorem, the law of cosines, and the law of sines, effectively reducing the learning difficulty of complex geometry and vector geometry and stimulating geometric inquiry ability. Furthermore, this framework enables learners to deeply understand the geometric essence of Peng's conjecture [9] and accurately grasp the structural logic of geometric proof based on complex identities [4].

Contrary to the viewpoint of Zhou and Lin [10], who denied the theoretical value of planar vector division due to its limitation to two-dimensional space, this paper argues that although vector quotient is only applicable to plane geometry, it effectively clarifies the essential origin of plane coordinate systems and complex number systems. As a fundamental concept connecting vector algebra and complex geometry, vector quotient deserves a formal position in high-school mathematics textbooks and can significantly improve the integrity and self-consistency of elementary geometric operational systems.

This study also raises an important open theoretical problem worthy of further exploration. In previous research, Peng et al. [4] directly applied real-number polynomial rules to vector expressions in geometric derivation without rigorously defining vector multiplication. In standard mathematical systems, it remains questionable whether algebraic derivation based on undefined operations is rigorous and legitimate.

In fact, pure vector product and vector square are merely formal algebraic symbols without self-consistent geometric definitions or independent geometric interpretations. Such formal vector expressions can obtain valid geometric configurations and transformation meanings only after being converted into vector quotient fractions and interpreted through complex similarity transformations. Therefore, directly applying conventional vector algebraic formulas belongs to a special state of "valid formal operation without explicit geometric meaning".

For instance, the vector difference-of-squares factorization $\mathbf{A}^2 - \mathbf{B}^2 = (\mathbf{A} + \mathbf{B})(\mathbf{A} - \mathbf{B})$ is formally executable, while its geometric rationality cannot be guaranteed without vector quotient reconstruction. This leads to a core theoretical question: although intrinsic vector multiplication is undefined in plane vector theory, can formal vector expressions fully follow real-number rules for polynomial expansion and factorization? Whether all algebraically valid vector polynomial transformations correspond to uniquely interpretable geometric meanings within

the vector quotient system remains an unsolved foundational problem in elementary geometry.

4.1. Future Work. Future research can generalize the vector quotient framework from two-dimensional Euclidean planes to higher-dimensional Euclidean spaces and unified multi-dimensional geometric transformation systems. Subsequent studies can further explore the rationality and mathematical rigor of applying real-number polynomial operational rules to formal vector expressions, and establish rigorous criteria for distinguishing formal algebraic validity and geometric interpretability of vector polynomial identities.

In addition, the vector quotient method can be extended to automated geometric theorem proving, providing intuitive, coordinate-free, and geometrically interpretable proof strategies for mechanical geometric reasoning. From the perspective of mathematics education, further curriculum design and teaching experiments can be carried out to promote the popularization and systematization of vector quotient concepts in high-school mathematics teaching.

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