

A Delayed Geometric Response in Cluster Mergers: *Predictive Evidence from the Bullet Cluster and Abell 520*

ALEJANDRO REY¹

¹ *Independent Researcher, Argentina*
ORCID: 0009-0007-5052-3917

ABSTRACT

When intracluster gas is braked by a collision, does the gravitational configuration stop with it? We test a finite-response picture in which the pre-impact geometry continues along the direction traced by the galaxies while the shocked gas builds a new central baryonic concentration; the displaced pattern is expected to return toward that concentration. We use gravitational lensing shear, the coherent distortion of background galaxy images, as the observable. Public X-ray, optical, and lensing measurements provide four linked tests. In Abell 520, we use lensing measured outside the collision to forecast the shear inside the central shocked region; the forecast improves when forward displacement is allowed ($\Delta\chi^2 = 8.63$ and 5.66 ; $p = 0.004$ and 0.006). In the Bullet Cluster, we compare a luminosity-weighted galaxy template with a template built from the X-ray gas-mass map on reserved background sources; the galaxy template gives higher held-out shear variance (held-out $R^2 = 0.03167$ versus 0.01374 ; ratio 2.30). In the Bullet Cluster and MACS J0025, we test whether the lensing–gas offset follows the galaxy–gas direction; all four vectors are consistent with pointing toward the galaxy side ($p = 0.00335$). Across four phase-ordered core mergers classified as outbound (moving apart after first passage) or returning (moving back toward the central region), the signed ordering reverses and the phase-dependent fit outperforms a phase-blind constant ($\Delta\text{BIC} = 27.50$). Two additional published systems extend the sign comparison to six mergers, with all six signs matching the predicted sequence (sign test $p = 0.015625$). The same ordering supports the outbound interpretation of the conflicting phase assignments in Abell 1240. Together these results link spatial displacement, direction, and phase reversal within a common delayed geometric response to the collision.

Keywords: galaxies: clusters: general — gravitational lensing: weak — hydrodynamics — gravitation

1. INTRODUCTION

The Bullet Cluster makes the aftermath of a cluster collision unusually clear: X-ray gas shocks and slows, galaxies continue approximately ballistically, and the main lensing structure lies ahead of the gas, near the galaxies (Markevitch et al. 2004; Clowe et al. 2006). The standard reading assigns the displaced lensing signal to collisionless matter travelling with the galaxies (Markevitch & Vikhlinin 2007; Randall et al. 2008). We ask whether a gravitational configuration with memory of its baryonic source produces the observed ordering.

The Theory of Inertial Delayed Effects (TIDE) (Rey 2026d) supplies the dynamical hypothesis: effective gravitational geometry takes finite time to adjust when the baryonic distribution changes, retaining a transient imprint of its previous configuration. In a merger, the field therefore does not stop instantaneously with the shocked gas. The displaced lensing pattern initially advances along the direction marked by the galaxies, while compressed gas builds a new central gravitational well toward which the pattern later shifts inward. The field remains continuous across the collision region: it is a measurable lensing peak or fitted centre that moves. Galaxies mark the incoming direction, while the evolving field carries the displaced signal.

This picture makes a sequence of tests possible. In Abell 520 we fit shear outside the collision centre and ask what the fit predicts for sources held out inside it. In the Bullet Cluster we ask whether shocked gas or the transported galaxy distribution better predicts shear in withheld sky regions. Bullet and MACS J0025 then test the direction of the offset against an axis fixed from gas and galaxies alone. Finally, published orbital phases test the change in signed gas–lensing order between outbound and returning mergers.

These tests continue a sequence of studies of the same residual gravitational geometry using galaxy rotation curves, cluster hydrostatics, and galaxy–galaxy weak lensing (Rey 2026a,b,c). Here we follow that geometry into cluster collisions, where its baryonic source is abruptly rearranged.

2. WHY THE FIELD ADVANCES—AND RETURNS

Before core passage, gas, galaxies, and the gravitational configuration move as one system. The shock sharply decelerates the gas while the galaxies continue along their incoming orbits. The field retains a transient contribution from its earlier configuration as compressed gas sources a new central contribution (Figure 1); these are overlapping responses of one continuous geometry. We use the standard weak-field mapping from the projected lensing potential to shear and convergence, together with a finite adjustment to the evolving baryonic source. Equation (1) expresses that response; a restoring law for the displaced centre supplies the transport-and-return closure in Appendix A.

We express the finite adjustment compactly as

$$\tau_{\text{eff}}(\mathbf{x}, t) \frac{D\mathcal{G}}{Dt} + \mathcal{G} = \mathcal{G}_{\text{eq}}[\rho_{\text{b}}(t)]. \quad (1)$$

For a scalar weak-field representation, we write $\mathcal{G} = \Phi_{\text{L}}/c^2$, where Φ_{L} is the effective lensing potential $(\Phi + \Psi)/2$ of the two scalar metric potentials, with a fixed potential reference. Thus $\mathcal{G}$ and $\mathcal{G}_{\text{eq}}$ are dimensionless and τ_{eff} has units of time. The equilibrium term is sourced by the current baryonic distribution ρ_{b} . The derivative $D/Dt = \partial_t + \mathbf{v}_{\mathcal{G}} \cdot \nabla$ follows the transported pattern, whose effective velocity is $\mathbf{v}_{\mathcal{G}}$. A finite local response time allows the pre-collision configuration to persist after the gas is braked.

In the thin-lens approximation, the line-of-sight projection of Φ_{L} gives the angular lensing potential ψ ; its second derivatives give convergence κ and shear γ (Bartelmann & Schneider 2001). This is the connection to the observables below. Paper I uses the radial gradient of the dynamical potential; here we measure the projected lensing field. In the equal-metric-potential weak-field limit, the two are representations of the same gravitational potential.

For the measurements below, shear and convergence locate the maximum or fitted centre of the projected lensing field. We use *gas* for the X-ray-emitting intracluster plasma, *galaxies* for the optical member positions, and *lensing centre* for that projected maximum or fitted centre. *Outbound* denotes systems moving apart after first passage, whereas *returning* denotes systems moving back toward the central baryonic concentration. Section 3 defines the data and fixed measurement rules used to compare these references.

Conceptual picture

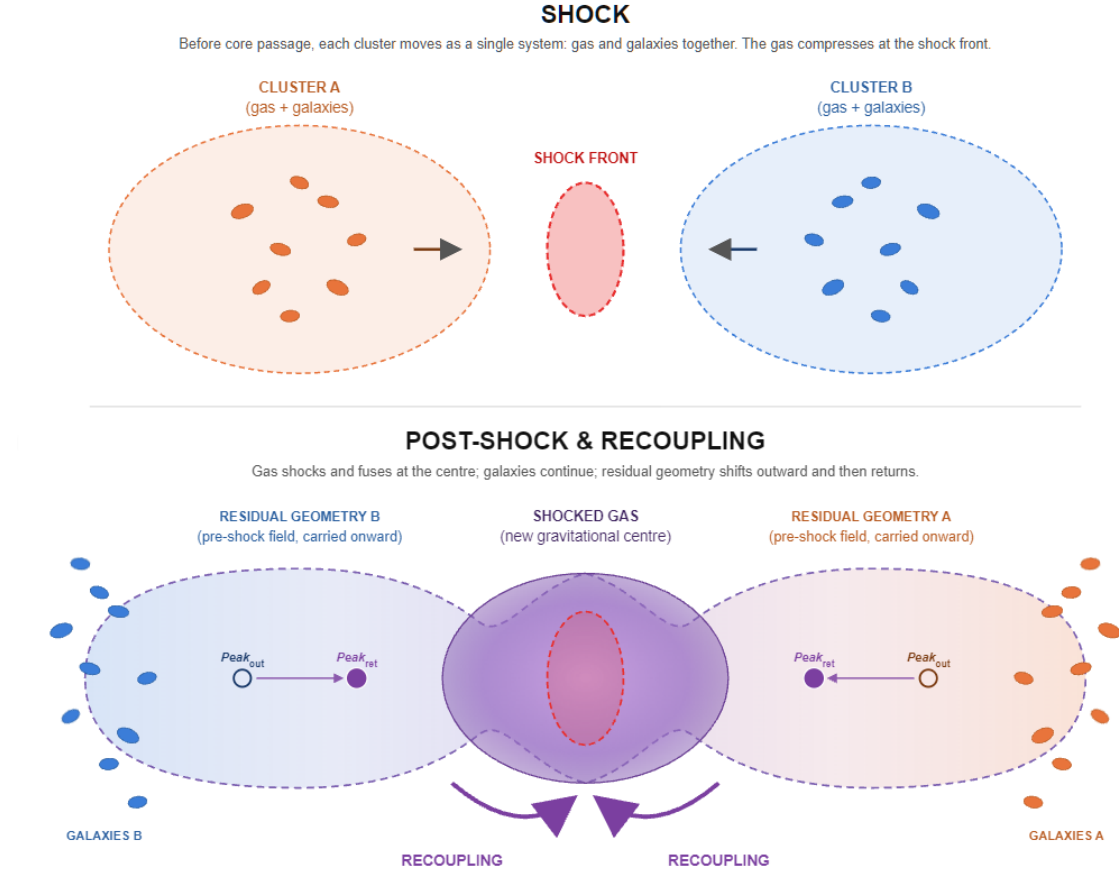


Figure 1. Physical picture at core passage and afterward. At the shock (top), gas and galaxies approach in a common configuration. Afterward (bottom), galaxies continue outward, shocked gas forms a central baryonic concentration, and the pre-collision geometry retains a displaced effective centre while remaining connected to the collision region. Open and filled markers indicate the outbound and returning peak positions; the arrows illustrate source-driven recoupling. The diagram summarizes the physical sequence tested in this work.

3. DATA AND VALIDATION STRATEGY

We combine public weak-lensing source catalogues or convergence maps, X-ray gas measurements, and optical cluster-member positions. The main spatial systems were chosen for traceable public data, identifiable gas and galaxy references, and lensing positions reconstructed from lensing data alone. Abell 520 supplies an exterior-to-centre shear prediction from the Clowe et al. (2012) and Jee et al. (2014a) catalogues (hereafter C12 and J14): two source-catalogue reconstructions. The Bullet Cluster supplies the shear-template comparison (Clowe et al. 2006; Barrena et al. 2002); MACS J0025 supplies the second merger for the directional test (Bradač et al. 2008). The phase sample comprises Bullet, Abell 56, MACS J1149, and El Gordo, classified from published merger dynamics and radio-relic information (Dawson 2013; Wittman et al. 2023; Golovich et al. 2016; Ng et al. 2015). RM J1508 and NGC 4839/Coma

provide an outbound and returning sign-consistency extension, respectively (Stancioli et al. 2023; HyeonHan et al. 2026; Sasaki et al. 2016; Lyskova et al. 2019). Table 1 summarizes the observables and role assigned to each system.

Table 1. Systems and their roles in the analysis

System	Observables	Primary role
Bullet Cluster ^{C06,B02}	shear, κ , X-ray, galaxies	template; direction; field
MACS J0025 ^{B08}	gas, lensing, optical peaks	directional replication
ZwCl 0008 ^{G17}	gas, lensing, optical peaks	pair separation
Abell 520 ^{C12,J14a}	two shear catalogues; gas; galaxies	exterior prediction; galaxy-lock
Abell 56 ^{W23}	phase; gas/lensing positions	turnover test (Sec. 4.4)
El Gordo ^{J14b,N15}	phase; gas/lensing/galaxy maps	turnover; recoupling extension
MACS J1149 ^{G16}	phase; gas-lensing order	turnover test (Sec. 4.4)
RM J1508 ^{S23}	outbound phase; offset sign	turnover sign extension
NGC 4839 ^{H26,S16,L19}	returning phase; offset sign	turnover sign extension
Abell 1240 ^{W19,C22}	phase clocks; X-ray; lensing peaks	phase adjudication
PSZ2 G181 ^{A25}	number/light maps	morphology (data package)

Reference tags. C06: Clowe et al. (2006); B02: Barrena et al. (2002); B08: Bradač et al. (2008); G17: Golovich et al. (2017); C12: Clowe et al. (2012); J14a: Jee et al. (2014a); W23: Wittman et al. (2023); J14b: Jee et al. (2014b); N15: Ng et al. (2015); G16: Golovich et al. (2016); S23: Stancioli et al. (2023); H26: HyeonHan et al. (2026); S16: Sasaki et al. (2016); L19: Lyskova et al. (2019); A25: Ahn et al. (2025). W19: Wittman (2019); C22: Cho et al. (2022).

3.1. Defining direction without using the lensing map

For each merger branch, gas and galaxy positions alone define the outward axis,

$$\hat{\mathbf{u}} = \frac{\mathbf{x}_{\text{gal}} - \mathbf{x}_{\text{gas}}}{|\mathbf{x}_{\text{gal}} - \mathbf{x}_{\text{gas}}|}. \quad (2)$$

The galaxy position sets the direction; the lensing measurement does not enter that choice. For a fitted lobe centre or a published lensing centroid, as specified in each test, we measure its projected location along this axis with

$$f_{\parallel} = \frac{(\mathbf{x}_{\text{lens}} - \mathbf{x}_{\text{gas}}) \cdot \hat{\mathbf{u}}}{|\mathbf{x}_{\text{gal}} - \mathbf{x}_{\text{gas}}|}, \quad \beta_{\text{G}} = 1 - f_{\parallel}. \quad (3)$$

An exactly galaxy-locked lensing centre has $\beta_{\text{G}} = 0$; a centre between gas and galaxies has $0 < \beta_{\text{G}} < 1$.

Angular tests preserve the two arms of a merger as a single unit. Pair-separation tests assign equal weight to each merger rather than to each component. Spatial hold-outs reserve background sources or cells, whereas smoothing scales and amplitudes are selected only inside the corresponding training region.

The analysis uses three distinct validation labels. A *holdout* contains measurements reserved from fitting. A *replication between mergers* tests the same geometric relation in a different cluster. A *stress test* probes stability across analysis choices within one

observation. The Bullet and A520 holdouts provide the main predictive comparisons. The directional test adds replication in MACS J0025. The four-system phase curve evaluates turnover with measured normalized displacements; two further mergers extend the sign comparison. Gas containment and pair separation provide additional spatial checks. Appendix A collects the dynamical closure.

4. RESULTS

4.1. Abell 520: predicting the shear inside the shock

Can lensing measured outside A520’s collision region predict the shear inside it? A transported pattern places its fitted centres ahead of the gas, toward the galaxies, while still extending across the shocked region. We test that prediction separately with the C12 and J14 source catalogues (Clowe et al. 2012; Jee et al. 2014a).

Each gas peak is represented by a smooth lensing component. Its centre is allowed to move along the independently defined gas–galaxy axis; the control uses the same model but forbids that forward displacement. We fit only the outer sources and ask which version predicts the reserved central shear. In each catalogue we fit two circular Gaussian convergence components and a constant external shear. In angular coordinates $\boldsymbol{\theta}$, the model is

$$\kappa(\boldsymbol{\theta}) = \sum_{j=1}^2 a_j \exp\left[-\frac{|\boldsymbol{\theta} - \mathbf{c}_j|^2}{2s_j^2}\right], \quad \mathbf{c}_j = \boldsymbol{\theta}_{\text{gas},j} + d_j \hat{\mathbf{u}}_j + b_j \hat{\mathbf{v}}_j, \quad (4)$$

where $\hat{\mathbf{v}}_j$ is perpendicular to the gas–galaxy axis. The amplitudes a_j are dimensionless; widths s_j and shifts d_j, b_j are in arcseconds. The Gaussian profiles determine the two shear components analytically, and the predicted source distortion is the reduced shear $\mathbf{g} = (\boldsymbol{\gamma}^{(1)} + \boldsymbol{\gamma}^{(2)} + \boldsymbol{\gamma}_{\text{ext}})/(1 - \kappa)$.

Only sources at $60'' \leq r \leq 220''$ enter the fit. The fits use 2266 outer sources in C12 and 3677 in J14; 214 and 353 central sources, respectively, are reserved for evaluation. Amplitudes, widths, and both longitudinal and transverse component shifts are fitted outside the centre. The control fixes only the forward, longitudinal shifts to zero and refits the other parameters on the same outer sources. The alternative therefore has ten fitted parameters and the longitudinally fixed control has eight. Both signs of d_j are allowed: the predicted outward direction is tested, not imposed. A wild bootstrap under the control calibrates the reserved-centre improvement, with the statistic set to zero unless both fitted d_j are positive. Table 2 separates the information used to construct the prediction from the measurements used to evaluate it.

Table 2. A520 prediction design. Positions share one tangent plane; r is measured from the fixed field centre.

Stage	Information or operation
Fixed references	Chandra gas maxima; galaxy tracers; two axes; inner $60''$ holdout; outer $220''$ limit.
Outer calibration	Two amplitudes; two widths; four centre shifts; external shear; control-residual noise.
Reserved prediction	Central shear: 214 C12 and 353 J14 sources; no holdout shapes enter the fit.
Chance control	499 source-level wild-bootstrap refits and holdout evaluations.

The forward model improves the reserved-centre fit by $\Delta\chi^2 = 8.63$ in C12 and 5.66 in J14, with conditional bootstrap $p = 0.004$ and 0.006. All four fitted component centres lie between gas and galaxies, at $f_{\parallel} = 0.632$ – 0.963 ; the gas also lies within the radius enclosing 90% of each fitted component. Together, the two catalogue reconstructions show that an outward-shifted pattern inferred from the outskirts predicts shear inside the region excluded from the fit (Table 3; Figure 2).

Table 3. Abell 520: exterior fit and reserved central-shear prediction. $f_{\parallel}$ is the fitted-centre position along the gas-to-galaxy baseline. “Gas containment” means the gas lies within the fitted component’s 90% enclosed-radius contour. C12 and J14 are the two A520 catalogues; $\Delta\chi^2$ and p are their holdout results.

Catalogue	Branch	$f_{\parallel}$	Gas containment	$\Delta\chi^2$	p
C12	NE	0.963	yes	8.63	0.004
C12	SW	0.632	yes		
J14	NE	0.757	yes	5.66	0.006
J14	SW	0.825	yes		

The forward ordering and gas containment persist in all 48 tested physical variants; their largest gas–centre separation is 0.825 times the fitted 90% enclosed radius. The ordering also survives the tested X-ray references. Appendix B summarizes these checks. Signed-signal injections recover both outward signs in 98/100 C12 and 99/100 J14 realizations. The stricter archived null-threshold detections and the zero-shift and inward injections are reported there.

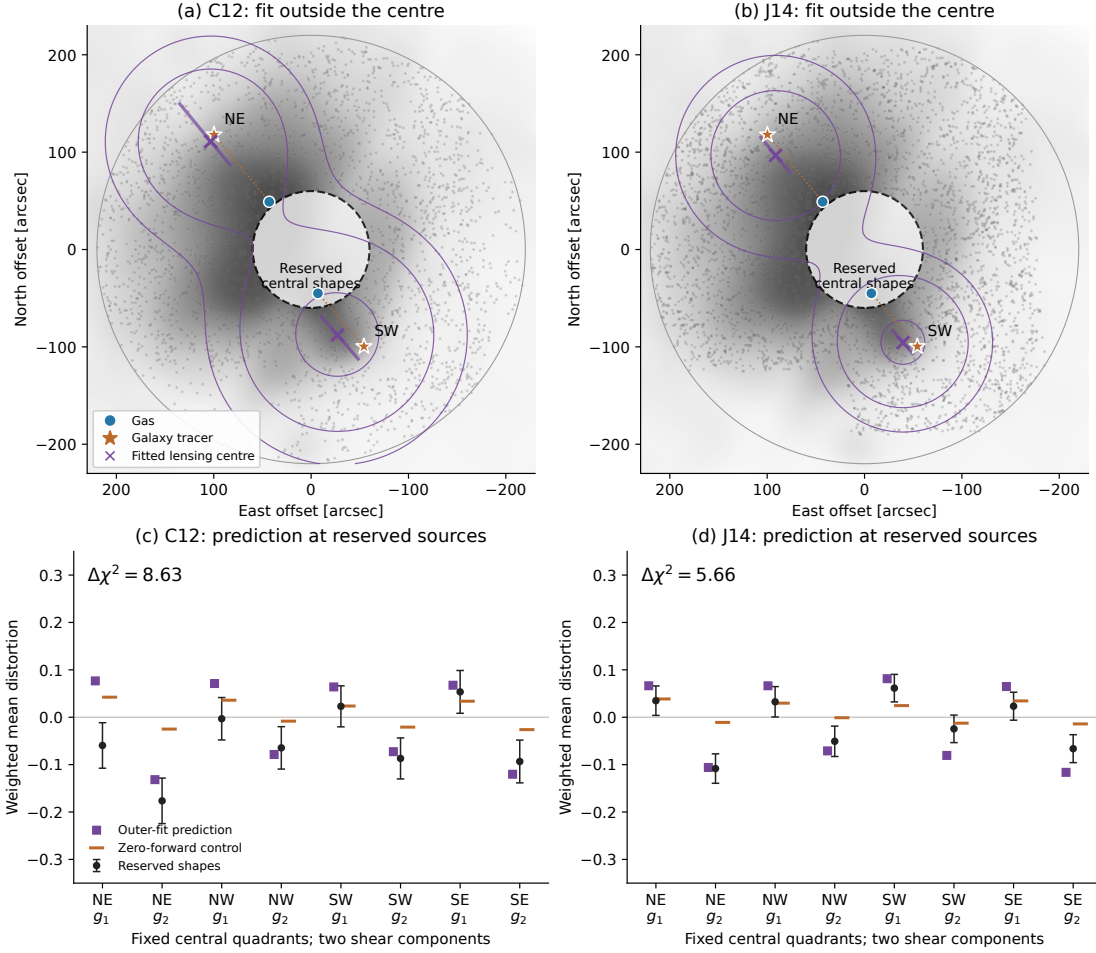


Figure 2. Abell 520 exterior-to-centre prediction. Top: Chandra brightness, gas maxima (circles), galaxy tracers (stars), and convergence contours and centres (crosses) fitted using only the outer source shapes. The dashed circle marks the 60'' central holdout. Purple segments show 95% conditional bootstrap intervals along each gas–galaxy axis, with the transverse position held at its fitted value. Bottom: observed mean source distortions in four fixed central quadrants, compared with the outer-fit prediction and the zero-forward-shift control. Bars are one-standard-error shape-noise estimates using the outer noise scale and catalogue weights. Binning is for display; the reported $\Delta\chi^2$ uses all individual reserved sources.

4.2. Bullet Cluster: does the lensing pattern follow the shocked gas or the galaxies?

The Bullet Cluster tests the morphology of the transported signal. Does the shear follow the gas that shocked, or the cluster structure marked by galaxies that continued outward? We build separate shear templates from the released gas-mass map and from the *I*-band light of 72 spectroscopic members, using 2750 background-source shapes on their common grid (Clowe et al. 2006; Barrena et al. 2002). A linear lensing conversion (Kaiser & Squires 1993) supplies each template’s shear pattern. Amplitude and smoothing are fitted on training regions and evaluated on sky cells

kept out of that fit. In other words, we ask which map better predicts the observed lensing distortion in regions that were not used to tune the model.

In five-fold spatial cross-validation, the held-out coefficient of determination is $R^2 = 0.03167$ for the galaxy-light template and 0.01374 for shocked gas: 2.30 times as much (Table 4). Higher held-out R^2 means that the template reproduces more of the observed shear variation in the reserved regions. The galaxy template wins in 20 of 25 evaluation cells and all five outer folds. Adding the gas map to the optical template increases held-out R^2 by only 0.00079 . Grouped residual-sign tests give conditional $p = 0.03125$ for five folds or strips and $p = 0.0625$ for four broad regions. The measured shear therefore follows the galaxy-marked, transported morphology more closely than the stalled gas in this spatial prediction (Figure 3).

Table 4. Bullet Cluster shear-template prediction.

Template	Held-out R^2	Relative note
Gas	0.01374	baseline
Galaxies	0.03167	2.30× gas
Gas + galaxies	0.03245	+0.00079 over galaxies
Published κ	0.04482	reconstruction reference

Note. Published κ is the released convergence reconstruction, shown as a reference.

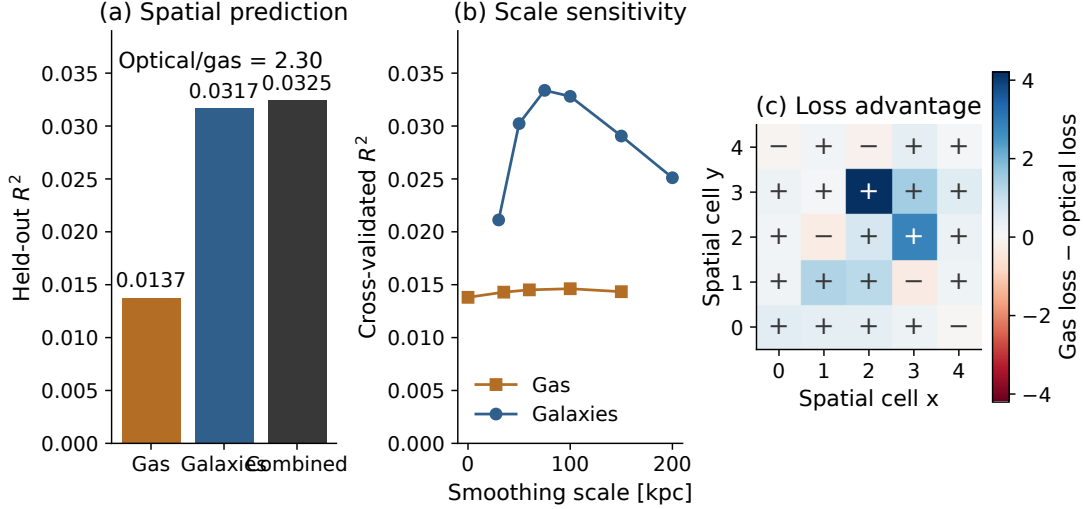


Figure 3. Bullet Cluster template competition. Left: held-out variance explained by each template. Middle: smoothing-scale response. Right: paired optical-minus-gas prediction advantage across 25 spatial cells; positive cells favour the optical template.

4.3. Do the lensing offsets point toward the galaxies?

A displacement can be large yet point anywhere. The transport picture makes a sharper prediction: the gas-to-lensing vector points toward the galaxies on each side of a merger. We test the angle between that vector and the gas-to-galaxy axis fixed in Equation (2), using published positions in the Bullet Cluster and MACS J0025 (Clowe et al. 2006; Bradač et al. 2008). A gas-to-lensing vector pointing toward the galaxies has an angle near 0° ; a vector pointing in the opposite direction would lie near 180° .

The two mergers contribute four arms. We weight each merger equally and rotate its two arms together in the random-orientation control, preserving the internal geometry while removing any preferred overall orientation. All four measured vectors point toward the galaxy side of the gas. Their merger-weighted mean cosine is 0.97955, meaning that the directions are nearly parallel; the common-orientation null gives $p = 0.00335$. Propagating published position uncertainties across 6000 coordinate realizations leaves all four arms on the galaxy side in 97.05–97.25% of draws under the tested cross-arm correlations. MACS J0025 thus repeats the Bullet’s directional ordering in a separate merger (Figure 4).

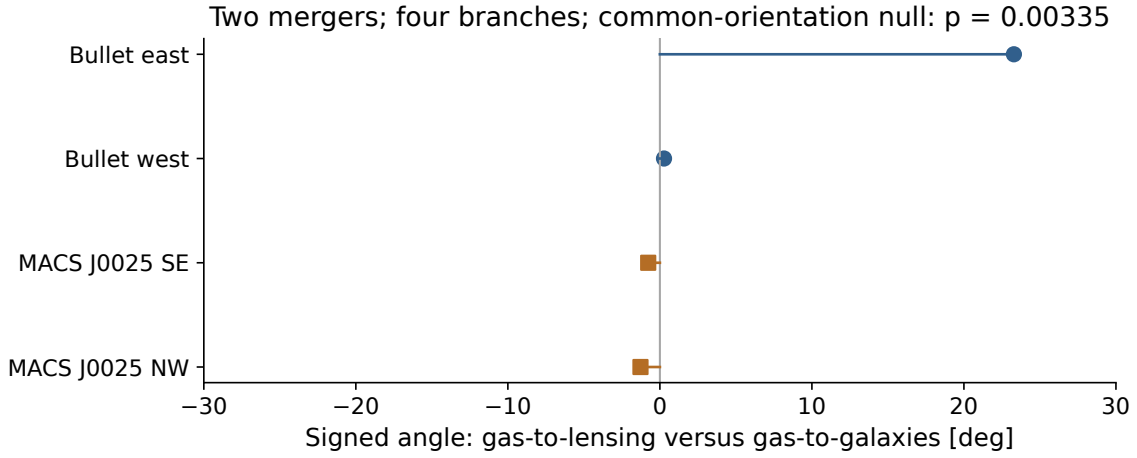


Figure 4. Angles between gas-to-lensing and gas-to-galaxy vectors in the two-merger sample. The outward axis uses gas and galaxies alone; all four central estimates point toward the galaxy side.

4.4. Turnover: does the lensing centre return toward the gas?

The first three tests examine the outward stage. Recoupling adds a distinct phase diagnostic: a lensing centre initially beyond the slowed gas later appears on its central side as the field readjusts toward the post-shock baryonic well. Four merger snapshots sample the outbound and returning stages. The Bullet Cluster and Abell 56 are classified as outbound; MACS J1149 and El Gordo as returning. Here, “outbound” means that the subclusters are still moving apart after first passage, whereas “returning” means that they have passed apocentre and are moving back toward the central region. Published dynamics and radio-relic constraints fix these phase labels before the gas–lensing offsets enter (Dawson 2013; Wittman et al. 2023; Golovich et al. 2016; Ng et al. 2015; Jee et al. 2014b).

Published dynamical analyses first classify each merger as outbound or returning. Gas and galaxy positions then fix the collision centre, branch, and outward axis. The projected pair separation and the common definition of D fix the normalization, after which the published lensing position is projected onto that axis. The phase-test README and input table in the reproducibility package list every phase source, position reference, pair scale, and reported position error used in this construction.

For each system the signed axial offset is divided by its projected subcluster separation, $D = d_{\parallel}/s_{\text{pair}}$. This normalization places mergers of different physical size on the same scale. A positive D places the lensing centre farther outward than the gas; a negative D reverses that order. Both outbound systems have $D > 0$ and both returning systems have $D < 0$. Their means change from +0.23685 outbound to −0.09001 returning (Table 5; Figure 5).

Table 5. Phase-ordered gas–lensing displacement. The four core systems have normalized D measurements; RM J1508 and NGC 4839/Coma extend the comparison with the published sign. Published dynamics fix every phase before the tested gas–lensing ordering enters.

Merger	Phase classification	Measured result	Expected sign
Bullet Cluster	outbound	+0.236	$D > 0$
Abell 56	outbound	$+0.237 \pm 0.094$	$D > 0$
MACS J1149	returning	−0.096	$D < 0$
El Gordo	returning	-0.084 ± 0.042	$D < 0$
RM J1508	outbound	positive sign	$D > 0$
NGC 4839/Coma	returning	negative sign	$D < 0$

The key result is the sign reversal: the four core measurements are positive in outbound systems and negative in returning systems. A two-state fit, evaluated by leaving out one system at a time, reduces the prediction error from RMSE = 0.21798 for a phase-blind constant to 0.00884; a damped continuous curve gives 0.02539. The same models improve the Akaike and Bayesian information criteria (AIC and

BIC) as well (two-state $\Delta\text{AIC} = 26.88$, $\Delta\text{BIC} = 27.50$; damped $\Delta\text{AIC} = 18.66$, $\Delta\text{BIC} = 19.28$). The exact probability of four specified signs under independent random signs is $p = 0.0625$; permuting the two outbound and two returning labels gives $p = 0.1667$ for their ordering.

The same fixed sign rule extends to two further published systems. RM J1508 is observed 96–236 Myr after pericentre and has the outbound sign (Stancioli et al. 2023). The NGC 4839 group is returning after first apocentre; its X-ray peak lies approximately $1'$ farther outward than its weak-lensing centre, giving the returning sign (Lyskova et al. 2019; Sasaki et al. 2016; HyeongHan et al. 2026). All six systems therefore follow the phase ordering. The exact one-sided probability of six matching random signs is $p = 1/64 = 0.015625$; permuting three outbound and three returning labels gives $p = 1/20 = 0.05$. The four measured amplitudes determine the phase curve in Figure 5; RM J1508 and NGC 4839/Coma expand the sign statistic to six systems.

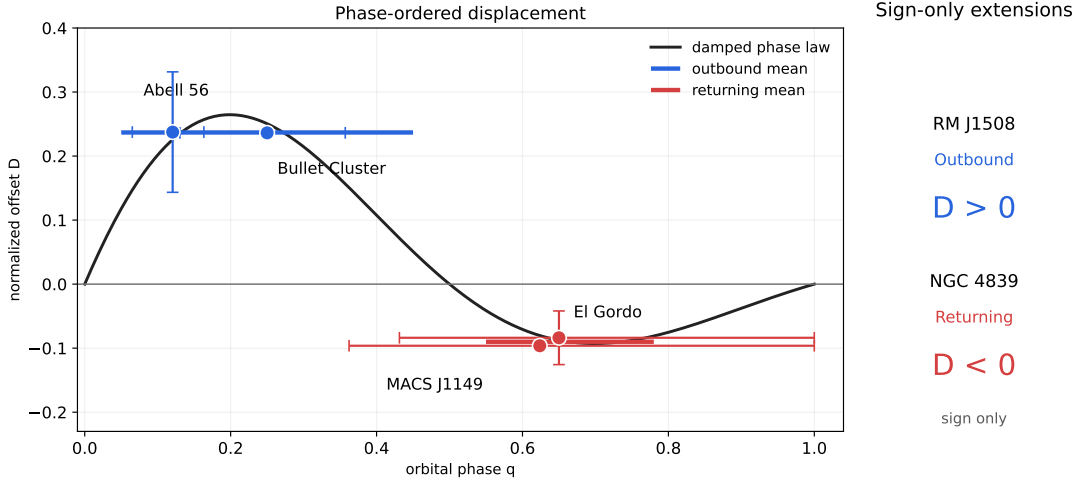


Figure 5. Phase-ordered signed displacement. Points are the four merger snapshots; horizontal intervals show published orbital-phase ranges and vertical errors are plotted where reported. Positive D places lensing beyond gas along the outward axis; negative D places it on the central side. Thick segments mark the phase-group means and the thin curve is the fitted damped description. The separate right-hand panel shows the two sign-only extensions; neither is assigned a numerical phase or included in the curve fit.

This phase-linked reversal is the turnover signal in the observed ordering. Within the finite-response picture, it marks the outward pattern yielding to the restoring contribution of the new central source. The damped phase law gives a compact description of that sequence; its construction is given in Appendix A.

4.4.1. Resolving the Abell 1240 phase disagreement

Abell 1240 supplies a direct use of the turnover sign as a phase discriminator. A cosmological-analog analysis assigns an 84% probability to the outbound solution, whereas a radio-relic-constrained reconstruction favors return by odds of 37.5:1 (Wittman 2019; Cho et al. 2022). These alternatives predict opposite signs for the same observable D . Here $D > 0$ means that the lensing centre lies outward of the gas along the fixed merger axis; $D < 0$ is the returning-side ordering.

We fix the midpoint, pair separation, and two outward axes from the published galaxy-luminosity peaks. Diffuse gas maxima are measured from the public 0.5–2 keV Chandra ObsID 4961 events after point-source masking. The centres and two-dimensional covariance ellipses of the published weak-lensing peaks are then projected onto those fixed axes. The statistic is the direct projected sign of this displacement. We propagate 1000 Poisson realizations of the X-ray map and 100000 gas/lensing draws, and repeat the measurement at Gaussian smoothing scales of 15, 20, 25, and 30 arcsec.

Table 6. Abell 1240 signed gas–lensing displacement. The probability and intervals propagate the X-ray and published lensing-centroid uncertainties.

Branch	Primary D	Median D	95% interval	$P(D > 0)$
A1240N	+0.496	+0.454	[+0.131, +0.638]	0.99957
A1240S	+0.373	+0.369	[+0.115, +0.620]	0.99776

Both branches give $D > 0$ (Table 6; Figure 6). The minimum branch probability for the outbound sign is 0.99776, and all eight branch-by-smoothing measurements remain positive. Both branches therefore select the outbound cosmological-analog solution for Abell 1240. The same pre-defined sign criterion resolves the published dynamical disagreement and requires no new phase-curve amplitude.

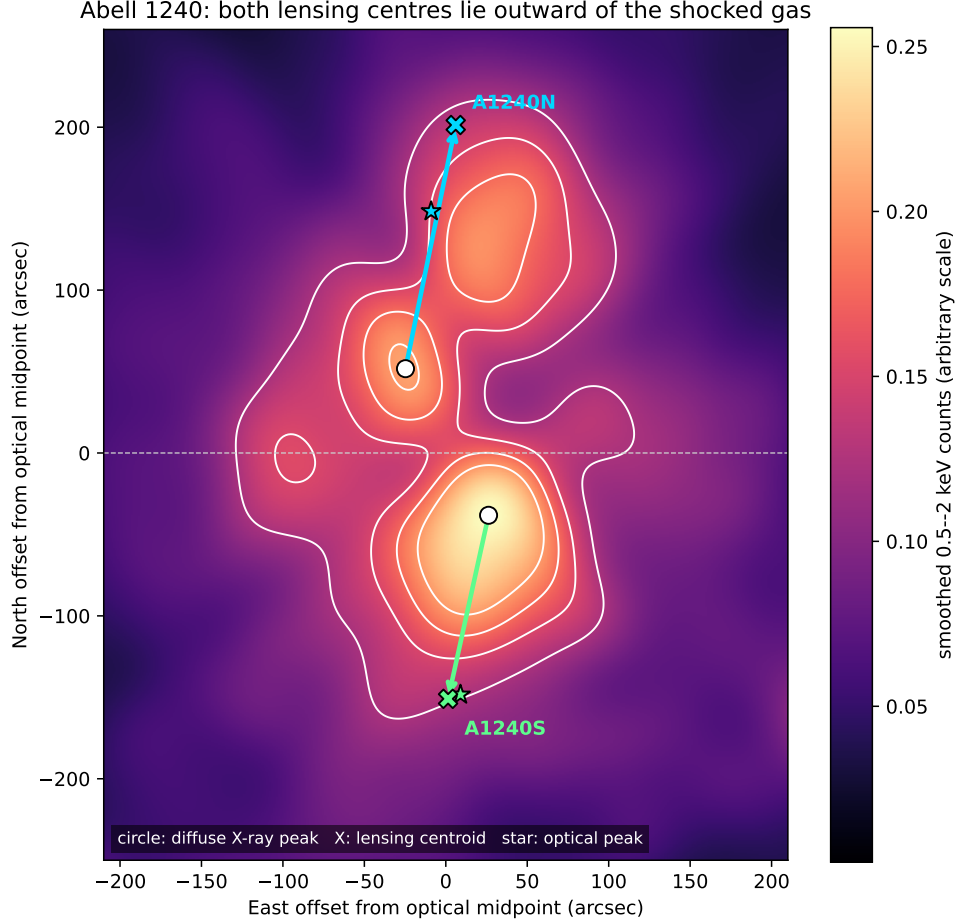


Figure 6. Abell 1240 phase adjudication. The background is the point-source-masked Chandra 0.5–2 keV count map smoothed with a 20-arcsec Gaussian; white contours are X-ray isophotes of the same map. Stars, circles, and crosses mark published optical peaks, diffuse X-ray maxima, and published weak-lensing centroids, respectively. Coloured arrows join gas maxima to lensing centroids; both point outward.

4.5. Where are the galaxies relative to the lensing centre?

This final spatial check completes the ordering: galaxies mark the outgoing trajectory while the lensing centre lies inward of them. In A520 we measure their relative location with $\beta_G = 1 - f_{||}$: exact coincidence gives zero, whereas a positive value places the fitted lensing centre inward of the galaxy tracer. All four A520 median values are positive (Figure 7). The conditional bootstrap places both arms inside the open gas–galaxy interval in 0.558 of C12 draws and 0.910 of J14 draws.

In the returning merger El Gordo, the resolved southeast number-weighted galaxy centre lies farther outward than the published lensing/luminosity core by 0.286 of the core-pair separation (Jee et al. 2014b). Together, A520 and El Gordo place the galaxy tracers outward of the lensing centres, completing the delayed spatial ordering.

Two further spatial checks locate the gas within this picture. The two Bullet gas peaks project inside the connected 90% regions of the forward lensing structures, and

Observable departure from a lensing centre fixed to the galaxies

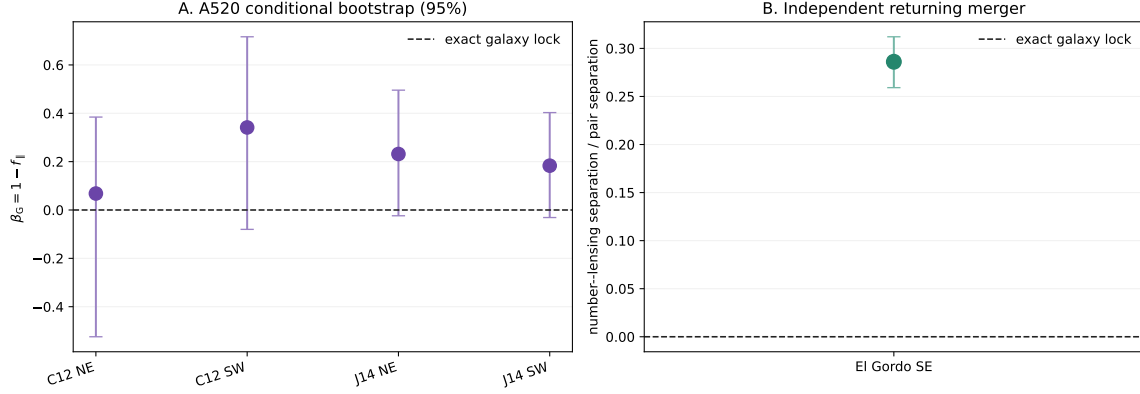


Figure 7. Lensing-centre separation from galaxy tracers. A520 shows $\beta_G = 1 - f_{\parallel}$ with 95% conditional bootstrap intervals; El Gordo shows outward number–lensing separation normalized by the lensing-core pair, with 16–84% digitization intervals. Each panel states its normalization; the dashed line denotes exact projected coincidence.

the lensing centres are farther apart than the gas in Bullet, MACS J0025, and ZwCl 0008 (merger-level sign $p = 0.125$). MACS J0025 contributes its common central gas maximum. These measurements show projected gas–field overlap; their extraction and null constructions are documented in the gas-containment and pair-separation modules of the reproducibility package.

5. INTERPRETATION

The tests follow the same physical sequence at different observational scales. The A520 holdout locates an advanced component using shear outside the shock and predicts the reserved centre. In Bullet, the galaxy-marked transported morphology predicts the shear better than gas. Bullet and MACS J0025 share the same outward vector ordering; the phase-classified sample adds its later reversal. A fitted lensing centre moves relative to gas and galaxies while the full field remains connected to the central collision region.

The post-shock gas builds the new compact baryonic source. The retained configuration responds over finite time and is redirected toward that source. Projected shear and lensing positions trace this transported geometry and its source-driven return.

5.1. Geometric evidence and particle-halo comparison

The standard collisionless interpretation assigns the displaced lensing signal to halo mass crossing with the galaxies (Markevitch & Vikhlinin 2007; Randall et al. 2008). The tests here make a different construction. The spatial predictors use observed gas, galaxy light and positions, and background-source shapes to predict the lensing observables. They require no particle-halo density profile, halo mass, concentration, interaction law, or cluster-specific orbital condition; the calibrated quantities describe the effective field itself.

This merger result extends the same geometric program developed in Papers I–III for galaxy rotation curves, cluster hydrostatics, and weak lensing (Rey 2026a,b,c). The new question is dynamical: after the gas is braked, does the effective field continue along the galaxy-defined trajectory and then return toward the new baryonic source? The tests answer that question through displacement, direction, and phase ordering using the observed baryonic structure.

6. CONCLUSIONS

The effective gravitational field in a cluster collision does not stop with the shocked gas: its lensing pattern extends toward the galaxy-defined outgoing side. A520 shows that a forward-shifted pattern fitted outside the shock predicts the reserved central shear. In Bullet, the galaxy-marked structure explains 2.30 times the held-out shear variance explained by shocked gas. Bullet and MACS J0025 show the predicted offset direction, while the phase sample changes from outward to inward gas–lensing order. RM J1508 and NGC 4839/Coma extend that sign sequence to six systems ($p = 0.015625$), and the same rule selects the outbound branch of Abell 1240.

These observations give finite-response geometry a concrete empirical form: transport after the shock, a lensing centre that can separate from its galaxy tracers, and a phase-associated return toward the new central baryonic source. The spatial predictions use the measured baryonic structure and lensing data without particle-halo templates.

DATA AND CODE AVAILABILITY

The accompanying reproducibility package contains the public input catalogues and maps, extraction rules, analysis scripts, numerical outputs, signed-recovery simulations, figure sources, and this manuscript. The archived release is available at Zenodo: [doi:10.5281/zenodo.23272604](https://doi.org/10.5281/zenodo.23272604). The main README links each test to its methods, input provenance, uncertainty calculations, and sensitivity controls. Each test README provides the commands and output files needed to reproduce its results. A SHA-256 inventory identifies the packaged files. The observational inputs are attributed to the studies cited above.

APPENDIX

A. A MINIMAL LAW FOR THE DISPLACED CENTRE

The field law in Equation (1) describes finite adjustment along a moving pattern. We close its centre motion with an effective restoring law. To represent the position of a fitted lensing feature, let $\mathbf{R} = \mathbf{x}_{\text{lens}} - \mathbf{x}_c$, where $\mathbf{x}_c$ is the new central baryonic minimum. A restoring potential with effective damping gives

$$\ddot{\mathbf{x}}_{\text{lens}} = -\nabla\Phi_c(\mathbf{x}_{\text{lens}}, t) - 2\Gamma(\dot{\mathbf{x}}_{\text{lens}} - \dot{\mathbf{x}}_c). \quad (\text{A1})$$

Here Φ_c is the potential sourced by the compact post-shock baryonic concentration and Γ is the effective damping rate. Near a slowly moving central minimum, $-\nabla\Phi_c \simeq -\Omega_c^2 \mathbf{R}$, so the axial displacement obeys

$$\ddot{R} + 2\Gamma\dot{R} + \Omega_c^2 R = 0, \quad R(0) = 0, \quad \dot{R}(0) = v_{\text{in}}, \quad (\text{A2})$$

where Ω_c is the local restoring frequency and v_{in} is the inherited outward velocity. That inherited motion produces the initial advance and the central gradient produces the return. For an underdamped response, $R(t) \propto e^{-\Gamma t} \sin(\omega_d t)$ with $\omega_d^2 = \Omega_c^2 - \Gamma^2$, where ω_d is the damped angular frequency. Expressing merger time as phase $q = \text{TSP}/T$, where TSP is the time since pericentre and T the adopted orbital timescale for each system, and choosing the one-cycle closure $\omega_d T = 2\pi$ gives the fitted phase description

$$D(q) = Ae^{-\lambda q} \sin(2\pi q), \quad \lambda = \Gamma T, \quad (\text{A3})$$

where A absorbs the normalized incoming velocity. The four-system fit gives $A = 0.423$ and $\lambda = 2.096$, with leave-one-system-out RMSE 0.02539 against 0.21798 for the phase-blind constant. The force law concerns the centre relative to the baryonic minimum. The observed D uses the gas peak as the available baryonic reference and normalizes the projected separation by the subcluster pair. The comparison therefore tests the predicted phase ordering, using the damped curve as its two-parameter description. The dimensional time scales T and τ_{eff} are local to each merger.

B. ROBUSTNESS AND ADDITIONAL SPATIAL CHECKS

A520 recovery and reference robustness. Synthetic signals with known outward shifts of $(60, 40)''$ are detected above the archived 95th-percentile null threshold in 91/100 C12 and 99/100 J14 draws. The corresponding zero-shift cases produce 0/100 and 2/100 outward detections. The same signed fits are also tested with inward injections; the packaged signed-recovery module gives all recovered centres, biases and sign frequencies. The 48 physical variants of the exterior fit preserve both forward displacement and gas containment; the largest gas–centre separation is 0.825 of the fitted 90% enclosed radius. A fit restricted to the independently defined gas–galaxy

axis also improves reserved-centre χ^2 over gas-centred components: $410.14 \rightarrow 403.49$ in C12 and $613.99 \rightarrow 611.57$ in J14. Allowing transverse motion changes those losses by less than one in either catalogue. These checks preserve the longitudinal forward signal across reference and model variants.

Bullet spatial dependence. The optical-minus-gas predictive advantage has the same sign in all five outer folds, in horizontal and vertical strips, and in four broad regions. Repeating the complete fit under a gas-only wild-bootstrap null gives conditional $p = 0.03125$ with five folds or strips and $p = 0.0625$ with four broad regions. A 25-cell sign test gives $p = 0.000305$. The positive held-out contrast persists at every tested spatial partition.

Direction and phase. We propagated position errors through 2000 realizations at each of three assumed correlations between a merger’s arms ($\rho = -0.5, 0, +0.5$). Across these assumptions, 97.05–97.25% of realizations retain all four outward signs, and 96.95–97.70% retain a joint orientation $p < 0.05$. An alternative Bayesian analysis of these same two merger orientations gives $B_{10} = 8.612$ for its primary prior, ranging from 2.547 to 23.569 over the reported prior and correlation choices. In the phase comparison, published outbound and return classifications define the two groups, and leave-one-system-out errors quantify prediction across the four snapshots.

The test READMEs in the reproducibility package give the full null constructions and the pair-separation and Bullet gas-overlap measurements cited in Section 4.5.

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