

Lie Algebra, Global Inversion, and the Case for Ontological Two-Sidedness

Stephen P Smith

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Abstract. The effectiveness of mathematics in describing nature has long invited philosophical explanation. This paper examines whether Lie algebra embodies organizational principles that extend beyond its traditional role in symmetry analysis. Beginning with the fundamental properties of Lie brackets, including antisymmetry, bilinearity, the Jacobi identity, and the derived relation $[X, -X] = 0$, it is argued that Lie algebra provides a formal grammar of relational coherence. Particular attention is given to the invariance of Lie-algebraic structure under global reflection $X \mapsto -X$, which preserves brackets and relational organization while remaining indifferent to overall orientation. This observation motivates the claim that relational structure alone may not uniquely determine ontological orientation. Connections are explored with CPT symmetry, electromagnetism, active inference, developmental biology, and multiscale regulation, while carefully distinguishing mathematical results from heuristic interpretation. The paper does not claim that Lie algebra proves ontological two-sidedness or derives physical symmetries. Rather, it argues that Lie-theoretic structures exhibit a remarkable compatibility with coherent organization across multiple domains and therefore leave two-sided ontological interpretations admissible. In this view, the effectiveness of mathematics may arise because both mathematics and nature instantiate common principles of relational coherence and regulated organization.

Keywords: Biology, Coherence, CPT Symmetry, Electromagnetism, Free Energy Principle, Global Orientation, Middle-Term, Organization, Passive Transformation, Template, Reflection, Regulation.

1. Introduction

The enduring effectiveness of mathematics in the sciences is often presented as a mystery. Since Wigner's (1960) famous essay on the unreasonable effectiveness of mathematics, many have wondered why abstract mathematical structures developed for their own internal reasons should later prove indispensable for describing nature.

One possible answer is that some mathematical structures succeed because they embody deep principles of organization that recur throughout reality itself. Among the strongest candidates is Lie algebra (cf. Erdmann and Wildon 2006). Its importance in modern physics is unquestionable, but its significance may extend beyond symmetry groups and gauge theories. It may reveal a more general grammar of coherence.

At the heart of Lie algebra stand several remarkably simple relations:

$$[X, X] = 0, \quad [X, -X] = 0, \quad [X, Y] = -[Y, X],$$

together with the Jacobi identity

$$[X, [Y, Z]] + [Y, [Z, X]] + [Z, [X, Y]] = 0.$$

To these may be added bilinearity,

$$[aX + bY, Z] = a[X, Z] + b[Y, Z],$$

which allows more complex relational expressions to be decomposed into simpler ones.

Taken together, these relations form a striking system of constraints.

The first relation,

$$[X, X] = 0,$$

states that a generator has no discrepancy with itself. If the bracket is interpreted as measuring relational distinction or misalignment, then perfect self-correspondence generates no residual discrepancy.

Similarly,

$$[X, -X] = 0,$$

expresses exact cancellation between a generator and its opposite orientation. A generator and its perfect reflection close without remainder.

Antisymmetry, $[X, Y] = -[Y, X]$, interpreted relationally introduces reciprocity: every distinction admits two orientations, and relations possess no intrinsically preferred direction.

Finally, the Jacobi identity constrains the coherence of nested relations, ensuring that local distinctions remain consistent within a larger relational structure. Individual pairwise relations may be nonzero, but their nested organization must remain globally coherent.

These observations do not change the mathematical meaning of Lie algebra. They simply suggest that the algebra embodies a powerful organizational grammar.

2. Bilinearity and Incremental Organization

The role of bilinearity deserves special attention.

Lie algebras arise naturally from infinitesimal transformations because infinitesimal generators inhabit a vector space. Consequently,

$$[aX + bY, Z] = a[X, Z] + b[Y, Z]$$

and similarly for the second argument.

This property means that local discrepancies can be decomposed into constituent discrepancies and recombined predictably.

Such behavior is essential if large-scale organization is to emerge through incremental adjustment.

Development, learning, active inference, and biological adaptation do not solve global problems all at once. They proceed through successive local corrections. This capacity requires “additivity,” as recognized by Smith (2024), so that biology can navigate both development and evolution pulling “itself up from its bootstraps”. Serendipitously, most genetic variation is found as additive genetic variation (Hill et al., 2008), but this may not be accidental.

Heuristically, this suggests that bilinearity is compatible with a broader organizational principle:

$$\textit{global coherence} = \textit{accumulated local regulation}.$$

Whether in differential geometry, active inference, or development, successful organization seems to depend on the ability to build large structures from small adjustments.

3. Lie Algebra and the Mirror Problem

One consequence of Lie algebra is especially striking.

Suppose every generator is reflected simultaneously:

$$R: X \mapsto -X.$$

Under simultaneous sign reversal of all generators, bilinearity gives $[-X, -Y] = [X, Y]$. Thus the defining relational expressions retain their form under the global reversal, even

though $X \mapsto -X$ is not, in general, a Lie-algebra automorphism. Likewise, antisymmetry remains unchanged. The Jacobi identity remains unchanged. The relational structure expressed by these bracket relations survives the simultaneous reflection. This is one of the strongest observations to emerge from the present discussion.

Lie algebra determines relations among generators, yet it does not intrinsically privilege one global orientation over its opposite. In that sense Lie algebra resembles a perfect mirror. It sees relations. It preserves coherence. Yet it cannot determine which side of a globally reflected pair should be regarded as ontologically primary. As Smith (2022) notes, this is the I-Am-That-I-Am condition, where the sides of a two-sided mirror cannot be distinguished.

This observation does not prove ontological two-sidedness as developed by Smith (2021). However, it does demonstrate something important:

Relational structure alone does not determine ontological orientation.

This is closely related to another recurring principle:

formal correspondence $\neq$ ontological identity.

A successful correspondence between mathematics and observation does not automatically settle what ultimately exists.

4. CPT Symmetry and Global Inversion

At this point it is important to distinguish two different notions of CPT.

In standard quantum field theory, CPT refers to charge conjugation, parity inversion, and time reversal acting on fields and states within a fixed theory.

The CPT theorem is a rigorous result arising from assumptions such as Lorentz invariance, locality, and the framework of relativistic quantum field theory. Nothing in the present discussion alters that theorem. The claim is not that CPT symmetry is secretly derivable from Lie algebra. A different notion has been explored here.

By ‘global CPT-like inversion’ I mean, in this paper, a passive or global reversal of orientation across an entire relational structure. This resembles what Susskind and Friedman (2017) describe as a passive transformation, where the description changes while the relational content remains the same.

The global orientation toggles with a simultaneous sign reversal of all generators in the relational structure. Under this broader interpretation, this global inversion resembles

$R: X \mapsto -X$, for all generators simultaneously. The Lie-algebraic relations accommodate such a simultaneous reflection naturally. Although $[X, -X] = 0$ is not an independent Lie-algebra axiom, its derived status makes it a useful illustrative template for discussing globally reflected descriptions.

Indeed, modern gauge theories, Lorentz symmetry, and relativistic quantum field theory are already deeply Lie-theoretic. The interesting question is therefore not whether Lie algebra proves CPT, but why Lie-theoretic structures are so compatible with CPT-compatible physics.

The answer may lie in a shared organizational principle. Lie algebra allows opposites. Lie algebra allows reversals. Lie algebra allows local asymmetries. What it does not allow is incoherence.

A similar statement can be made about many physical symmetries. Thus, the correspondence may be summarized as:

Local asymmetry is permitted; global incoherence is not.

5. Electromagnetism and Passive Symmetry

This distinction becomes particularly visible in electromagnetism.

As shown in the Appendix, Maxwell's equations may be shown to remain invariant under charge inversion, parity inversion, and time reversal taken separately. This is a stronger statement than merely preserving a combined CPT symmetry.

These transformations must be distinguished conceptually. Passive coordinate transformations change the description of a physical configuration, whereas charge conjugation reverses the charge assignments and associated electromagnetic fields. In each case, the relevant question is whether Maxwell's equations retain their form under the specified transformation. The distinction matters here because the present argument concerns the preservation of relational structure, not the derivation of physical symmetries from Lie algebra.

Such an analysis encourages viewing physical theories less as collections of isolated objects and more as coherent relational structures capable of surviving global changes of orientation. This convention can now be formalized for a Lie algebraic understanding of global inversion and electromagnetism.

a) Reflection Symmetry and an Enlarged Organizational Space

The separate invariance of Maxwell's equations under charge, parity, and time inversion suggests a broader organizational interpretation. Before introducing an enlarged space, it is useful to recall that electromagnetism already possesses an underlying Lie-theoretic structure through the gauge-covariant derivatives

$$D_\mu = \partial_\mu + iA_\mu,$$

whose commutator yields

$$[D_\mu, D_\nu] = iF_{\mu\nu}.$$

The field tensor is antisymmetric in its spacetime indices, $F_{\mu\nu} = -F_{\nu\mu}$. More fundamentally, the commutator of gauge-covariant derivatives provides a direct connection to Lie-algebraic structure. This offers a natural starting point for examining how antisymmetry and relational consistency enter the electromagnetic description. The Jacobi identity for the covariant derivatives is reflected in the Bianchi identity discussed in the Appendix, thereby ensuring consistency of the relational structure.

A second algebraically equivalent sector may be obtained by reversing the charge orientation,

$$[D_\mu, D_\nu] = -iF_{\mu\nu}.$$

The two sectors differ only by overall charge orientation and may therefore be regarded as redundant descriptions related by global reflection. Mathematically, the simplest enlarged construction is the direct sum

$$g = g_+ \oplus g_-,$$

where g_+ and g_- represent algebraically equivalent sectors interpreted as opposite charge orientations $q = +1$ and $q = -1$, respectively. In the proposed construction, this enlarged space permits both orientations to be represented simultaneously. The redundancy is intentional: it makes explicit a mirroring relation that would otherwise remain implicit in a description restricted to one orientation. Relations such as

$$[X, -X] = 0 \text{ \& \, } [X, Y] = [-X, -Y]$$

then serve as formal templates for considering how relational structure may be preserved under global reversal.

This produces an eight-dimensional space containing both orientations simultaneously. The construction introduces no new electromagnetic predictions and should be understood primarily as a bookkeeping device. Its significance is conceptual rather than physical.

b) Collapse Back to Maxwell Theory

Once a particular charge orientation is selected, attention is restricted to one of the two redundant sectors, recovering the familiar four-dimensional description used by Maxwell's equations. Again, the significance of the enlarged space is not that it produces new electromagnetic predictions, but that it makes explicit the reflection principle already implicit in the separate C, P, and T symmetries.

From this perspective, the enlarged construction remains indifferent to which global orientation is chosen. The relational structure survives the restriction to either sector, and the resulting Maxwell description inherits the same coherence conditions present in the larger bilateral space.

Maxwell's equations may therefore be viewed as the four-dimensional projection of an orientation-symmetric framework in which opposite charge sectors coexist formally, even though only one orientation is retained in a particular description. Heuristically, CPT symmetry provides a template through which relational visibility may cohere.

Because the two sectors are related by direct sum rather than by additional dynamics, no new electromagnetic phenomena are predicted. The enlarged construction serves only to make explicit a reflection relation already tolerated by the algebraic structure. In this sense the four-dimensional Maxwell description may be regarded as one admissible orientation within a larger bilateral framework.

6. Friston, Markov Blankets, and Agency

Friston (2010) and Friston et al. (2023) employ the free-energy principle to describe how agents maintain themselves through continual regulation across a Markov blanket that separates internal and external states, while defining the statistical relations through which the states interact. In this respect, it offers a useful analogy with the relational organization considered within Lie algebra: distinctions are meaningful within a larger structure that constrains how the parts relate. The analogy is organizational rather than mathematical, and does not imply that a Markov blanket is itself a Lie bracket. Internal and external states are distinguished, yet neither side possesses meaning independently of the boundary that relates them. The free-energy principle is therefore concerned not with isolated substances but with relational organization across a two-sided interface. Within the free-energy framework, adaptive systems can be described in terms of minimizing variational free energy, which provides an upper bound on

sensory surprise. Their interactions with the environment occur through the statistical dependencies represented by the Markov blanket. The relevance to the present argument is that sustained organization depends on relations across a boundary, rather than on internal states considered in isolation.

The significance of the Markov blanket lies not primarily in its informational role but in its regulatory role. The blanket constitutes a persistent boundary through which local interactions remain coordinated with larger-scale organization. In this respect it resembles the function served by the Jacobi identity within Lie algebra: local relations remain admissible, but global incoherence is excluded.

The free-energy principle is fundamentally concerned with maintaining coherence across a two-sided boundary. The Markov blanket separates agent and environment while simultaneously relating them. Thus, the agent's world is accessed through relational organization rather than through unrestricted direct access to external states.

The agent may exhibit considerable local autonomy while remaining constrained by the larger relational context in which it is embedded. In this respect, the free-energy principle resonates strongly with Koestler's (1967) conception of the Janus-faced holon. Like the holon, the agent functions simultaneously as a coherent whole and as a part of a wider organization. Local freedom and global coherence therefore appear not as opposites but as complementary requirements of adaptive organization.

The comparison with Lie algebra can now be stated more precisely. A Markov blanket maintains a statistical distinction between internal and external states while permitting their interaction within a larger organized system. Lie algebra, by contrast, imposes formal constraints on relations among generators, with the Jacobi identity ensuring algebraic consistency. Their structures are not equivalent, but both provide ways of examining how differentiated elements participate in coherent organization.

From the perspective developed here, the parallel raises a philosophical question: can relational coherence be maintained without the relations themselves determining which side is ontologically primary? The free-energy principle does not answer that question, nor does it establish a two-sided ontology. Nevertheless, it provides a useful interdisciplinary comparison in which boundary maintenance, local adjustment, and larger-scale organization can be considered together.

These parallels remain heuristic and should not be confused with Friston's mathematical formalism. Their value lies in suggesting a common organizational problem across otherwise distinct domains, not in demonstrating a common underlying mechanism.

7. Development, Evolution, and Regulated Emergence

The same organizational challenge appears throughout biology. Embryos develop incrementally. Organisms learn incrementally. Evolution explores adaptive possibilities incrementally.

Coherent structures do not appear all at once. They bootstrap themselves through local modification.

This observation weakens the appeal of naive emergence as a complete explanation. Generation alone does not explain coherence. Production alone does not explain regulation. The persistence and organization of complex systems require additional explanatory principles.

Increasingly, biology points toward multiscale regulatory organization, in which causation and control are distributed across interacting levels rather than residing exclusively in the genome. This trend is reflected in Evolution "On Purpose": Teleonomy in Living Systems (Corning et al. 2023), which presents teleonomy, or evolved purposiveness, as a real feature of living systems and argues for a broader framework that integrates molecular genetics with development, physiology, behavior, niche construction, and other higher-level regulatory processes. Some examples follow.

Shapiro (2022) emphasizes cellular information processing and genome restructuring. Nobel (2008, 2012) and Nobel et al. (2019) emphasize causation across multiple levels of biological organization, where DNA is recognized only as a template. Levin (2021, 2023) emphasizes bioelectric pattern formation, collective cellular decision-making, and large-scale developmental regulation. These researchers disagree on many details, but they share a common theme: biological order is not adequately described as an unregulated accumulation of local events. Regulation is central. Not peripheral.

8. The Middle-Term and Ontological Two-Sidedness

This leads naturally to the question of a middle-term. The present argument does not prove the existence of a universal regulator. Nor does Lie algebra by itself establish Smith's (2021) account of ontological two-sidedness. Those claims would exceed what the mathematics demonstrates.

A more modest conclusion is both stronger and more defensible. Lie algebra describes relations. Lie algebra preserves coherence. Lie algebra remains remarkably indifferent to global orientation. Consequently, the mathematics itself does not uniquely license a one-sided ontology.

A deeper organizing principle therefore remains an admissible possibility. Such a principle would not compete with empirical science. Rather, it would help explain how coherent empirical relations arise and persist in the first place. From this perspective, nature is not merely a collection of isolated objects and not merely a collection of generated structures. Nature is a nested holarchy in which local distinctions participate in larger coherent wholes.

The reflection relation, $[X, -X] = 0$, need not be interpreted merely as a formal consequence of antisymmetry and bilinearity. More generally, it may be viewed as a candidate template for admissible global reflection, one that extends beyond the specific charge, parity, and time inversions discussed in the present work. The separate C, P, and T invariances of Maxwell's equations provide only the simplest illustration of the idea. Richer possibilities arise in non-Abelian gauge theories, where Lie-algebraic structure enters more deeply through gauge fields, curvature, and the consistency conditions associated with Jacobi and Bianchi identities. These developments trace back to the gauge-theoretic framework introduced by Yang and Mills (1954). Likewise, if developmental regulation is ultimately guided by coherent multiscale constraints, as suggested by the bioelectric morphogenesis framework of Levin (2019, 2021), then Lie-theoretic notions of reflection, compatibility, and closure may prove useful far beyond their traditional role in fundamental physics. The present discussion therefore should be regarded only as a first exploration of a broader organizational principle whose implications remain largely unexplored.

Lie algebra does not prove this vision. Yet its reciprocity, its symmetry under global reflection, its compatibility with modern physics, and its insistence on coherence suggest that reality may be organized less by isolated substances and more by relational order.

In that sense, the unreasonable effectiveness of mathematics may not be unreasonable at all. Mathematics succeeds because both mathematics and nature may instantiate the same underlying principles of coherence. Lie algebra suggests that relational coherence alone does not uniquely determine ontological orientation; consequently, two-sided ontologies remain admissible.

Appendix: Demonstrating that Maxwell's Equations are Invariant to Time, Parity and Charge Inversions Taken Separately

The appendix is not evidence for ontological two-sidedness. It is supplied only as an illustration that fundamental physics often tolerates global inversions without loss of

relational structure. With the passive coordinate and field transformations defined below, Maxwell's equations retain their form under each of the three transformations considered separately. This should be distinguished from establishing the corresponding active physical symmetries.

1. Maxwell's Equations

$$(1) \vec{\nabla} \cdot \mathbf{B} = 0$$

$$(2) \vec{\nabla} \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \vec{0}$$

$$(3) \vec{\nabla} \cdot \mathbf{E} = \rho$$

$$(4) \vec{\nabla} \times \mathbf{B} - \frac{\partial \mathbf{E}}{\partial t} = \mathbf{j}$$

Where $\mathbf{B}$ and $\mathbf{E}$ are 3-vectors representing the magnetic and electric fields, respectively. The electric charge density is given by ρ , and the current density is represented by the 3-vector $\mathbf{j}$. The components of $\mathbf{B}$ and $\mathbf{E}$ are located inside an anti-symmetric matrix $\mathbf{F}$, which is a rank-2 tensor in 4 dimensions and transforms accordingly. $\mathbf{F}$, or $F_{\mu\nu}$, is called the field tensor, and its covariant components are indicated in the following matrix.

$$F_{\mu\nu} = \mathbf{F} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ +E_x & 0 & +B_z & -B_y \\ +E_y & -B_z & 0 & +B_x \\ +E_z & +B_y & -B_x & 0 \end{pmatrix}$$

When ρ and $\mathbf{j}$ are concatenated into a 4-vector $\mathbf{J}$, they also make a rank-1 tensor J^μ that transforms accordingly, and is represented by:

$$J^\mu = \mathbf{J} = \begin{pmatrix} \rho \\ \mathbf{j} \end{pmatrix}$$

One advantage to moving to tensors is how they simplify Maxwell's equations in symbolic terms. Equations (1) and (2), become the 64 equations represented by the Bianchi identity:

$$\partial_\sigma F_{\mu\nu} + \partial_\mu F_{\nu\sigma} + \partial_\nu F_{\sigma\mu} = 0$$

All but four of the equations are redundant, so this redundancy is the cost of the apparent symbolic simplicity. Equations (3) and (4) show even more simplicity when they are represented by these additional four tensor equations that are written using the contravariant components, $F^{\mu\nu}$:

$$\partial_\nu F^{\mu\nu} = J^\mu$$

CPT is an acronym for charge (C), parity (P) and time (T), and together they form a discrete symmetry. Maxwell's equations not only respect CPT symmetry, they are symmetric with each of these (C, P and T) when taken separately to invariantly transform Maxwell's equations back into themselves.

The matrix notation used in the following sections serves as a compact representation of the underlying tensor transformations. Because only flat Minkowski spacetime and linear discrete symmetries are considered, explicit distinction between covariant and contravariant components is suppressed where it does not affect the symmetry analysis. Index raising and lowering are accomplished by the constant Minkowski metric, and the results obtained are independent of the particular tensor representation employed.

2. Time Reversal Symmetry

First consider time reversal, which is accomplished by making a new set of coordinates that are marked by the prime notation, and defining them by transforming the original coordinates (t, x, y, z) :

$$t' = -t, \quad x' = x, \quad y' = y, \quad z' = z.$$

In matrix notation the time reversal transformation is given by:

$$\mathbf{v}' = \mathbf{D}_t \mathbf{v},$$

$$\text{where } \mathbf{v} = \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}, \quad \mathbf{v}' = \begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} \quad \text{and} \quad \mathbf{D}_t = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}.$$

Note that $\mathbf{D}_t$ is very simple, given that $\mathbf{D}_t = \mathbf{D}_t^T = \mathbf{D}_t^{-1}$, which implies that the needed tensor transformations required when changing coordinates to the time-reverse frame are transparently achieved by simple matrix multiplications. This gives the following:

$\mathbf{F}' = \mathbf{D}_t \mathbf{F} \mathbf{D}_t$ and $\mathbf{J}' = \mathbf{D}_t \mathbf{J}$; and in particular,

$$\begin{aligned}\mathbf{E}' &= -\mathbf{E} \\ \mathbf{B}' &= \mathbf{B} \\ \rho' &= -\rho \\ \mathbf{j}' &= \mathbf{j}.\end{aligned}$$

First derivatives, given by $\vec{\nabla}$ and $\partial/\partial t$, transform in a similar way as indicated below.

$$\left(\frac{\partial}{\partial t'} \right)_{\vec{\nabla}'} = \mathbf{D}_t \left(\frac{\partial}{\partial t} \right)_{\vec{\nabla}} \text{ and in particular, } \frac{\partial}{\partial t'} = -\frac{\partial}{\partial t} \text{ and } \vec{\nabla}' = \vec{\nabla}.$$

This transformation can now be substituted into equations (1), (2), (3) and (4), and by dropping all the prime demarcations this gives the following set.

$$\begin{aligned}(5) \quad \vec{\nabla} \cdot \mathbf{B} &= 0 \\ (6) \quad -\vec{\nabla} \times \mathbf{E} - \frac{\partial \mathbf{B}}{\partial t} &= \vec{0} \\ (7) \quad -\vec{\nabla} \cdot \mathbf{E} &= -\rho \\ (8) \quad \vec{\nabla} \times \mathbf{B} - (-1)(-1) \frac{\partial \mathbf{E}}{\partial t} &= \mathbf{j}\end{aligned}$$

Equation (8) is no different to (4), as the multiplication of -1 by itself cancels. Equations (6) and (7) are also the same as (2) and (3) respectively, as multiplying both sides of the respective equation by -1 restores the equivalence. Equations (5) to (8) are essentially the unchanged Maxwell's equations which are now found invariant under time reversal.

3. Parity Symmetry

The same type of transformation of coordinates can be conducted to demonstrate parity invariance. A spatial reflection can be achieved by reversing any single spatial coordinate, but parity inversion is more conveniently represented by reversing all three spatial coordinates as indicated by the transformation matrix $\mathbf{D}_{xyz}$:

$$\mathbf{v}' = \mathbf{D}_{xyz} \mathbf{v},$$

$$\text{where } \mathbf{v} = \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}, \mathbf{v}' = \begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} \text{ and } \mathbf{D}_{xyz} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}.$$

The details of this transformation are given below.

$$\mathbf{F}' = \mathbf{D}_{xyz} \mathbf{F} \mathbf{D}_{xyz} \text{ and } \mathbf{J}' = \mathbf{D}_{xyz} \mathbf{J}; \text{ and in particular,}$$

$$\mathbf{E}' = -\mathbf{E}$$

$$\mathbf{B}' = \mathbf{B}$$

$$\rho' = \rho$$

$$\mathbf{j}' = -\mathbf{j}.$$

$$\left(\frac{\partial}{\partial t'} \right) = \mathbf{D}_{xyz} \left(\frac{\partial}{\partial t} \right), \text{ and in particular, } \frac{\partial}{\partial t'} = \frac{\partial}{\partial t} \text{ and } \vec{\nabla}' = -\vec{\nabla}.$$

This transformation can now be applied to equations (1), (2), (3) and (4), but it is better to apply it to the equivalent Maxwell Equations (4), (6), (7) and (8) to automatically keep track of the sign alternations as the transformations are combined into a single time and parity transformation. This produces the following equations.

$$(9) - \vec{\nabla} \cdot \mathbf{B} = 0$$

$$(10) - (-)(-)\vec{\nabla} \times \mathbf{E} - \frac{\partial \mathbf{B}}{\partial t} = \vec{0}$$

$$(11) - (-)(-)\vec{\nabla} \cdot \mathbf{E} = -\rho$$

$$(12) - \vec{\nabla} \times \mathbf{B} - (-1)\frac{\partial \mathbf{E}}{\partial t} = -\mathbf{j}$$

These equations are again equivalent to (1), (2), (3), and (4), as demonstrated by multiplying both sides of these equations by -1. Maxwell's equations are found invariant under parity reversal taken alone, which is a stronger condition than combined invariance found by making time and parity reversals together.

4. Charge Reversal Symmetry

The charge is not a coordinate. Therefore, the reversal of charge is a different type of transformation. Nevertheless, reversing charge includes reversing the direction of the electric field, the polarity of the magnetic field, and negating both the charge density and the current density. The derivatives, $\partial/\partial t$ and $\vec{\nabla}$, are unchanged. These switches are indicated below.

$$\begin{aligned}\rho' &= -\rho \\ \mathbf{j}' &= -\mathbf{j} \\ \mathbf{E}' &= -\mathbf{E} \\ \mathbf{B}' &= -\mathbf{B} \\ \frac{\partial}{\partial t'} &= \frac{\partial}{\partial t} \\ \vec{\nabla}' &= \vec{\nabla}\end{aligned}$$

Applying these substitutions to (9), (10), (11) and (12), returns Maxwell's equations exactly:

$$(13) \vec{\nabla} \cdot \mathbf{B} = 0$$

$$(14) \vec{\nabla} \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \vec{0}$$

$$(15) \vec{\nabla} \cdot \mathbf{E} = \rho$$

$$(16) \vec{\nabla} \times \mathbf{B} - \frac{\partial \mathbf{E}}{\partial t} = \mathbf{j}$$

Maxwell's equations are found invariant under charge inversion by itself, which is a stronger condition than joint CPT invariance.

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