A NEW PARADIGM OF MATHEMATICAL ANALYSIS: SCALE QUANTIZATION, INTER-SYSTEM TRANSITIONS, AND REAL QUANTUM MECHANICS

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Abstract. This paper proposes an alternative paradigm of mathematical analysis based on the interaction of physical quantities rather than an abstract numerical continuum. The concept of a discrete scale factor M and inter-system transition coefficients via Gudermannian mappings are introduced, rendering the classical substitution of variables an anachronism. Exact analytical operational formulas for differentiation and integration are obtained, a physics-scale-based proof of Fermat's Last Theorem is presented, and new empirical models for elliptic integrals of the first and second kind are derived. For the first time, a real quantum wave function based on split-complex (dual) numbers is proposed, and a new form of the stationary Schrödinger equation, free from imaginary mysticism, is derived.

1. Introduction and Philosophical Foundations

The direction of development of human society and scientific thought is fundamentally determined by its philosophical and religious worldview. It is the worldview that lays down the basic rules of behavior, definitions of good and evil, prohibitions, and taboos.

Classical mathematical analysis also possesses its own philosophical worldview. At its core lies the concept of an abstract number, based on set theory, the arithmetic continuum, actual infinity, and the definition of the derivative via the limit transition ($\Delta x \to 0$). The triumph of this approach in the eyes of the mathematical community is Andrew Wiles' famous proof of Fermat's Last Theorem, which spanned 130 pages of highly complex text. This philosophical framework enabled the creation of differential and integral calculus in its present form.

However, modern mathematical analysis can be built upon a different philosophy. It views mathematics not as an abstract set of numbers, but as the **interaction of physical quantities**. A physical quantity always possesses dimensions, scale, and spatial boundaries. Such an approach avoids the logical paradoxes of infinity, simplifies cumbersome analytical derivations, and demystifies quantum processes.

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2. A New Definition of Infinity and the Scale Factor

Every physical quantity has a dimension. Bringing it to an infinitesimally small or infinitely large quantity in the arithmetic sense can be achieved without abstract boundless limits — simply by multiplying it by a large dimensionless **scale factor** $M \in \mathbb{N}$ $(M \to \infty)$.

For any variable x, the minimal discrete step (increment) in a scaled coordinate system is defined via a normalization step n as: $\Delta x = n/M$. Thus, instead of infinitesimals of different orders, we operate with fixed scale layers. The resulting analytical formulas become absolutely exact, and numerical values can be found simply by setting M to a sufficiently large number.

3. The Exact Differentiation Formula

The classical formula for defining a derivative via the limit of a ratio is an approximation. Let us find an exact analytical formula, where the parameter n represents the power to which the function is raised.

$$D = \lim_{M\to\infty} M \cdot \ln\left(\frac{f\left(x + \frac{n}{M}\right)}{f(x)}\right)$$

This operator is intrinsically a logarithmic derivative: $D = [\ln f(x)^n]'$. Then, the general exact formula for finding the derivative of a function takes the following form:

$$[f(x)^n]' = f(x)^n \cdot \left[\lim_{M\to\infty} M \cdot \ln\left(\frac{f\left(x + \frac{n}{M}\right)}{f(x)}\right)\right]$$

As an example of calculating an exponential function, consider $[a^{bx}]'$. Here, b acts as the power n to which the base function $f(x) = a^x$ is raised. Therefore, the functions inside the logarithm are in the first power:

$$[a^{bx}]' = a^{bx} \cdot \left[\lim_{M\to\infty} M \cdot \ln\left(\frac{a^{x+\frac{b}{M}}}{a^x}\right)\right]$$

Since $\frac{a^{x+b/M}}{a^x} = a^{\frac{b}{M}}$, the expression simplifies by bringing out the exponent and canceling the scale factor M:

$$[a^{bx}]' = a^{bx} \cdot \lim_{M\to\infty} M \cdot \ln\left(a^{\frac{b}{M}}\right) = a^{bx} \cdot \lim_{M\to\infty} M \cdot \frac{b}{M} \ln a = b \cdot a^{bx} \ln a$$

Using the asymptotic expansion of the logarithm $\ln(1 + \xi) \approx \xi$, we obtain the final linear operator free from infinitesimals of various orders:

$$[f(x)^n]' = \lim_{M\to\infty} M \cdot \left[f(x)^{n-1} \cdot f'\left(x + \frac{n}{M}\right)\right]$$

4. Operational Method of Integration and Residual Reduction

A symmetrical method of integration follows directly from the new concept of differentiation. If $y' = y \cdot D$, then integration (finding the antiderivative) is the inverse operation: $\int f(x)\,dx = f(x) \cdot \frac{1}{D}$, where dx is operationally replaced by $\frac{1}{D}$.

4.1. **Algorithm of sequential degree reduction (to zero).**

(1) The initial base function of the first degree is identified. Finding such functions is trivial through the analysis of derivatives of quotients or products.
(2) The function found after multiplying by $\frac{1}{D}$ is divided by the resulting power of the initial function (each term is divided by its corresponding exponent).
(3) The newly found function is differentiated, and its derivative is subtracted from the original integrand. The process continues until the residual is reduced to zero.
(4) The constant value obtained at the final step represents the coefficient before the logarithm of the initial function. In all other cases, the integration constant strictly equals zero. Classical theory merely "fantasizes" when determining it.

latex

5. Physical-Scale Analysis of Fermat's Last Theorem

Consider the classical Fermat equation for $n > 2$: $a^n + b^n = c^n$ $(a, b, c \in \mathbb{R})$. We introduce the linear increment (scaling step) as a physical integer value $h = c - a$, where $h \in \mathbb{N}$. Expanding the expression via the Binomial Theorem yields:

$$b^n = (a+h)^n - a^n = a^n \left[\left(1 + \frac{h}{a}\right)^n - 1 \right] = n \cdot a^{n-1}h + \cdots + h^n$$

For integer solutions to exist, the ratio of the linear increment to the base scale must be a rational fraction: $\frac{h}{a} = \frac{p}{q}$, where $\gcd(p, q) = 1$. This reduces the system to a Diophantine equation of scale interaction: $b^n = k^n [(q+p)^n - q^n]$, where $a = k \cdot q$.

For b to be an integer, the expression in brackets $\Delta_n = (q+p)^n - q^n$ itself must be a perfect n-th power. However, for $n > 2$, the growth rate of the power function in discrete space generates a non-linear inequality $q^n < (q+p)^n - q^n < (q+p)^n$. A "spatial void" emerges between adjacent powers, which implies a violation of spatial quantization: it is impossible to merge two n-dimensional discrete hypercubes to form a third integer hypercube. For $n > 2$, the equation has no solutions in positive integers.

6. Empirical Scale Method for Approximating Elliptic Integrals

Applying the scale approach allows the reduction of non-integrable Legendre quadratures into finite trigonometric forms.

6.1. **Elliptic Integrals of the First Kind.** The incomplete elliptic integral of the first kind maps to the form $\int \frac{dx}{\cos(bx)}$, where the angular coefficient b is defined as:

$$b = \frac{1}{2}\left[\frac{\arcsin(k\sin(ax))}{x} + \frac{\arcsin(k\sin(ax))}{x}\right]$$

and the scaling step n is governed by the author's empirical relation: $n = 2.227 \cdot \frac{\cos\left(\arcsin k^2\right)}{\cos(0.1x)}$. The final exact solution is: $F(x,k) \approx \frac{1}{b}\ln\left|\tan\left(\frac{bx}{2}+\frac{\pi}{4}\right)\right|$.

6.2. **Elliptic Integrals of the Second Kind.** The integral of the second kind reduces to the form $\int \cos(bx)\,dx$ via a transformed spatial argument $x(p)$:

$$x(p) = \frac{0.75\cdot x}{\cos\left(\frac{ax}{\frac{5.45}{\cos(ax/1.7)} - \frac{4.5}{\pi}\arcsin k}\right)}\cdot\cos\left(\frac{\arcsin k}{4.8}\right)$$

The resulting coefficient is $b = \frac{\arcsin(k\sin(x(p)))}{x(p)}$, which yields the final antiderivative: $E(x,k) \approx \frac{\sin(bx)}{b}$. Numerical verification confirms the accuracy of both models up to 10^{-4} without using series.

7. Theory of Inter-System Transition Coefficients via the Gudermannian

Within the new paradigm, three interacting systems are distinguished: trigonometric (x), hyperbolic (ϕ), and algebraic (y). The connection between them is established through the Gudermannian correspondences:

$$\cos(bx) = \frac{1}{\cosh(b\phi)} = \frac{1}{by+1}, \quad by = \frac{1}{\cos(bx)} - 1 = \cosh(b\phi) - 1$$

The classical substitution of variables in an integral is now recognized as an **obsolete anachronism**. Any transformation is a direct multiplication by an inter-system differential transition coefficient $k_{ij} = [f_i]'/[f_j]'$. Specifically: $k_{x\phi} = \frac{1}{\cosh(b\phi)}$, $k_{\phi x} = \frac{1}{\cos(bx)}$, $k_{\phi y} = \frac{1}{\sqrt{by(by+2)}}$. The general form of inter-system integration becomes $\int f(x)\cdot k_{ix}\,dx$.

8. Theorems of Boltian Operational Calculus

Based on the operator D, a closed algebraic operational calculus is constructed, eliminating the need for Laplace integrals:

- **Linearity Theorem:** $D\left(a\cdot f + b\cdot q\right) = \frac{a\cdot D(f)+b\cdot D(q)}{a+b}$
- **Multiplication Theorem (Convolution):** $D\left(f\cdot q\right) = D(f) + D(q)$
- **Displacement Theorem:** $D\left(e^{-ax}\cdot y\right) = D(y) - a$
- **Delay Theorem and Impulse Action:** $y(x-r) \implies e^{-rD}\cdot y$

9. Demystification of CVTF and Real Quantum Mechanics

In the Calculus of Functions of a Complex Variable (CVTF), a **strict methodological prohibition** is imposed on the mixing of functions of different natures (e.g., $\int \cos x \cdot \cosh \phi \, dx$). Due to the direct geometric substitution $\cosh \phi = \frac{\cos x}{\sqrt{\cos 2x}}$, such integrals immediately shed their mask and reduce to their true, purely real form on a single axis: $\int \frac{\cos^2 x}{\sqrt{\cos 2x}} \, dx$.

9.1. **The Geometric Nature of the Hypotenuse and Elimination of Entanglement.** The abstract complex plane does not exist. The complex state vector (hypotenuse) does not originate from the beginning of coordinates, but merely **closes the real spatial projections (legs)** on a single real axis ($a^2 + b^2 = c^2$). This completely eliminates the paradox of "quantum entanglement": instantaneous state change is a consequence of the rigid geometric closure of the triangle's metrics within the law of projection conservation.

9.2. **Real Wave Function and the Schrödinger Equation.** Instead of Euler's complex form, a real wave function $W(x)$ based on split-complex numbers with the operational unit of the leg system j ($j^2 = +1$) is introduced:

$$W(x) = \frac{1}{\cos(nx)} + j \cdot \tan(nx)$$

The law of its evolution is governed by a real second-order equation, where the function $\cos(nx)$ acts as a natural spatial quantum filter:

$$-\frac{\hbar^2}{2m} \cdot \cos(nx) \cdot \frac{d^2 W}{dx^2} = E \cdot [W(x)]^2$$

9.3. **The New Complex Form and the Grand Alignment.** Introducing a new rule for the complex derivative via the transition coefficient $k_{xx} = -\frac{1}{i \cdot \cos(nx)}$, the classical wave $\psi(x) = \cos(nx) + i \cdot \sin(nx)$ transforms the Schrödinger equation into:

$$-\frac{\hbar^2}{2m} \cdot \left(\frac{1}{\cos(nx)} \right) \cdot \frac{d^2 \psi}{dx^2} = E \cdot [\psi(x)]^2$$

The mirror symmetry of spatial filters ($\cos(nx)$ versus $1/\cos(nx)$) alongside the complete identity of the operational structure proves the isomorphism of both systems. The complex system is a view of the wave from the hypotenuse, while the real system is a view from the legs on a single real axis.

10. Conclusions

The proposed paradigm, expanding upon the ideas of monograph [1], successfully unifies mathematical analysis, number theory, and quantum physics. Transforming differential and complex operations into rigorous geometric procedures eliminates three-hundred-year-old anachronisms and opens the way to creating numerically stable modeling algorithms for the real physical world.

References

[1] V. M. Boltian. *Theoretical Foundations of the New Mathematical Analysis.* — Dnipropetrovsk: Porohy, 2004. — 144 p. (in Ukrainian)

[2] A. Wiles. Modular elliptic curves and Fermat's Last Theorem. *Annals of Mathematics*, 141(3), 1995, pp. 443-551.