Gravity Without a Fundamental Graviton

A Toroidal Global-to-Local Construction:
Self-Calibration, Emergent Geometry, and Its Physical Limits

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Abstract

Can gravitational geometry arise as a collective response without postulating a fundamental graviton? We investigate this possibility within the Principium Geometricum (PG) program, in which local sectors are restrictions of a global state and toroidal phase and breathing variables are proposed as underlying degrees of freedom. A local normalization $\Phi = Z\Theta$ yields $K = K_0 Z^2$ and $\beta = \beta_0 Z$, preserving β^2/K. An exact change-of-variable test shows, however, that this identity alone produces no new gravitational observable: when all gradient terms are retained, Z cancels from the physical potential $\Psi = \beta\Theta$. A physical toroidal mechanism must therefore modify the source energy or introduce independently derived interactions. Conditional on universal clock coupling, the source is total energy, and a static gapless sector yields a Poisson equation and inverse-square acceleration. A reciprocal exponential metric reproduces the static weak-field coefficients $\gamma = \beta = 1$, but its exterior Einstein tensor is nonzero at nonlinear order. The construction thus provides an effective framework and a precise set of consistency tests, rather than a completed quantum theory of gravity. Its central open problem is to derive source energies, constitutive coefficients, and gravitational dynamics from the same microscopic theory. A fundamental graviton is not assumed; emergent graviton-like excitations are neither derived nor excluded.

1 The question: what must be fundamental?

The PG program asks whether gravitational geometry can be a collective property of an underlying toroidal state rather than a microscopic starting point. Its proposed ordering is

$$\Omega_G \longrightarrow \text{local phase and breathing sectors} \longrightarrow \text{effective geometry.} \tag{1}$$

The hypothesis concerns the origin of the effective description. It does not deny the observed particle-like behavior of matter, nor does it establish the absence of quantized gravitational excitations. If such excitations emerge, they would belong to the collective description rather than being postulated as its microscopic constituents.

The analogy with pressure is useful but limited. A macroscopic response need not have a fundamental particle of its own. Nevertheless, its microscopic dynamics must determine its energy, constitutive relations, and propagating disturbances. The same requirement applies here: replacing particles with geometric structures does not remove the need for a quantitative dynamics.

This perspective has precedents. General relativity admits a low-energy quantum effective-field-theory treatment [1, 2]; other approaches investigate quantum geometry or thermodynamic origins of gravitational equations [3, 4]. Neither emergence nor the absence of a fundamental graviton is by itself a novelty claim. The specific PG proposal is the toroidal global-to-local mechanism. This paper tests what its present effective formulation actually establishes.

The results below are conditional on explicit assumptions. We derive a normalization identity, identify its observational limitation, recover a static weak-field sector, and compare a metric ansatz with Schwarzschild geometry. We do not derive a microscopic quantum state space, the observed value of Newton's constant, or the Standard Model.

2 Global-to-local restriction and invariant coupling

Let the restriction of a global state to a region U be denoted by $\mathfrak{E}_{\Omega_G}(U)$. A collective scalar Φ is expressed locally as

$$\Phi = Z_U \Theta_U, \qquad Z_U > 0. \tag{2}$$

The effective continuum coordinates used below are assumed to exist at this stage; their emergence from a pre-geometric state is not derived here.

We use signature $(-,+,+,+)$. For the field-action formulas, time and action units are chosen with $c = 1$; explicit factors of c are restored in the gravitational identifications. With $K_0 > 0$, take

$$S_\Phi = \int \mathrm{d}^4x \left[-\frac{K_0}{2} \partial_\mu \Phi \partial^\mu \Phi - \beta_0 \Phi \rho_E \right], \tag{3}$$

where ρ_E is the energy-density source in the effective description. This sign convention gives positive free-field kinetic energy and the static action used below.

If Z_U is constant within a patch, substitution gives

$$K_U = K_0 Z_U^2, \qquad \beta_U = \beta_0 Z_U, \qquad \boxed{\frac{\beta_U^2}{K_U} = \frac{\beta_0^2}{K_0}}. \tag{4}$$

For a varying Z_U, retaining only these terms is an approximation. It requires the neglected contributions involving $\Theta_U \partial Z_U$ to be small relative to those involving $Z_U \partial \Theta_U$, not merely a qualitative claim of slow variation.

On overlapping patches,

$$\Theta_V = g_{UV} \Theta_U, \quad g_{UV} = Z_U / Z_V, \quad g_{UV} g_{VW} g_{WU} = 1. \tag{5}$$

These are compatibility identities for local normalizations. They do not independently determine the physical state or the coefficients K_0, β_0.

3 The exact test: normalization is not an observable

Write $Z = e^\sigma$. The exact static form of Eq. (3) is

$$S_{\text{stat}} = -\int \mathrm{d}t\, \mathrm{d}^3x \left[\frac{K_0 e^{2\sigma}}{2} |\nabla\Theta + \Theta\nabla\sigma|^2 + \beta_0 e^\sigma \Theta \rho_E \right]. \tag{6}$$

Define the physical potential by

$$\Psi = \beta\Theta = \beta_0 e^\sigma \Theta = \beta_0 \Phi. \tag{7}$$

Then

$$\nabla\Theta + \Theta\nabla\sigma = \frac{e^{-\sigma}}{\beta_0} \nabla\Psi, \tag{8}$$

so the same action becomes

$$\boxed{S_{\text{stat}} = -\int \mathrm{d}t\, \mathrm{d}^3x \left[\frac{K_0}{2\beta_0^2} |\nabla\Psi|^2 + \Psi \rho_E \right].} \tag{9}$$

The calibration field has disappeared from this sector. At fixed ρ_E, fixed global coefficients, and fixed physical boundary conditions, changing Z alone does not change Ψ.

This is a substantive restriction on the proposed mechanism. The invariant ratio is exact, but it does not by itself demonstrate gravitational self-regulation or a new constitutive observable. Such an observable requires a change in the physical source or a coupling that cannot be removed by Eq. (2).

For comparison, treating the truncated local action with only $e^{2\sigma}|\nabla\Theta|^2$ as an independent theory would give

$$\nabla^2\Psi - \left[\nabla^2\sigma + |\nabla\sigma|^2\right]\Psi = \frac{\beta_0^2}{K_0}\rho_E. \tag{10}$$

That correction is not a prediction of the exact action above. It would require a separate derivation of the truncated action as physical dynamics.

4 A candidate saturation sector

To investigate a physical breathing response, introduce the additional effective action

$$S_\sigma = \int \mathrm{d}^4x \left[-\frac{f_\sigma^2}{2}(\partial\sigma)^2 - V(\sigma) + g_X\sigma X_G\right], \quad V = \frac{1}{2}f_\sigma^2 m_\sigma^2\sigma^2. \tag{11}$$

For a prescribed source X_G independent of σ, variation gives

$$(\Box - m_\sigma^2)\sigma = -\frac{g_X}{f_\sigma^2}X_G. \tag{12}$$

In the static limit this is $(-\nabla^2 + m_\sigma^2)\sigma = g_X X_G/f_\sigma^2$. When gradients are negligible,

$$\sigma \simeq \frac{g_X}{f_\sigma^2 m_\sigma^2}X_G, \qquad Z \simeq \exp\left(\frac{g_X X_G}{f_\sigma^2 m_\sigma^2}\right). \tag{13}$$

Here m_σ is an inverse-length parameter in the stated units. If X_G depends on σ, its variation contributes additional terms.

A possible descriptor of linked structures is

$$X_G(x) = \sum_{\langle ab\rangle} |Lk_{ab}|\, E_{ab}\, \mathcal{C}(\Omega_{ab})W_{ab}(x), \quad \Omega_{ab} = |\omega_a - \omega_b|. \tag{14}$$

The linking number Lk_{ab} is topological, E_{ab} is an interaction-energy scale, $\mathcal{C}$ is a response function, and W_{ab} supplies spatial weighting. Their definitions, dimensions, and normalization must make $g_X\sigma X_G$ an action-density contribution. Equation (14) is an ansatz, not a derived microscopic law.

In particular, zero linking number does not imply zero interaction. Equation (14) can describe a linkage-weighted contribution, but cannot automatically be identified with the full energy density. Adding S_σ also does not undo the cancellation in Eq. (9): the route by which the breathing sector enters the gravitational source or other observables must be specified explicitly.

5 Universal clock coupling and the Newtonian sector

Assume an internal system with variables q^A experiences a universal clock factor $\mathrm{d}\tau = N\mathrm{d}t$, where $N = e^{\beta\Theta}$. Its action is

$$S_m = \int \mathrm{d}t\, NL(q, \dot{q}/N). \tag{15}$$

Writing $v^A = \dot{q}^A/N$, $p_A = \partial L/\partial v^A$, and $H = p_A v^A - L$, one obtains

$$\frac{\partial(NL)}{\partial N} = -H, \qquad \frac{\partial L_t}{\partial\Theta} = -\beta NH. \tag{16}$$

At the background $N = 1$, the source coefficient is therefore

$$q_g = \beta E. \tag{17}$$

This result is conditional on the common clock coupling. Extending it to relativistic fields, pressure, stresses, and gravitational self-energy requires a complete matter action.

Varying Eq. (9) gives

$$\nabla^2\Psi = \frac{\beta_0^2}{K_0}\rho_E, \qquad \boxed{G_{\rm PG} = \frac{c^4}{4\pi}\frac{\beta_0^2}{K_0}}. \tag{18}$$

For nonrelativistic matter, $\rho_E \simeq \rho_m c^2$ and $\Phi_N = c^2\Psi$, so $\nabla^2\Phi_N = 4\pi G_{\rm PG}\rho_m$. For a localized spherical source with energy E_s, vanishing potential at infinity gives

$$\Psi(R) = -\frac{\beta_0^2 E_s}{4\pi K_0 R} = -\frac{G_{\rm PG}E_s}{c^4 R}. \tag{19}$$

The weak-field acceleration is $-c^2\nabla\Psi$. Thus the inverse-square radial dependence follows from the assumed three-dimensional gapless Poisson sector. Its normalization and source strength remain independent tasks.

6 A reciprocal metric and its limits

A purely conformal ansatz $g_{\mu\nu} = e^{2\Psi}\eta_{\mu\nu}$ gives the wrong spatial weak-field sign for the desired Newtonian potential. We instead examine

$$\boxed{\mathrm{d}s^2 = -e^{2\Psi}c^2\mathrm{d}t^2 + e^{-2\Psi}\mathrm{d}\boldsymbol{x}^2.} \tag{20}$$

This is a metric ansatz, not a metric derived from the microscopic loop action. The exponential geometry has prior literature [6, 7]; it is not a novelty claim of PG.

Let $u = G_{\rm PG}M/(c^2R)$ and $\Psi = -u$, where R is the isotropic radial coordinate. Then

$$g_{00} = -1 + 2u - 2u^2 + \tfrac{4}{3}u^3 + \mathcal{O}(u^4), \tag{21}$$

$$g_{ij} = \left(1 + 2u + 2u^2 + \tfrac{4}{3}u^3 + \mathcal{O}(u^4)\right)\delta_{ij}. \tag{22}$$

These expansions match the static coefficients conventionally identified with $\gamma_{\rm PPN} = \beta_{\rm PPN} = 1$. They do not establish a full parametrized post-Newtonian theory, including moving sources and preferred-frame parameters [5].

For the static metric, using $x^0 = ct$ and the curvature convention yielding the usual positive Einstein coupling, direct calculation gives

$$\mathcal{R} = 2e^{2\Psi}\left[\nabla^2\Psi - |\nabla\Psi|^2\right], \tag{23}$$

$$G_{00} = e^{4\Psi}\left[2\nabla^2\Psi - |\nabla\Psi|^2\right], \tag{24}$$

$$G_{ij} = \delta_{ij}|\nabla\Psi|^2 - 2\partial_i\Psi\partial_j\Psi. \tag{25}$$

The derivatives here are those of the flat spatial coordinates of the ansatz. At first order, $G_{00}^{(1)} = 2\nabla^2\Psi = 8\pi G_{\rm PG}\rho_E/c^4$.

Outside the source, $\nabla^2\Psi = 0$ but $G_{\mu\nu}$ need not vanish. With $\mu = G_{\rm PG}M/c^2$,

$$\mathcal{R}_{\rm ext} = -2e^{-2\mu/R}\frac{\mu^2}{R^4}, \qquad G_{00}^{\rm ext} = -e^{-4\mu/R}\frac{\mu^2}{R^4}. \tag{26}$$

Consequently the exponential metric is not Schwarzschild vacuum geometry. The difference occurs at nonlinear order, even though the leading static coefficients agree.

One can define $T^{\rm eff}_{\mu\nu} = c^4G_{\mu\nu}/(8\pi G_{\rm PG})$ as a diagnostic. For a static unit observer $u^\mu = (e^{-\Psi}, 0, 0, 0)$, its exterior energy density is

$$T^{\rm eff}_{\mu\nu}u^\mu u^\nu = -\frac{c^4}{8\pi G_{\rm PG}}e^{2\Psi}|\nabla\Psi|^2. \tag{27}$$

Quantity	Schwarzschild, isotropic coordinates	Reciprocal exponential metric
Static γ, β	$1, 1$	$1, 1$
u^2 coefficient in g_{ij}	$3/2$	2
u^3 coefficient in g_{00}	$3/2$	$4/3$
Exterior Einstein tensor	Zero	Generally nonzero

Table 1: A metric comparison, not a complete dynamical equivalence test.

It is negative for a nontrivial exterior field when $G_{\mathrm{PG}} > 0$. This diagnostic tensor cannot therefore simply be equated with a positive loop-energy density. A dynamical derivation would have to explain the effective stress and its relation to the microscopic energy; alternatively, the effective equations need not take Einstein form.

The spatial volume element is $\mathrm{d}V_{\mathrm{proper}} = e^{-3\Psi}\mathrm{d}^3x$. For $\Psi < 0$, it exceeds the flat volume of the same coordinate cell. This is not, by itself, a demonstration of physical compression or "space collapse." Any such interpretation must specify an operational comparison of regions and distances.

7 From toroidal energy to a gravitational source

A concrete effective model for the proposed topology is a unit field $\boldsymbol{n} : \mathbb{R}^3 \to S^2$ with fixed value at infinity. Its Hopf sectors admit the energy functional

$$E[\boldsymbol{n}] = \int \mathrm{d}^3x \left[\frac{\chi}{2} \sum_i |\partial_i \boldsymbol{n}|^2 + \frac{\kappa}{4} \sum_{i,j} |\partial_i \boldsymbol{n} \times \partial_j \boldsymbol{n}|^2 \right], \quad \chi, \kappa > 0. \tag{28}$$

Such models and their linked configurations are established tools [8, 9]; adopting them is not itself a new quantization of gravity. Here they supply a candidate energetic description for the PG program.

For a fixed dimensionless profile dilated to size L, Eq. (28) gives

$$E(L) = A\chi L + \frac{B\kappa}{L}, \qquad L_* = \sqrt{\frac{B\kappa}{A\chi}}, \quad E_* = 2\sqrt{AB\chi\kappa}. \tag{29}$$

The transformation $(\chi, \kappa) \mapsto (\lambda\chi, \lambda\kappa)$ preserves L_* but multiplies E_* by λ. Geometry alone therefore does not fix the absolute source energy within this effective model.

There is nevertheless a calculable relative response. For a uniform dilation $s = 1 + \epsilon > 0$ from scale equilibrium,

$$\boxed{\frac{\Delta E}{E_*} = \frac{1}{2}\left(s + s^{-1} - 2\right) = \frac{\epsilon^2}{2(1+\epsilon)}.} \tag{30}$$

This preserves the Hopf class and does not represent a change of linkage. During a time-dependent oscillation, kinetic energy must be included; a varying potential-energy contribution is not a varying conserved total energy.

For multiple structures, the required calculation is an energy family $E(d, \varphi, \ldots)$ at fixed total topological charge. The radial and phase responses are

$$F_d = -\left.\frac{\partial E}{\partial d}\right|_{\varphi}, \qquad \mathcal{T}_\varphi = -\left.\frac{\partial E}{\partial \varphi}\right|_{d}. \tag{31}$$

An attractive channel is not automatically a universal gravitational interaction. Likewise, a repulsive microscopic channel does not by itself exclude an attractive collective field sourced by total energy. Numerical trial profiles require convergence checks, a specified ansatz, and relaxation analysis before claims about stable states or universal forces can be made.

If the loop energy consistently enters the universal source, without double counting, a static change obeys

$$\Delta\Psi(\boldsymbol{x}) = -\frac{\beta_0^2}{4\pi K_0}\int \frac{\Delta\rho_E^{\text{loops}}(\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|}\,\mathrm{d}^3x'. \tag{32}$$

This is the proposed physical bridge: a computed change of source energy, rather than a removable normalization, affects the observable potential. The bridge remains conditional until both sectors are derived and coupled consistently.

8 What would make the proposal predictive?

For an isolated source in the static weak-field limit, the gravitational parameter is

$$\boxed{\mu_s^{\text{PG}} = \frac{c^2\beta_0^2}{4\pi K_0}E_s^{\text{PG}}.} \tag{33}$$

Predicting it requires both the source energy and the coupling normalization. Inserting an astronomically inferred mass through $E_s = Mc^2$, or fitting the coefficient to orbital data, does not achieve a prediction independent of those data.

Similarly, $\alpha_U = k_e A_P$ is a definition, not a derivation of the gravitational scale. If $A_P = \hbar G/c^3$, measured gravitational information enters through A_P. This is legitimate for a model using G as an input, but cannot establish an independent derivation of G. A separately derived area scale would need its own dynamics and normalization.

The next requirements are therefore specific:

1. Derive the energetic and coupling coefficients from a specified microscopic theory, distinguishing universal constants from properties of a state.
2. Derive how the loop and breathing energies enter the gravitational source, including stresses, binding, and self-interaction.
3. Establish the time-dependent metric sector, its constraints, propagation, and polarizations; the static ansatz alone does not do this.
4. Obtain a PG-specific observable that survives field redefinitions, then compare it with data not used to determine the prediction.

Matching a few static coefficients is insufficient to establish agreement with the full experimental content of gravity [5]. Quantization also requires more than discrete topology: states, observables, and quantum dynamics must be defined.

9 Conclusion

The PG program can be formulated without assuming a fundamental graviton. Its current effective construction connects global-to-local normalization, universal clock coupling, a gapless static potential, and a candidate reciprocal geometry. Within their stated assumptions, these ingredients yield a universal coupling ratio, an energy source, and inverse-square weak-field acceleration.

The tests also identify where the proposal remains incomplete. Exact normalization changes cancel from the physical potential at fixed source. The reciprocal metric is not a vacuum Einstein solution beyond linear order. Toroidal geometry can determine relative energetic responses without fixing the absolute energy scale. None of these gaps is closed by the absence of fundamental particles.

The decisive problem is now concrete: derive source energy, gravitational coupling, and collective dynamics from one microscopic construction. A successful result would explain why the same underlying state generates both local matter-like structures and a universal gravitational response. Until that derivation and its tests are available, PG is an effective emergent-gravity proposal with a toroidal microscopic hypothesis, not an established quantum theory of gravity.

A Spherical check of the Einstein tensor

With $c = 1$ and isotropic radius r, Eq. (20) reads

$$\mathrm{d}s^2 = -e^{2\Psi(r)}\mathrm{d}t^2 + e^{-2\Psi(r)}\left(\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2\right). \tag{34}$$

The curvature scalar and nonzero diagonal Einstein components are

$$\mathcal{R} = \frac{2e^{2\Psi}}{r}\left[r\Psi'' + 2\Psi' - r(\Psi')^2\right], \tag{35}$$

$$G_{tt} = \frac{e^{4\Psi}}{r}\left[2r\Psi'' + 4\Psi' - r(\Psi')^2\right], \tag{36}$$

$$G_{rr} = -(\Psi')^2, \qquad G_{\theta\theta} = r^2(\Psi')^2, \tag{37}$$

$$G_{\phi\phi} = r^2\sin^2\theta\,(\Psi')^2. \tag{38}$$

For $\Psi = -\mu/r$, $\Psi' = \mu/r^2$ and $\Psi'' = -2\mu/r^3$, giving $G_{tt} = -e^{-4\mu/r}\mu^2/r^4$.

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This manuscript was developed with substantial assistance from OpenAI ChatGPT for drafting, algebraic analysis, literature searches, and technical editing. The PG conceptual program and physical interpretation are attributed to the human author. No AI system is listed as an author. Scientific responsibility and final approval of the manuscript rest with the human author.

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