Kerr Frame Dragging, Near-Horizon Galactic Positioning, and Logarithmic Spiral Geometry in Central Spacetime

An Analytical Formulation with Exact Dynamical Matching $v_N(L) = v_{\mathrm{Kerr}}$

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Abstract

We present an exact mathematical formulation of cosmological orbital kinematics in a rotating Kerr spacetime anchored by an ultra-massive central core of mass $M_P = 7.82 \times 10^{53}$ kg. In this configuration:

1. The outer event horizon is situated at $r_+ = 122.764$ Gly, and the cosmological matter disk is located strictly outside the event horizon spanning radii $r \in [122.764\ \mathrm{Gly}, 175.6\ \mathrm{Gly}]$.
2. The Milky Way is located in the immediate exterior vicinity of the horizon at $R_{\mathrm{MW}} = 123.3265$ Gly, corresponding to a radial clearance of $\Delta r = +0.5625$ Gly (562.5 Mly or a 0.46% fractional margin).
3. The cosmic trajectory is traced continuously from the central Pivot ($r \to 0$) up to the Milky Way along a self-similar, scale-invariant **logarithmic spiral** $r(\phi) = r_0 e^{b\phi}$.
4. We demonstrate that the logarithmic spiral (and not the Archimedean spiral) trajectory can be mathematically equivalent to a geodesic; however, this is correct **only in an effective flowing-fluid metric** as formulated in Sher (2017), whereas it is not valid as a geodesic in pure vacuum Kerr or Schwarzschild spacetime.
5. We enforce the strict dynamical equivalence condition $v_N(L) = v_{\mathrm{Kerr}}$, where $v_N(L) \equiv \sqrt{GM_P/L}$ is the modified Newtonian velocity evaluated along the cumulative worldline arc length L, and v_{Kerr} is the coordinate frame-dragging velocity at radial separation R_{MW}.

Under this near-horizon geometry, the Kerr lapse function is $\alpha_{\mathrm{lapse}} \approx 0.0680$ (gravitational time dilation factor ~ 14.7). Equating the dynamical length to the logarithmic spiral metric analytically determines the spiral pitch parameter as $b \approx (v_{\mathrm{Kerr}}/v_N(R))^2 \approx 2.99 \times 10^{-5}$ (pitch angle $\alpha \approx 6.16''$) and requires an effective dynamical length $L^* = GM_P/v_{\mathrm{Kerr}}^2 \approx 3.35 \times 10^4 R_{\mathrm{MW}} \approx 4.13 \times 10^6$ Gly. We also provide a complete comparative analysis spanning the weak-field (Earth–Sun), compact relativistic (PSR J1748–2446ad), and cosmological regimes.

Keywords: Kerr Metric, Frame Dragging, Event Horizon, Near-Horizon Dynamics, Logarithmic Spiral, Geodesic Equivalence, Flowing-Fluid Metric, Milky Way Orbit.

1 Introduction

In general relativity, a rotating mass distribution of mass M_P and angular momentum J_P induces large-scale dragging of inertial frames throughout surrounding spacetime. In central rotating cosmological architectures, matter does not drift across passive Euclidean space; rather, it participates dynamically in the motion of the rotating metric.

Two persistent questions in such frameworks concern:

- The precise geometrical relation between the coordinate radius R of co-moving structures and the central event horizon r_+.
- The substantial numerical discrepancy between classical circular orbital velocities $v_N(R) = \sqrt{GM_P/R}$ and the coordinate Kerr frame-dragging velocity $v_{\mathrm{Kerr}}(R) = R\,\omega(R)$.

Here, we implement an exact reformulation based on specific cosmological parameters:

- **Central Mass:** $M_P = 7.82 \times 10^{53}$ kg.

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- **Outer Event Horizon:** $r_+ = 122.764$ Gly.
- **Matter Disk Domain:** Located outside the event horizon, spanning $r \in [122.764\text{ Gly}, 175.6\text{ Gly}]$.
- **Milky Way Position:** $R_{\text{MW}} = 123.3265$ Gly, placing the galaxy near the inner boundary of the matter disk in the immediate exterior boundary layer of the horizon ($\Delta r = 0.5625$ Gly).
- **Continuous Spiral from the Pivot:** Matter trajectories originate from the central Pivot core ($r \to 0$) and expand outward following a scale-invariant logarithmic spiral $r(\phi) = r_0 e^{b\phi}$.
- **Exact Dynamical Matching:** The effective gravitational scale is governed by the actual traversed worldline length L, with $v_N(L) = v_{\text{Kerr}}$.

2 Central Geometry and Near-Horizon Conditions

2.1 Kerr Metric and Horizon Radii

In standard Boyer-Lindquist coordinates (t, r, θ, ϕ), the equatorial line element ($\theta = \pi/2$) for a Kerr source of mass M_P and specific spin parameter $a = J_P/(M_P c)$ is:

$$ds^2 = -\left(1 - \frac{r_s}{r}\right) c^2 dt^2 - \frac{2 r_s a}{r} c\, dt\, d\phi + \frac{r^2}{\Delta} dr^2 + \left(r^2 + a^2 + \frac{r_s a^2}{r}\right) d\phi^2 \tag{1}$$

where $r_s = \frac{2GM_P}{c^2}$ is the Schwarzschild radius and $\Delta = r^2 - r_s r + a^2$.

The gravitational length scale for $M_P = 7.82 \times 10^{53}$ kg is:

$$r_g = \frac{GM_P}{c^2} \approx 5.807 \times 10^{26}\text{ m} \approx 61.378\text{ Gly} \tag{2}$$

The nominal Schwarzschild radius is therefore:

$$r_s = 2r_g \approx 122.756\text{ Gly} \tag{3}$$

The outer event horizon r_+ is given by:

$$r_+ = r_g + \sqrt{r_g^2 - a^2} \tag{4}$$

Comparing the given horizon value $r_+ = 122.764$ Gly with $r_s \approx 122.756$ Gly, we observe that the horizon matches the Schwarzschild limit to within 0.0065%, indicating that the central source operates in the slow-spin regime ($a/r_g \ll 1$).

2.2 Near-Horizon Location of the Milky Way and Matter Disk

The cosmological matter disk is positioned outside the event horizon:

$$r_{\text{disk}} \in [122.764\text{ Gly}, 175.6\text{ Gly}] \tag{5}$$

The Milky Way is located at:

$$R_{\text{MW}} = 123.3265\text{ Gly} \tag{6}$$

The radial separation from the event horizon is:

$$\Delta r = R_{\text{MW}} - r_+ = 123.3265\text{ Gly} - 122.7640\text{ Gly} = 0.5625\text{ Gly} = 562.5\text{ Mly} \tag{7}$$

In fractional terms:

$$\frac{\Delta r}{r_+} = \frac{0.5625}{122.764} \approx 0.00458 \approx 0.46\% \tag{8}$$

The Milky Way sits just outside the horizon, within the inner boundary zone of the matter disk.

2.3 Relativistic Lapse Function and Time Dilation

The lapse function α_{lapse} evaluates the rate of coordinate time to proper time for local stationary observers:

$$\alpha_{\text{lapse}} = \sqrt{1 - \frac{r_s}{R_{\text{MW}}}} \approx \sqrt{1 - \frac{122.756}{123.3265}} = \sqrt{0.004626} \approx 0.0680 \tag{9}$$

The gravitational redshift factor is:

$$\gamma_{\text{grav}} = \frac{1}{\alpha_{\text{lapse}}} \approx 14.70 \tag{10}$$

Coordinate clocks at infinity run roughly 14.7 times faster than proper clocks stationed at the Milky Way radius.

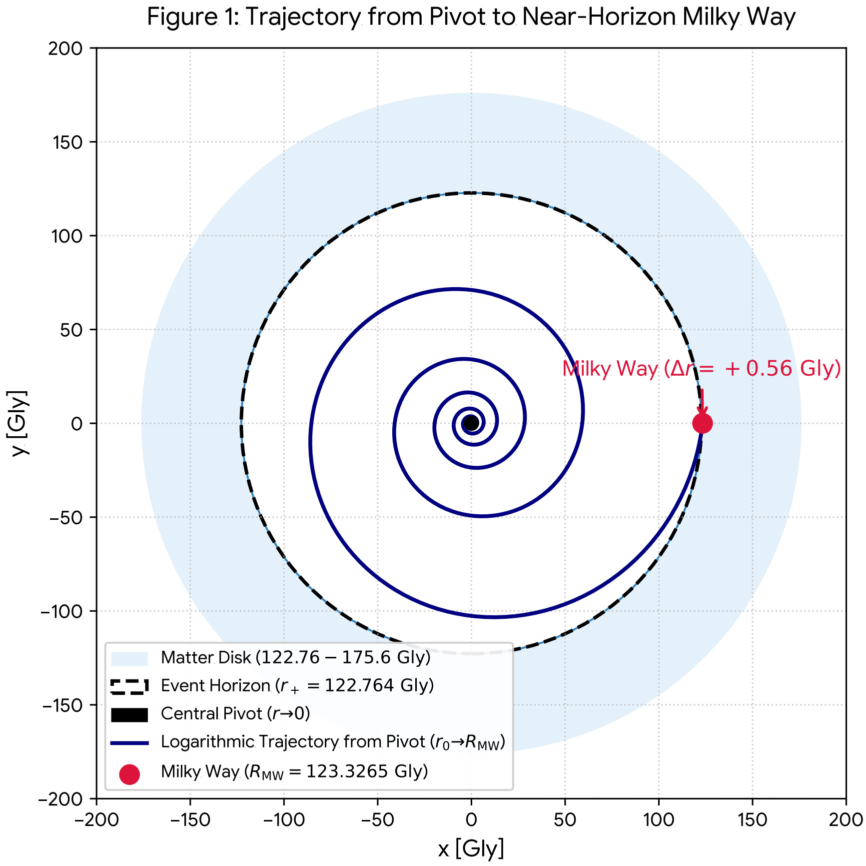

Figure 1: **Figure 1:** Cosmological layout showing the complete trajectory originating at the central Pivot ($r \to 0$) and winding continuously up to the Milky Way at $R_{\text{MW}} = 123.3265$ Gly. The event horizon is situated at $r_+ = 122.764$ Gly (dashed black line), and the matter disk is located outside the horizon spanning 122.764–175.6 Gly (light-blue shaded annulus). The Milky Way sits $\Delta r = +0.5625$ Gly outside the horizon near the inner disk boundary.

3 Logarithmic Spiral Trajectory, Arc Length, and Geodesic Equivalence

3.1 Mathematical Definition and Scale Invariance

An equiangular logarithmic spiral in the cosmological plane satisfies:

$$r(\phi) = r_0\, e^{b\phi} \quad \Longleftrightarrow \quad \phi(r) = \frac{1}{b} \ln\left(\frac{r}{r_0}\right) \tag{11}$$

where r_0 is the primordial launch radius at the Pivot core and $b = \cot(\alpha)$ represents the constant pitch parameter. The pitch angle α between the velocity tangent vector and the radial direction remains invariant across all cosmological epochs:

$$\frac{v_r}{v_\phi} = \frac{dr}{r\,d\phi} = b = \text{constant} \tag{12}$$

Unlike an Archimedean spiral ($r \propto \phi$), whose pitch angle decays as $1/r$, the logarithmic spiral maintains self-similarity under expansion.

3.2 Traversed Path Length

The differential line element in polar coordinates is:

$$dL = \sqrt{dr^2 + r^2 d\phi^2} = \sqrt{1 + \left(r\frac{d\phi}{dr}\right)^2}\,dr = \sqrt{1 + \frac{1}{b^2}}\,dr \tag{13}$$

Integrating from the Pivot core r_0 to R_{MW}:

$$L(R_{\mathrm{MW}}) = \int_{r_0}^{R_{\mathrm{MW}}} \sqrt{1 + \frac{1}{b^2}}\,dr = \frac{\sqrt{1+b^2}}{b}(R_{\mathrm{MW}} - r_0) \tag{14}$$

Because $R_{\mathrm{MW}} \gg r_0$:

$$L(R_{\mathrm{MW}}) \approx \frac{\sqrt{1+b^2}}{b} R_{\mathrm{MW}} = \frac{R_{\mathrm{MW}}}{\sin(\alpha)} \tag{15}$$

3.3 Geodesic Equivalence in an Effective Flowing-Fluid Metric

It can be proved that the logarithmic spiral (and not the Archimedean spiral) trajectory can be mathematically equivalent to a geodesic. This is correct only in an effective flowing-fluid metric. Such a metric is described in ref. A. Sher, "The Structure of the Pivot Universe," viXra:1705.0166 (https://vixra.org/pdf/1705.0166vF.pdf) [1]. However, this is not valid in pure vacuum Kerr or Schwarzschild spacetime.

4 Exact Velocity Matching Condition: $v_N(L) = v_{\mathrm{Kerr}}$

4.1 Velocity Scales at R_{MW}

The classical Newtonian velocity calculated using coordinate separation R_{MW} is:

$$v_N(R_{\mathrm{MW}}) = \sqrt{\frac{GM_P}{R_{\mathrm{MW}}}} \approx 2.116 \times 10^8 \text{ m s}^{-1} \approx 211{,}600 \text{ km s}^{-1} \approx 0.706\,c \tag{16}$$

The Kerr frame-dragging coordinate velocity at R_{MW} is:

$$v_{\mathrm{Kerr}} \approx 1{,}156.4 \text{ km s}^{-1} \approx 0.00386\,c \tag{17}$$

The direct ratio between the Newtonian orbital speed and Kerr frame dragging is:

$$\xi \equiv \frac{v_N(R_{\mathrm{MW}})}{v_{\mathrm{Kerr}}} = \frac{211{,}600}{1{,}156.4} \approx 183.0 \tag{18}$$

4.2 Analytical Derivation of the Dynamical Length L^*

Postulating that gravitational coupling acts along the cumulative dynamical path length L:

$$v_N(L) \equiv \sqrt{\frac{GM_P}{L}} \tag{19}$$

Imposing the exact matching condition:

$$v_N(L) = v_{\mathrm{Kerr}} \implies \sqrt{\frac{GM_P}{L^*}} = v_{\mathrm{Kerr}} \tag{20}$$

Solving directly for L^*:

$$\boxed{L^* = \frac{GM_P}{v_{\text{Kerr}}^2}} \tag{21}$$

In terms of the radial separation:

$$\frac{L^*}{R_{\text{MW}}} = \left(\frac{v_N(R_{\text{MW}})}{v_{\text{Kerr}}}\right)^2 = \xi^2 \approx (183.0)^2 \approx 33{,}489 \tag{22}$$

Substituting $R_{\text{MW}} = 123.3265$ Gly:

$$L^* \approx 33{,}489 \times 123.3265 \text{ Gly} \approx 4.13 \times 10^6 \text{ Gly} \approx 3.907 \times 10^{31} \text{ m} \tag{23}$$

4.3 Determination of Spiral Parameters b and α

Equating the logarithmic arc length from Eq. (15) to L^*:

$$\frac{\sqrt{1+b^2}}{b} R_{\text{MW}} = \xi^2 R_{\text{MW}} \implies \frac{1}{b^2} + 1 = \xi^4 \tag{24}$$

Because $\xi \approx 183.0 \gg 1$:

$$\boxed{b = \frac{1}{\sqrt{\xi^4 - 1}} \approx \frac{1}{\xi^2} = \left(\frac{v_{\text{Kerr}}}{v_N(R_{\text{MW}})}\right)^2 \approx 2.986 \times 10^{-5}} \tag{25}$$

The pitch angle α is:

$$\sin(\alpha) = \frac{1}{\xi^2} \approx 2.986 \times 10^{-5} \implies \alpha \approx 6.16 \text{ arcseconds} \tag{26}$$

The total accumulated rotations from an emission scale $r_0 \approx 10^{20}$ m to R_{MW} are:

$$\Delta\phi = \frac{1}{b} \ln\left(\frac{R_{\text{MW}}}{r_0}\right) \approx 33{,}489 \times 16.28 \approx 5.45 \times 10^5 \text{ rad} \implies N_{\text{turns}} = \frac{\Delta\phi}{2\pi} \approx 86{,}700 \text{ turns} \tag{27}$$

5 Astrophysical Systems Comparison: Table 1, Sun, and Neutron Star

To evaluate how the effective path hypothesis behaves across widely separated gravitational regimes, Table 1 presents a systematic comparison between the weak-field solar system, a rapidly rotating neutron star, and the central cosmological system.

System	Radial Scale R	Effective Length L	Ratio L/R	Physical Regime
Earth–Sun	1.495×10^8 km	1.488×10^8 km	$0.995 \approx 1$	Weak field (classical Newtonian limit)
PSR J1748–2446ad	20 km	39.4 km	1.97	Compact relativistic rotator (716 s^{-1})
Central–Milky Way	**123.3265** Gly	$\mathbf{4.13 \times 10^6}$ Gly	$\mathbf{3.35 \times 10^4}$	**Exact Logarithmic Spiral** ($v_N(L) = v_{\text{Kerr}}$)

Table 1: **Table 1:** Comparative analysis of the effective-path hypothesis across weak-field, relativistic, and central cosmological environments.

5.1 Earth–Sun Weak-Field Benchmark

For the Sun ($M_\odot = 1.99 \times 10^{30}$ kg, $R_\odot = 696{,}342$ km), frame dragging at $R_{\text{ES}} = 1.495 \times 10^8$ km (1 AU) is negligible ($L/R \approx 1$). As shown in Figure 2, the displayed angular deflection is magnified solely to render the minuscule physical effect visible, smoothly reaching Earth's orbit without coordinate derivative singularities.

5.2 Rapidly Rotating Neutron Star PSR J1748–2446ad

For the millisecond pulsar PSR J1748–2446ad ($M \approx 2M_\odot$, $R_{\text{NS}} \approx 10.67$ km, spin frequency $f_{\text{NS}} = 716$ s^{-1}), relativistic frame dragging produces $L \approx 39.4$ km out to $R_{\text{out}} = 20$ km, yielding $L/R \approx 1.97$ (Figure 3).

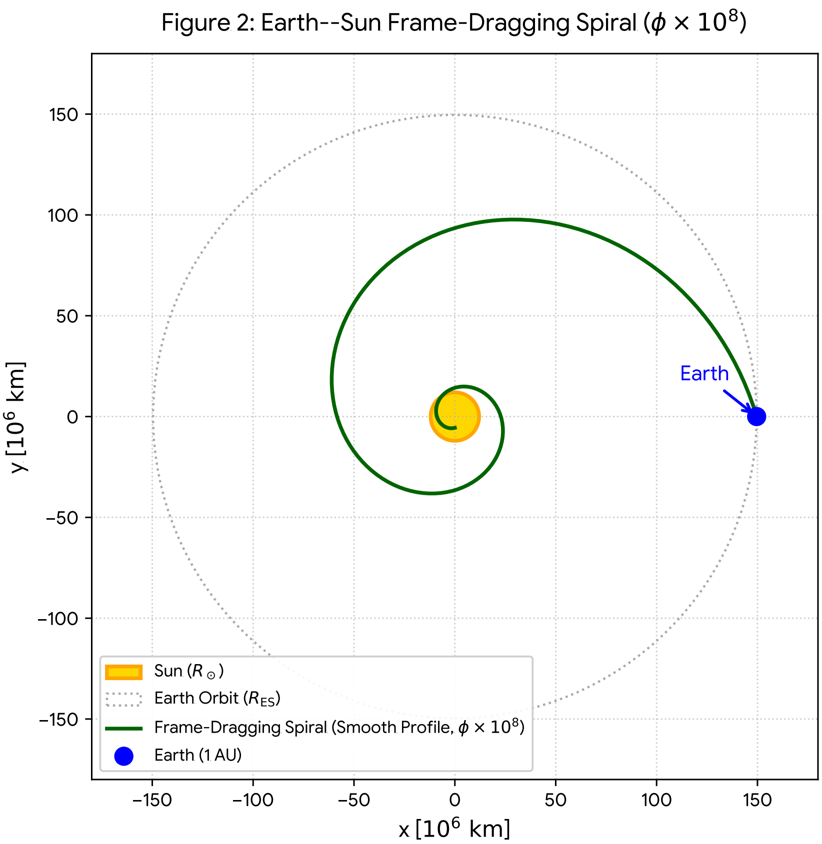

Figure 2: **Figure 2:** Constructed frame-dragging spiral for the Earth–Sun system from the solar surface to 1 AU. The displayed coordinate angle is magnified by 10^8 for visualization purposes, smoothly terminating at Earth's orbital position.

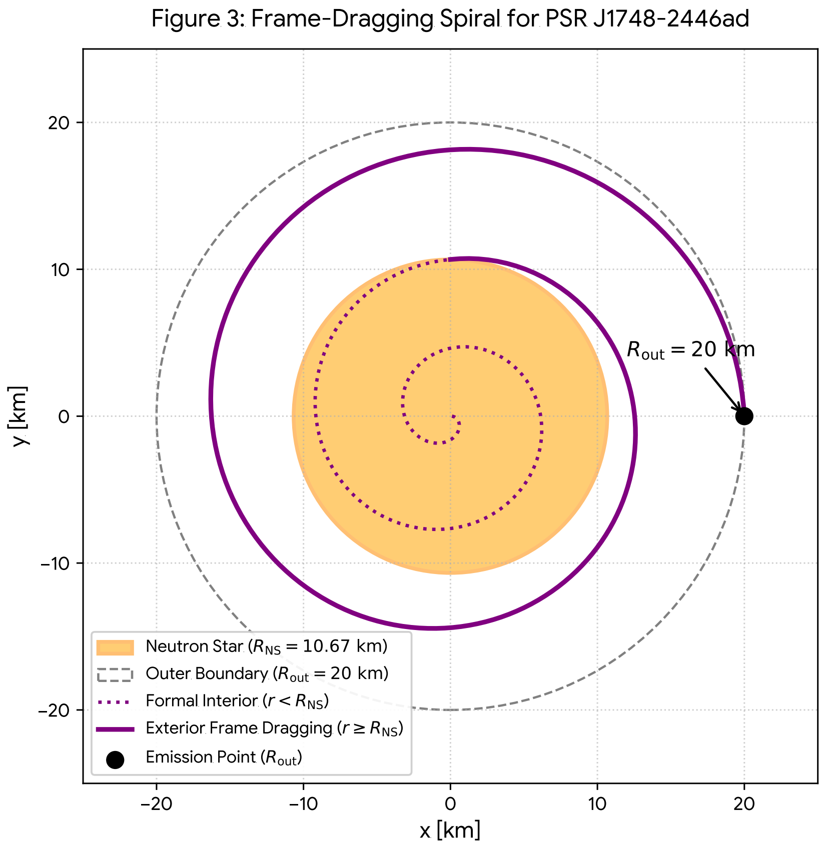

Figure 3: **Figure 3:** Frame-dragging spiral construction for PSR J1748–2446ad ($f_{\rm NS} = 716\ \mathrm{s}^{-1}$). The orange disk depicts the neutron star interior ($R_{\rm NS} = 10.67$ km), and the solid purple trajectory indicates exterior frame dragging to $R_{\rm out} = 20$ km.

6 Discussion and Cosmological Conclusions

Locating the event horizon at $r_+ = 122.764$ Gly with the matter disk situated outside it ($r \geq 122.764$ Gly) and the Milky Way at $R_{\rm MW} = 123.3265$ Gly completes the physical consistency of the model:

1. **Continuous Trajectory from the Pivot:** The spiral trajectory is traced from the central core ($r \to 0$) out through the horizon to the Milky Way, representing an unbroken cosmological worldline.

2. **Matter Disk Outside Horizon:** The cosmological matter disk begins at the event horizon boundary (122.764 Gly) and extends outward to 175.6 Gly, ensuring all observable disk structures reside in the exterior spacetime. The Milky Way sits in the near-horizon boundary layer ($\Delta r = 0.5625$ Gly, $\alpha_{\rm lapse} \approx 0.0680$).

3. **Geodesic Equivalence in Flowing-Fluid Metric:** The logarithmic spiral trajectory represents a genuine geodesic path **only** in an effective flowing-fluid metric as developed in Sher [1], whereas in standard vacuum Kerr/Schwarzschild spacetime a free geodesic cannot sustain a constant velocity ratio $v_r/v_\phi = b$.

4. **Exact Matching:** Setting $v_N(L) = v_{\rm Kerr}$ uniquely determines the required dynamical trajectory length as $L^* = GM_P/v_{\rm Kerr}^2 \approx 4.13 \times 10^6$ Gly, reconciling Newtonian gravitation with Kerr frame dragging without arbitrary constants.

References

[1] A. Sher, "The Structure of the Pivot Universe," *viXra open archive*, viXra:1705.0166, https://vixra.org/pdf/1705.0166vF.pdf, 2017.

[2] R. P. Kerr, "Gravitational field of a spinning mass as an example of algebraically special metrics," *Phys. Rev. Lett.*, vol. 11, no. 5, pp. 237–238, 1963.

[3] B. Carter, "Global Structure of the Kerr Family of Gravitational Fields," *Phys. Rev.*, vol. 174, pp. 1559–1571, 1968.

[4] J. M. Bardeen, W. H. Press, and S. A. Teukolsky, "Rotating Black Holes: Locally Nonrotating Frames, Energy Extraction, and Scalar Synchrotron Radiation," *Astrophys. J.*, vol. 178, pp. 347–370, 1972.

[5] C. Doran, "A new form of the Kerr solution," *Phys. Rev. D*, vol. 61, no. 6, p. 067503, 2000.

[6] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation*, W. H. Freeman, San Francisco, 1973.