A Physics Knowledge Graph from $\LaTeX$ Notation to Lean Conjectures

Brent S. Hartshorn brenthartshorn@proton.me

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Abstract

Physics notation is ambiguous, this paper reports what happens when a knowledge base of 97 equations, 136 symbols and 64 physical quantities is made to declare what each of its letters denotes, and is then allowed to connect itself. Equations become graph nodes; edges follow shared *quantities* rather than shared characters; and two equations stating the same quantity are joined by transitivity into one annotated formula. Each statement is checked to be a well-formed proposition over an axiomatic `Real` by a Calculus of Constructions micro-kernel, then emitted as Lean 4.

$$
\left.
\begin{aligned}
\overbrace{E}^{\text{energy}} &= \underbrace{mc^2}_{\text{mass-energy}} = \underbrace{h\nu}_{\text{Planck}} = \underbrace{k_B T}_{\text{thermal}} \\
\overbrace{F}^{\text{force}} &= \underbrace{ma}_{\text{Newton II}} = \underbrace{G\frac{m_1 m_2}{r^2}}_{\text{Newton grav.}} = \underbrace{\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}}_{\text{Coulomb}} = \underbrace{\frac{W}{d}}_{\text{work}} = \underbrace{-(kx)}_{\text{Hooke}} \\
\overbrace{m}^{\text{mass}} &= \underbrace{\frac{E}{c^2}}_{\text{mass-energy}} = \underbrace{\frac{F}{a}}_{\text{Newton II}} = \underbrace{\frac{2E_k}{v^2}}_{\text{kinetic}} = \underbrace{\frac{p}{v}}_{\text{momentum}} = \underbrace{\frac{U}{hg}}_{\text{grav. p.e.}} = \underbrace{\rho V}_{\text{density}} = \underbrace{\frac{h}{\lambda_C c}}_{\text{Compton}} \\
\overbrace{G}^{\text{gravitational constant}} &= \underbrace{\frac{Fr^2}{m_1 m_2}}_{\text{Newton grav.}} = \underbrace{\frac{r_s c^2}{2M}}_{\text{Schwarzschild}} = \underbrace{\frac{k_B c^3 A}{4S\hbar}}_{\text{Bekenstein-Hawking}} = \underbrace{\frac{gr^2}{M}}_{\text{surface grav.}} = \underbrace{\frac{\ell_P^2 c^3}{\hbar}}_{\text{Planck length}} = \underbrace{\frac{v_e^2 r}{2M}}_{\text{escape vel.}} \\
\overbrace{h}^{\text{planck constant}} &= \underbrace{\frac{E}{\nu}}_{\text{Planck}} = \underbrace{\lambda p}_{\text{de Broglie}} = \underbrace{\lambda_C mc}_{\text{Compton}} \\
\overbrace{T}^{\text{temperature}} &= \underbrace{\frac{PV}{nR}}_{\text{ideal gas}} = \underbrace{\frac{\lambda_D^2 ne^2}{\epsilon_0 k_B}}_{\text{Debye}} = \underbrace{\frac{E}{k_B}}_{\text{thermal}} \\
\overbrace{v}^{\text{speed}} &= \underbrace{\frac{p}{m}}_{\text{momentum}} = \underbrace{f\lambda}_{\text{wave speed}} = \underbrace{H_0 D}_{\text{Hubble}} \\
\overbrace{g}^{\text{gravitational field}} &= \underbrace{\frac{U}{hm}}_{\text{grav. p.e.}} = \underbrace{\frac{GM}{r^2}}_{\text{surface grav.}} = \underbrace{\frac{P}{h\rho}}_{\text{hydrostatic}} \\
\overbrace{\alpha'}^{\text{regge slope}} &= \underbrace{\frac{1}{2T\pi}}_{\text{tension}} = \underbrace{\frac{J-\alpha_0}{M^2}}_{\text{Regge}} = \underbrace{\ell_s^2}_{\text{string length}} = \underbrace{\left(\frac{1}{4T_H k_B \pi}\right)^2}_{\text{Hagedorn}}
\end{aligned}
\right\}
\begin{array}{l}\text{one conjecture:}\\ \text{37 equations, 9 quantities}\end{array}
$$

Figure 1: The library, conjoined. 37 equations from nine quantities, each `\underbraced` with the equation it came from and `\overbraced` with the quantity it determines. Generated by `rosettaphys.grand()`; the corresponding Lean proposition, with all 43 variables bound once across the whole conjunction, is Appendix C.

1 Introduction

Nothing in Figure 1 was written down. Each equation entered the library on its own terms, described in its own words, with no reference to the others; the rows are what the library implies once you ask which of its equations are about the same thing. Producing them needed no physics—only a willingness to say what m means.

Here is what a knowledge base of physics equations looks like once it knows what its own letters mean. Every row is a physical quantity; every term is what some equation in the library says that quantity equals; the side brace gathers the rows into a single claim.

Three earlier papers built a translator, an explorer and a kernel. RosettaMath [1] reads a subset of LaTeX and writes Python, in the subset it reads. `lean4.py` [2] is a Calculus of Constructions [4] checker whose statement language is LaTeX, so a theorem about a Python function is written `\forall x \in \text{Nat}, x = x`. The most recent [3] hands the same terms to Lean 4 [5], so a second kernel can be asked what the first already answered.

All three treat notation as something to be *translated*. This paper treats it as something to be *interrogated*, starting from an observation the project's own documentation has made since the beginning, in an article titled "Overloaded notation":

> The single hardest thing about reading physics is that the alphabet ran out long ago. One glyph carries many meanings and the reader is expected to infer which from context alone.

If equations are to be linked automatically, something must decide when two of them talk about the same thing. The obvious answer—that they share a symbol—is wrong, and wrong in a way that yields confident nonsense rather than errors. Section 4 is an account of how wrong.

The contribution is threefold: a graph over **quantities** rather than glyphs, in which equations declare what their letters denote and may be joined only where they agree (Section 2); an algebra that rearranges equations to expose a shared quantity, notable mostly for what it **refuses** (Section 3); and a route from a composed statement to a Lean conjecture in which the brace annotations survive as provenance (Section 7).

This document is generated by `rosettapaper.py` from the module it describes. Every count, table and displayed equation below is a function call at build time, because a paper about a knowledge base is a second copy of it, and the second copy is the one that goes stale.

2 A graph over quantities

The nodes are 97 equations, 136 symbols, 19 prose articles and 64 physical quantities—the last being things like *energy* and *microstate count*, each with a dimension in the usual $M/L/T/I/K/N$ basis. The graph has 316 nodes and 1217 edges.

Most edges are derived. A signature already names the symbols an equation is built from, and two equations mentioning the same quantity are related whether or not anyone noticed, so the graph grows on its own as entries are added. The edges that cannot be derived are the interesting ones—that the time-independent Schrödinger equation specialises the time-dependent one is a fact about physics, not notation—and 46 stated relations are stated explicitly.

3 An algebra, and what it refuses

Joining needs both equations to state the pivot. Requiring them to state it *directly*—alone on one side of the equals—yielded only five joins, so the module grows a small expression algebra: a term type, a reader, a printer, and `isolate`, which peels operations off one side and applies their inverses to the other.

This is the project's third LaTeX reader. `rosettaui` parses for *shape*, to draw a fraction stacked; `lean4` parses *types*, where juxtaposition is application; this parses *algebra*, where juxtaposition is multiplication. The three disagree about what a space between two letters means, so one parser with a mode flag would need telling which dialect it was reading anyway.

Relation	Count	What it records
`denotes`	200	equation to the quantity one of its letters means
`uses`	305	equation to a symbol in its signature
`shares`	598	two equations with a quantity in common
`mentions`	54	article to a symbol it discusses
`specialises`	21	a special case of a more general law
`defines`	19	one equation giving another its meaning
`limit-of`	1	what a law becomes in some limit
`cites`	14	article to an equation it discusses
`field`	5	equations of the same kind

Table 1: The edge vocabulary. `denotes` did not exist in the first version: without it an equation is connected to the *characters* it contains, with it to the quantities it is about. The difference between those two graphs is Section 4.

3.1 The first bug

Nothing in the first version of the reader stopped it parsing $\nabla^2\psi$ as *nabla squared times psi*. Having done so, solving the time-independent Schrödinger equation for ψ meant dividing by it, and the library duly reported

$$\frac{-\left(\frac{\hbar^2}{2m}\nabla^2\psi\right)+V\psi}{\psi}=h\nu$$

as a join between the Schrödinger equation and the Planck relation. It parses, it renders, and it would have come back from the kernel a well-formed proposition. It means nothing at all.

The repair is a table of notation that is not scalar algebra—derivatives, tensor indices, summations, ambiguous signs—checked on the token stream before the parser starts believing things. An equation containing any of it is refused for rearrangement but stays in the graph, keeping its signature and its readings. 32 equations of 97 are refused on these grounds.

Refused because it contains	Count	For example
a partial derivative	8	Schrodinger equation (time-dependent)
a gradient, divergence or curl	8	Schrodinger equation (time-independent)
the d'Alembert operator	3	Klein-Gordon equation
a summation	3	Shannon entropy
an integral	3	Gauss-Bonnet theorem
a change in a quantity, not a factor	2	Heisenberg uncertainty principle
an indexed tensor component	1	Einstein field equations
an ambiguous sign	1	Quadratic formula
a conditional bar	1	Bayes' theorem
a divergence bar	1	Kullback-Leibler divergence
a dimension, not a factor	1	Kakeya conjecture

Table 2: What the algebra declines to read, and why.

3.2 What isolate will not do

Two further refusals would otherwise produce true-looking falsehoods. An even power is refused outright: $x^2 = 4$ gives $x = \pm 2$, and a chain built on the wrong branch is a false statement that typechecks. So is a quantity occurring twice, since isolating it needs terms collected first.

The simplifier is held to the same standard. It flattens nested fractions and drops multiplication by one, which hold for every value. It does *not* cancel the b in ab/b, because that needs $b \neq 0$ and the equation does not say so.

Of 74 pairwise joins the library yields, 58 required rearrangement, and each derivation records the fact—a quoted equation and a rearranged one are believable to different degrees.

4 The second bug, and the readings table

With the algebra refusing what it should, the library produced 114 joins. Among them was this one:

$$\exp\left(\frac{S}{k_B}\right) = Fd$$

Boltzmann's W counts microstates. The W in $W = Fd$ is an energy. The letters match, so the graph joined them, and the result is a false statement that parses, typechecks and renders beautifully. The same fault produced a join between the Planck relation and gravitational potential energy on h, where one h is Planck's constant and the other is a height.

There is no clever fix, and the absence of one is the point. No amount of reading the notation recovers the distinction, because the distinction is not in the notation—it lives in the surrounding prose and the reader's training, exactly as "Overloaded notation" said. The only repair is to write it down.

Each equation therefore carries a `reads` table mapping symbols to quantities, and a pivot requires both equations to agree. An equation that has declared nothing agrees with nothing, and the join is refused: silence is not agreement. The library declares 200 readings across 67 equations.

Because the readings are declared, the collisions can be *computed*: any two equations sharing a letter and disagreeing about it mark a place where physics reused a character. There are 26 distinct collisions.

Letter	One reading	The other	First seen in
c	speed of light	length	Mass-energy equivalence
a	acceleration	length	Newton's second law
W	microstate count	energy	Boltzmann entropy
k	boltzmann constant	spring constant	Boltzmann entropy
S	entropy	action	Boltzmann entropy
h	planck constant	length	Planck relation
$lambda$	wavelength	inflation factor	de Broglie relation
A	area	patch area	Bekenstein-Hawking entropy
A	area	tile area	Bekenstein-Hawking entropy
n	amount	number density	Ideal gas law
R	gas constant	resistance	Ideal gas law
V	volume	voltage	Ideal gas law
P	pressure	power	Ideal gas law
n	amount	covering degree	Ideal gas law
n	amount	hausdorff dimension	Ideal gas law
T	temperature	string tension	Ideal gas law
n	number density	covering degree	Plasma frequency
n	number density	hausdorff dimension	Plasma frequency
g	gravitational field	tile frequency	Gravitational potential energy
g	gravitational field	genus	Gravitational potential energy
Q	charge	energy	Capacitor charge
C	capacitance	dimensionless	Capacitor charge
D	length	spacetime dimension	Hubble's law
A	patch area	tile area	Inflation of area
g	tile frequency	genus	Spectre reciprocal pair
n	covering degree	hausdorff dimension	Riemann-Hurwitz (unbranched)

Table 3: Letters this library uses for two different physical quantities, generated by comparing declared readings. Available as `make collisions`.

Gating joins on agreement cut the library from 114 statements to 74—40 statements discarded. The ones this paper first caught that way, Boltzmann's W against a work done and Planck's h against a height, were plainly false; the table above is the list of places the same fault can recur.

5 Composition, and the braces that carry it

Transitivity is the only inference this module makes: if two equations both say what E is, whatever each says E equals must equal the other.

The join keeps its provenance. The `\underbrace` under each side names the equation it came from; the `\overbrace` names the quantity eliminated to make it. Neither is decoration—they are what lets a reader, and the Lean bridge of Section 7, recover the derivation from the formula.

$$\overbrace{\underbrace{k_B \log W}_{\text{Boltzmann entropy}} = \underbrace{\frac{k_B c^3 A}{4G\hbar}}_{\text{Bekenstein-Hawking entropy}}}^{\text{both equal the entropy}}$$

Boltzmann entropy gives $k_B \log W$ for the entropy. Bekenstein-Hawking entropy gives $\frac{k_B c^3 A}{4G\hbar}$ for the same quantity. Every expression here denotes the entropy, so they are all equal.

$$\overbrace{\underbrace{\frac{PV}{nR}}_{\text{Ideal gas law}} = \underbrace{\frac{E}{k_B}}_{\text{Thermal energy}}}^{\text{both equal the temperature}}$$

Ideal gas law gives $\frac{PV}{nR}$ for the temperature. Thermal energy gives $\frac{E}{k_B}$ for the same quantity. Every expression here denotes the temperature, so they are all equal. (Ideal gas law and Thermal energy had to be rearranged to say so; the join is only as good as that rearrangement.)

5.1 Families

Pairs undersell the graph. Where three or more equations determine the same quantity they chain into one statement, as in Figure 1. 17 quantities do so; the largest is the gravitational constant, fixed six ways:

$$G = \underbrace{\frac{Fr^2}{m_1 m_2}}_{\text{Newton's law of gravitation}}$$
$$G = \underbrace{\frac{r_s c^2}{2M}}_{\text{Schwarzschild radius}}$$
$$G = \underbrace{\frac{k_B c^3 A}{4S\hbar}}_{\text{Bekenstein-Hawking entropy}}$$
$$G = \underbrace{\frac{gr^2}{M}}_{\text{Surface gravity}}$$
$$G = \underbrace{\frac{\ell_P^2 c^3}{\hbar}}_{\text{Planck length}}$$
$$G = \underbrace{\frac{v_e^2 r}{2M}}_{\text{Escape velocity}}$$

Each of these six was added on its own terms with no reference to the others. The chain is not in the knowledge base; it is what the knowledge base implies, and finding it required nothing but agreeing about what the letters mean.

6 A group that is not physics

Every entry so far has been a physical law, and every quantity has had a dimension that constrains it. That is a comfortable place for a method like this one to be tested, because dimensional analysis quietly catches a great deal: an energy cannot be joined to a length whatever the letters say.

So the library was pointed at something it was not built for. 33 entries were added across three new fields—tilings, extremal geometry, string theory—chosen because they share a subject rather than a discipline. A minimal surface minimises area given a boundary. A Kakeya set minimises measure given a direction in every direction. A covering surface has its total curvature fixed by a whole number it cannot escape. And the Nambu-Goto action, which is where physics re-enters, says that a string extremises the area of the sheet it sweeps: the same variational problem Plateau posed about soap films, moved into a spacetime where the signature puts a minus sign under the root.

Field	Entries	Refused by the algebra	Readings declared
Tilings	16	0	16
Extremal geometry	10	3	10
String theory	7	2	7

Table 4: The new group. "Refused" counts entries the algebra of Section 3 declines to rearrange—integrals, sums and the dimension operator. It does so for 15% of this group against 42% of the rest of the library: the refusals cluster in the handful of statements that are genuinely integrals (Gauss-Bonnet, Plateau, Nambu-Goto), while the tilings are plain algebra over a quadratic field and are refused nothing at all.

6.1 What happens when the dimensions stop helping

17 of the library's 64 physical quantities are dimensionless, and most of the new ones are among them. A genus, a tile count, a Hausdorff dimension, an Euler characteristic and a spacetime dimension are all the pure number 1, and no amount of dimensional checking will tell any of them from any other. The readings table is therefore carrying the whole load: the name is the only thing keeping a count of handles apart from a count of sheets.

This produced a new bug, and one a level below the bug of Section 4. Bekenstein-Hawking entropy is stated in terms of a horizon area. A Penrose inflation multiplies the area of a patch by the square of the golden ratio. Both say A, both declared A to be an *area*, both are L^2, and the graph duly joined them:

$$\frac{S \cdot 4G\hbar}{k_B c^3} = \frac{A'}{\phi^2}$$

Nothing here is false. A black hole whose horizon happened to have the area of an inflated tiling patch would satisfy it. It is simply about nothing, and no check in the module could have said so—the letters match, the dimensions match, and the declared readings agree. Agreement about the *quantity* was not enough, because a tiling patch and an event horizon are areas of different things, and a patch can be made as large as you like.

The repair is the same repair as before, applied one level down: say more. The tiling entries now declare their A to be a *patch area*, which is an area that is joinable to no other, and the statement disappears. That this had to be done by hand, again, is the finding. There is no depth at which declaring what a letter means becomes unnecessary; there is only the depth at which the current library has stopped being wrong.

6.2 What it found

Set against that, the group also produced joins of the kind the method is for. The mean curvature is defined as the average of the principal curvatures; a surface is minimal when the mean curvature vanishes. Neither entry was written with the other in mind, and together they say what minimality actually constrains:

$$\overbrace{\underbrace{\frac{\kappa_1+\kappa_2}{2}}_{\text{Mean curvature}} = \underbrace{0}_{\text{Minimal surface condition}}}^{\text{both equal the mean curvature}}$$

Mean curvature gives $\frac{\kappa_1+\kappa_2}{2}$ for the mean curvature. Minimal surface condition gives 0 for the same quantity. Every expression here denotes the mean curvature, so they are all equal.

The string entries went further and formed a family. Four equations—the tension, the Regge trajectory, the string length and the Hagedorn temperature—each determine the Regge slope, and none of them was entered with the others in view. They chain, and the result is now row nine of Figure 1:

$$\begin{aligned} \alpha' &= \underbrace{\frac{1}{2T\pi}}_{\text{String tension}} \\ \alpha' &= \underbrace{\frac{J-\alpha_0}{M^2}}_{\text{Regge trajectory}} \\ \alpha' &= \underbrace{\ell_s^2}_{\text{String length}} \\ \alpha' &= \underbrace{\left(\frac{1}{4T_H k_B \pi}\right)^2}_{\text{Hagedorn temperature}} \end{aligned}$$

The last of those is where the group ought to rejoin the rest of the library, and does not. The Hagedorn entry declares its left side to be a *temperature*—the same quantity the gas laws are about—and carries the Boltzmann constant besides. It joins to nothing outside the string cluster. The obstruction is not physical but orthographic: it writes T_H where the gas laws write T, and a subscript is part of a name in this algebra, deliberately, because that is what keeps m_1 and m_2 apart.

So the library can see the connection and cannot act on it. It records the collision—T is a temperature in one entry and a string tension in another—and it can be asked for both readings, but no join follows, because joining needs a shared *symbol* as well as a shared quantity.

6.3 The repair that does not work

The obvious response is to pivot on quantities directly: join any two equations that declare the same quantity, whatever letters they use. That was implemented and measured before being rejected. It takes the library from 74 to 240 pairwise statements. Two specimens of what it adds:

$$\begin{aligned} mc^2 &= \tfrac{1}{2}mv^2 \\ \tfrac{1}{2}mv^2 &= k_B T \end{aligned}$$

The first identifies a rest energy with a kinetic one. The second is worse, because it is nearly equipartition and a reader skimming might not stop: the true relation carries a factor of three halves, and the library has produced something that looks like a law and is not one.

The diagnosis is that requiring a shared letter was never really about letters. It was a proxy for the two equations meaning the same *instance* of a quantity, and for everything except a constant that proxy is all there is. Two equations both about "an energy" are not thereby about the same energy.

Section 4 established that a glyph is not a quantity; this establishes the converse limit, that a quantity is not an instance, and the readings table as designed cannot express the difference.

There is exactly one case where the fold is safe, and the module already had the vocabulary for it. A *constant* has one instance in the universe, so the ϵ_0 of Coulomb's law and the ϵ_0 of the Debye length are the same number and equating them assumes nothing. Folding a subscript is therefore permitted when, and only when, the quantity is declared a constant and the equation contains exactly one symbol with that base—Newton's law of gravitation has m_1 and m_2, so m names neither and the fold is refused. This recovers six joins, all of them about the permittivity, and makes it a family of four:

$$\overbrace{\underbrace{\frac{q_1 q_2}{4Fr^2\pi}}_{\text{Coulomb's law}} = \underbrace{\frac{\lambda_D^2 n e^2}{T k_B}}_{\text{Debye length}}}^{\text{both equal the permittivity}}$$

That is the whole of what the permissive rule was right about. The remaining 166 statements were classified rather than characterised, since "most of them are wrong" is not a measurement. Truth is the wrong test—every join in this library is an equation that holds only where both sides really do equal the pivot, and that is as true of the gravitational family as of anything here. The test applied instead is whether the transitivity step is *licensed*: are the two equations about the same instance of the quantity?

	Count	
Unlicensed	129	different symbols, quantity not a constant: the join asserts two differently named things are one instance
Already known	37	the pivot quantity is a family already, so the library knows it is multiply determined and suppresses the pairs deliberately
Novel	0	licensed, and about a quantity no family covers

Table 5: Every statement the permissive rule adds, classified by `quantity_join_census()`. The bottom row is the one that would have mattered.

The bottom row is empty, and that is the result. Not that the permissive rule is mostly wrong—that it offers the library nothing it wants. Three quarters of what it adds is unwarranted and the remaining quarter is already recorded as a family, suppressed on purpose because a pivot on π or on a mass is true and uninformative.

The audit did find one thing, though not where it was looking. That bottom row was not empty at first: it held $\omega/2\pi = v/\lambda$, which relates an angular frequency to a wave speed and is exactly the sort of statement the library exists to produce. It was being suppressed because f appeared on a list of letters too common to pivot on—a list written before the readings table existed, when it had to keep out ambiguous pivots as well as uninformative ones. `agree()` does the first job properly now. Measured against the current library, m on that list suppresses twenty-one statements, k and π eight each, r three, and all of those are vacuous or unwarranted; f suppressed one good one and nothing else, so f is gone and the join is in. Five of the remaining entries suppress nothing at all.

6.4 The edge of the method

The tilings were included for a reason that has nothing to do with area. A set of Wang tiles either tiles the plane or does not, and Berger proved in 1966 that no procedure decides which. The statement is finite, perfectly precise, and beyond computation altogether.

Nothing in this paper touches it. The kernel of Section 7 asks whether a statement is a well-formed proposition, which is decidable; it never asks whether the proposition is true, which in general is not. The library can hold the domino problem as prose and can hold the arithmetic consequence—that Penrose rhombs occur in an irrational ratio, so no tiling of them repeats—as an equation it can join. It cannot hold the theorem. Knowing which of the three is being offered is the difference between a knowledge base and a claim to have automated mathematics.

The same group supplies the opposite extreme, which is worth putting beside it. The monotile entries pin names to closed arithmetic—the inflation factor is $4+\sqrt{15}$, the commonest metatile frequency is $4-\sqrt{15}$—and once a name has a number, any other entry written entirely in such names has nothing left to be true about. It can simply be evaluated. The library declares seven constants this way, which makes four entries decidable, and they are checked on every run:

$$\mu^2 = 8\mu - 1$$
$$g\mu = 1$$

Four statements out of 97 is not a result about physics. It is a result about where the boundary sits. Everything else here is a conjecture because it quantifies over variables and the world decides; these are not, because they quantify over nothing. That is also the only place in the library where a mistyped surd can be caught automatically—$4+\sqrt{14}$ parses, typechecks, renders and joins exactly as well as the right answer, and is caught here and nowhere else.

7 Conclusion: From a composed statement to a Lean conjecture

`lean4.py` already reads LaTeX, but reads the language of types and stops at the first `^`. The bridge supplies the other half: a walk over the algebra's term type into kernel expressions over an axiomatic `Real`.

What is claimed matters. The kernel is told `Real` is a type and that `Real.add`, `Real.mul` and the rest are operations on it. It is *not* told the field axioms, so it cannot prove $mc^2 = h\nu$, and nothing here pretends otherwise. It checks that the statement is well formed: every symbol bound, both sides of the equality in one type, the result a `Prop`. That is the right claim for a conjecture—one provable from the notation alone would not be one.

Constants are declared rather than quantified, using the same table that stops `eq2py` treating c as an argument. This is not cosmetic: `forall pi : Real` is strictly stronger than the equation, and false for almost every π.

The brace labels survive the trip, so the generated Lean carries the name of the equation each side came from:

```
-- \overbrace{\underbrace{k_{B} \log W}_{\text{Boltzmann entropy}} = \underbrace{\frac{k_{B} c^{3} A
    }{4 G \hbar}}_{\text{Bekenstein-Hawking entropy}}}^{\text{both equal the entropy}}
--   Boltzmann entropy : k_{B} \log W
--   Bekenstein-Hawking entropy : \frac{k_{B} c^{3} A}{4 G \hbar}
--   joined on S
theorem Boltzmann_entropy_Bekenstein_Hawking_entropy :
    forall A W : Real,
      k_B * Real.log W = k_B * c ^ 3 * A / (4 * G * hbar) := by sorry
```

All 74 statements the library composes are emitted this way, and so is Figure 1 itself: conjoining its rows and binding the shared variables once over the whole gives a single proposition with 43 binders and 37 conjuncts, which the micro-kernel accepts as a `Prop`. The complete file is Appendix C.

It has not been checked by Lean 4 itself. The preamble declares the type-class instances an axiomatic `Real` needs—`Add`, `Mul`, `HPow`, `OfNat`—so the infix output elaborates, but that is a prediction rather than a result.

Reproducibility — Source Code

https://github.com/brentharts/RosettaMath

```
python3 rosettaphys.py              the census: nodes, edges, joins, collisions
python3 rosettaphys.py --selftest   252 entries, the graph, and the algebra
python3 rosettalean.py              every join, as a checked conjecture
python3 rosettalean.py --selftest   the bridge, end to end
python3 rosettapaper.py             regenerate this document
make test                           all of the above, plus the kernel
make collisions                     the table in Section 4
```

A The readings table

What each equation declares its letters to denote. An equation absent from this table takes part in no joins.

Equation	Statement	Readings
Mass-energy equivalence	$E = mc^2$	E: energy, c: speed of light, m: mass
Newton's second law	$F = ma$	F: force, a: acceleration, m: mass
Newton's law of gravitation	$F = G\frac{m_1 m_2}{r^2}$	F: force, G: gravitational constant, m_1: mass, m_2: mass, r: length
Coulomb's law	$F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$	F: force, *epsilon_0*: permittivity, *pi*: pi, q_1: charge, q_2: charge, r: length
Boltzmann entropy	$S = k_B \log W$	S: entropy, W: microstate count, k_B: boltzmann constant
Planck relation	$E = h\nu$	E: energy, h: planck constant, *nu*: frequency
de Broglie relation	$\lambda = \frac{h}{p}$	h: planck constant, *lambda*: wavelength, p: momentum
Schwarzschild radius	$r_s = \frac{2GM}{c^2}$	G: gravitational constant, M: mass, c: speed of light, r_s: length
Bekenstein-Hawking entropy	$S = \frac{k_B c^3 A}{4G\hbar}$	A: area, G: gravitational constant, S: entropy, c: speed of light, *hbar*: reduced planck constant, k_B: boltzmann constant
Ideal gas law	$PV = nRT$	P: pressure, R: gas constant, T: temperature, V: volume, n: amount
Stefan-Boltzmann law	$j = \sigma T^4$	T: temperature, j: flux, *sigma*: stefan-boltzmann constant
Plasma frequency	$\omega_p = \sqrt{\frac{ne^2}{\epsilon_0 m_e}}$	e: elementary charge, *epsilon_0*: permittivity, m_e: mass, n: number density, *omega_p*: angular frequency
Debye length	$\lambda_D = \sqrt{\frac{\epsilon_0 k_B T}{ne^2}}$	T: temperature, e: elementary charge, *epsilon_0*: permittivity, k_B: boltzmann constant, *lambda_D*: length, n: number density
Pythagorean theorem	$a^2 + b^2 = c^2$	a: length, b: length, c: length
Kinetic energy	$E_k = \frac{1}{2}mv^2$	E_k: energy, m: mass, v: speed
Momentum	$p = mv$	m: mass, p: momentum, v: speed
Work done by a force	$W = Fd$	F: force, W: energy, d: length

Equation	Statement	Readings
Gravitational potential energy	$U = mgh$	U: energy, g: gravitational field, h: length, m: mass
Surface gravity	$g = \frac{GM}{r^2}$	G: gravitational constant, M: mass, g: gravitational field, r: length
Hooke's law	$F = -kx$	F: force, k: spring constant, x: length
Angular frequency	$\omega = 2\pi f$	f: frequency, *omega*: angular frequency, *pi*: pi
Wave speed	$v = f\lambda$	f: frequency, *lambda*: wavelength, v: speed
Ohm's law	$V = IR$	I: current, R: resistance, V: voltage
Electrical power	$P = VI$	I: current, P: power, V: voltage
Capacitor charge	$Q = CV$	C: capacitance, Q: charge, V: voltage
Density	$\rho = \frac{m}{V}$	V: volume, m: mass, *rho*: density
Hydrostatic pressure	$P = \rho gh$	P: pressure, g: gravitational field, h: length, *rho*: density
Thermal energy	$E = k_B T$	E: energy, T: temperature, *k_B*: boltzmann constant
Heat capacity	$Q = mc_p \Delta T$	Q: energy, T: temperature, *c_p*: dimensionless, m: mass
Compton wavelength	$\lambda_C = \frac{h}{mc}$	c: speed of light, h: planck constant, *lambda_C*: wavelength, m: mass
Planck length	$\ell_P = \sqrt{\frac{\hbar G}{c^3}}$	G: gravitational constant, c: speed of light, *ell_P*: length, *hbar*: reduced planck constant
Escape velocity	$v_e = \sqrt{\frac{2GM}{r}}$	G: gravitational constant, M: mass, r: length, *v_e*: speed
Hubble's law	$v = H_0 D$	D: length, *H_0*: hubble parameter, v: speed
Coulomb potential energy	$U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$	U: energy, *epsilon_0*: permittivity, *pi*: pi, *q_1*: charge, *q_2*: charge, r: length
Golden ratio	$\phi = \frac{1+\sqrt{5}}{2}$	*phi*: golden ratio
Inflation quadratic	$\lambda^2 = \lambda + 1$	*lambda*: inflation factor
Penrose tile ratio	$N_f = \phi N_t$	*N_f*: tile count, *N_t*: tile count, *phi*: golden ratio
Inflation of area	$A' = \phi^2 A$	A: patch area, *A_prime*: patch area, *phi*: golden ratio
Spectre inflation quadratic	$\mu^2 = 8\mu - 1$	*mu*: inflation factor
Spectre inflation factor	$\mu = 4 + \sqrt{15}$	*mu*: inflation factor
Spectre reciprocal pair	$g\mu = 1$	g: tile frequency, *mu*: inflation factor
Spectre triplet frequency	$g = 4 - \sqrt{15}$	g: tile frequency
Spectre frequency ratio	$v_\Theta = \frac{v_\Gamma}{\mu}$	*mu*: inflation factor, *v_Gamma*: tile frequency, *v_Theta*: tile frequency
Spectre count recurrence	$N'' = 8N' - N$	N: tile count, *N_prime*: tile count, *N_prime_prime*: tile count
Spectre tile area	$A = 2\sqrt{3}a^2 + 3ab + \sqrt{3}b^2$	A: tile area, a: length, b: length
Hat tile area	$A_H = 8\sqrt{3}$	*A_H*: tile area
Hat inflation quadratic	$\kappa^2 = 7\kappa - 1$	*kappa*: inflation factor
Hat inflation factor	$\kappa = \frac{7+3\sqrt{5}}{2}$	*kappa*: inflation factor
Hat constant as a golden power	$\kappa = \phi^4$	*kappa*: inflation factor, *phi*: golden ratio
Hat chirality ratio	$\frac{N_u}{N_r} = \kappa$	*N_r*: tile count, *N_u*: tile count, *kappa*: inflation factor
Euler characteristic of a closed surface	$\chi = 2 - 2g$	*chi*: euler characteristic, g: genus
Riemann-Hurwitz (unbranched)	$\chi_C = n\chi_B$	*chi_B*: euler characteristic, *chi_C*: euler characteristic, n: covering degree
Gauss-Bonnet theorem	$\int_M K\,dA + \oint_{\partial M} k_g\,ds = 2\pi\chi$	K: gaussian curvature, *chi*: euler characteristic, *k_g*: geodesic curvature, *pi*: pi
Mean curvature	$H = \frac{\kappa_1+\kappa_2}{2}$	H: mean curvature, *kappa_1*: principal curvature, *kappa_2*: principal curvature

Equation	Statement	Readings
Minimal surface condition	$H = 0$	H: mean curvature
Minimal surface equation	$(1+u_y^2)u_{xx} - 2u_x u_y u_{xy} + (1+u_x^2)u_{yy} = 0$	*u_x*: slope, *u_y*: slope
Catenoid	$r = c\cosh\left(\frac{z}{c}\right)$	c: length, r: length, z: length
Area functional	$A = \int\int \sqrt{1+\lvert\nabla u\rvert^2}\,dx\,dy$	A: area
Kakeya conjecture	$\dim_H K = n$	n: hausdorff dimension
Perron tree bound	$A \leq \frac{C}{\log n}$	A: area, C: dimensionless, n: count
Nambu-Goto action	$S = -T\int d^2\sigma\sqrt{-\det(h_{ab})}$	S: action, T: string tension
String tension	$T = \frac{1}{2\pi\alpha'}$	T: string tension, *alpha_prime*: regge slope, *pi*: pi
Regge trajectory	$J = \alpha' M^2 + \alpha_0$	J: spin, M: mass, *alpha_0*: dimensionless, *alpha_prime*: regge slope
String length	$\ell_s = \sqrt{\alpha'}$	*alpha_prime*: regge slope, *ell_s*: length
Genus expansion	$Z = \sum_g g_s^{-\chi} Z_g$	Z: partition function, *chi*: euler characteristic, *g_s*: string coupling
Critical dimension	$D = 26$	D: spacetime dimension
Hagedorn temperature	$T_H = \frac{1}{4\pi\sqrt{\alpha'}k_B}$	*T_H*: temperature, *alpha_prime*: regge slope, *k_B*: boltzmann constant, *pi*: pi

B Entries derived rather than quoted

Most of the library can be checked against a textbook. Twelve entries cannot: their values were computed, and an entry that says only which Wikipedia article explains the idea does not say where its number came from. Those entries carry a `source` naming the matrix or function it was computed from.
`https://github.com/brentharts/spectre`
`https://github.com/brentharts/CICY`

The field exists because of a near-miss. Two of the substitutions below give different inflation factors for the aperiodic monotile—ϕ^4 for the hat and $4+\sqrt{15}$ for the spectre—and both are right, because they are different substitutions on different numbers of species. Written without provenance the two entries read as a contradiction, and the first draft of the tilings group made exactly that mistake.

Entry	Computed from
Spectre inflation quadratic	`brentharts/spectre: chirality_e8.substitution_matrix(), nine species`
Spectre inflation factor	`brentharts/spectre: chirality_e8.substitution_matrix(), nine species`
Spectre reciprocal pair	`brentharts/spectre: chirality_e8.substitution_matrix(), nine species`
Spectre triplet frequency	`brentharts/spectre: chirality_e8.substitution_matrix(), nine species`
Spectre frequency ratio	`brentharts/spectre: chirality_e8.substitution_matrix(), nine species`
Spectre count recurrence	`brentharts/spectre: spectre_tile_counter.py`
Spectre tile area	`brentharts/spectre: spectre.get_spectre_points()`
Hat tile area	`brentharts/spectre: spectre.get_spectre_points()`
Hat inflation quadratic	`brentharts/CICY: pyCICY.monotile.SUBSTITUTION, four metatiles`
Hat inflation factor	`brentharts/CICY: pyCICY.monotile.inflation_factor()`
Hat constant as a golden power	`brentharts/CICY: pyCICY.monotile.hat_chirality(), is_phi4`
Hat chirality ratio	`brentharts/CICY: pyCICY.monotile.hat_chirality()`

C The generated Lean 4 source

Produced by `python3 rosettalean.py --lean`. `Real` and its operations are declared, not defined: this file asks Lean whether the statements are well-formed propositions, which is the question the micro-kernel already answered. Proving them needs the field axioms and the physics, and neither is claimed.

```
-- Generated by rosettalean.py from the RosettaMath knowledge base.
--
-- Real and its operations are declared, not defined: this file asks Lean
-- whether the statements below are well formed propositions, which is the
-- same question lean4.py's micro-kernel was asked.  Proving them needs the
-- field axioms and the physics, and neither is claimed here.

namespace RosettaPhys

axiom Real : Type
axiom Real.add : Real -> Real -> Real
axiom Real.sub : Real -> Real -> Real
axiom Real.mul : Real -> Real -> Real
axiom Real.div : Real -> Real -> Real
axiom Real.neg : Real -> Real
axiom Real.sqrt : Real -> Real
axiom Real.log : Real -> Real
axiom Real.exp : Real -> Real
axiom Real.sin : Real -> Real
axiom Real.cos : Real -> Real
axiom Real.tan : Real -> Real
axiom Real.abs : Real -> Real
axiom Real.pow : Real -> Nat -> Real
axiom Real.rpow : Real -> Real -> Real
axiom Real.lit : Nat -> Real
axiom Real.le : Real -> Real -> Prop
axiom Real.lt : Real -> Real -> Prop
axiom Real.ge : Real -> Real -> Prop
axiom Real.gt : Real -> Real -> Prop

-- The printer below emits infix * + - / ^ <= and Lean
-- resolves those through type classes, so an axiomatic
-- Real needs the instances spelled out or none of it
-- elaborates.
instance : Add Real := Add.mk Real.add
instance : Sub Real := Sub.mk Real.sub
instance : Mul Real := Mul.mk Real.mul
instance : Div Real := Div.mk Real.div
instance : Neg Real := Neg.mk Real.neg
instance : LE Real := LE.mk Real.le
instance : LT Real := LT.mk Real.lt
instance : HPow Real Nat Real := HPow.mk Real.pow

-- so that a numeral in an equation means a real number.
-- Without it `4 * pi` does not elaborate: Real is an
-- axiom here and carries no arithmetic instances of its own.
instance (n : Nat) : OfNat Real n := OfNat.mk (Real.lit n)

-- physical constants: named, not quantified
axiom G : Real
axiom N_A : Real
axiom c : Real
axiom epsilon_0 : Real
axiom h : Real
axiom hbar : Real
axiom k_B : Real
axiom m_e : Real
axiom m_n : Real
axiom m_p : Real
axiom mu_0 : Real
axiom pi : Real

-- \overbrace{\underbrace{m c^{2}}_{\text{Mass-energy equivalence}} = \underbrace{h \nu}_{\text{Planck relation}}}^{\text{both equal the energy}}
--   Mass-energy equivalence : m c^{2}
--   Planck relation : h \nu
--   joined on E
theorem Mass_energy_equivalence_Planck_relation :
    forall m nu : Real,
      m * c ^ 2 = h * nu := by sorry

-- \overbrace{\underbrace{m c^{2}}_{\text{Mass-energy equivalence}} = \underbrace{k_{B} T}_{\text{Thermal energy}}}^{\text{both equal the energy}}
--   Mass-energy equivalence : m c^{2}
--   Thermal energy : k_{B} T
--   joined on E
theorem Mass_energy_equivalence_Thermal_energy :
    forall T m : Real,
      m * c ^ 2 = k_B * T := by sorry

-- \overbrace{\underbrace{m a}_{\text{Newton's second law}} = \underbrace{G \frac{m_{1} m_{2}}{r^{2}}}_{\text{Newton's law of gravitation}}}^{\text{both equal the force}}
--   Newton's second law : m a
--   Newton's law of gravitation : G \frac{m_{1} m_{2}}{r^{2}}
--   joined on F
theorem Newton_s_second_law_Newton_s_law_of_gravitation :
    forall a m m_1 m_2 r : Real,
      m * a = G * (m_1 * m_2 / r ^ 2) := by sorry

-- \overbrace{\underbrace{m a}_{\text{Newton's second law}} = \underbrace{\frac{1}{4 \pi \epsilon_{0}} \frac{q_{1} q_{2}}{r^{2}}}_{\text{Coulomb's law}}}^{\text{both equal the force}}
--   Newton's second law : m a
--   Coulomb's law : \frac{1}{4 \pi \epsilon_{0}} \frac{q_{1} q_{2}}{r^{2}}
--   joined on F
theorem Newton_s_second_law_Coulomb_s_law :
```

```
    forall a m q_1 q_2 r : Real,
      m * a = 1 / (4 * pi * epsilon_0) * (q_1 * q_2 / r ^ 2) := by sorry

-- \overbrace{\underbrace{m a}_{\text{Newton's second law}} = \underbrace{\frac{W}{d}}_{\text{Work done by a force}}}^{\text{both equal the force}}
--   Newton's second law : m a
--   Work done by a force : \frac{W}{d}
--   joined on F
theorem Newton_s_second_law_Work_done_by_a_force :
    forall W a d m : Real,
      m * a = W / d := by sorry

-- \overbrace{\underbrace{m a}_{\text{Newton's second law}} = \underbrace{-\left( k x \right)}_{\text{Hooke's law}}}^{\text{both equal the force}}
--   Newton's second law : m a
--   Hooke's law : -\left( k x \right)
--   joined on F
theorem Newton_s_second_law_Hooke_s_law :
    forall a k m x : Real,
      m * a = Real.neg (k * x) := by sorry

-- \overbrace{\underbrace{G \frac{m_{1} m_{2}}{r^{2}}}_{\text{Newton's law of gravitation}} = \underbrace{\frac{1}{4 \pi \epsilon_{0}} \frac{q_{1} q_{2}}{r^{2}}}_{\text{Coulomb's law}}}^{\text{both equal the force}}
--   Newton's law of gravitation : G \frac{m_{1} m_{2}}{r^{2}}
--   Coulomb's law : \frac{1}{4 \pi \epsilon_{0}} \frac{q_{1} q_{2}}{r^{2}}
--   joined on F
theorem Newton_s_law_of_gravitation_Coulomb_s_law :
    forall m_1 m_2 q_1 q_2 r : Real,
      G * (m_1 * m_2 / r ^ 2) = 1 / (4 * pi * epsilon_0) * (q_1 * q_2 / r ^ 2) := by sorry

-- \overbrace{\underbrace{\frac{F r^{2}}{m_{1} m_{2}}}_{\text{Newton's law of gravitation}} = \underbrace{\frac{r_{s} c^{2}}{2 M}}_{\text{Schwarzschild radius}}}^{\text{both equal the gravitational constant}}
--   Newton's law of gravitation : \frac{F r^{2}}{m_{1} m_{2}}
--   Schwarzschild radius : \frac{r_{s} c^{2}}{2 M}
--   joined on G
theorem Newton_s_law_of_gravitation_Schwarzschild_radius :
    forall F M m_1 m_2 r r_s : Real,
      F * r ^ 2 / (m_1 * m_2) = r_s * c ^ 2 / (2 * M) := by sorry

-- \overbrace{\underbrace{\frac{F r^{2}}{m_{1} m_{2}}}_{\text{Newton's law of gravitation}} = \underbrace{\frac{k_{B} c^{3} A}{4 S \hbar}}_{\text{Bekenstein-Hawking entropy}}}^{\text{both equal the gravitational constant}}
--   Newton's law of gravitation : \frac{F r^{2}}{m_{1} m_{2}}
--   Bekenstein-Hawking entropy : \frac{k_{B} c^{3} A}{4 S \hbar}
--   joined on G
theorem Newton_s_law_of_gravitation_Bekenstein_Hawking_entropy :
    forall A F S m_1 m_2 r : Real,
      F * r ^ 2 / (m_1 * m_2) = k_B * c ^ 3 * A / (4 * S * hbar) := by sorry

-- \overbrace{\underbrace{G \frac{m_{1} m_{2}}{r^{2}}}_{\text{Newton's law of gravitation}} = \underbrace{\frac{W}{d}}_{\text{Work done by a force}}}^{\text{both equal the force}}
--   Newton's law of gravitation : G \frac{m_{1} m_{2}}{r^{2}}
--   Work done by a force : \frac{W}{d}
--   joined on F
theorem Newton_s_law_of_gravitation_Work_done_by_a_force :
    forall W d m_1 m_2 r : Real,
      G * (m_1 * m_2 / r ^ 2) = W / d := by sorry

-- \overbrace{\underbrace{\frac{F r^{2}}{m_{1} m_{2}}}_{\text{Newton's law of gravitation}} = \underbrace{\frac{g r^{2}}{M}}_{\text{Surface gravity}}}^{\text{both equal the gravitational constant}}
--   Newton's law of gravitation : \frac{F r^{2}}{m_{1} m_{2}}
--   Surface gravity : \frac{g r^{2}}{M}
--   joined on G
theorem Newton_s_law_of_gravitation_Surface_gravity :
    forall F M g m_1 m_2 r : Real,
      F * r ^ 2 / (m_1 * m_2) = g * r ^ 2 / M := by sorry

-- \overbrace{\underbrace{G \frac{m_{1} m_{2}}{r^{2}}}_{\text{Newton's law of gravitation}} = \underbrace{-\left( k x \right)}_{\text{Hooke's law}}}^{\text{both equal the force}}
--   Newton's law of gravitation : G \frac{m_{1} m_{2}}{r^{2}}
--   Hooke's law : -\left( k x \right)
--   joined on F
theorem Newton_s_law_of_gravitation_Hooke_s_law :
    forall k m_1 m_2 r x : Real,
      G * (m_1 * m_2 / r ^ 2) = Real.neg (k * x) := by sorry

-- \overbrace{\underbrace{\frac{F r^{2}}{m_{1} m_{2}}}_{\text{Newton's law of gravitation}} = \underbrace{\frac{\ell_{P}^{2} c^{3}}{\hbar}}_{\text{Planck length}}}^{\text{both equal the gravitational constant}}
--   Newton's law of gravitation : \frac{F r^{2}}{m_{1} m_{2}}
--   Planck length : \frac{\ell_{P}^{2} c^{3}}{\hbar}
--   joined on G
theorem Newton_s_law_of_gravitation_Planck_length :
    forall F ell_P m_1 m_2 r : Real,
      F * r ^ 2 / (m_1 * m_2) = ell_P ^ 2 * c ^ 3 / hbar := by sorry

-- \overbrace{\underbrace{\frac{F r^{2}}{m_{1} m_{2}}}_{\text{Newton's law of gravitation}} = \underbrace{\frac{v_{e}^{2} r}{2 M}}_{\text{Escape velocity}}}^{\text{both equal the gravitational constant}}
--   Newton's law of gravitation : \frac{F r^{2}}{m_{1} m_{2}}
--   Escape velocity : \frac{v_{e}^{2} r}{2 M}
--   joined on G
theorem Newton_s_law_of_gravitation_Escape_velocity :
    forall F M m_1 m_2 r v_e : Real,
      F * r ^ 2 / (m_1 * m_2) = v_e ^ 2 * r / (2 * M) := by sorry

-- \overbrace{\underbrace{\frac{q_{1} q_{2}}{4 F r^{2} \pi}}_{\text{Coulomb's law}} = \underbrace{\frac{n e^{2}}{\omega_{p}^{2} m_{e}}}_{\text{Plasma frequency}}}^{\text{both equal the permittivity}}
--   Coulomb's law : \frac{q_{1} q_{2}}{4 F r^{2} \pi}
--   Plasma frequency : \frac{n e^{2}}{\omega_{p}^{2} m_{e}}
```

```
--   joined on \epsilon
theorem Coulomb_s_law_Plasma_frequency :
    forall F e n omega_p q_1 q_2 r : Real,
      q_1 * q_2 / (4 * F * r ^ 2 * pi) = n * e ^ 2 / (omega_p ^ 2 * m_e) := by sorry

-- \overbrace{\underbrace{\frac{q_{1} q_{2}}{4 F r^{2} \pi}}_{\text{Coulomb's law}} = \underbrace{\frac{\lambda_{D}^{2} n e^{2}}{T k_{B}}}_{\text{Debye length}}}^{\text{both equal the permittivity}}
--   Coulomb's law : \frac{q_{1} q_{2}}{4 F r^{2} \pi}
--   Debye length : \frac{\lambda_{D}^{2} n e^{2}}{T k_{B}}
--   joined on \epsilon
theorem Coulomb_s_law_Debye_length :
    forall F T e lambda_D n q_1 q_2 r : Real,
      q_1 * q_2 / (4 * F * r ^ 2 * pi) = lambda_D ^ 2 * n * e ^ 2 / (T * k_B) := by sorry

-- \overbrace{\underbrace{\frac{1}{4 \pi \epsilon_{0}} \frac{q_{1} q_{2}}{r^{2}}}_{\text{Coulomb's law}} = \underbrace{\frac{W}{d}}_{\text{Work done by a force}}}^{\text{both equal the force}}
--   Coulomb's law : \frac{1}{4 \pi \epsilon_{0}} \frac{q_{1} q_{2}}{r^{2}}
--   Work done by a force : \frac{W}{d}
--   joined on F
theorem Coulomb_s_law_Work_done_by_a_force :
    forall W d q_1 q_2 r : Real,
      1 / (4 * pi * epsilon_0) * (q_1 * q_2 / r ^ 2) = W / d := by sorry

-- \overbrace{\underbrace{\frac{1}{4 \pi \epsilon_{0}} \frac{q_{1} q_{2}}{r^{2}}}_{\text{Coulomb's law}} = \underbrace{-\left( k x \right)}_{\text{Hooke's law}}}^{\text{both equal the force}}
--   Coulomb's law : \frac{1}{4 \pi \epsilon_{0}} \frac{q_{1} q_{2}}{r^{2}}
--   Hooke's law : -\left( k x \right)
--   joined on F
theorem Coulomb_s_law_Hooke_s_law :
    forall k q_1 q_2 r x : Real,
      1 / (4 * pi * epsilon_0) * (q_1 * q_2 / r ^ 2) = Real.neg (k * x) := by sorry

-- \overbrace{\underbrace{\frac{q_{1} q_{2}}{4 F r^{2} \pi}}_{\text{Coulomb's law}} = \underbrace{\frac{q_{1} q_{2}}{4 U r \pi}}_{\text{Coulomb potential energy}}}^{\text{both equal the permittivity}}
--   Coulomb's law : \frac{q_{1} q_{2}}{4 F r^{2} \pi}
--   Coulomb potential energy : \frac{q_{1} q_{2}}{4 U r \pi}
--   joined on \epsilon
theorem Coulomb_s_law_Coulomb_potential_energy :
    forall F U q_1 q_2 r : Real,
      q_1 * q_2 / (4 * F * r ^ 2 * pi) = q_1 * q_2 / (4 * U * r * pi) := by sorry

-- \overbrace{\underbrace{k_{B} \log W}_{\text{Boltzmann entropy}} = \underbrace{\frac{k_{B} c^{3} A}{4 G \hbar}}_{\text{Bekenstein-Hawking entropy}}}^{\text{both equal the entropy}}
--   Boltzmann entropy : k_{B} \log W
--   Bekenstein-Hawking entropy : \frac{k_{B} c^{3} A}{4 G \hbar}
--   joined on S
theorem Boltzmann_entropy_Bekenstein_Hawking_entropy :
    forall A W : Real,
      k_B * Real.log W = k_B * c ^ 3 * A / (4 * G * hbar) := by sorry

-- \overbrace{\underbrace{\frac{E}{\nu}}_{\text{Planck relation}} = \underbrace{\lambda p}_{\text{de Broglie relation}}}^{\text{both equal the planck constant}}
--   Planck relation : \frac{E}{\nu}
--   de Broglie relation : \lambda p
--   joined on h
theorem Planck_relation_de_Broglie_relation :
    forall E lambda nu p : Real,
      E / nu = lambda * p := by sorry

-- \overbrace{\underbrace{h \nu}_{\text{Planck relation}} = \underbrace{k_{B} T}_{\text{Thermal energy}}}^{\text{both equal the energy}}
--   Planck relation : h \nu
--   Thermal energy : k_{B} T
--   joined on E
theorem Planck_relation_Thermal_energy :
    forall T nu : Real,
      h * nu = k_B * T := by sorry

-- \overbrace{\underbrace{\frac{E}{\nu}}_{\text{Planck relation}} = \underbrace{\lambda_{C} m c}_{\text{Compton wavelength}}}^{\text{both equal the planck constant}}
--   Planck relation : \frac{E}{\nu}
--   Compton wavelength : \lambda_{C} m c
--   joined on h
theorem Planck_relation_Compton_wavelength :
    forall E lambda_C m nu : Real,
      E / nu = lambda_C * m * c := by sorry

-- \overbrace{\underbrace{\frac{h}{\lambda}}_{\text{de Broglie relation}} = \underbrace{m v}_{\text{Momentum}}}^{\text{both equal the momentum}}
--   de Broglie relation : \frac{h}{\lambda}
--   Momentum : m v
--   joined on p
theorem de_Broglie_relation_Momentum :
    forall lambda m v : Real,
      h / lambda = m * v := by sorry

-- \overbrace{\underbrace{\frac{h}{p}}_{\text{de Broglie relation}} = \underbrace{\frac{v}{f}}_{\text{Wave speed}}}^{\text{both equal the wavelength}}
--   de Broglie relation : \frac{h}{p}
--   Wave speed : \frac{v}{f}
--   joined on \lambda
theorem de_Broglie_relation_Wave_speed :
    forall f p v : Real,
      h / p = v / f := by sorry

-- \overbrace{\underbrace{\lambda p}_{\text{de Broglie relation}} = \underbrace{\lambda_{C} m c}_{\text{Compton wavelength}}}^{\text{both equal the planck constant}}
--   de Broglie relation : \lambda p
```

```
--   Compton wavelength : \lambda_{C} m c
--   joined on h
theorem de_Broglie_relation_Compton_wavelength :
    forall lambda lambda_C m p : Real,
      lambda * p = lambda_C * m * c := by sorry

-- \overbrace{\underbrace{\frac{r_{s} c^{2}}{2 M}}_{\text{Schwarzschild radius}} = \underbrace{\frac{k_{B} c^{3} A}{4 S \hbar}}_{\text{Bekenstein-Hawking
      entropy}}}^{\text{both equal the gravitational constant}}
--   Schwarzschild radius : \frac{r_{s} c^{2}}{2 M}
--   Bekenstein-Hawking entropy : \frac{k_{B} c^{3} A}{4 S \hbar}
--   joined on G
theorem Schwarzschild_radius_Bekenstein_Hawking_entropy :
    forall A M S r_s : Real,
      r_s * c ^ 2 / (2 * M) = k_B * c ^ 3 * A / (4 * S * hbar) := by sorry

-- \overbrace{\underbrace{\frac{r_{s} c^{2}}{2 G}}_{\text{Schwarzschild radius}} = \underbrace{\frac{g r^{2}}{G}}_{\text{Surface gravity}}}^{\text{both
      equal the mass}}
--   Schwarzschild radius : \frac{r_{s} c^{2}}{2 G}
--   Surface gravity : \frac{g r^{2}}{G}
--   joined on M
theorem Schwarzschild_radius_Surface_gravity :
    forall g r r_s : Real,
      r_s * c ^ 2 / (2 * G) = g * r ^ 2 / G := by sorry

-- \overbrace{\underbrace{\frac{r_{s} c^{2}}{2 M}}_{\text{Schwarzschild radius}} = \underbrace{\frac{\ell_{P}^{2} c^{3}}{\hbar}}_{\text{Planck length
      }}}^{\text{both equal the gravitational constant}}
--   Schwarzschild radius : \frac{r_{s} c^{2}}{2 M}
--   Planck length : \frac{\ell_{P}^{2} c^{3}}{\hbar}
--   joined on G
theorem Schwarzschild_radius_Planck_length :
    forall M ell_P r_s : Real,
      r_s * c ^ 2 / (2 * M) = ell_P ^ 2 * c ^ 3 / hbar := by sorry

-- \overbrace{\underbrace{\frac{r_{s} c^{2}}{2 G}}_{\text{Schwarzschild radius}} = \underbrace{\frac{v_{e}^{2} r}{2 G}}_{\text{Escape velocity}}}^{\text{
      both equal the mass}}
--   Schwarzschild radius : \frac{r_{s} c^{2}}{2 G}
--   Escape velocity : \frac{v_{e}^{2} r}{2 G}
--   joined on M
theorem Schwarzschild_radius_Escape_velocity :
    forall r r_s v_e : Real,
      r_s * c ^ 2 / (2 * G) = v_e ^ 2 * r / (2 * G) := by sorry

-- \overbrace{\underbrace{\frac{k_{B} c^{3} A}{4 S \hbar}}_{\text{Bekenstein-Hawking entropy}} = \underbrace{\frac{g r^{2}}{M}}_{\text{Surface gravity
      }}}^{\text{both equal the gravitational constant}}
--   Bekenstein-Hawking entropy : \frac{k_{B} c^{3} A}{4 S \hbar}
--   Surface gravity : \frac{g r^{2}}{M}
--   joined on G
theorem Bekenstein_Hawking_entropy_Surface_gravity :
    forall A M S g r : Real,
      k_B * c ^ 3 * A / (4 * S * hbar) = g * r ^ 2 / M := by sorry

-- \overbrace{\underbrace{\left( \frac{S 4 G \hbar}{A k_{B}} \right)^{\frac{1}{3}}}_{\text{Bekenstein-Hawking entropy}} = \underbrace{\frac{h}{\lambda_{C
      } m}}_{\text{Compton wavelength}}}^{\text{both equal the speed of light}}
--   Bekenstein-Hawking entropy : \left( \frac{S 4 G \hbar}{A k_{B}} \right)^{\frac{1}{3}}
--   Compton wavelength : \frac{h}{\lambda_{C} m}
--   joined on c
theorem Bekenstein_Hawking_entropy_Compton_wavelength :
    forall A S lambda_C m : Real,
      Real.rpow (S * 4 * G * hbar / (A * k_B)) (1 / 3) = h / (lambda_C * m) := by sorry

-- \overbrace{\underbrace{\frac{k_{B} c^{3} A}{4 S G}}_{\text{Bekenstein-Hawking entropy}} = \underbrace{\frac{\ell_{P}^{2} c^{3}}{G}}_{\text{Planck
      length}}}^{\text{both equal the reduced planck constant}}
--   Bekenstein-Hawking entropy : \frac{k_{B} c^{3} A}{4 S G}
--   Planck length : \frac{\ell_{P}^{2} c^{3}}{G}
--   joined on \hbar
theorem Bekenstein_Hawking_entropy_Planck_length :
    forall A S ell_P : Real,
      k_B * c ^ 3 * A / (4 * S * G) = ell_P ^ 2 * c ^ 3 / G := by sorry

-- \overbrace{\underbrace{\frac{k_{B} c^{3} A}{4 S \hbar}}_{\text{Bekenstein-Hawking entropy}} = \underbrace{\frac{v_{e}^{2} r}{2 M}}_{\text{Escape
      velocity}}}^{\text{both equal the gravitational constant}}
--   Bekenstein-Hawking entropy : \frac{k_{B} c^{3} A}{4 S \hbar}
--   Escape velocity : \frac{v_{e}^{2} r}{2 M}
--   joined on G
theorem Bekenstein_Hawking_entropy_Escape_velocity :
    forall A M S r v_e : Real,
      k_B * c ^ 3 * A / (4 * S * hbar) = v_e ^ 2 * r / (2 * M) := by sorry

-- \overbrace{\underbrace{\frac{P V}{n R}}_{\text{Ideal gas law}} = \underbrace{\frac{\lambda_{D}^{2} n e^{2}}{\epsilon_{0} k_{B}}}_{\text{Debye length
      }}}^{\text{both equal the temperature}}
--   Ideal gas law : \frac{P V}{n R}
--   Debye length : \frac{\lambda_{D}^{2} n e^{2}}{\epsilon_{0} k_{B}}
--   joined on T
theorem Ideal_gas_law_Debye_length :
    forall P R V e lambda_D n : Real,
      P * V / (n * R) = lambda_D ^ 2 * n * e ^ 2 / (epsilon_0 * k_B) := by sorry

-- \overbrace{\underbrace{\frac{n R T}{P}}_{\text{Ideal gas law}} = \underbrace{\frac{m}{\rho}}_{\text{Density}}}^{\text{both equal the volume}}
--   Ideal gas law : \frac{n R T}{P}
--   Density : \frac{m}{\rho}
--   joined on V
theorem Ideal_gas_law_Density :
    forall P R T m n rho : Real,
      n * R * T / P = m / rho := by sorry

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-- \overbrace{\underbrace{\frac{n R T}{V}}_{\text{Ideal gas law}} = \underbrace{\rho g h}_{\text{Hydrostatic pressure}}}^{\text{both equal the pressure}}
--   Ideal gas law : \frac{n R T}{V}
--   Hydrostatic pressure : \rho g h
--   joined on P
theorem Ideal_gas_law_Hydrostatic_pressure :
    forall R T V g n rho : Real,
      n * R * T / V = rho * g * h := by sorry

-- \overbrace{\underbrace{\frac{P V}{n R}}_{\text{Ideal gas law}} = \underbrace{\frac{E}{k_{B}}}_{\text{Thermal energy}}}^{\text{both equal the temperature}}
--   Ideal gas law : \frac{P V}{n R}
--   Thermal energy : \frac{E}{k_{B}}
--   joined on T
theorem Ideal_gas_law_Thermal_energy :
    forall E P R V n : Real,
      P * V / (n * R) = E / k_B := by sorry

-- \overbrace{\underbrace{\frac{n e^{2}}{\omega_{p}^{2} m_{e}}}_{\text{Plasma frequency}} = \underbrace{\frac{\lambda_{D}^{2} n e^{2}}{T k_{B}}}_{\text{Debye length}}}^{\text{both equal the permittivity}}
--   Plasma frequency : \frac{n e^{2}}{\omega_{p}^{2} m_{e}}
--   Debye length : \frac{\lambda_{D}^{2} n e^{2}}{T k_{B}}
--   joined on \epsilon
theorem Plasma_frequency_Debye_length :
    forall T e lambda_D n omega_p : Real,
      n * e ^ 2 / (omega_p ^ 2 * m_e) = lambda_D ^ 2 * n * e ^ 2 / (T * k_B) := by sorry

-- \overbrace{\underbrace{\frac{n e^{2}}{\omega_{p}^{2} m_{e}}}_{\text{Plasma frequency}} = \underbrace{\frac{q_{1} q_{2}}{4 U r \pi}}_{\text{Coulomb potential energy}}}^{\text{both equal the permittivity}}
--   Plasma frequency : \frac{n e^{2}}{\omega_{p}^{2} m_{e}}
--   Coulomb potential energy : \frac{q_{1} q_{2}}{4 U r \pi}
--   joined on \epsilon
theorem Plasma_frequency_Coulomb_potential_energy :
    forall U e n omega_p q_1 q_2 r : Real,
      n * e ^ 2 / (omega_p ^ 2 * m_e) = q_1 * q_2 / (4 * U * r * pi) := by sorry

-- \overbrace{\underbrace{\frac{\lambda_{D}^{2} n e^{2}}{\epsilon_{0} k_{B}}}_{\text{Debye length}} = \underbrace{\frac{E}{k_{B}}}_{\text{Thermal energy}}}^{\text{both equal the temperature}}
--   Debye length : \frac{\lambda_{D}^{2} n e^{2}}{\epsilon_{0} k_{B}}
--   Thermal energy : \frac{E}{k_{B}}
--   joined on T
theorem Debye_length_Thermal_energy :
    forall E e lambda_D n : Real,
      lambda_D ^ 2 * n * e ^ 2 / (epsilon_0 * k_B) = E / k_B := by sorry

-- \overbrace{\underbrace{\frac{\lambda_{D}^{2} n e^{2}}{T k_{B}}}_{\text{Debye length}} = \underbrace{\frac{q_{1} q_{2}}{4 U r \pi}}_{\text{Coulomb potential energy}}}^{\text{both equal the permittivity}}
--   Debye length : \frac{\lambda_{D}^{2} n e^{2}}{T k_{B}}
--   Coulomb potential energy : \frac{q_{1} q_{2}}{4 U r \pi}
--   joined on \epsilon
theorem Debye_length_Coulomb_potential_energy :
    forall T U e lambda_D n q_1 q_2 r : Real,
      lambda_D ^ 2 * n * e ^ 2 / (T * k_B) = q_1 * q_2 / (4 * U * r * pi) := by sorry

-- \overbrace{\underbrace{\frac{p}{m}}_{\text{Momentum}} = \underbrace{f \lambda}_{\text{Wave speed}}}^{\text{both equal the speed}}
--   Momentum : \frac{p}{m}
--   Wave speed : f \lambda
--   joined on v
theorem Momentum_Wave_speed :
    forall f lambda m p : Real,
      p / m = f * lambda := by sorry

-- \overbrace{\underbrace{\frac{p}{m}}_{\text{Momentum}} = \underbrace{H_{0} D}_{\text{Hubble's law}}}^{\text{both equal the speed}}
--   Momentum : \frac{p}{m}
--   Hubble's law : H_{0} D
--   joined on v
theorem Momentum_Hubble_s_law :
    forall D H_0 m p : Real,
      p / m = H_0 * D := by sorry

-- \overbrace{\underbrace{\frac{W}{d}}_{\text{Work done by a force}} = \underbrace{-\left( k x \right)}_{\text{Hooke's law}}}^{\text{both equal the force}}
--   Work done by a force : \frac{W}{d}
--   Hooke's law : -\left( k x \right)
--   joined on F
theorem Work_done_by_a_force_Hooke_s_law :
    forall W d k x : Real,
      W / d = Real.neg (k * x) := by sorry

-- \overbrace{\underbrace{\frac{U}{h m}}_{\text{Gravitational potential energy}} = \underbrace{\frac{G M}{r^{2}}}_{\text{Surface gravity}}}^{\text{both equal the gravitational field}}
--   Gravitational potential energy : \frac{U}{h m}
--   Surface gravity : \frac{G M}{r^{2}}
--   joined on g
theorem Gravitational_potential_energy_Surface_gravity :
    forall M U m r : Real,
      U / (h * m) = G * M / r ^ 2 := by sorry

-- \overbrace{\underbrace{\frac{U}{m g}}_{\text{Gravitational potential energy}} = \underbrace{\frac{P}{\rho g}}_{\text{Hydrostatic pressure}}}^{\text{both equal the length}}
--   Gravitational potential energy : \frac{U}{m g}
--   Hydrostatic pressure : \frac{P}{\rho g}
--   joined on h
theorem Gravitational_potential_energy_Hydrostatic_pressure :
    forall P U g m rho : Real,
      U / (m * g) = P / (rho * g) := by sorry
```

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-- \overbrace{\underbrace{m g h}_{\text{Gravitational potential energy}} = \underbrace{\frac{1}{4 \pi \epsilon_{0}} \frac{q_{1} q_{2}}{r}}_{\text{Coulomb potential energy}}}^{\text{both equal the energy}}
--   Gravitational potential energy : m g h
--   Coulomb potential energy : \frac{1}{4 \pi \epsilon_{0}} \frac{q_{1} q_{2}}{r}
--   joined on U
theorem Gravitational_potential_energy_Coulomb_potential_energy :
    forall g m q_1 q_2 r : Real,
      m * g * h = 1 / (4 * pi * epsilon_0) * (q_1 * q_2 / r) := by sorry

-- \overbrace{\underbrace{\frac{G M}{r^{2}}}_{\text{Surface gravity}} = \underbrace{\frac{P}{h \rho}}_{\text{Hydrostatic pressure}}}^{\text{both equal the gravitational field}}
--   Surface gravity : \frac{G M}{r^{2}}
--   Hydrostatic pressure : \frac{P}{h \rho}
--   joined on g
theorem Surface_gravity_Hydrostatic_pressure :
    forall M P r rho : Real,
      G * M / r ^ 2 = P / (h * rho) := by sorry

-- \overbrace{\underbrace{\frac{g r^{2}}{M}}_{\text{Surface gravity}} = \underbrace{\frac{\ell_{P}^{2} c^{3}}{\hbar}}_{\text{Planck length}}}^{\text{both equal the gravitational constant}}
--   Surface gravity : \frac{g r^{2}}{M}
--   Planck length : \frac{\ell_{P}^{2} c^{3}}{\hbar}
--   joined on G
theorem Surface_gravity_Planck_length :
    forall M ell_P g r : Real,
      g * r ^ 2 / M = ell_P ^ 2 * c ^ 3 / hbar := by sorry

-- \overbrace{\underbrace{\frac{g r^{2}}{G}}_{\text{Surface gravity}} = \underbrace{\frac{v_{e}^{2} r}{2 G}}_{\text{Escape velocity}}}^{\text{both equal the mass}}
--   Surface gravity : \frac{g r^{2}}{G}
--   Escape velocity : \frac{v_{e}^{2} r}{2 G}
--   joined on M
theorem Surface_gravity_Escape_velocity :
    forall g r v_e : Real,
      g * r ^ 2 / G = v_e ^ 2 * r / (2 * G) := by sorry

-- \overbrace{\underbrace{\frac{\omega}{2 \pi}}_{\text{Angular frequency}} = \underbrace{\frac{v}{\lambda}}_{\text{Wave speed}}}^{\text{both equal the frequency}}
--   Angular frequency : \frac{\omega}{2 \pi}
--   Wave speed : \frac{v}{\lambda}
--   joined on f
theorem Angular_frequency_Wave_speed :
    forall lambda omega v : Real,
      omega / (2 * pi) = v / lambda := by sorry

-- \overbrace{\underbrace{f \lambda}_{\text{Wave speed}} = \underbrace{H_{0} D}_{\text{Hubble's law}}}^{\text{both equal the speed}}
--   Wave speed : f \lambda
--   Hubble's law : H_{0} D
--   joined on v
theorem Wave_speed_Hubble_s_law :
    forall D H_0 f lambda : Real,
      f * lambda = H_0 * D := by sorry

-- \overbrace{\underbrace{\frac{V}{R}}_{\text{Ohm's law}} = \underbrace{\frac{P}{V}}_{\text{Electrical power}}}^{\text{both equal the current}}
--   Ohm's law : \frac{V}{R}
--   Electrical power : \frac{P}{V}
--   joined on I
theorem Ohm_s_law_Electrical_power :
    forall P R V : Real,
      V / R = P / V := by sorry

-- \overbrace{\underbrace{I R}_{\text{Ohm's law}} = \underbrace{\frac{Q}{C}}_{\text{Capacitor charge}}}^{\text{both equal the voltage}}
--   Ohm's law : I R
--   Capacitor charge : \frac{Q}{C}
--   joined on V
theorem Ohm_s_law_Capacitor_charge :
    forall C I Q R : Real,
      I * R = Q / C := by sorry

-- \overbrace{\underbrace{\frac{P}{I}}_{\text{Electrical power}} = \underbrace{\frac{Q}{C}}_{\text{Capacitor charge}}}^{\text{both equal the voltage}}
--   Electrical power : \frac{P}{I}
--   Capacitor charge : \frac{Q}{C}
--   joined on V
theorem Electrical_power_Capacitor_charge :
    forall C I P Q : Real,
      P / I = Q / C := by sorry

-- \overbrace{\underbrace{\frac{m}{V}}_{\text{Density}} = \underbrace{\frac{P}{h g}}_{\text{Hydrostatic pressure}}}^{\text{both equal the density}}
--   Density : \frac{m}{V}
--   Hydrostatic pressure : \frac{P}{h g}
--   joined on \rho
theorem Density_Hydrostatic_pressure :
    forall P V g m : Real,
      m / V = P / (h * g) := by sorry

-- \overbrace{\underbrace{\frac{h}{\lambda_{C} m}}_{\text{Compton wavelength}} = \underbrace{\left( \frac{\hbar G}{\ell_{P}^{2}} \right)^{\frac{1}{3}}}_{\text{Planck length}}}^{\text{both equal the speed of light}}
--   Compton wavelength : \frac{h}{\lambda_{C} m}
--   Planck length : \left( \frac{\hbar G}{\ell_{P}^{2}} \right)^{\frac{1}{3}}
--   joined on c
theorem Compton_wavelength_Planck_length :
    forall ell_P lambda_C m : Real,
      h / (lambda_C * m) = Real.rpow (hbar * G / ell_P ^ 2) (1 / 3) := by sorry
```

```
-- \overbrace{\underbrace{\frac{\ell_{P}^{2} c^{3}}{\hbar}}_{\text{Planck length}} = \underbrace{\frac{v_{e}^{2} r}{2 M}}_{\text{Escape velocity}}}^{\text{both equal the gravitational constant}}
--   Planck length : \frac{\ell_{P}^{2} c^{3}}{\hbar}
--   Escape velocity : \frac{v_{e}^{2} r}{2 M}
--   joined on G
theorem Planck_length_Escape_velocity :
    forall M ell_P r v_e : Real,
      ell_P ^ 2 * c ^ 3 / hbar = v_e ^ 2 * r / (2 * M) := by sorry

-- \overbrace{\underbrace{\frac{1 + \sqrt{5}}{2}}_{\text{Golden ratio}} = \underbrace{\frac{N_{f}}{N_{t}}}_{\text{Penrose tile ratio}}}^{\text{both equal the golden ratio}}
--   Golden ratio : \frac{1 + \sqrt{5}}{2}
--   Penrose tile ratio : \frac{N_{f}}{N_{t}}
--   joined on \phi
theorem Golden_ratio_Penrose_tile_ratio :
    forall N_f N_t : Real,
      (1 + Real.sqrt 5) / 2 = N_f / N_t := by sorry

-- \overbrace{\underbrace{4 + \sqrt{15}}_{\text{Spectre inflation factor}} = \underbrace{\frac{1}{g}}_{\text{Spectre reciprocal pair}}}^{\text{both equal the inflation factor}}
--   Spectre inflation factor : 4 + \sqrt{15}
--   Spectre reciprocal pair : \frac{1}{g}
--   joined on \mu
theorem Spectre_inflation_factor_Spectre_reciprocal_pair :
    forall g : Real,
      4 + Real.sqrt 15 = 1 / g := by sorry

-- \overbrace{\underbrace{4 + \sqrt{15}}_{\text{Spectre inflation factor}} = \underbrace{\frac{v_{\Gamma}}{v_{\Theta}}}_{\text{Spectre frequency ratio}}}^{\text{both equal the inflation factor}}
--   Spectre inflation factor : 4 + \sqrt{15}
--   Spectre frequency ratio : \frac{v_{\Gamma}}{v_{\Theta}}
--   joined on \mu
theorem Spectre_inflation_factor_Spectre_frequency_ratio :
    forall v_Gamma v_Theta : Real,
      4 + Real.sqrt 15 = v_Gamma / v_Theta := by sorry

-- \overbrace{\underbrace{\frac{1}{\mu}}_{\text{Spectre reciprocal pair}} = \underbrace{4 - \sqrt{15}}_{\text{Spectre triplet frequency}}}^{\text{both equal the tile frequency}}
--   Spectre reciprocal pair : \frac{1}{\mu}
--   Spectre triplet frequency : 4 - \sqrt{15}
--   joined on g
theorem Spectre_reciprocal_pair_Spectre_triplet_frequency :
    forall mu : Real,
      1 / mu = 4 - Real.sqrt 15 := by sorry

-- \overbrace{\underbrace{\frac{1}{g}}_{\text{Spectre reciprocal pair}} = \underbrace{\frac{v_{\Gamma}}{v_{\Theta}}}_{\text{Spectre frequency ratio}}}^{\text{both equal the inflation factor}}
--   Spectre reciprocal pair : \frac{1}{g}
--   Spectre frequency ratio : \frac{v_{\Gamma}}{v_{\Theta}}
--   joined on \mu
theorem Spectre_reciprocal_pair_Spectre_frequency_ratio :
    forall g v_Gamma v_Theta : Real,
      1 / g = v_Gamma / v_Theta := by sorry

-- \overbrace{\underbrace{\frac{7 + 3 \sqrt{5}}{2}}_{\text{Hat inflation factor}} = \underbrace{\phi^{4}}_{\text{Hat constant as a golden power}}}^{\text{both equal the inflation factor}}
--   Hat inflation factor : \frac{7 + 3 \sqrt{5}}{2}
--   Hat constant as a golden power : \phi^{4}
--   joined on \kappa
theorem Hat_inflation_factor_Hat_constant_as_a_golden_power :
    forall phi : Real,
      (7 + 3 * Real.sqrt 5) / 2 = phi ^ 4 := by sorry

-- \overbrace{\underbrace{\frac{7 + 3 \sqrt{5}}{2}}_{\text{Hat inflation factor}} = \underbrace{\frac{N_{u}}{N_{r}}}_{\text{Hat chirality ratio}}}^{\text{both equal the inflation factor}}
--   Hat inflation factor : \frac{7 + 3 \sqrt{5}}{2}
--   Hat chirality ratio : \frac{N_{u}}{N_{r}}
--   joined on \kappa
theorem Hat_inflation_factor_Hat_chirality_ratio :
    forall N_r N_u : Real,
      (7 + 3 * Real.sqrt 5) / 2 = N_u / N_r := by sorry

-- \overbrace{\underbrace{\phi^{4}}_{\text{Hat constant as a golden power}} = \underbrace{\frac{N_{u}}{N_{r}}}_{\text{Hat chirality ratio}}}^{\text{both equal the inflation factor}}
--   Hat constant as a golden power : \phi^{4}
--   Hat chirality ratio : \frac{N_{u}}{N_{r}}
--   joined on \kappa
theorem Hat_constant_as_a_golden_power_Hat_chirality_ratio :
    forall N_r N_u phi : Real,
      phi ^ 4 = N_u / N_r := by sorry

-- \overbrace{\underbrace{\frac{\kappa_{1} + \kappa_{2}}{2}}_{\text{Mean curvature}} = \underbrace{0}_{\text{Minimal surface condition}}}^{\text{both equal the mean curvature}}
--   Mean curvature : \frac{\kappa_{1} + \kappa_{2}}{2}
--   Minimal surface condition : 0
--   joined on H
theorem Mean_curvature_Minimal_surface_condition :
    forall kappa_1 kappa_2 : Real,
      (kappa_1 + kappa_2) / 2 = 0 := by sorry

-- \overbrace{\underbrace{\frac{1}{2 T \pi}}_{\text{String tension}} = \underbrace{\frac{J - \alpha_{0}}{M^{2}}}_{\text{Regge trajectory}}}^{\text{both equal the regge slope}}
--   String tension : \frac{1}{2 T \pi}
--   Regge trajectory : \frac{J - \alpha_{0}}{M^{2}}
--   joined on \alpha'
```

```
theorem String_tension_Regge_trajectory :
    forall J M T alpha_0 : Real,
      1 / (2 * T * pi) = (J - alpha_0) / M ^ 2 := by sorry

-- \overbrace{\underbrace{\frac{1}{2 T \pi}}_{\text{String tension}} = \underbrace{\ell_{s}^{2}}_{\text{String length}}}^{\text{both equal the regge slope}}
--   String tension : \frac{1}{2 T \pi}
--   String length : \ell_{s}^{2}
--   joined on \alpha'
theorem String_tension_String_length :
    forall T ell_s : Real,
      1 / (2 * T * pi) = ell_s ^ 2 := by sorry

-- \overbrace{\underbrace{\frac{1}{2 T \pi}}_{\text{String tension}} = \underbrace{\left( \frac{1}{4 T_{H} k_{B} \pi} \right)^{2}}_{\text{Hagedorn temperature}}}^{\text{both equal the regge slope}}
--   String tension : \frac{1}{2 T \pi}
--   Hagedorn temperature : \left( \frac{1}{4 T_{H} k_{B} \pi} \right)^{2}
--   joined on \alpha'
theorem String_tension_Hagedorn_temperature :
    forall T T_H : Real,
      1 / (2 * T * pi) = (1 / (4 * T_H * k_B * pi)) ^ 2 := by sorry

-- \overbrace{\underbrace{\frac{J - \alpha_{0}}{M^{2}}}_{\text{Regge trajectory}} = \underbrace{\ell_{s}^{2}}_{\text{String length}}}^{\text{both equal the regge slope}}
--   Regge trajectory : \frac{J - \alpha_{0}}{M^{2}}
--   String length : \ell_{s}^{2}
--   joined on \alpha'
theorem Regge_trajectory_String_length :
    forall J M alpha_0 ell_s : Real,
      (J - alpha_0) / M ^ 2 = ell_s ^ 2 := by sorry

-- \overbrace{\underbrace{\frac{J - \alpha_{0}}{M^{2}}}_{\text{Regge trajectory}} = \underbrace{\left( \frac{1}{4 T_{H} k_{B} \pi} \right)^{2}}_{\text{Hagedorn temperature}}}^{\text{both equal the regge slope}}
--   Regge trajectory : \frac{J - \alpha_{0}}{M^{2}}
--   Hagedorn temperature : \left( \frac{1}{4 T_{H} k_{B} \pi} \right)^{2}
--   joined on \alpha'
theorem Regge_trajectory_Hagedorn_temperature :
    forall J M T_H alpha_0 : Real,
      (J - alpha_0) / M ^ 2 = (1 / (4 * T_H * k_B * pi)) ^ 2 := by sorry

-- \overbrace{\underbrace{\ell_{s}^{2}}_{\text{String length}} = \underbrace{\left( \frac{1}{4 T_{H} k_{B} \pi} \right)^{2}}_{\text{Hagedorn temperature}}}^{\text{both equal the regge slope}}
--   String length : \ell_{s}^{2}
--   Hagedorn temperature : \left( \frac{1}{4 T_{H} k_{B} \pi} \right)^{2}
--   joined on \alpha'
theorem String_length_Hagedorn_temperature :
    forall T_H ell_s : Real,
      ell_s ^ 2 = (1 / (4 * T_H * k_B * pi)) ^ 2 := by sorry


end RosettaPhys
```

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