Unification by SL(4,C) Twistors

N. B. Cook

July 31, 2026

Abstract

Dr Woit's 14 July 2026 dated "Notes on Wick Rotation and Chiral Field Theories", page 11, Section 5 "Twistors and chiral conformal field theories in four dimensions" states: "Things are simplest when one complexifies, in which case the complexified four dimensional conformal group is the group Spin(6,C) = SL(4,C). Twistor space T = C4 is one of the half-spinor representations of Spin(6,C), or equivalently, the fundamental representation of SL(4,C)." If SL(4,C) is taken to be the global symmetry, its real forms SU(4), SL(4,R) and SU(2,2) offer three different ways to derive particle physics. SU(2,2) is the usual Penrose twistor, SU(4) is the basis of our recent unification paper, and SL(4,R), the split real form of the global symmetry, is Dr Wilson's unification group. It is shown that SU(4) and SL(4,R) both offer feasible alternative routes to unification, justifying the twistor unification approach and therefore getting rid of superstring theory. Details of mechanisms referred to, and prediction successes justifying them, may be found in the references. Fig. 1 is a flowchart of the basic steps.

1. Global Complex Symmetry

$$SL(4,\mathbb{C}) \cong SO(6,\mathbb{C})$$

Real forms:

$$SU(4), \qquad SL(4,\mathbb{R}), \qquad SU(2,2).$$

Spin groups:

$$Spin(6) = SU(4), \qquad Spin(3,3) = SL(4,\mathbb{R}), \qquad Spin(4,2) = SU(2,2).$$

Twistor chain:

$$SU(2,2) \subset SL(4,\mathbb{C}) \supset SL(4,\mathbb{R}),\ SU(4).$$

2. Compact Real Form: SU(4)

Maximal subgroup:

$$SU(4) \supset SU(3) \times U(1)_G.$$

Fundamental representation:

$$\mathbf{4} \to \mathbf{3}_{(+1)} \oplus \mathbf{1}_{(-3)}.$$

Generators:

$$G = \mathrm{diag}(1, 1, 1, -3), \qquad X = \mathrm{diag}(1, 1, -2, 0).$$

3. SU(3) and SU(2) Emergence

$$SU(3) \supset SU(2) \times U(1)_X$$

$$\mathbf{3} \to \mathbf{2}_{(+1)} \oplus \mathbf{1}_{(-2)}.$$

Hypercharge:

$$Y = aX + bG.$$

Electroweak group:

$$SU(2) \times U(1)_Y.$$

Electric charge:

$$Q = T_3 + \frac{Y}{2}.$$

4. Emergent SU(3) Strong Sector

Neutrino–antineutrino vacuum shells:

$$\mathcal{C}_{\nu\bar{\nu}} \cong SL(3, \mathbb{R}), \qquad \mathfrak{sl}(3, \mathbb{R}) = \{A \in M_3(\mathbb{R}) \mid \mathrm{tr}(A) = 0\}.$$

Compactification:

$$SL(3, \mathbb{R}) \longrightarrow SU(3)_{\mathrm{eff}}.$$

Gluons:

$$g^a \sim (\nu\bar{\nu})^a, \qquad a = 1, \ldots, 8.$$

5. Split Real Form: SL(4,R)

$$SL(4,\mathbb{R}) \cong Spin(3,3)$$

Vacuum polarisation defines effective metric:

$$g_{\mu\nu}^{\text{eff}} = \left.\frac{\partial^2 \Pi}{\partial p^\mu \partial p^\nu}\right|_{p=0}, \qquad g_{\mu\nu}^{\text{eff}} p^\mu p^\nu = 0.$$

Signature:

$$(+,+,+,+) \to (-,+,+,+).$$

Lorentz group:

$$SO(3,1) \subset SL(4,\mathbb{R}).$$

6. Spin Group SL(2,C) and Dirac Algebra

$$Spin(3,1) = SL(2,\mathbb{C})$$

$$\mathfrak{so}(3,1) \otimes \mathbb{C} = \mathfrak{su}(2) \oplus \mathfrak{su}(2).$$

Weyl components:

$$\psi_L \sim (2,0), \qquad \psi_R \sim (0,2).$$

Gamma matrices:

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}.$$

7. Vacuum-Polarisation Mass Operator

$$M_{\text{vac}}(x) = \langle H_{\text{vac}}(x) \rangle, \qquad M_{\text{vac}} = \begin{pmatrix} M_L & 0 \\ 0 & M_R \end{pmatrix}.$$

Corrected Dirac equation:

$$(i\gamma^\mu \partial_\mu - M_{\text{vac}})\psi = 0.$$

8. CKM Matrix from SU(4) Shell Eigenmodes

Vacuum shell eigenmodes:

$$\mathcal{S}\psi_n = \lambda_n \psi_n, \qquad \lambda_n \propto m_n.$$

Overlap integrals:

$$V_{ij} = \langle \psi_i | \psi_j \rangle.$$

CKM structure:

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1-a & a & a^2 \\ a & 1-a & a \\ a^2 & a & 1-a \end{pmatrix}.$$

9. U(1)_G Vacuum Field: Gravity and Dark Energy

Vacuum energy density:

$$\rho_G = \langle 0 | G | 0 \rangle.$$

Local curvature:

$$F_\mu = -\partial_\mu \rho_G(x).$$

Newton limit:

$$F = \frac{G m_1 m_2}{r^2}, \qquad G = \sigma_G P_G.$$

Homogeneous mode:

$$\rho_\Lambda = \rho_G^{\mathrm{hom}}.$$

10. Twistor Real Form SU(2,2)

$$SU(2,2) = Spin(4,2)$$

Twistor space:

$$\mathbb{T} = \mathbb{C}^4, \qquad PT = \mathbb{CP}^3.$$

Spacetime points:

$$\mathrm{Gr}(2,4,\mathbb{C}) \subset PT.$$

Embedding:

$$SU(2,2) \subset SL(4,\mathbb{C}) \supset SL(4,\mathbb{R}),\ SU(4).$$

11. Final Unified Structure

$$SU(4) \to SL(4,\mathbb{R}) \to SU(2,2)$$

$$SU(4) \supset SU(3) \times U(1)_G \supset SU(2) \times U(1)_Y.$$

All forces arise from vacuum polarisation within the SU(4)–SL(4,R)–SU(2,2) chain.

12. Wilson's SL(4,R) Real–Form Basis

The split real form of the global symmetry,

$$SL(4,\mathbb{R}) = Spin(3,3),$$

provides an alternative real–form decomposition parallel to the compact $SU(4)$ branch. Wilson's SL(4,R) unification begins with the identification

$$Spin(3,3) \supset SO(3,3),$$

interpreted as a six–dimensional spacetime symmetry with three spatial and three temporal directions (D. R. Lunsford, 2003).

12.1 Decomposition to the Electroweak Sector

The maximal compact subgroup of $SO(3,3)$ is

$$SO(3,3) \supset SO(4) = SU(2)_L \times SU(2)_R.$$

The left factor $SU(2)_L$ is identified with the weak isospin group. The right factor $SU(2)_R$ becomes dynamically suppressed by the infinite magnetic field self–inductance of one-way massless charge transfer, cancelling the net charge transfer term of the Yang-Mills field strength equation, which turns it into an effective Maxwell field strength equation, thus providing the mechanism for yielding an effective Abelian subgroup from SU(2):

$$SU(2)_R \longrightarrow U(1)_{\mathrm{EM}}.$$

Thus Wilson's electroweak chain is:

$$SL(4,\mathbb{R}) \;\supset\; SO(3,3) \;\supset\; SU(2)_L \times SU(2)_R \;\longrightarrow\; SU(2)_L \times U(1)_{\mathrm{EM}}.$$

12.2 Neutrino–Antineutrino Shell Geometry

Wilson proposed gauge bosons can be composed of neutrino–antineutrino pairs, so we identify the strong sector with the split real form

$$SL(3,\mathbb{R}),$$

arising from neutrino–antineutrino composite shells. The Lie algebra is

$$\mathfrak{sl}(3,\mathbb{R}) = \{A \in M_3(\mathbb{R}) \mid \mathrm{tr}(A) = 0\}.$$

Compactification of the split form yields the effective colour group:

$$SL(3,\mathbb{R}) \longrightarrow SU(3)_{\mathrm{eff}},$$

with gluons represented as composite excitations

$$g^a \sim (\nu\bar{\nu})^a, \qquad a = 1, \ldots, 8.$$

12.3 U(1)_G Vacuum Field

The Abelian factor associated with the diagonal generator of $SL(4,\mathbb{R})$ is identified with the vacuum–energy field:

$$U(1)_G,$$

producing both gravitational attraction (reaction flux) and cosmological repulsion (homogeneous mode). The Newtonian limit arises from

$$F = \frac{Gm_1m_2}{r^2}, \qquad G = \sigma_G\, \rho_G,$$

where ρ_G is the vacuum energy density.

12.4 Summary of Wilson's Real–Form Chain

Wilson's SL(4,R) pathway is therefore:

$$SL(4,\mathbb{R}) \ \supset \ SO(3,3) \ \supset \ SU(2)_L \times SU(2)_R \ \longrightarrow \ SU(2)_L \times U(1)_{\mathrm{EM}}$$

$$SL(4,\mathbb{R}) \ \supset \ SL(3,\mathbb{R}) \ \longrightarrow \ SU(3)_{\mathrm{eff}}$$

$$SL(4,\mathbb{R}) \ \supset \ U(1)_G.$$

This real–form chain is algebraically parallel to the compact SU(4) decomposition:

$$SU(4) \supset SU(3) \times U(1)_G \supset SU(2) \times U(1)_Y,$$

and both arise as real forms of the global symmetry

$$SL(4,\mathbb{C}).$$

SU(4) and SL(4,R) real form chains of the global twistor symmetry produce parallel unification routes. Figure 1 summarises both chains side-by-side and shows that the compact and split real forms produce structurally parallel decompositions:

$$\begin{aligned} SU(4) &\longrightarrow SU(3) \times U(1)_G \longrightarrow SU(2) \times U(1)_Y, \\ SL(4,R) &\longrightarrow SO(3,3) \longrightarrow SU(2)_L \times U(1)_{\mathrm{EM}}. \end{aligned}$$

Both yield the approximation of the Standard Model gauge structure (while additionally predicting dark energy, gravitation, mixing angles, couplings and masses),

$$SU(3) \times SU(2) \times U(1),$$

and both arise as real forms of the global twistor symmetry

$$SL(4,\mathbb{C}) = \mathrm{Spin}(6,\mathbb{C}).$$

References

Cook, N. B., Derivation of the Correct Chiral Dirac Equation for Leptons and Quarks from Su(4), Motivated by Critical Discussion with Robert a Wilson, 2026. https://vixra.org/pdf/2607.0118v1.pdf

Cook, N. B., Understanding Euclidean Quantum Mechanics and the Path Integral of Quantum Force Fields, 2026 https://vixra.org/pdf/2607.0096v1.pdf

Cook, N. B., Lecture Notes for Teaching Mechanistic Quantum Field Theory Axiomatically, 2026 https://vixra.org/pdf/2606.0090v1.pdf

Cook, N. B., The Final Theory, viXra:2606.0012, 2026. https://vixra.org/pdf/2606.0012v2.pdf

Cook, N. B., U(1) X SU(2) X SU(3) Quantum Gravity Successes, 2011. https://vixra.org/pdf/1111.0111v1.pdf

Cook, N. B., Einstein's Rank-2 Tensor Compression of Maxwell's Equations Does not Turn Them Into Rank-2 Spacetime Curvature, 2013. https://vixra.org/pdf/1301.0187v1.

Cook, N. B., The Quantum Gravity Lagrangian, 2013. https://vixra.org/pdf/1301.0188v1.pdf

Cook, N. B., Understanding Confirmed Predictions in Quantum Gravity, 2013. https://vixra.org/pdf/1302.0004v2.pdf

Cook, N. B., An Improved Diagram for Explaining Quantum Gravity, 2013. https://vixra.org/pdf/1304.0175v1.pdf

Cook, N. B., Quantum Gravity is a Result of U(1) Repulsive Dark Energy, 2013. https://vixra.org/pdf/1305.0012v2.pdf

Cook, N. B., Planck Data and Correction of Frampton's Repulsive Gravity, 2013. https://vixra.org/pdf/1305.0018v1.pdf

Cook, N. B., Massless electroweak field propagator predicts mass gap, 2014, https://vixra.org/pdf/1408.0151v1.pdf

Cook, N. B., Derivation of G Using Quantum Gravity, 2026. https://vixra.org/pdf/2511.0122v2.pdf

Lunsford, D. R., Gravitation and Electrodynamics Over So(3,3), 2003. https://cds.cern.ch/record/688763/files/ext-2003-090.pdf

Wilson, R. A., Hidden Assumptions: Weyl Groups, 2026. https://robwilson1.wordpress.com/2026/0 groups/

Woit, P., Notes on Wick Rotation and Chiral Field Theories, Preliminary Version July 14, 2026 https://www.math.columbia.edu/ woit/twistorunification/chiralwick-sketch.pdf

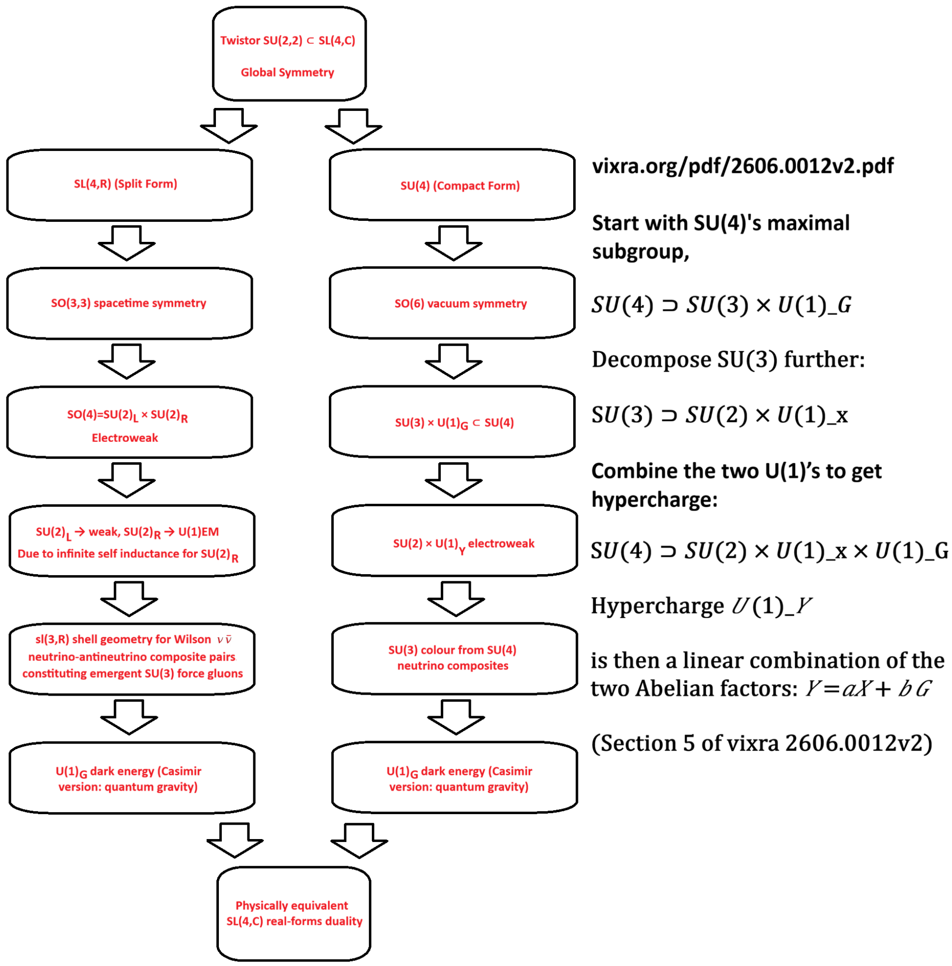

Figure 1: Twistor unification flowchart