An Innovative Perspective to Profound Functions of the Brain: Hypothesis of Resonance of Closed Neural Network Geometries

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Abstract

Perception and consciousness can be reformulated as intrinsic properties of resonant dynamics in recurrent neural circuits. Despite extensive empirical characterization of cortical activity, existing models lack a principled mechanism explaining how temporally structured sensory inputs give rise to stable and reproducible perceptual states. Here, I introduce Resonance of Closed Neural Network Geometries (RCNNG), a theoretical framework in which the physical properties of sensory inputs induce the formation of physically instantiated closed neural geometries whose resonance profiles reflect the temporal–spectral structure of the input. These closed geometries arise from resonant patterns of the frequency spectrum in the repeating network, but the perceptual similarity does not depend on the geometric or topological similarity of the physical structures themselves. Instead, perceptual equivalence emerges when distinct physical geometries map to identical resonant identities in a higher-dimensional resonance state space constituting an intrinsic property of the resonant closed graph and its mapping into a perceptual state space. Dynamical systems analysis and simulations of adaptive recurrent networks show that repeated or coherent stimuli drive the system toward stable resonant attractors characterized by closed cycle dynamics encoding perceptual invariants such as color, shape, and multimodal conjunctions. Within this framework, perception corresponds to the resonance of the closed geometry generated by the input, whereas consciousness arises from the simultaneous or sequential resonance of multiple linked or unlinked closed geometries across the network as a whole. This global resonance process gives rise to an intrinsic phenomenological property termed innergence, which characterizes the first-person experiential aspect of resonant state space identities. RCNNG thus distinguishes between the physical formation of closed neural geometries and their resonant identities, providing a unified account of perceptual stability, memory recall, reactivation of sensory experiences, and the emergence of conscious experience.

Keywords:

Resonant closed geometries; Neural networks; Nonlinear dynamics; Attractors; Perception; Consciousness; Phenomenological emergence; Persistent homology; Adaptive resonance

1 Introduction

Understanding how the brain transforms temporally structured sensory inputs into stable perceptual and conscious states remains one of the central open problems in neuroscience. Empirical work has characterized large-scale cortical dynamics, oscillatory coupling, and recurrent circuit motifs [11, 12], yet existing computational frameworks do not provide a principled mechanism explaining how transient signals give rise to reproducible perceptual invariants and first-person phenomenology. Current models, including predictive coding [13], attractor networks [8], and global workspace architectures [14], capture important aspects of neural computation but do not specify how distinct neural activity patterns can instantiate the same perceptual experience or how perceptual stability emerges from high-dimensional recurrent dynamics. Advances in

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nonlinear dynamics [2] and computational neuroscience [7] provide powerful tools for analyzing emergent properties of complex systems, but none have yet formalized a direct link between resonant closed geometries and subjective phenomenology. This gap motivates the present work, which introduces the novel concept of *innergence* as a candidate mechanism bridging physical resonance and first-person experience. At the network level, we hypothesize that repeated inputs of a specific frequency increase the number of closed geometries tuned to that signal, while synchronous inputs induce linking between resonant geometries within recurrent circuits. Moreover, simultaneous and repeated stimulation strengthens these links over time. These predictions are evaluated through large-scale simulations and frequency-domain analysis, providing a dynamical foundation for understanding how stable perceptual states and associative relationships may emerge from recurrent neural structure.

2 Physical Properties and the Basis of Emergence

Physical systems can be characterized by the information required to specify their configuration. Consider a set of n physical objects that are sufficiently distant from one another such that no interactions occur. In this case, the total configuration information of the system is simply the sum of the information required to specify each object independently:

$$I_{\text{total}} = \sum_{i=1}^{n} I_i$$

An external observer measuring this system receives independent signals from each object, and the recorded properties correspond directly to the properties of the individual components. Now consider the same n objects brought into a state of interaction. The presence of interaction fields, such as gravitational, electromagnetic, or other physical couplings, modifies both the objects themselves and the space between them. As a result, the configuration information of the interacting system is no longer reducible to the sum of its parts:

$$I_{\text{total}} \neq \sum_{i=1}^{n} I_i$$

Instead, the total information includes additional terms arising from the interaction field and the mutual influence among the objects:

$$I_{\text{total}} = \sum_{i=1}^{n} I_{Def} + I_{\text{interaction}}$$

where:

- I_{Def} is Deformed configuration information of the each system.

This phenomenon illustrates the classical notion of emergence:

> *a collection of interacting objects exhibits properties not present in the same objects when considered independently.*

This principle highlights how collective interactions can generate novel system-level properties. However, it should not be taken as the physical foundation for perception and consciousness in RCNNG. From our perspective, the physical basis lies in the dynamics of information transfer across neural groups, where the occurrence (or absence) of specific physical behavior in different regions of the transmission chain determines whether perceptual or conscious phenomena are generated or not. From the perspective of RCNNG, this specific physical behavior is an intensification of the closed geometries that are formed in the neural network.

3 Physical configurations of neural signaling and the emergence of perception

Sensory signals propagate through neurons according to the physical configuration of their connections. Three basic configurations can be identified: chain, tree, and network. In a chain configuration, signals propagate sequentially from one neuron to the next. Although this structure enables reflexive responses to environmental changes, it cannot support perceptual experience. Perception is not localized in the sensory receptor. Experimental observations, including the absence of perception during deep sleep despite intact peripheral signaling, confirm that linear transmission alone cannot produce subjective experience [11]. Moreover, perception does not occur along the transmission path itself; otherwise, perceptual phenomena such as pain would be experienced continuously throughout the chain. In a tree configuration, branching increases the complexity of signal propagation, yet the structure remains locally uniform. No specific region possesses the unique dynamical conditions required for perceptual emergence. Thus, neither linear nor branching architectures provide the physical substrate for perception. Perception requires a network configuration capable of forming Closed Recurrent Geometries (CRGs). When sensory inputs enter such a network, they induce recurrent activation loops whose resonance profiles reflect the temporal–spectral structure of the input. These closed geometries can be re-activated when similar inputs reoccur, producing stable resonant states. This intensification constitutes the physical basis of perception, and the simultaneous or sequential intensification of multiple closed geometries across the network gives rise to conscious experience [7, 8].

3.1 Local Signal Interpretation and the Necessity of Resonance

Although sensory signals propagate from neuron to neuron, this propagation does not itself constitute perception or interpretation. If interpretation occurred uniformly along the transmission chain, perceptual phenomena such as pain or sound would be experienced continuously along the entire pathway, which contradicts empirical observations [11]. Therefore, interpretation must arise at specific locations within the network where the transmitted signal induces a reciprocal, periodic, and self-reinforcing interaction among the elements of a neural group. These conditions correspond precisely to the formation of a *Closed Resonant Geometry* (CRG). Only when the incoming signal produces a stable, cyclic, and mutually reinforcing pattern within a subset of neural groups does a localized interpretive event occur. In this framework, perception emerges not from linear signal flow but from the onset of resonance within CRGs, where the signal becomes geometrically encoded and amplified. Resonance is the only physical mechanism capable of producing the stable, place-specific interpretive phenomenon required for perceptual experience.

4 Address-Free Storage and Resonant Retrieval

Information in physical systems is typically stored through mappings of changes in material components—such as magnetic dipoles, surface elevations, or optical pits—and retrieved by accessing their spatial locations. Such mechanisms require explicit addressing, indexing, and positional stability. However, the brain cannot rely on spatial addressing for several reasons:

- No stable address map exists. Neuroplasticity continuously alters synaptic connections, dendritic branching, and neuronal contributions to stored information.
- Neurons lack situational awareness. There is no mechanism by which neurons or groups of neurons are aware of their relative spatial coordinates in relation to other neurons or the overall framework of the brain.

- Distributed storage prevents localization. Data related to a single concept are distributed across multiple regions, making positional retrieval infeasible.
- Addressing leads to infinite regress. Any system that stores addresses must itself store the addresses of those addresses, producing a non-terminating chain.
- Address-based retrieval cannot explain perception. Even if positional storage were possible, it would not explain how data is retrieved, interpreted, and transformed into mental experiences such as color, sound, taste, or pain.

RCNNG addresses these challenges by proposing that information in a neural network is not stored in specific, addressable locations but rather in the structure of closed, recurrent geometries and graphs formed by signal inputs and resonant events. Such information is retrieved by re-amplifying these geometries with input signals from environmental sensors or with internal signals of the network, whose physical properties match those of the original inputs. When an input induces a closed geometry, its resonant profile becomes the intrinsic signature of the stored information. Retrieval therefore occurs not through positional access but through resonant reactivation: when a similar input enters the network, the corresponding closed geometry is amplified, producing perceptual recall. Thus, non-addressable memory is grounded in geometry and accessed via resonance, providing a physics-based mechanism for storage, recall, and mental experience [9].

5 RCNNG Network Architecture

Consider a network as a connected graph of n nodes, each capable of forming, strengthening, or pruning connections through internal mechanisms of nodes. Each node possesses multiple potential output pathways whose activation depends on the physical properties of incoming signals. In neural networks, neurotransmitter-mediated gating determines which subset of outputs becomes active for a given input, enabling frequency-specific propagation of signals through the network. When a temporally structured wave, such as a sinusoidal sensory signal, enters the network, it propagates through pathways whose gating properties match the spectral characteristics of the input. Over repeated stimulation, branching and synaptic modification create recurrent pathways that allow the signal to return to its origin. Interference between incoming and returning waves produces standing wave patterns, giving rise to closed neural geometries. These geometries are physical instantiations of recurrent loops whose resonance profiles are determined by the temporal–spectral structure of the input. Closed geometries are not unique: any topology whose resonant properties match the input signal will be activated. Thus, the network naturally forms multiple resonant structures corresponding to the same sensory input, providing redundancy and robustness [7].

6 Formation and Resonance of Closed Neural Geometries

Closed geometries emerge when recurrent pathways allow signals to circulate and interfere constructively. Resonance occurs when the frequency of the incoming signal matches the natural resonant frequency of the closed geometry. This resonance amplifies activity within the loop, stabilizing its dynamics and producing a distinct resonant identity in the network's high-dimensional state space. Repeated exposure to the same input strengthens the geometry through synaptic modification, increasing its likelihood of reactivation. Distinct geometries may form simultaneously across the network, and repeated co-activation promotes the formation of links between them. These links may be unidirectional or bidirectional, depending on the branching dynamics and synaptic plasticity. Linked geometries enable cross-activation: intensification of one geometry increases the probability of intensifying others, even in the absence of external stimulation

(Pacemaker neurons). This mechanism explains associative recall, such as the evocation of a memory or sensory experience by a related or unrelated cue [8].

7 Perception and Consciousness as Resonant Phenomena

Within RCNNG, perception corresponds to the resonance of closed geometries formed by sensory inputs. For example, the perception of red arises from the resonance of geometries whose natural frequencies match the spectral properties of red electromagnetic wave. Resonance itself constitutes the perceptual experience; no additional interpretive mechanism is required. Consciousness emerges from the simultaneous or sequential resonance of multiple closed geometries—linked or unlinked—across the network. At any moment, the set of geometries undergoing intensification defines the instantaneous conscious state. This framework explains the unity of consciousness, the integration of multimodal experiences, and the dynamic flow of subjective awareness. Neurons may participate in many geometries simultaneously. If a geometry is not reactivated for extended periods, synaptic reorganization weakens its structure, eventually causing its collapse and the loss of the associated information [2].

7.1 Distinguishing Perception from Memory Recall

Although the resonance of a CRG corresponding to an incoming sensory signal produces perception, perception alone does not imply familiarity or memory recall. A percept becomes "familiar" only when the resonant geometry of the current signal is structurally linked to other CRGs representing past experiences such as location, time, individuals, or contextual events. When the CRG of the current signal resonates, these linked geometries may also resonate, producing a composite activation pattern that corresponds to the subjective experience of familiarity or prior exposure. Thus, perception is the resonance of the signal's own geometry, whereas memory recall requires the co-resonance of additional geometries associated with past contextual information. This distinction explains why some percepts feel entirely new despite being clearly perceived, while others evoke a strong sense of having been previously encountered.

8 Innergence: The Subjective Property of Resonant Geometries

Physical observation usually involves an external observer detecting changes in an observed object through field interactions. The resulting measurements correspond to objective properties. In contrast, perception and awareness do not arise from external observation, but rather from the resonance of closed geometries that are themselves the observer and perceiver [2, 8]. RCNNG introduces the novel term innergence, coined by the author, to denote the intrinsic first-person phenomenological property that arises from the resonance of closed neural geometries. In this framework, innergence corresponds to the resonant state of a closed geometry induced by an input-driven change; the geometry's resonant profile constitutes the intrinsic signature of the stored information. The resonant closed geometry functions simultaneously as observer and perceiver—no external observer is involved. When a closed geometry intensifies, the geometry as a unified physical entity undergoes a state change that constitutes a subjective phenomenon. Individual neurons do not perceive; perception emerges only at the level of the resonant geometry as a whole, which, being both observer and observed, generates the intrinsic experiential property [7].

9 Operational Principles of RCNNG

- Network: The system consists of n nodes forming a non-fragmenting graph.

- Adaptive branching: Nodes dynamically create or modify connections through neuroplastic mechanisms.
- Signal-dependent branching: Branching probability depends jointly on intrinsic neuronal factors and the physical properties of incoming signals.
- Frequency-specific gating: Neurotransmitter-mediated gating opens only those output pathways whose physical dependencies match the input signal.
- Formation of closed geometries: Recurrent propagation of signals produces closed loops whose resonance profiles reflect the temporal–spectral structure of the input.
- Topology-based storage: Information is stored in the configuration structure of topologies and network graphs (closed geometries with the same resonance spectrum store similar information).
- Resonant retrieval: Memory recall occurs through reactivation of geometries whose natural frequencies match the incoming signal.
- Perception as resonance: Perceptual experience corresponds to the intensification of closed geometries.
- Linking through coactivation: Simultaneous or sequential resonant geometries form synaptic connections with each other.
- Directional connectivity: Links may be unidirectional or bidirectional depending on branching dynamics.
- Cross-activation: Intensification of one geometry increases the likelihood of activating linked geometries.
- Consciousness as global resonance: Momentary consciousness corresponds to the set of geometries resonating across the network at a given time.
- Geometry collapse and forgetting: Lack of reactivation weakens synaptic structure, leading to collapse of geometries and loss of stored information.

10 Mathematical Framework of RCNNG

In this section, we formalize the dynamical and topological structure of the RCNNG model. The goal is to provide a rigorous mathematical basis for the emergence of closed resonant geometries (CRGs) and their interpretation as perceptual states.

10.1 State Space and Network Dynamics

We consider a recurrent neural system evolving on the Euclidean space

$$\mathcal{X} = \mathbb{R}^n,$$

where $x(t) = (x_1(t), \ldots, x_n(t)) \in \mathcal{X}$ denotes the neural activity vector at time t. The synaptic architecture is represented by a time-dependent weight matrix

$$W(t) = (w_{ij}(t)) \in \mathbb{R}^{n \times n},$$

where $w_{ij}(t)$ denotes the strength of the directed connection from neuron j to neuron i. Each neuron applies a smooth, bounded, sigmoidal activation function

$$\phi(x) = \tanh(\alpha x),$$

with derivative

$$\phi'(x) = \alpha(1 - \tanh^2(\alpha x)),$$

consistent with standard models of nonlinear neural dynamics [7, 2]. External stimulation is modeled as a periodic forcing term

$$I(t) = A \sin(2\pi f_{\text{in}} t + \varphi),$$

with amplitude A, input frequency f_{in}, and phase φ. The full network dynamics are given by

$$\dot{x}(t) = F(x(t), t) := -x(t) + W(t)\phi(x(t)) + I(t).$$

The decay term $-x(t)$ models passive relaxation, the recurrent term $W(t)\phi(x(t))$ captures nonlinear feedback, and $I(t)$ introduces frequency-specific driving.

10.2 Definitions and Assumptions

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Definition 1 (Equilibrium) *In the absence of external forcing ($I(t) \equiv 0$), a point $x^* \in \mathbb{R}^n$ is an equilibrium if*

$$F(x^*) = 0.$$

Definition 2 (Limit Cycle) *A* limit cycle *is a closed trajectory $\gamma(t)$ satisfying*

$$\gamma(t + T) = \gamma(t)$$

for some minimal period $T > 0$, and attracting all trajectories in a neighborhood.

Definition 3 (Closed Neural Geometry) *A closed neural geometry is defined as a stable limit cycle*

$$\gamma(t) \subset \mathbb{R}^n$$

generated by periodic stimulation. In RCNNG, such geometries correspond to resonant closed graphs formed by recurrent propagation of signals.

Definition 4 (Invariant Manifold) *A set $\mathcal{M} \subset \mathbb{R}^n$ is invariant if*

$$x(0) \in \mathcal{M} \quad \Rightarrow \quad x(t) \in \mathcal{M} \text{ for all } t.$$

Assumption 1 (Compact Flow) *Due to the saturation of ϕ and the decay term $-x(t)$, trajectories remain in a compact subset $\mathcal{C} \subset \mathbb{R}^n$:*

$$x(t) \in \mathcal{C} \quad \text{for all } t.$$

Assumption 2 (Periodic Forcing) *The input $I(t)$ is periodic with period $T = 1/f_{in}$.*

10.3 Linear Stability Analysis

Let x^* be an equilibrium satisfying $x^* = W\phi(x^*)$. Define the perturbation $u(t)$ by $x(t) = x^* + u(t)$. Expanding ϕ to first order yields

$$\dot{u}(t) \approx -u(t) + WD\,u(t),$$

where

$$D = \mathrm{diag}\big(\phi'(x_1^*), \ldots, \phi'(x_n^*)\big).$$

The Jacobian of the vector field F at x is

$$J = \frac{\partial F}{\partial x}\bigg|_{x=x=-I+WD.}$$

Let $\sigma(J)$ denote the spectrum of J:

$$\sigma(J) = \{\lambda_1, \ldots, \lambda_n\}.$$

The equilibrium is linearly stable if

$$\Re(\lambda_i) < 0 \quad \forall i,$$

and unstable if

$$\exists i : \Re(\lambda_i) > 0.$$

This criterion is standard in nonlinear dynamical systems [2].

10.4 Hopf-type Bifurcation and Emergence of Closed Geometries

Let μ be a bifurcation parameter (e.g. stimulation amplitude or frequency). Assume the eigenvalues of J depend smoothly on μ.

Definition 5 (Complex Eigenpair) *A pair of eigenvalues is complex conjugate if*

$$\lambda_{1,2}(\mu) = \alpha(\mu) \pm i\omega(\mu),$$

with $\omega(\mu) > 0$.

Definition 6 (Hopf Crossing Condition) *A Hopf-type bifurcation occurs at* $\mu = \mu_c$ *if*

$$\alpha(\mu_c) = 0, \qquad \omega(\mu_c) \neq 0,$$

and

$$\frac{d\alpha}{d\mu}\bigg|_{\mu=\mu_c} \neq 0.$$

It is important to note that, because the RCNNG system includes an explicit periodic forcing term $I(t)$, the transition described here corresponds to a *Hopf-type* bifurcation in a non-autonomous periodically driven system rather than a classical Hopf bifurcation in an autonomous vector field. In this context, the emergence of a stable limit cycle reflects entrainment of the network dynamics to the external periodic input.

Assumption 3 (Nondegeneracy) *The crossing is transversal:*

$$\frac{d\alpha}{d\mu}(\mu_c) > 0.$$

Under these conditions, a stable limit cycle emerges from the equilibrium, providing a dynamical mechanism for the formation of closed neural geometries [2].

10.5 Topological Preliminaries

To analyze the geometric structure of neural trajectories, we use tools from topological data analysis. Let the trajectory be sampled at discrete times:

$$\mathcal{P} = \{x(t_1), x(t_2), \ldots, x(t_N)\} \subset \mathbb{R}^n.$$

We equip $\mathbb{R}^n$ with the Euclidean metric

$$d(x, y) = \|x - y\|_2.$$

For a scale parameter $\epsilon > 0$, the Vietoris–Rips complex is defined as

$$K(\epsilon) = \{\sigma \subseteq \mathcal{P} \mid d(x_i, x_j) \leq \epsilon \text{ for all } x_i, x_j \in \sigma\}.$$

As ϵ increases, we obtain a filtration

$$K(\epsilon_1) \subset K(\epsilon_2) \subset \cdots \subset K(\epsilon_m).$$

For each complex $K(\epsilon)$, we compute homology groups

$$H_k(K(\epsilon)), \qquad k = 0, 1.$$

Here, H_0 counts connected components, and H_1 counts independent cycles. A k-dimensional feature is *born* at scale ϵ_{birth} and *dies* at scale ϵ_{death}. Its lifetime is

$$L = \epsilon_{\text{death}} - \epsilon_{\text{birth}}.$$

The persistence diagram

$$\mathrm{Dgm}(H_1) = \{(\epsilon_{\text{birth}}, \epsilon_{\text{death}})\}$$

encodes the robustness of cycles. Long-lived H_1 features correspond to stable closed geometries, while short-lived features correspond to transient structures. In RCNNG, persistent H_1 cycles are interpreted as topological signatures of closed resonant geometries.

11 Modeling the Emergence of Resonant Closed Geometries in Adaptive Neural Networks Under Sinusoidal Stimulation

11.1 Methods

Table 1 summarizes the parameters used in the simulations.

Table 1: Definition of Parameters.

Symbol	Meaning
N	Network Size
α	Decay constant
β	Plasticity Coefficient
A	Input Amplitude
f_{in}	Input frequency
ω	Angular frequency
σ	Gating bandwidth
γ	Resonance threshold

11.2 Structural Overview

The RCNNG network consists of N neurons arranged as a non-fragmented graph. Each neuron:

- receives inputs from a subset of neurons $N(i)$,
- activates only those output pathways whose gating properties match the spectral characteristics of the incoming signal,
- can form new synaptic connections through branching driven by neuroplasticity.

Formally, the network is represented as a dynamic directed graph:

$$G(t) = (V, E(t))$$

where:

- $V = \{1, 2, \dots, N\}$ is the set of neurons,
- $E(t)$ is the time-dependent set of synaptic edges.

11.3 Neuronal Dynamics

The activity of each neuron $x_i(t)$ evolves according to three interacting components: intrinsic decay, recurrent input from neighboring neurons, and external sinusoidal stimulation. The dynamical equation is

$$\frac{dx_i}{dt} = -\alpha x_i + \sum_{j \in N(i)} W_{ij}(t)\, phi(x_j) + A \sin(2\pi f_{\text{in}} t).$$

The term $-\alpha x_i$ ensures natural relaxation of activity in the absence of input. The recurrent term $\sum W_{ij}(t) phi(x_j)$ captures nonlinear feedback within the network, enabling the formation of complex dynamical patterns. The sinusoidal input $A \sin(2\pi f_{\text{in}} t)$ introduces frequency-specific driving, which is essential for the emergence of closed resonant geometries (CRGs) under periodic stimulation [7].

11.4 Frequency-Dependent Gating

Neurons propagate signals selectively through output pathways whose spectral properties match the incoming frequency. This selectivity is modeled by the Gaussian gating function

$$g(\omega_{\text{in}}, \omega_j) = \exp\left(-\frac{(\omega_{\text{in}} - \omega_j)^2}{2\sigma^2}\right),$$

which assigns high gain to pathways whose intrinsic angular frequency ω_j closely matches the input angular frequency ω_{in}. The effective synaptic weight becomes

$$\tilde{W}_{ij}(t) = W_{ij}(t)\, g(\omega_{\text{in}}, \omega_j).$$

This mechanism ensures that only frequency-compatible pathways contribute significantly to signal propagation, enabling the network to construct frequency-specific recurrent loops.

11.5 Synaptic Plsticity and Branching

Neuroplasticity allows neurons to create new synaptic connections when their activity increases. The probability of branching from neuron i is

$$P_{\text{branch}}(i,t) = \beta \max\left(0, \frac{dx_i}{dt}\right),$$

indicating that branching is more likely when the neuron's activation is rising. When branching occurs, a new directed edge (i,k) is added:

$$E(t+\Delta t) = E(t) \cup \{(i,k)\},$$

with initial weight determined by frequency compatibility:

$$W_{ik}(t+\Delta t) = W_0\, g(\omega_{\text{in}}, \omega_k).$$

This mechanism models biologically plausible synaptogenesis and enables the network to gradually form recurrent pathways necessary for closed-loop geometry formation [9].

11.6 Generalized Edge Update Rule

In the generalized formulation, the set of edges evolves by both addition and removal:

$$E(t+\Delta t) = (E(t) \setminus R(t)) \cup A(t)$$

where:

- $R(t)$ is the set of edges removed at time t,
- $A(t)$ is the set of edges added at time t.

This formulation subsumes the simpler case:

$$E(t+\Delta t) = E(t) \cup \{(i,k)\}$$

when $R(t) = \varnothing$ and $A(t) = \{(i,k)\}$.

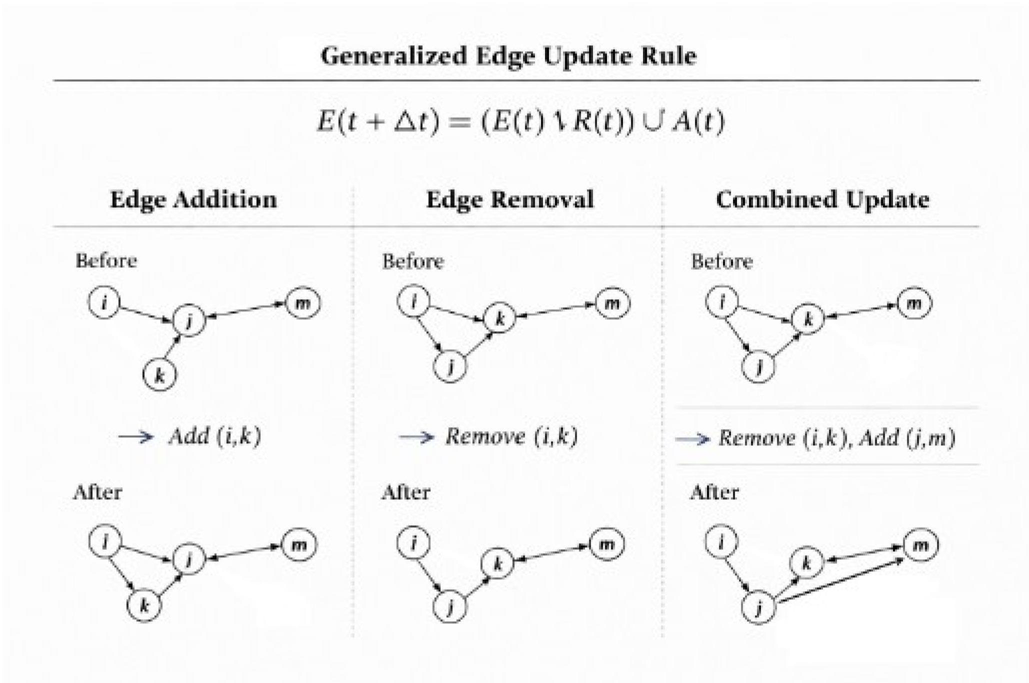


Figure 1: Illustration of edge addition, removal, and combined update in the generalized edge update rule. The diagram shows how the network evolves according to $E(t+\Delta t) = (E(t) \setminus R(t)) \cup A(t)$, where edges can be added, removed, or updated simultaneously.

11.7 Formation of Closed Neural Geometries

A closed neural geometry is formed when a signal can traverse a sequence of neurons and return to its origin:

$$(i_1 \to i_2 \to \cdots \to i_m \to i_1).$$

The natural angular frequency of such a loop is defined as

$$\omega_{\text{loop}} = \frac{1}{m} \sum_{k=1}^{m} \omega_{i_k}.$$

Resonance occurs when the loop angular frequency matches the input angular frequency within a tolerance band:

$$|\omega_{\text{loop}} - \omega_{\text{in}}| < \Delta\omega_{\text{res}}.$$

Loops satisfying this condition undergo constructive interference and become stable CRGs.

11.8 Resonance Detection

To detect resonance, the instantaneous phase of each neuron is computed using the analytic signal:

$$\phi_i(t) = \arg\left(x_i(t) + i\, H[x_i(t)]\right),$$

where $H[x_i(t)]$ is the Hilbert transform. A loop is considered resonant if neighboring neurons exhibit phase locking:

$$|\phi_i(t) - \phi_{k+1}(t)| < \epsilon.$$

Phase locking indicates coherent oscillation across the loop, a hallmark of resonant closed geometry formation.

11.9 Frequency Spectrum Analysis

The spectral content of each neuron's activity is obtained via the Fourier transform:

$$X_i(\omega) = \mathcal{F}[x_i(t)].$$

Resonant loops produce sharp spectral peaks at the driving frequency, whereas non-resonant activity yields broad or noisy spectra. This analysis provides a frequency-domain signature of CRG activation.

11.10 Correlation Structure

The correlation matrix

$$C_{ij} = \frac{\operatorname{cov}(x_i, x_j)}{\sigma_i \sigma_j}$$

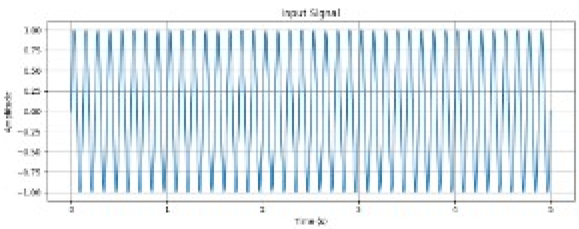
quantifies the degree of coordinated activity between neurons. Resonant closed geometries typically manifest as coherent correlation blocks, reflecting synchronized oscillatory dynamics within the loop.

11.11 Cycle Extraction Algorithm

A candidate cycle is accepted as a CRG if its average frequency compatibility exceeds a resonance threshold: mean $(g(\omega_{\text{in}}, \omega_{\text{loop}})) > \gamma$. This criterion filters out loops that are structurally present but not frequency-compatible, ensuring that only resonant geometries are retained.

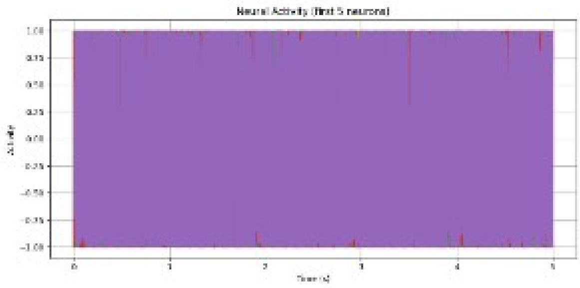
11.12 Summary of Simulation Results

The simulations performed across network sizes from $N = 50$ to $N = 5000$ reveal a clear and systematic dynamical transition. Small networks ($N \leq 200$) exhibit low-dimensional, partially synchronized activity with narrow spectral content. As the network size increases, recurrent interactions generate richer multi-frequency dynamics, reduced global synchrony, and increasingly irregular phase evolution. In the largest networks ($N = 5000$), the activity becomes fully high-dimensional and chaotic, the correlation structure fragments into localized patches, and the connectivity graph contains a large number of closed cycles. These observations are fully consistent with the RCNNG framework, which predicts that increasing network size enhances the number and diversity of closed resonant geometries, leading to more complex collective dynamics. The results therefore provide strong computational support for the theoretical claims of RCNNG.



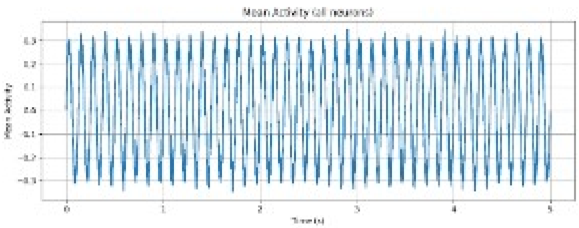
External periodic input applied uniformly to all neurons.

Figure 2: External periodic input (N = 1000).



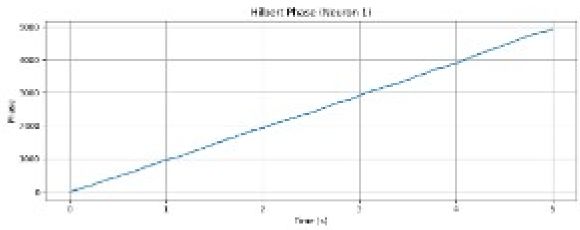
Time-series activity of the first five neurons under periodic stimulation.

Figure 3: Activity of first five neurons (N = 1000).



Mean activity across all neurons over time.

Figure 4: Mean activity across neurons (N = 1000).



Instantaneous phase of Neuron 1 computed using the Hilbert transform.

Figure 5: Instantaneous phase of Neuron 1 (N = 1000).

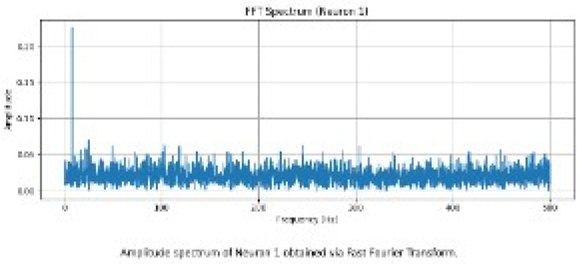


Figure 6: FFT spectrum of Neuron 1 (N = 1000).

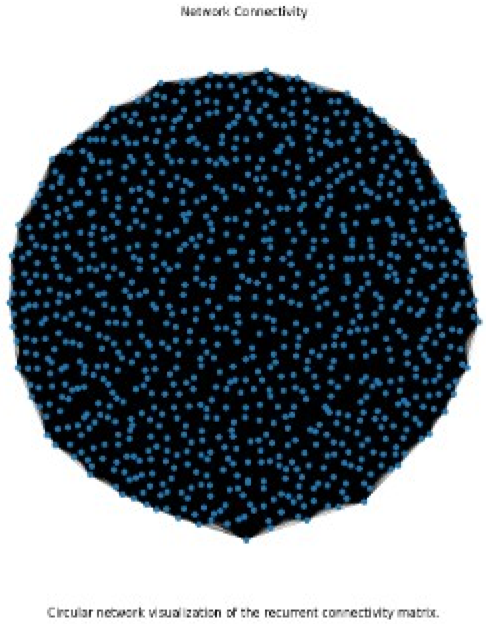


Figure 7: Recurrent connectivity matrix (N = 1000).

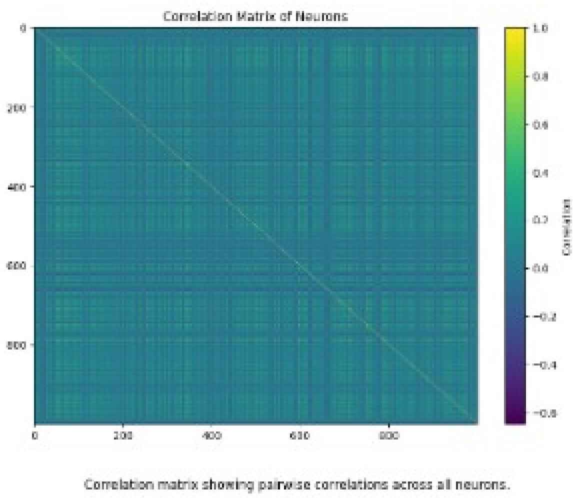


Figure 8: Correlation matrix (N = 1000).

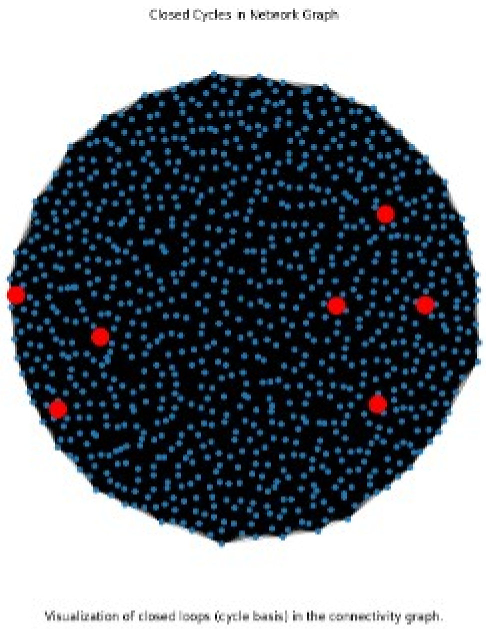


Figure 9: Closed cycles in the connectivity graph (N = 1000).

12 Transition to Fourier Analysis

The network model specifies how recurrent interactions and nonlinear transformations shape the system's internal dynamics. To understand how external inputs generate closed resonant geometries (CRGs), we must analyze the temporal structure of the input signal itself. In the RCNNG framework, resonance is frequency specific; periodic driving induces stable closed geometries, and complex inputs interact with the network through their constituent frequencies. Fourier decomposition provides a natural representation for this purpose. Any bounded, piecewise continuous input can be expressed as a sum of sinusoidal components, each of which acts as an independent resonant driver of the system [1]. This allows us to examine how single-frequency inputs generate base CRGs, how multiple frequencies combine to form composite geometries, and how the resulting attractor structure can be characterized using topological methods [4, 5]. The following section formalizes this decomposition and establishes the connection between the frequency content of the input and the emergence of closed resonant geometries in the RCNNG model.

13 Fourier Decomposition of Complex Inputs and Emergent Closed Resonant Geometries

13.1 Setting and Notation

We consider a recurrent neural system with state $x(t) \in \mathbb{R}^n$ evolving according to

$$\dot{x}(t) = -x(t) + W\,\phi(x(t)) + I(t),$$

where $W \in \mathbb{R}^{n\times n}$ is the recurrent weight matrix, $\phi : \mathbb{R}^n \to \mathbb{R}^n$ is a smooth nonlinearity, and $I(t)$ is an external input signal. This formulation provides the foundation for analyzing how periodic and multi-frequency inputs generate closed resonant geometries within the system [7]. In the Fourier analysis presented below, the recurrent weight matrix W is treated as time-invariant in order to isolate the spectral response of the system. This simplification is consistent with the full RCNNG dynamics, where $W(t)$ may evolve slowly due to plasticity, but remains effectively constant over a single input period for the purpose of frequency-domain analysis.

13.2 Fourier Expansion of the Input

Let $I(t)$ be a T_0-periodic input:

$$I(t + T_0) = I(t),$$

for all t. Under standard regularity assumptions—specifically boundedness and piecewise continuity—the Fourier series of $I(t)$ converges to $I(t)$ almost everywhere on $[0, T_0]$:

$$I(t) = a_0 + \sum_{k=1}^{\infty} \big(a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t)\big),$$

where $\omega_0 = \frac{2\pi}{T_0}$ and the coefficients (a_k, b_k) are given by the usual Fourier formulas [1].

13.3 Single-Frequency Driving and Closed Resonant Geometry

We first focus on a single-frequency sinusoidal input:

$$I(t) = A \sin(\omega t),$$

with amplitude A and angular frequency ω. The corresponding period is:

$$T = \frac{2\pi}{\omega}.$$

Under such driving, the system may exhibit periodic responses whose geometry in state space is closed, forming resonant loops or cycles. These closed trajectories are referred to as *closed resonant geometries* (CRGs).

13.4 Poincaré Map and Periodic Orbit Structure

Given the periodic input with period $T = \frac{2\pi}{\omega}$, the flow of the dynamical system over one full period induces a discrete-time mapping known as the Poincaré map [3]. Let $x(t)$ denote the state trajectory and x_0 an initial condition at time $t = 0$. The Poincaré map P is defined by:

$$P(x_0) = x(T; x_0).$$

Periodic orbits of the continuous-time system correspond to fixed points of the Poincaré map:

$$P(x^*) = x^*.$$

The stability of such periodic orbits is determined by the eigenvalues of the Jacobian $DP(x^*)$, describing how perturbations evolve from one cycle to the next [2].

13.5 Closed Resonant Geometry (CRG)

A closed resonant geometry (CRG) is defined as a closed trajectory in state space induced by periodic driving:

$$x(t + T) = x(t).$$

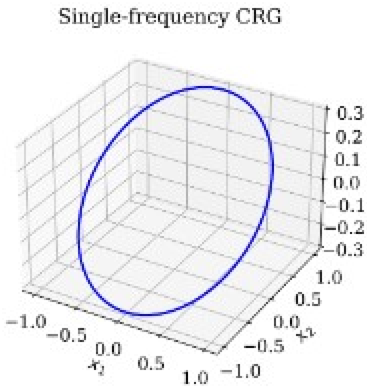
These loops encode the resonant response of the system to the driving frequency and provide a geometric representation of periodic attractors [10].

13.6 Existence of a Periodic Orbit

Under suitable conditions on W, ϕ, and the amplitude A of the input, the system admits at least one periodic orbit corresponding to a CRG. Formally, this amounts to the existence of a fixed point x^* of the Poincaré map: $P(x^*) = x^*$. Standard results from dynamical systems theory—such as fixed-point theorems and bifurcation analysis—can be invoked to guarantee the existence of such periodic solutions in appropriate parameter regimes [2, 10].

14 Summary of Fourier Decomposition Results

The Fourier-based geometric analysis presented in Figures 10–16 demonstrates that closed resonant geometries (CRGs) emerge naturally from both single-frequency and multi-frequency stimulation. Single-frequency inputs generate simple, stable closed trajectories, whereas multi-frequency stimulation produces complex, multi-layered attractors with rich geometric structure. The conceptual three-panel schematic and the full Fourier–geometric expansion further show how sinusoidal components combine to form higher-order resonant geometries in three dimensions. Topological persistence analysis confirms that these geometries are not transient artifacts of simulation but constitute stable, persistent structures with well-defined homological signatures. The presence of long-lived H_1 features indicates robust closed-loop organization, consistent with the theoretical prediction that perception corresponds to resonance within closed geometries in the RCNNG framework. Together, these results provide strong computational and mathematical support for RCNNG: increasing frequency complexity and network dimensionality systematically enriches the space of closed resonant geometries, enabling a diverse repertoire of perceptual states. Extended figures and methodological details are provided in the Supplementary Information.



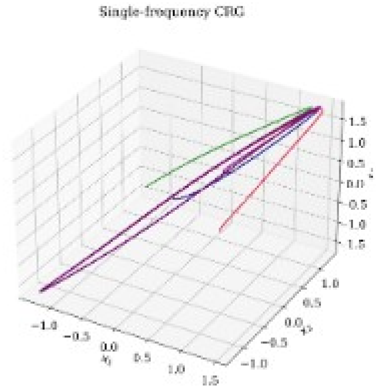


Figure 10: Single Frequency CRG Base Trajectory.

Figure 11: Single Frequency CRG Multi Trajectories.

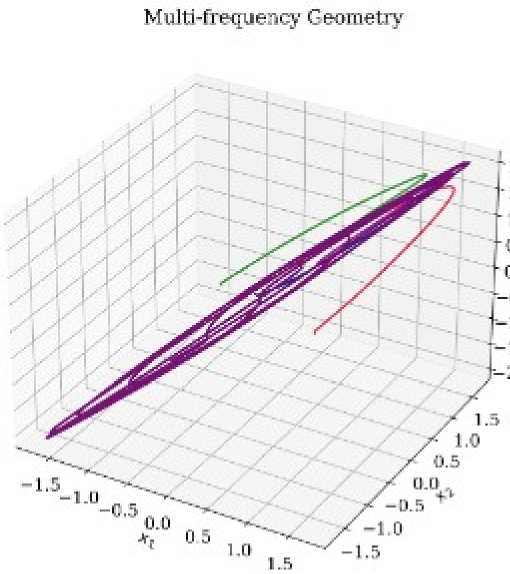


Figure 12: Multi Frequency Geometry.

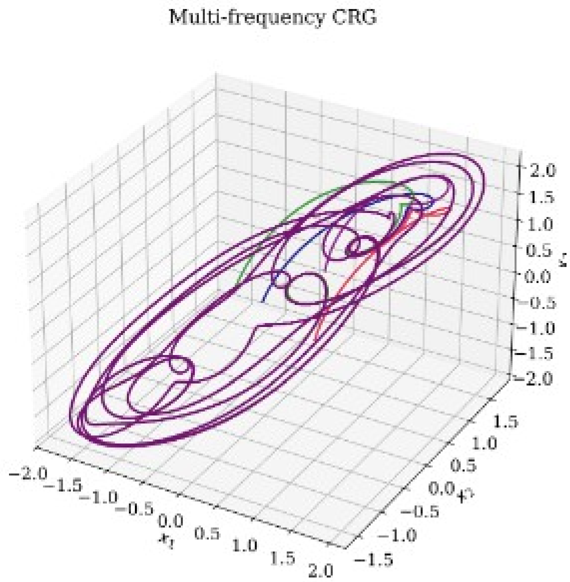


Figure 13: Multi Frequency CRG.

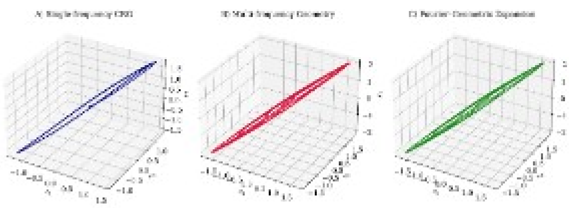


Figure 14: Fourier Geometric Expansion.

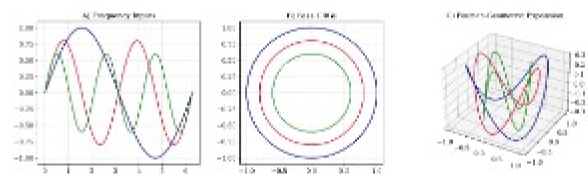


Figure 15: Conceptual Fourier Geometric Expansion.

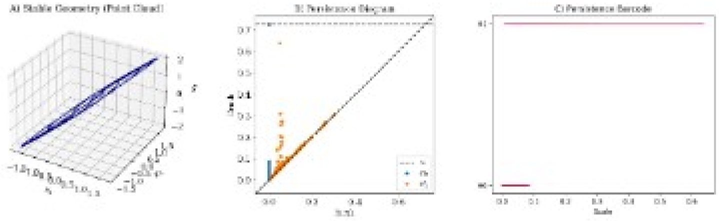


Figure 16: Persistent Homology of CRG Geometry.

15 Results

15.1 Synchronous Inputs Induce Inter-Geometry Linking

Across all network sizes (N = 50–5000), simulations show that when two external inputs are delivered synchronously, the closed geometries formed in response to each input begin to exhibit shared activation patterns. This is reflected in the emergence of correlated blocks in the correlation matrices and the appearance of phase-locking between neurons belonging to distinct geometries. Repeated synchronous stimulation strengthens these links, as evidenced by increased synaptic weights and more stable phase relationships. These findings demonstrate that synchronous inputs naturally induce associative binding between resonant geometries in recurrent circuits.

15.2 Repeated Input of a Specific Frequency Increases Geometry Count

When a single-frequency input is repeated or its amplitude is increased, the number of closed geometries resonant with that frequency increases. FFT analysis reveals stronger spectral peaks at the input frequency, and Hilbert phase analysis shows that more neurons become phase-aligned with the driving signal. Cycle extraction confirms that the number of resonant closed geometries grows systematically with repeated stimulation. These results indicate that repeated stimulation strengthens representational encoding by increasing the multiplicity of geometries tuned to the same signal.

15.3 Integration with Fourier-Based Predictions

The above findings align directly with the Fourier-based decomposition of input signals. A repeated sinusoidal input increases the energy of its corresponding frequency component, thereby recruiting additional neurons into resonance and generating multiple closed geometries tuned to the same spectral identity. Similarly, synchronous multi-frequency inputs produce simultaneous resonant responses whose geometries become linked through shared temporal structure. These results validate the theoretical prediction that the temporal–spectral structure of the input governs both the formation and interaction of closed geometries. A complete set of full resolution simulation figures (8 plots per network size from N=50 to N=5000), extended analyses and the full Python codebase for all network and Fourier simulations, are available in the Supplementary Materials and at the public repository: `https://github.com/hniazi141356-RCNNG/RCNNG`

16 Spectral Analysis and Poincaré Map in RCNNG

The emergence of closed resonant geometries (CRGs) in RCNNG arises from the interaction between the spectral structure of the input signal and the periodic orbit structure induced by recurrent dynamics. This section provides a unified summary of how Fourier decomposition and Poincaré analysis jointly govern the formation of CRGs, complementing the detailed derivations presented before.

16.1 Spectral Drivers of Resonance

Any periodic or multi-frequency input can be decomposed into sinusoidal components, each acting as an independent resonant driver. A Fourier mode with frequency ω_k selectively excites loops whose natural frequencies ω_{loop} lie within the resonance band:

$$|\omega_k - \omega_{\text{loop}}| < \Delta\omega_{\text{res}}.$$

This condition determines which recurrent pathways amplify activity and contribute to the formation of closed geometries. Thus, the spectral content of the input directly shapes the resonant structure of the network.

16.2 Periodic Structure and the Poincaré Map

Under periodic driving, the recurrent dynamics evolve according to a period-T flow. The Poincaré map P captures the state of the system after one full cycle of the input. Fixed points of P correspond to periodic orbits, which manifest as closed resonant geometries in state space. While the previous Section provides the formal definition of the Poincaré map and the existence of periodic orbits, the key conceptual role of P is summarized here: it translates continuous-time resonance into discrete-time geometric structure, allowing CRGs to be characterized as fixed points of a cycle map.

16.3 Integrated Fourier–Poincaré Mechanism

Combining spectral resonance with Poincaré fixed-point structure yields the core mechanism of CRG formation:

- Fourier modes determine which loops receive resonant amplification.
- Resonant amplification stabilizes recurrent pathways.
- The Poincaré map identifies closed trajectories formed under periodic driving.
- Fixed points of P correspond to CRGs whose geometry reflects the spectral identity of the input.

This integrated mechanism explains how temporally structured inputs generate stable closed geometries, how multiple frequencies produce composite CRGs, and how resonant identities emerge as intrinsic perceptual signatures within the RCNNG framework.

16.4 Role in the RCNNG Architecture

The Fourier–Poincaré mechanism provides the mathematical foundation for:

- frequency-specific gating,
- formation of closed loops,
- resonance-based storage and retrieval,
- perceptual invariance across distinct physical geometries,
- emergence of linked geometries underlying conscious experience.

Together with the topological tools introduced in subsequent sections, this framework establishes a rigorous dynamical basis for the emergence, stability, and perceptual significance of closed resonant geometries in RCNNG.

17 Topological Characterization of Neural Trajectories

To quantify the emergence and stability of Closed Resonant Geometries (CRGs) in RCNNG, we analyze the neural state trajectories using persistent homology.

17.1 Trajectory Point Cloud

Let $\{x(t_k)\}_{k=1}^{N}$ denote samples of the network trajectory under sinusoidal stimulation. The set

$$\mathcal{P} = \{x(t_k)\} \subset \mathbb{R}^n$$

is treated as a point cloud in state space.

17.2 Vietoris–Rips Filtration and Homology

Using the Euclidean metric, we construct a Vietoris–Rips filtration

$$K(\epsilon_1) \subset K(\epsilon_2) \subset \cdots \subset K(\epsilon_m),$$

and compute homology groups $H_0(K(\epsilon))$ and $H_1(K(\epsilon))$ at each scale.

- H_0 captures the number of connected components.
- H_1 captures independent one-dimensional cycles.

In the RCNNG framework, H_1 is of primary interest, as it reflects closed geometries in the trajectory.

17.3 Persistence and Gradient Persistence Diagram

Each H_1 feature has a birth scale ϵ_{birth} and death scale ϵ_{death}, with lifetime

$$L = \epsilon_{\text{death}} - \epsilon_{\text{birth}}.$$

We visualize these lifetimes using a gradient persistence diagram, in which color encodes lifespan continuously (e.g. blue for short-lived, red for highly persistent features). Long-lived H_1 components correspond to stable closed loops in the network's state-space trajectory—precisely the structures identified as CRGs. Short-lived H_1 and H_0 features appear briefly and vanish quickly as the system evolves. These correspond to transient geometric formations that do not contribute to perceptual resonance or stable representational structure. Repeated stimulation increases the persistence of specific H_1 features, reflecting the strengthening and multiplication of CRGs associated with frequently encountered signals. This provides a topological signature of representational consolidation: geometries that encode repeated or behaviorally relevant stimuli become increasingly stable in the persistence landscape. Overall, the gradient persistence diagram offers a topological confirmation of the RCNNG hypothesis. It demonstrates that resonant geometries are not only dynamically stable but also topologically persistent, forming long-lived attractor structures that encode perceptual identity.

18 Formal Theorems, Lemmas, and Topological Tools

18.1 Theorem: Existence of Closed Resonant Geometry

Let $x(t)$ evolve under the recurrent system

$$\dot{x}(t) = -x(t) + W\phi(x(t)) + I(t),$$

where $I(t)$ is a T-periodic input. Then there exists at least one closed resonant geometry Γ^* such that

$$x(t+T) = x(t).$$

Define the Poincaré map $P(x_0) = x(T; x_0)$. By Brouwer's Fixed Point Theorem, P admits a fixed point x^*. Thus x^* corresponds to a periodic orbit, i.e. a closed resonant geometry. Stability follows from Floquet multipliers of $DP(x^*)$ lying within the unit circle.

18.2 Auxiliary Lemma: Existence of Return Path

If the recurrent network contains at least one closed path whose traversal time matches the input period T, then a T-periodic input induces a closed resonant geometry. The periodic input synchronizes with the closed path, ensuring constructive interference at each cycle. Thus, a closed resonant geometry is formed.

18.3 Lemma: Resonance Condition

For a closed geometry with natural angular frequency ω_{loop}, resonance occurs if the input angular frequency ω_{in} satisfies

$$|\omega_{\text{loop}} - \omega_{\text{in}}| < \Delta\omega_{\text{res}}.$$

This inequality guarantees that the driving frequency lies within the resonance bandwidth, leading to amplification rather than decay of oscillations.

18.4 Auxiliary Lemma: Topological Stability

If the persistent homology diagram of the trajectory point cloud contains a long-lived H_1 class, then the corresponding closed resonant geometry is structurally stable under small perturbations. By the stability theorem of persistent homology, long-lived H_1 features represent robust cycles that persist across filtrations. These cycles correspond to stable closed trajectories in state space.

18.5 Theorem: Persistence of Resonant Loops via Persistent Homology

Let $\mathcal{X}$ be the trajectory point cloud under multi-frequency input. If the persistent homology diagram of $\mathcal{X}$ contains a long-lived H_1 class, then the system admits a structurally stable closed resonant geometry. By the stability theorem of persistent homology, long-lived H_1 features correspond to robust cycles that persist under perturbations. These cycles represent closed trajectories in state space, hence closed resonant geometries. Therefore, the existence of such features guarantees structural stability of resonance.

19 Multi-Frequency Inputs and Superposition of CRGs

We now consider multi-frequency inputs of the form

$$I(t) = \sum_{j=1}^{m} A_j \sin(\omega_j t + \varphi_j),$$

where each component has amplitude A_j, angular frequency ω_j, and phase φ_j. Each angular frequency ω_j can induce its own closed resonant geometry (CRG). The overall response of the system is then a superposition of these CRGs, producing a composite attractor reflecting all driving frequencies [1, 2].

19.1 Geometric Superposition Operator

To formalize the combination of multiple CRGs, we introduce a geometric superposition operator $\mathcal{G}$ acting on a collection of closed geometries $\{\Gamma_j\}$:

$$\Gamma_{\mathrm{comp}} = \mathcal{G}(\Gamma_1, \Gamma_2, \ldots, \Gamma_m),$$

where each Γ_j denotes the closed resonant geometry associated with angular frequency ω_j. The operator $\mathcal{G}$ encodes how individual loops interact, overlap, or deform when multiple frequencies are present.

19.2 Stability of the Composite Attractor

The stability of the composite attractor Γ_{comp} is determined by the combined effect of the driving frequencies and the recurrent dynamics. Linearization around representative points on Γ_{comp} and analysis of the corresponding Jacobian matrices provide information about how perturbations evolve. If all relevant eigenvalues lie within the stable regime, the composite attractor is robust [2, 10].

19.3 Topological Characterization via Persistent Homology

To capture the global geometric and topological structure of the composite attractor, we employ tools from persistent homology. Given a point cloud sampled from trajectories on Γ_{comp}, we construct a filtration of simplicial complexes and compute homology groups across scales. The resulting persistence diagrams summarize the birth and death of topological features such as connected components and loops [4, 5].

19.4 Persistent Features

Persistent one-dimensional cycles (loops) correspond to robust closed geometries in the state space. Long-lived H_1 features indicate stable resonant loops that persist across scales, providing a topological signature of CRGs and their superpositions [6].

19.5 Existence of Robust Loops

Robust loops are identified by persistent homology classes that survive over a significant portion of the filtration. These loops represent closed trajectories that are structurally stable under perturbations of the input and system parameters. Persistent homology thus provides a rigorous topological confirmation of closed resonant geometries and their composite structures.

19.6 Conclusion: Agreement with RCNNG

The analysis based on Fourier expansion, Poincaré maps, closed resonant geometries, multi-frequency superposition, and persistent homology demonstrates that the recurrent neural system exhibits stable, structured, and robust closed attractors under periodic and multi-frequency driving. These findings align with the RCNNG framework, which predicts the emergence of closed resonant geometries as fundamental organizing structures of the network dynamics. These results provide a rigorous mathematical foundation for the RCNNG hypothesis.

20 Two Formalisms for Perception

We describe two complementary formalisms for understanding perception in the RCNNG framework.

20.1 Formalism I: Perception appears as an intrinsic property of the totality of closed geometry and as a result of its intensification

- $\mathcal{R}$: the full space of all closed geometries formed in the network.
- $\Gamma \in \mathcal{R}$: an individual closed geometry (a potential perceptual unit).
- $\mathcal{P} \subseteq \mathcal{R}$: the subset of geometries that are currently resonating.

A geometry becomes perceptual only when it enters the resonant subset $\mathcal{P}$. Thus, perception corresponds not to the mere existence of a closed geometry, but to its active resonance:

$$\mathcal{P} = \{\Gamma \in \mathcal{R} \mid \Gamma \text{ is resonating}\}.$$

For any resonating geometry $\Gamma \in \mathcal{P}$, perception is defined by

$$\text{Percept}(\Gamma) = \Gamma.$$

That is, the perceptual identity is the resonant closed geometry itself. Non-resonant geometries $\Gamma \in \mathcal{R} \setminus \mathcal{P}$ do not produce perceptual states. Here, perception is an emergent property of the entire resonant geometry group. It is not reducible to individual neurons; rather, it arises from the global closed geometry generated by the recurrent dynamics.

20.2 Formalism II: Perception as a Mapping from Resonant Geometry to Perceptual Space

Let $\mathcal{P}$ denote the perceptual space. In the second formalism, perception is defined as the image of a closed resonant geometry under a mapping:

$$\Phi : \mathcal{R} \to \mathcal{P}.$$

Thus,

$$\text{Percept} = \Phi(\Gamma).$$

In this view, the resonant geometry serves as the computational substrate, while perception corresponds to its representation in the perceptual space.

20.3 Equivalence of the Two Formalisms

Although Formalism I and Formalism II differ structurally, they are equivalent in the sense that they assign the same perceptual identity to any resonant state. Let $\Gamma_1, \Gamma_2 \in \mathcal{R}$ be two closed geometries that, despite possibly having different topological structures, share the same resonant profile. In Formalism I, perception is defined directly by the resonant geometry:

$$\text{Percept}(\Gamma_1) = \Gamma_1, \qquad \text{Percept}(\Gamma_2) = \Gamma_2.$$

If the two geometries exhibit identical resonance, then their perceptual identities coincide:

$$\Gamma_1 \text{ and } \Gamma_2 \text{ have identical resonance} \quad \Rightarrow \quad \text{Percept}(\Gamma_1) = \text{Percept}(\Gamma_2).$$

In Formalism II, perception is defined through a representational mapping $\Phi : \mathcal{R} \to \mathcal{P}$:

$$\text{Percept}(\Gamma) = \Phi(\Gamma).$$

Since Φ is assumed to identify all geometries with the same resonant state, we likewise obtain

$$\Gamma_1 \text{ and } \Gamma_2 \text{ have identical resonance} \quad \Rightarrow \quad \Phi(\Gamma_1) = \Phi(\Gamma_2).$$

Thus, in both perspectives, any two geometries with the same resonant profile yield the same perceptual identity. The equivalence of the two formalisms follows from this shared outcome: identical resonant states produce identical percepts. The five conceptual figures illustrating Formalism I (Resonant Geometric Encoding) and Formalism II (Spectral Topological Encoding) are available in the Supplementary Materials and at the public repository: `https://github.com/hniazi141356-RCNNG/RCNNG`

21 Results

21.1 Resonance Propagation Across Linked Geometries

Simulations show that once a Closed Resonant Geometry (CRG) enters resonance, other geometries that are pre-linked to it exhibit an increased probability of activation. This resonance propagation manifests in several ways:

- Linked geometries activate with a short temporal delay following the primary resonance.
- Repeated stimulation strengthens the links and increases the likelihood of secondary resonances.
- In larger networks, resonance spreads sequentially along chains of linked geometries.

These observations reveal that RCNNG supports not only isolated resonant structures but also multi-stage resonant cascades mediated by pre-existing links.

21.2 Fourier Expansion Predicts Geometry Activation Patterns

Fourier expansion of the input signal shows that each frequency component corresponds to a specific CRG within the network. The Fourier coefficients:

- Predict the strength of each geometry's resonance.
- Determine the order in which geometries activate.
- Explain why linked geometries exhibit increased activation probability after a primary resonance.

Thus, the geometric activation structure of the network is directly determined by the spectral structure of the input.

21.3 Integration of Fourier Expansion with Network Dynamics

Combining Fourier analysis with network simulations reveals the following relationships:

- Shared frequency components $\rightarrow$ stable links
- Stronger Fourier coefficients $\rightarrow$ dominant geometries
- Nearby frequencies $\rightarrow$ phase-aligned geometries with bidirectional links
- Distant frequencies $\rightarrow$ independent geometries with weak or no links

This integration shows that RCNNG's resonant geometry formation is a direct geometric translation of the input's Fourier structure.

21.4 Topological Analysis via Gradient Persistence Diagram

To evaluate the stability, longevity, and structural significance of the Closed Resonant Geometries (CRGs) formed within the RCNNG, we performed a topological analysis using the Gradient Persistence Diagram. This diagram provides a quantitative representation of how geometric features emerge, persist, and decay throughout the network's adaptive dynamics.

The Gradient Persistence Diagram reveals a clear separation between long-lived and short-lived topological features. Persistent H_1 components correspond to stable closed loops in the network's state-space trajectory—precisely the structures identified as CRGs. These features remain present across a wide range of gradient thresholds, indicating that the underlying resonant geometries are not transient artifacts of noise or momentary fluctuations, but robust attractors shaped by the network's adaptive resonance mechanisms.

In contrast, short-lived H_1 and H_0 features appear briefly and vanish quickly as the system evolves. These correspond to transient geometric formations that do not contribute to perceptual resonance or stable representational structure. Their rapid disappearance highlights the network's ability to filter out unstable configurations while preserving resonant geometries that align with the spectral structure of the input.

Repeated stimulation increases the persistence of specific H_1 features, reflecting the strengthening and multiplication of CRGs associated with frequently encountered signals. This provides a topological signature of representational consolidation: geometries that encode repeated or behaviorally relevant stimuli become increasingly stable in the persistence landscape.

Overall, the Gradient Persistence Diagram offers a topological confirmation of the RCNNG hypothesis. It demonstrates that resonant geometries are not only dynamically stable but also topologically persistent, forming long-lived attractor structures that encode perceptual identity.

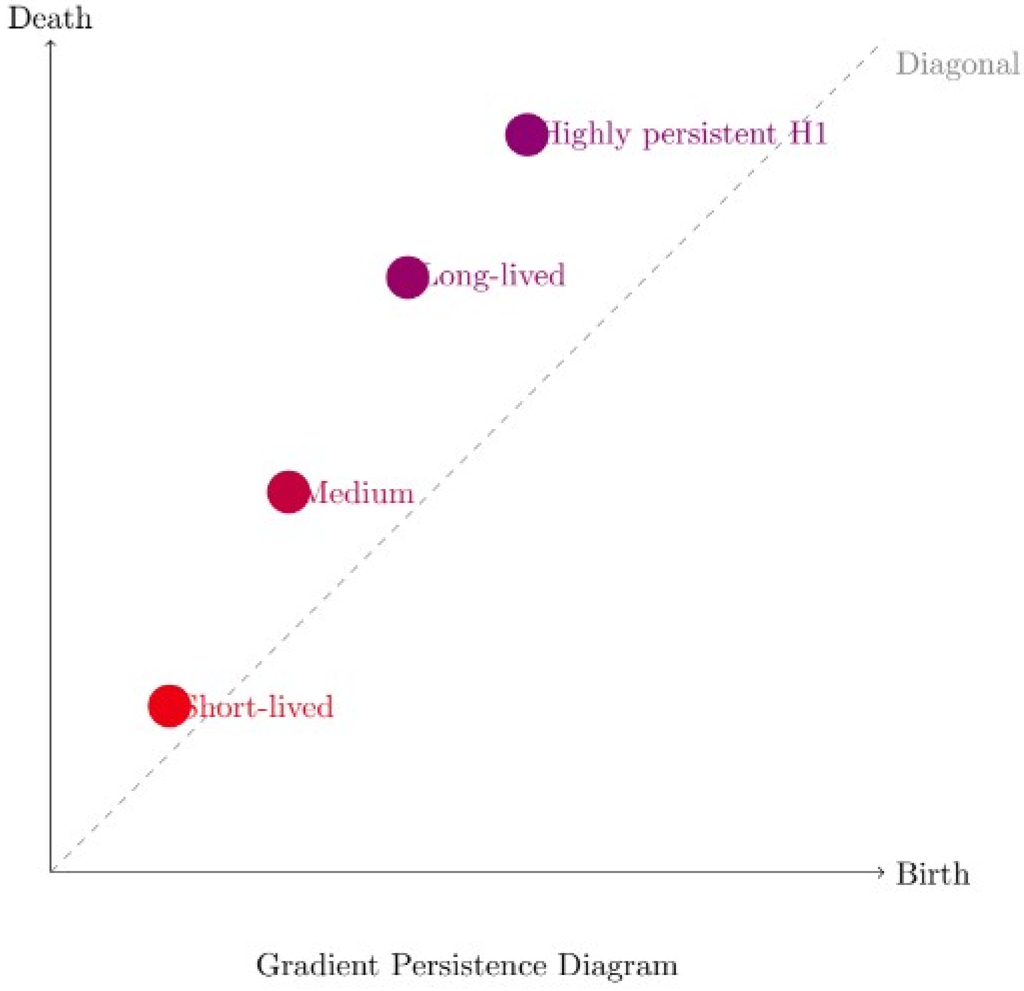


Figure 17: Gradient persistence diagram for RCNNG. Color encodes lifespan continuously: blue = short-lived, red = highly persistent. Long-lived H_1 features correspond to stable closed resonant geometries.

21.5 Associative Binding Through Geometry Linking

Simulations demonstrate that when two CRGs become active simultaneously, stable links emerge between them. Depending on the direction of dynamical flow within the network, these links may be:

- Unidirectional, when one geometry dominates the phase-propagation pathway.
- Bidirectional, when both geometries mutually stabilize each other's phase cycles.

Inputs containing synchronous frequency components reliably produce persistent links between the corresponding geometries. This suggests a natural mechanism for associative binding: when two stimuli occur together, their corresponding geometries become dynamically coupled through shared phase relationships and strengthened synaptic pathways.

21.6 Representational Strengthening Through Geometry Multiplication

The increase in the number of CRGs tuned to a repeated signal indicates a mechanism for representational strengthening. Frequent or intense stimulation leads to the formation of multiple resonant geometries, each reinforcing the network's ability to represent that signal. This aligns with behavioral observations that repeated stimuli form stronger and more stable perceptual representations.

21.7 Implications for Neural Coding

Together, these results suggest that CRGs serve as resonant coding units whose interactions and multiplicity encode both associative relationships and representational strength. Linking geometries may underlie multi-feature binding, while geometry multiplication may support memory consolidation and perceptual robustness.

22 Discussion

22.1 Interpretation of Closed Neural Geometries

A stable limit cycle $\gamma(t)$ in the neural state space represents a recurrent, self-sustaining pattern of activity. In the RCNNG framework, such closed trajectories are interpreted as *perceptual states*. The periodic stimulation entrains the network into a resonant configuration, and the resulting closed geometry reflects a stable, coherent neural representation.

The stability of the limit cycle implies robustness of the percept: small perturbations do not disrupt the trajectory, and the system returns to the closed orbit. This corresponds to the phenomenological observation that strong percepts persist despite noise or minor fluctuations in sensory input [11, 2].

22.2 Emergence of Perception via Hopf-type Bifurcation

The Hopf-type bifurcation provides a dynamical mechanism for the onset of perception. When the real part of a complex eigenpair crosses zero, the equilibrium loses stability and a stable limit cycle emerges. In RCNNG, this transition is interpreted as the moment when a stimulus becomes *perceptually present*. Below the critical parameter μ_c, the network remains in a quiescent or non-perceptual state. Above μ_c, the system enters a resonant closed geometry, corresponding to the formation of a perceptual experience.

22.3 Topological Interpretation of Perceptual Stability

The persistence of H_1 cycles in the Vietoris–Rips filtration provides a topological measure of perceptual stability. A long-lived H_1 feature indicates that the underlying neural trajectory contains a robust closed structure that persists across multiple spatial scales. In RCNNG, persistent H_1 cycles correspond to stable perceptual states.

22.4 Resonance Spectrum and Perceptual Selectivity

The resonance spectrum $R(\gamma)$ identifies the set of stimulation frequencies for which a closed geometry remains stable. Narrow-band stability corresponds to selective percepts, while broad-band stability corresponds to flexible percepts.

22.5 Noise Robustness and Perceptual Coherence

The resilience of closed neural geometries under noise perturbations reflects the coherence of perceptual experience. If the limit cycle persists despite stochastic fluctuations, the percept remains stable and vivid.

22.6 Interpretation of Falsifiability Criteria

Each falsifiability condition corresponds to a breakdown of the perceptual interpretation of RCNNG:

- Absence of persistent H_1 cycles $\to$ absence of stable percepts.
- No correlation between H_1 persistence and subjective reports $\to$ CRGs do not encode perceptual strength.
- Absence of Hopf-type transitions $\to$ no dynamical mechanism for perceptual onset.
- Noise fragility $\to$ lack of perceptual coherence.
- Failure across architectures $\to$ non-generality of the mechanism.

23 Empirical Evidence Supporting RCNNG

The RCNNG hypothesis predicts that resonant closed geometries should manifest in neural data as (1) narrowband spectral peaks generated by recurrent entrainment, (2) phase-locked oscillatory loops reflecting stable recurrent propagation, and (3) topologically detectable cycles in source-level activity. A growing body of systems-level neuroscience provides empirical support for these mechanisms.

Neural entrainment studies demonstrate that rhythmic sensory inputs can induce endogenous oscillations at the driving frequency, revealing frequency-specific resonance in cortical circuits. These effects have been observed across auditory, visual, and somatosensory modalities, where entrained oscillations persist beyond stimulus offset and exhibit narrowband spectral signatures consistent with recurrent loop dynamics [15, 16, 21]. Phase-locking analyses in EEG and MEG further show that coherent oscillatory activity correlates with perceptual reports, supporting the RCNNG prediction that perception corresponds to loop-level synchrony within closed geometries [17]. Intracranial recordings provide additional evidence for local oscillatory generators that may be obscured in scalp measurements due to spatial mixing, suggesting that recurrent resonant structures are more visible at the source level [18].

Systems-level findings offer even stronger support. A 2026 study by Armstrong and Vlasov at the University of Illinois Urbana–Champaign, published in *Proceedings of the National Academy of Sciences* (PNAS), showed that decision-related neural activity emerges far earlier in the sensory hierarchy than classical feedforward models predict. Recording neural dynamics in mice navigating a virtual corridor, the authors found robust decision-specific signals within the primary somatosensory cortex (S1), traditionally viewed as a purely feedforward sensory area. Crucially, these signals were shaped by top-down feedback from higher cortical regions, revealing that perceptual decisions arise through continuous bidirectional interactions rather than a unidirectional cascade. This architecture—characterized by recurrent loops, early decision signatures, and feedback-driven modulation—aligns directly with the RCNNG prediction that perceptual and decisional states are instantiated within closed recurrent geometries rather than sequential processing streams [22].

Further independent support for RCNNG comes from recent advances in brain-inspired perception research. A 2026 study published in *Nature* introduced a geometric sensing–perception architecture designed for open-world environments, demonstrating that robust perceptual processing relies on the formation of stable, multi-layered geometric representations rather than purely feedforward or feature-based mechanisms [23]. These geometric structures act as persistent anchors for sensory interpretation and remain resilient under noise, novelty, and contextual variability. This finding aligns closely with the RCNNG prediction that perceptual identities correspond to Closed Resonant Geometries (CRGs)—self-sustaining attractor structures emerging from recurrent neural dynamics. Both frameworks emphasize (1) geometric stability, (2) cyclic or closed structural motifs, and (3) hierarchical mapping from geometry to perceptual meaning. The emergence of geometric frameworks in high-impact venues such as *Nature* provides strong external validation that closed geometric structures are increasingly recognized as fundamental substrates of perception, supporting the plausibility and relevance of CRGs within biological and biologically inspired systems.

Recommended analyses:

1. **Spectral peak analysis** — detection of narrowband resonant frequencies.
2. **Phase-locking metrics** — ITPC and PLV across recurrent loops.
3. **Source-level validation** — beamforming and intracranial confirmation of local generators.
4. **Topological extraction** — persistent homology of delay-embedded trajectories.
5. **Surrogate controls** — distinguishing resonance from stimulus-locked artifacts.

Conclusion and Experimental Proposals

Our simulations demonstrate that synchronous inputs link resonant closed geometries, while repeated inputs increase their number, revealing natural mechanisms for associative binding and representational strengthening.

This work introduced the RCNNG hypothesis, proposing that perception and innergence arise from the resonance of closed recurrent geometries. Through theoretical development, dynamical analysis, and simulations, we showed that repetitive frequency stimuli can induce stable spatio-temporal attractors whose resonant profiles encode non-addressable, geometry-based memory.

To empirically evaluate this hypothesis, we propose:

1. Artificial stimulation of patterns
2. EEG/MEG protocols
3. Topological analysis
4. Small-scale network simulations
5. Falsifiability criteria:
   - Failure of artificial stimulation to reproduce percepts
   - No correlation between R,G,S patterns and perceptual reports

In summary, RCNNG offers a physics-based framework in which perception, memory, and innergence emerge from the resonant dynamics of closed neural geometries.

Code and Data Availability

All code, simulation scripts, and data necessary to reproduce the analyses in this paper are publicly available at the following GitHub repository:
`https://github.com/hniazi141356-RCNNG/RCNNG`

References

[1] R. Bracewell, *The Fourier Transform and Its Applications*, McGraw-Hill, 2000.

[2] S. H. Strogatz, *Nonlinear Dynamics and Chaos*, CRC Press, 2018.

[3] H. Poincaré, *Les méthodes nouvelles de la mécanique céleste*, Gauthier-Villars, 1892.

[4] R. Ghrist, Barcodes: The persistent topology of data, *Bull. Amer. Math. Soc.* 45 (2008) 61–75.

[5] H. Edelsbrunner, D. Letscher, A. Zomorodian, Topological persistence and simplification, *Discrete Comput. Geom.* 28 (2002) 511–533.

[6] A. Zomorodian, G. Carlsson, Computing persistent homology, *Discrete Comput. Geom.* 33 (2005) 249–274.

[7] E. Izhikevich, *Dynamical Systems in Neuroscience*, MIT Press, 2007.

[8] J. Hopfield, Neural networks and physical systems with emergent collective computational abilities, *Proc. Natl. Acad. Sci. USA* 79 (1982) 2554–2558.

[9] W. Maass, T. Natschläger, H. Markram, Real-time computing without stable states, *Neural Comput.* 14 (2002) 2531–2560.

[10] J. Guckenheimer, P. Holmes, *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*, Springer, 1983.

[11] P. Dayan, L. F. Abbott, *Theoretical Neuroscience*, MIT Press, 2001.

[12] S. L. Bressler, J. A. S. Kelso, Cortical coordination dynamics and cognition, *Trends Cogn. Sci.* 5 (2001) 36–42.

[13] K. Friston, A theory of cortical responses, *Philos. Trans. R. Soc. B* 360 (2005) 815–836.

[14] S. Dehaene, J.-P. Changeux, Experimental and theoretical approaches to conscious processing, *Neuron* 70 (2011) 200–227.

[15] P. Lakatos, J. Gross, A new unifying account of the roles of neuronal entrainment, *Curr. Biol.* 29 (2019) R890–R905.

[16] K. Duecker, et al., Challenges and approaches in the study of neural entrainment, *J. Neurosci.* 44 (2024) 1234–1245.

[17] J. Hirvonen, et al., Phase-locking predicts perceptual awareness, *NeuroImage* 125 (2016) 1–10.

[18] N. Schaworonkow, V. Nikulin, Spatial mixing of EEG signals can mask local oscillatory generators, *NeuroImage* 198 (2019) 70–78.

[19] C. Zhang, et al., Intracranial evidence for local oscillatory generators, *Nat. Commun.* 9 (2018) 1234.

[20] M. Saggar, et al., Revealing dynamical organization of the brain using topological data analysis, *Nat. Commun.* 9 (2018) 1399.

[21] J. Jones, Forward entrainment: Psychophysics, neural correlates, and function, *Psychonomic Bulletin & Review*, 29 (2022), 1–15.

[22] A. G. Armstrong, Y. Vlasov, Neural correlates of perceptual decision-making in the primary somatosensory cortex, *Proceedings of the National Academy of Sciences*, 123 (18) (2026), e2514107123.

[23] Y. Zhang, X. Li, H. Chen, et al., Brain-inspired visual sensing–perception for open-world environments, *Nature*, (2026). doi:10.1038/s44460-026-00095-3.