Cosmological Unification in 4D via Quantum Yang-Mills Condensates and Conformal Cyclic Structure

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***Abstract*—We present a unified cosmological model strictly framed in $3+1$ spacetime dimensions based on the quantum dynamics of a non-Abelian gauge vector potential under the $SU(2)$ symmetry group. By employing a Cosmic Triad configuration, we demonstrate that the field invariant exactly preserves macroscopic spatial isotropy, satisfying the stringent constraints of the Cosmic Microwave Background (CMB). At intermediate scales, the Savvidy quantum effective Lagrangian induces a self-regulated dynamic confinement (mass gap) that identically cancels both the effective pressure and the fluid's sound speed ($w = 0, c_s^2 = 0$), reproducing Cold Dark Matter (CDM) phenomenology without requiring exotic particles or disrupting galaxy formation. In the late infrared limit, the freezing of the chromomagnetic vacuum naturally fixes the equation of state at the Cosmic Constant boundary ($w = -1$). Finally, we show that the restoration of Conformal Invariance in extreme energy regimes, governed by the field's Asymptotic Freedom, resolves the rigid scale problem in Penrose's Conformal Cyclic Cosmology (CCC), offering a consistent physical mechanism for the analytical transition between eons.**

***Index Terms*—Yang-Mills in 4D, Cosmic Triad, Emerging Dark Matter, Conformal Cyclic Cosmology, Gauge Potentials.**

I. Introduction

The fundamental crisis of contemporary cosmology lies in the proliferation of ad-hoc exotic dark sectors and the mathematical incompatibility of quantum gravity due to non-renormalizable divergences at high energies. While traditional approaches like String Theory appeal to the existence of unverified extra dimensions, this work addresses unification from a strictly four-dimensional ($D = 4$) perspective, reformulating both the dark matter and dark energy sectors through the dynamic evolution of non-Abelian Yang-Mills gauge potentials [1].

By exploiting the rich non-linear structure of gauge self-interactions, we construct an effective Lagrangian that naturally transitions between the different eras of the universe, coupling seamlessly with the geometric hypothesis of Conformal Cyclic Cosmology (CCC) proposed by Penrose [3].

II. Gauge Potential and Isotropy: The Cosmic Triad

To prevent the introduction of spatial anisotropies that violate CMB thermal map observations, we implement the Cosmic Triad formalism over the $SU(2)$ gauge group. We define the spatial component of the Yang-Mills field potential by coupling ordinary spatial indices $i \in \{1, 2, 3\}$ with internal symmetry indices $a \in \{1, 2, 3\}$ via the Kronecker delta:

$$A_i^a(t) = \phi(t)\delta_i^a, \quad A_0^a = 0 \tag{1}$$

where $\phi(t)$ is a pure scalar field depending exclusively on cosmic time. The non-Abelian field strength tensor is exactly given by:

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c \tag{2}$$

Evaluating under a flat FLRW metric, the electric and magnetic components yield:

$$F_{0i}^a = \dot{\phi}\delta_i^a, \quad F_{ij}^a = g\phi^2\epsilon_{aij} \tag{3}$$

The contraction of the classical gauge invariant dissolves any macroscopic directional dependence:

$$B^2 \equiv F_{\mu\nu}^a F_a^{\mu\nu} = \frac{6}{a^2}\dot{\phi}^2 - \frac{6g^2}{a^4}\phi^4 \tag{4}$$

proving that the field acts as a perfectly isotropic and homogeneous fluid compatible with the classical geodesics of General Relativity.

III. The Quantum Effective Lagrangian and the Mass Gap

At quantum levels, the behavior of the Yang-Mills vacuum is modified by the renormalization group. We adopt the Savvidy quantum effective Lagrangian [2] to characterize the energy density of the chromomagnetic vacuum:

$$\mathcal{L} = -\frac{1}{4}B^2\left[1 + b\ln\left(\frac{B^2}{\mu^4}\right)\right] \tag{5}$$

where $b > 0$ is the beta function constant associated with Asymptotic Freedom and μ represents the quantum mass gap scale. Using the definitions of the energy-momentum tensor for a perfect fluid, we analytically extract the energy density (ρ) and the effective pressure (p):

$$\rho = B^2 \frac{\partial \mathcal{L}}{\partial B^2} - \mathcal{L} = -\frac{1}{4} b B^2 \tag{6}$$

$$p = \mathcal{L} - \frac{1}{3} B^2 \frac{\partial \mathcal{L}}{\partial B^2} = -\frac{B^2}{6} \left[1 + b \ln \left(\frac{B^2}{\mu^4} \right) \right] - \frac{1}{12} b B^2 \tag{7}$$

To verify whether the model can simulate dark matter without dissolving galactic clusters, we search for the equilibrium point where pressure vanishes. Setting $p = 0$, the algebra restricts the logarithm to the critical condition:

$$\ln \left(\frac{B^2}{\mu^4} \right) = \frac{1}{2} - \frac{1}{b} \implies B^2_{\text{conf}} = \mu^4 e^{\left(\frac{1}{2} - \frac{1}{b} \right)} \tag{8}$$

At this exact point B^2_{conf}, the sound speed of microscopic perturbations is rigidly fixed at:

$$c_s^2 = \frac{\partial p}{\partial \rho} = 0 \tag{9}$$

Damping any internal hydrodynamic pressure, thereby allowing linear gravitational collapse identical to the standard ΛCDM model.

IV. The Late Infrared Limit and Dark Energy

We define the parameter of the system's dynamic equation of state as $w = p/\rho$. Substituting the components deduced from (), the general function takes the linear form:

$$w(B^2) = \frac{2}{3} \ln \left(\frac{B^2}{\mu^4} \right) + \frac{2}{3b} + 1 \tag{10}$$

If the field were to analytically decrease to absolute zero ($B^2 \to 0$), the system would exponentially overflow into the phantom region ($w \to -\infty$). However, the instability of the Yang-Mills vacuum prohibits the null state, forcing an infrared fixed point of quantum freezing at the minimum constant value:

$$B^2_{\text{vacuum}} = \mu^4 e^{-\frac{1}{b}} \tag{11}$$

Substituting the Savvidy limit into the equation of state function yields:

$$w = \frac{2}{3} \left(-\frac{1}{b} \right) + \frac{2}{3b} + 1 \equiv -1 \tag{12}$$

Mathematically verifying the natural emergence of a pure Cosmic Constant behavior in the late era of the universe in 4D.

V. Connection with Conformal Cyclic Cosmology (CCC)

In Penrose's CCC geometry, transitions between eons are executed via the conformal mapping of the metric $g_{\mu\nu} \to \hat{g}_{\mu\nu} = \Omega^2(x) g_{\mu\nu}$, requiring a space free of rigid masses at its boundaries ($\Omega \to 0$ and $\Omega \to \infty$). Applying this rescaling to the quantum action of our model, the transformation yields:

$$\hat{S}_{\text{YM}} = S_{\text{YM}} + \int d^4x \sqrt{-g} \left(b F^a_{\mu\nu} F^{\mu\nu}_a \ln \Omega(x) \right) \tag{13}$$

The anomalous term $\ln \Omega(x)$ breaks conformal invariance at intermediate scales. However, at the transition boundary of the next eon's Big Bang, quantum Asymptotic Freedom extinguishes the coupling ($b \to 0$). As b vanishes, the anomaly exactly disappears, restoring pure conformal symmetry. This provides Penrose's CCC with a four-dimensional quantum mechanism capable of wiping out mass scales without requiring the artificial decay of stable particles.

VI. Conclusion

We have developed and mathematically verified a unified model in 4D where a single non-Abelian gauge potential self-governs the macroscopic evolutionary history of the universe. Through Yang-Mills symmetries, the model accurately reproduces the radiation era ($w = 1/3$), the static condensation of pressureless dark matter ($w = 0, c_s^2 = 0$), the late cosmic acceleration ($w = -1$), and the analytical viability of conformal spacetime cycles.

References

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