Thermodynamic Backreaction and the Holographic Bounds of Quantum Computational Complexity: A Phenomenological Toy-Model Approach

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Abstract

We establish a rigorous, non-equilibrium thermodynamic framework governing the dynamic reconstruction of bulk spacetime geometry from boundary quantum states within the AdS/CFT correspondence. Adopting a phenomenological toy-model approach, we bypass the microscopic ultraviolet (UV) complications to extract universal constraints on holographic processing. Utilizing the Complexity=Action (CA) conjecture, we model the temporal evolution of emergent bulk geometry as a manifestation of unitary quantum gates. By extending the generalized Landauer principle to the quantum regime, we derive the unavoidable entropy production rate ($\dot{S}_{\text{gen}}$) triggered by irreversible state transitions at the boundary. Incorporating this dissipative channel into the extended first law of black hole thermodynamics, we compute the explicit gravitational backreaction on the bulk geometry using a 3D BTZ black hole background. We prove that a holographic quantum rendering efficiency (η_{hq}) naturally emerges and vanishes as the computational rate saturates the fundamental Lloyd's bound. This dynamic constraint establishes an absolute upper bound on the Hawking temperature (T_{H}), forcing a finite-time geometric divergence at t_{crash}. Rather than a mere destruction of information, we show that this critical threshold triggers a topological phase transition, serving as a non-local birth mechanism for parallel spacetime domains within a holographic multiverse.

1 Introduction

The holographic paradigm, established via the Anti-de Sitter / Conformal Field Theory (AdS/CFT) correspondence, asserts that the quantum states of a boundary theory encode the gravitational dynamics of a bulk spacetime [Maldacena1998]. While static geometric properties are robustly mapped via the Ryu-Takayanagi formula, the physical mechanism underpinning the temporal evolution of the bulk remains an active area of research. Recent developments, particularly the Complexity=Action (CA) and Complexity=Volume (CV) conjectures, suggest that the growth of quantum computational complexity within the boundary state corresponds directly to the growth of bulk action or volume behind the horizon [BrownSusskind2016].

However, the physical execution of these quantum operations cannot be purely dissipationless under realistic non-equilibrium conditions. Any physical system acting as a computational medium must adhere to the laws of information thermodynamics [Landauer1961, SagawaUeda2008]. In this paper, we reformulate the dynamic projection of the bulk as a non-equilibrium thermodynamic process.

To isolate the fundamental physics without becoming entangled in the microscopic complexities of a complete ultraviolet (UV) description, we employ a **phenomenological toy-model approach**. This methodology allows us to derive universal thermodynamic speed limits on cosmic information processing, demonstrating that boundary state transitions generate a localized, irreversible heat flux that backreacts on the bulk Einstein geometry.

2 Quantum Complexity Evolution and Landauer Dissipation

Let $|\psi(t)\rangle$ denote the time-dependent quantum state of the boundary CFT, and $\mathcal{C}(t)$ represent its quantum computational complexity defined relative to a reference state. According to the Brown-Susskind theorem, the growth rate of complexity for a holographic system with total mass-energy M is fundamentally bounded by the Lloyd's bound [Lloyd2000]:

$$\frac{d\mathcal{C}}{dt} \leq \frac{2M}{\pi\hbar} \tag{1}$$

When interpreting the boundary theory as a physical computational architecture that reconstructs the emergent bulk, continuous state transitions inherently involve non-unitary noise, erasure, or state resets. According to the generalized quantum Landauer principle for non-equilibrium systems, the irreversible entropy generation rate $\dot{S}_{\text{gen}}$ associated with the information flux is bounded by:

$$\dot{S}_{\text{gen}} = \frac{1}{T_{\text{H}}}\frac{dQ_{\text{diss}}}{dt} \geq k_B \ln 2 \cdot \lambda \frac{d\mathcal{C}}{dt} \tag{2}$$

where T_{H} is the Hawking temperature of the emergent bulk black hole, dQ_{diss}/dt is the continuous rate of thermal dissipation injected into the system's thermal matrix, and λ $(0 < \lambda \leq 1)$ is a phenomenological efficiency parameter representing the intrinsic irreversibility of the underlying quantum gate operations.

3 Specific Framework: The 3D BTZ Black Hole System

To evaluate how this boundary computational dissipation affects the bulk geometry, we implement the framework of extended black hole thermodynamics within an explicit $\text{AdS}_3/\text{CFT}_2$ setup. We consider a static Banados-Teitelboim-Zanelli (BTZ) black hole [BTZ1992], whose metric is given by:

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\phi^2 \tag{3}$$

where the metric function f(r) is defined via the bulk curvature radius L and total mass M:

$$f(r) = \frac{r^2}{L^2} - 8G_{\text{N}}M \tag{4}$$

The event horizon radius is located at $r_+ = L\sqrt{8G_{\text{N}}M}$. The Bekenstein-Hawking entropy S_{BH} and the Hawking temperature T_{H} are written as explicit functions of M:

$$S_{\text{BH}} = \frac{\pi L}{\hbar}\sqrt{\frac{2M}{G_{\text{N}}}}, \quad T_{\text{H}} = \frac{\hbar r_+}{2\pi L^2} = \frac{\hbar}{2\pi L}\sqrt{8G_{\text{N}}M} \tag{5}$$

The continuous injection of Landauer computational heat dQ_{diss} acts as a non-equilibrium energy influx across the horizon. Integrating this dissipative component, the modified extended first law of black hole thermodynamics is formalized as:

$$dM = T_{\text{H}} dS_{\text{BH}} + V dP + \frac{dQ_{\text{diss}}}{dt} dt \tag{6}$$

where $P = -\Lambda/(8\pi G_{\text{N}})$ is the thermodynamic pressure. Via the semi-classical Einstein field equations, the computational dissipation rate dQ_{diss}/dt explicitly alters the expectation value of the energy-momentum tensor $\langle T_{\mu\nu} \rangle$.

4 Energy-Momentum Tensor Modification and Causal Backreaction

We model the computational dissipation $\dot{Q}_{\text{diss}}$ as an infalling null fluid near the horizon:

$$T^{\text{diss}}_{\mu\nu} = \rho_{\text{diss}} k_\mu k_\nu \tag{7}$$

where k^μ is an inward-directed null vector ($k^\mu k_\mu = 0$). Integrating the energy flux across the horizon yields $\dot{Q}_{\text{diss}} = 2\pi r_+ \cdot L \cdot \rho_{\text{diss}}$. The dynamic expansion of the horizon radius r_+ is driven by this gravitational influx:

$$\frac{dr_+}{dt} = \frac{2G_{\text{N}}}{\pi r_+} \dot{Q}_{\text{diss}} \tag{8}$$

Substituting the saturated Landauer bound (Eq. 2) and the Lloyd's bound (Eq. 1) into Eq. 8, and using the BTZ relations for M and T_{H}, we arrive at a closed, non-linear evolutionary equation for the horizon:

$$\frac{dr_+}{dt} = \left(\frac{G_{\text{N}} k_B \lambda \ln 2}{2\pi^3 L^4} \right) r_+^3 \tag{9}$$

Equation 9 reveals that the gravitational backreaction driven by boundary quantum processing scales cubically with the horizon size, leading to an inevitable non-linear runaway.

5 The Computational-Thermodynamic Singularity and Multiverse Genesis

Solving the differential equation (Eq. 9) with an initial horizon radius r_0 demonstrates that the bulk geometry encounters a finite-time divergence (Big Rip-style singularity):

$$r_+(t) = \frac{r_0}{\sqrt{1 - \frac{t}{t_{\text{crash}}}}}, \quad t_{\text{crash}} = \frac{\pi^3 L^4}{G_{\text{N}} k_B \lambda \ln 2 \cdot r_0^2} \tag{10}$$

5.1 Holographic Quantum Rendering Efficiency

We define the Holographic Quantum Rendering Efficiency (η_{hq}) as the fraction of mass-energy available to sustain stable, classical bulk configurations:

$$\eta_{\text{hq}} \equiv 1 - \frac{1}{M} \frac{dQ_{\text{diss}}}{dt} \leq 1 - \frac{2k_B T_{\text{H}} \ln 2 \cdot \lambda}{\pi \hbar} \tag{11}$$

As $t \to t_{\text{crash}}$, the temperature diverges ($T_{\text{H}} \to \infty$), causing η_{hq} to drop below zero.

5.2 Discussion: Post-Selection and Multiverse Birth

The critical transition $\eta_{\text{hq}} < 0$ implies a profound conceptual shift in the nature of emergent spacetime. Instead of a passive background, the bulk geometry is a real-time quantum rendering driven by asymptotic boundary measurements. When an observer attempts to "fix the future" boundary state—analytically equivalent to imposing a post-selection condition on the final state via the Horowitz-Maldacena proposal [HorowitzMaldacena2004]—the holographic mapping demands a severe fine-tuning of the bulk causal networks.

Physically, this post-selection forces a non-local counter-flow of energy-momentum back into the past bulk horizon. As the computation saturation approaches t_{crash}, the thermodynamic backreaction diverges, transforming the smooth interior geometry into a dynamic, non-local firewall [AMPS2013].

Crucially, this finite-time explosion does not signify the absolute annihilation of existence. Rather, the saturation of $\eta_{\text{hq}} < 0$ denotes the ultimate endpoint of the *original* classical spacetime description. At t_{crash}, the macroscopically entangled bulk undergoes a geometric phase transition (quantum decoherence). The extreme thermal energy injected by the observer's post-selection acts as a localized inflationary seed. Through a quantum topology change, the original bulk de-coheres and branches off, spawning a distinct, causally disconnected bubble universe. The observer's attempt to deterministicly anchor a single future forces the geometry to fracture, generating a parallel spacetime domain within a grander holographic multiverse.

6 Conclusion

By synthesizing non-equilibrium information thermodynamics with extended black hole geometry in AdS3, we proved that boundary quantum computation imposes a hard physical limit on bulk stability. The observer's act of fixing a future state triggers an instantaneous, cubic runaway reaction in the past bulk horizon, culminating at a finite-time boundary $t$crash. This mechanism elegantly shifts the interpretation of the multiverse from a passive anthropic landscape to an active, thermodynamically driven byproduct of quantum state rendering. Future work will focus on mapping the precise microstate transitions during the post-selection phase transition.

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