Extension of Lorentz Spacetime Symmetry to Four Sectors of Flat and Curved Spacetime

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Abstract

This work asks what is obtained when the observer-independent light cone is used to organise the complete orientation of a local clock–ruler frame, rather than only the ordinary future-timelike part of that orientation. The result is a four-sector description of one spacetime. The sectors are not parallel universes and do not provide four independent metrics. They are the four successive orientations met when a local time axis, a spatial axis, the opposite time axis, and the opposite spatial axis are followed around one extended Lorentz orbit.

The same distinction is applied separately to matter and geometry. A particle has a fixed sector relative to the local geometric frame and undergoes ordinary Lorentz boosts only inside that sector; it is never accelerated through the light boundary into another particle class. Curved geometry may instead require a neighbouring sector chart when one of its proper scales reaches a null limit. A tensorial scale construction identifies the affected direction, pairs it with the local clock, and replaces the failed extrapolation by the reciprocal scale of the adjacent chart while retaining the same general-relativistic solution.

In Schwarzschild, LTB, Kerr, and FLRW geometries this procedure turns the apparent zero of an inactive scale into an infinite proper scale measured from the neighbouring sector centre. Curvature regularity is then a result to be checked with invariant scalars, not an effect assigned to a tetrad rotation. Because every sector is a reading of one geometry and one field history, the same framework also constrains black–white-hole interpretations, cosmological time reversal, and closed timelike curves. The construction is presented as a coherent classical framework whose remaining dynamical and quantum conditions are stated explicitly.

Keywords: Lorentz symmetry; particle sectors; invariant light speed; sector atlas; reciprocal normalisation; curvature; one-field spacetime; black holes; closed timelike curves; CPT symmetry

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1 Physical starting point: one light cone and one spacetime

Special relativity begins with the empirical statement that every inertial observer measures the same vacuum light speed c [1]. The statement is not only about a number obtained by dividing distance by time. It says that all observers agree on the null boundary separating timelike and spacelike directions. Without that common boundary they would disagree on the causal type of the same displacement, and there would be no single causal spacetime shared by them.

The present construction takes this geometrical content as its only physical starting point. It does not begin with four universes or four independently chosen metrics. It begins with one four-dimensional manifold, one local null set and one physical history. The four sectors arise only when the complete orientation of a clock–ruler plane is followed around that common null set.

Physical principle 1.1 (Observer-independent local light speed)**.** *Every local observer measures the same null speed c, independently of the observer's worldline and of which admissible sector orientation supplies the local clock.*

1.1 The light cone precedes a numerical velocity

A light cone is a property of the Lorentzian geometry before any observer assigns coordinate slopes to it. An observer chooses a unit timelike clock axis and three orthogonal ruler axes; only after this choice is the cone read numerically as propagation at c. A different observer uses tilted axes, but an ordinary Lorentz transformation preserves the same null directions. In a spacetime diagram the clock and the corresponding ruler tilt together. The cone is not deformed by the observer's motion.

This order prevents a common misinterpretation. A tangent may have an infinite or apparently superluminal slope when divided by the time coordinate of a sector whose clock it does not use. That quotient is not automatically a locally measured speed. Physical velocity is defined only after the tangent has been resolved in its active real clock–ruler frame.

An ordinary Lorentz boost explores one connected orientation domain. As its clock approaches a null ray, the normalisation of a unit timelike vector diverges because a null vector cannot carry a massive observer's proper time. No finite boost crosses that boundary. The four-sector extension therefore does not permit ordinary matter to pass through c. It asks instead how the neighbouring orientation domain is described when the old clock itself has reached the edge of its valid chart.

1.2 Why one Lorentz plane has four sectors

Choose one clock direction and one spatial direction. Their two null lines divide the oriented plane into four connected non-null domains. Their centres are the future clock $+t$, a positive spatial direction $+n^a$, the opposite clock $-t$, and the reflected spatial direction $-n^a$. Following these centres gives the closed orientation cycle

$$+t \longrightarrow +n^a \longrightarrow -t \longrightarrow -n^a \longrightarrow +t.$$

The continuum of boosts inside one domain does not create further sectors. In three spatial dimensions the side centre n^a may be chosen from a two-sphere of unit spatial directions, but each choice has the same causal role and belongs to the same side-sector class.

The null rays are chart boundaries, not additional sectors and not worldlines of massive particles. When a geometric clock reaches one of them, the old normalisation ends. The adjacent chart reads a direction previously called spatial as its clock centre and supplies its own ordinary Lorentz boosts about that centre. The orientation can therefore be followed continuously even though no normalised clock is carried through the null ray by a single boost.

1.3 Geometry orientation and particle orientation are different data

The same fourfold structure has two uses which must not be conflated. The geometry has an active sector n_g, telling us which orientation and which regular chart currently represent the common field. A particle X has a relative sector s_X, telling us which extended Lorentz transformation acts on the local geometric coframe. Its ordinary Lorentz matrix $\Lambda_{(X)}$ then describes motion inside that fixed particle sector.

These operations occur in a definite order. The geometry first supplies its sector coframe. The particle then applies its own sector phase and ordinary Lorentz motion to the components read from that coframe. The sum $N_X = n_g + s_X \pmod 4$ appears only after those two independent operations have been composed. A geometry-chart transition changes n_g and hence the total phase N_X, but it changes neither s_X nor the particle's local boost. Conversely, ordinary acceleration changes $\Lambda_{(X)}$ but not the two sector labels.

This separation allows ordinary, side-oriented and oppositely oriented particles to coexist in one flat or curved geometry. They are not copies of one particle in different universes. They are distinct local sector readings of particle motion in one spacetime, all bounded by the same null set.

1.4 Why curvature must be tested after the sector construction

In flat spacetime the four sectors concern orientation alone. Curvature adds a second problem: the proper separations between neighbouring worldlines can focus, stretch or rotate. A sector continuation must therefore identify which physical scale reaches the null chart boundary and whether the neighbouring chart remains a regular representation of the same gravitational field.

This requirement cannot be replaced by a tetrad choice. A freely falling frame can remove the connection at one event, but it cannot remove a nonzero Riemann tensor. If invariant curvature diverges, the relative acceleration of nearby geodesics becomes unbounded and no regular neighbourhood remains in which the universal cone structure can be transported. A different particle sector also cannot cure this failure: it only changes the particle's local reading of the common tensor.

The curved construction below consequently has two stages. Proper scale tensors determine the direction whose inherited chart is failing and select the reciprocal scale of the neighbouring sector. Curvature invariants are then evaluated on that active domain. Finite curvature is not assumed from the phase i^{n_g}; it is the invariant criterion by which a proposed Schwarzschild, LTB, Kerr or FLRW continuation succeeds or fails. This distinction keeps the physical motivation—one regular causal geometry for all observers—separate from the calculation that must verify it.

2 Four-sector Lorentz kinematics in flat spacetime

The sectors must first be defined in flat spacetime. Otherwise their meaning would depend on a particular gravitational solution and regularisation would be built into the notation. In inertial coordinates the common starting point is

$$\mathrm{d}s^2 = \eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -c^2\mathrm{d}t^2 + \mathrm{d}\boldsymbol{x}^2, \qquad \mathrm{d}s^2 = 0 \text{ for light.} \tag{2.1}$$

2.1 One spacetime and one notation

Lower-case Latin indices $a, b, c, \ldots$ are abstract spacetime indices. Greek indices $\mu, \nu, \ldots$ are used only for coordinate components, and upper-case Latin indices $A, B, \ldots$ label an orthonormal Lorentz coframe with $\eta_{AB} = \mathrm{diag}(-1, 1, 1, 1)$. Capital indices $I, J = 1, 2, 3$ label its spatial components. Labels in parentheses enumerate principal spatial modes but are not tensor indices. Sector labels take values in $\mathbb{Z}_4$ and are never summed.

The symbol x denotes one selected direction in the old spatial $\mathbb{R}^3$. It can later be represented by a spherical radial ray r, but the side sector contains every possible spatial ray rather than one preferred axis.

Let

$$\vartheta^A = \xi^A{}_a\, \mathrm{d}x^a, \qquad g^{(0)}_{ab} = \eta_{AB}\xi^A{}_a \xi^B{}_b \tag{2.2}$$

be a real reference coframe and its metric. The coefficients $\xi^A{}_a(x)$ may already contain the local deformation of a curved solution; the sector construction will not multiply them by an additional physical strain.

Three pieces of information must remain distinct. The active geometry sector is $n_g(x)$. A particle X carries a sector s_X relative to that geometry. Its ordinary Lorentz matrix $\Lambda_{(X)}$ describes continuous motion inside the fixed particle sector. Only after the geometry and particle operations are composed do we use the total label $N_X = n_g + s_X \pmod 4$ relative to the reference coframe.

All these descriptions concern one event of one manifold. There is one gravitational metric field, one null set and one tensor $R^a{}_{bcd}$. The geometry has sector representatives and particles have different tetrad-component readouts, but no particle acquires a private metric or private curvature. A geometric centre is flat only if the common Riemann tensor vanishes there; a particle being at rest is a separate kinematic statement.

2.2 The ordinary Lorentz transformation and the tilt of the axes

Before extending the set of orientations, consider exactly what an ordinary boost of a particle does inside its already chosen sector. After the sector factor has selected the real local component system on which $\Lambda_{(X)}$ acts, choose one spatial direction x and write the orthonormal components in that time–space plane as $\mathrm{d}x^0 = c\,\mathrm{d}t$ and $\mathrm{d}x^1 = \mathrm{d}x$. A frame moving along that direction is related to the reference frame by

$$\begin{gathered}\mathrm{d}x'^A = \Lambda_{(X)}{}^A{}_B(\beta_X)\mathrm{d}x^B, \qquad |\beta_X| < 1, \\ \Lambda_{(X)}{}^A{}_B(\beta_X)\big|_{(0,1)} = \frac{1}{\sqrt{1-\beta_X^2}}\begin{pmatrix} 1 & -\beta_X \\ -\beta_X & 1 \end{pmatrix}.\end{gathered} \tag{2.3}$$

Here β_X is dimensionless and equals the particle's v_X/c only in its own active sector frame. The same real matrix is used in all four sectors after their clock and ruler axes have been chosen. It changes $\Lambda_{(X)}$ but not s_X.

The normalising factor in Eq. (2.3) is fixed rather than chosen. The unnormalised clock direction $e_0{}^a + \beta_X e_1{}^a$ has squared norm $-(1-\beta_X^2)$. Requiring the new clock to have unit timelike norm gives $\gamma_X^2(1-\beta_X^2) = 1$ and hence $\gamma_X = (1-\beta_X^2)^{-1/2}$. The same factor normalises the ruler axis so that the two new axes remain orthogonal. In basis-vector form the inverse action is

$$e'_0{}^a = \gamma_X(e_0{}^a + \beta_X e_1{}^a), \qquad e'_1{}^a = \gamma_X(\beta_X e_0{}^a + e_1{}^a). \tag{2.4}$$

Equation (2.4) is the geometric content of a boost. At $\beta_X = 0$ the primed and unprimed axes coincide. As positive β_X increases, both primed axes lean toward the same positive null ray; negative β_X sends them toward the other one. They do not rotate as a Euclidean right angle. Their Minkowski orthogonality is preserved while their appearance in the inherited coordinate diagram becomes increasingly narrow.

The light boundary remains fixed during this tilt. Substituting $\mathrm{d}x = \pm c\,\mathrm{d}t$ into Eq. (2.3) gives $\mathrm{d}x' = \pm c\,\mathrm{d}t'$. Thus every boosted observer assigns the same number c to the same two null directions. Equivalently, $(\Lambda_{(X)})^T\eta\Lambda_{(X)} = \eta$. An ordinary boost therefore preserves the particle's sector cone and cannot change its relative sector.

The limit $|\beta_X| \to 1$ now has an unambiguous meaning. Both axes approach a null direction projectively, while γ_X diverges because a null vector cannot be normalised to unit timelike length. No finite real boost carries the clock through that ray. This is the boundary of the ordinary Lorentz chart, not a point at which a massive body attains or exceeds the speed of light. A particle with $s_X = 1$ or 3 is therefore not an $s_X = 0$ particle continued through $|\beta_X| = 1$. It begins in a different sector and obeys the same bound relative to its own clock.

2.3 Four orientation sectors of one Lorentz plane

The ordinary boost has so far explored only the domain centred on $+t$. To see the missing orientations, keep the same two null lines and follow an oriented axis around the complete time–space plane. The null rays divide that plane into four open connected domains. Starting from $+t$, one meets a domain centred on a selected positive spatial direction, then the domain centred on $-t$, then the reflected spatial domain, and finally returns to $+t$. The complete cycle is

n	i^n	clock orientation	sector
0	1	$+t$	ordinary sector
1	i	$+n^a$	first side sector
2	-1	$-t$	PT reflection of sector 0
3	$-\mathrm{i}$	$-n^a$	reflected side sector
$4 \equiv 0$	1	$+t$	closed orientation cycle.

(2.5)

Thus the ordinary flat reference has its future cone centred on $+t$. The first side reference is the family of equally admissible flat cones centred on directions that were spatial in the ordinary projection; a particular observer selects one $+n^a$. The opposite sector is the PT reflection of the ordinary sector, and the second side sector is the reflected family $-n^a$. These are four views of one flat Lorentz geometry, not four flat spacetimes.

The entries in the table are the centres of the four domains, not the only directions allowed inside them. Every sector has its own finite $|\beta_X| < 1$ Lorentz boosts, and none of those boosts creates another sector. The discrete label records the connected orientation component, whereas β_X records continuous particle motion inside the chosen component. The separate projective parameter β_g introduced below tracks the geometry from one chart to the next.

The table describes orientations, not four kinds of matter and not four disconnected regions. At a fixed event the null cone is one geometric set. What changes from one entry to the next is the direction chosen to bisect that cone and to carry the observer's clock. In sector 0 this direction is the usual $+t$ axis. At the neighbouring null boundary that axis can no longer be normalised as timelike; sector 1 then reads one of the old spatial directions as its real clock direction. Repeating the same construction produces the reflected $-t$ and $-n^a$ orientations and finally returns to the starting frame. This is why the cycle completes the Lorentz description of one spacetime rather than adding three spacetimes.

The null rays are boundaries of the four descriptions, not additional sectors. At such a ray the old unit clock is undefined because its norm is zero. The neighbouring sector begins by assigning a new timelike centre on the other side of that boundary. This change concerns the frame used to read the geometry. It does not say that a massive tangent has crossed the null ray or changed from timelike to spacelike.

2.4 Geometry sector, particle transformation, and total orientation

The sector factors enter in two successive operations. First the geometry chooses its active coframe:

$$\vartheta^A_{(g)} = \mathrm{i}^{n_g}\vartheta^A = \mathrm{i}^{n_g}\xi^A{}_a\,\mathrm{d}x^a, \qquad g^{(n_g)}_{ab} = \eta_{AB}\vartheta^A_{(g)a}\vartheta^B_{(g)b} = \mathrm{i}^{2n_g}g^{(0)}_{ab}. \tag{2.6}$$

The phase i^{n_g} belongs to the geometry coframe, while its square appears in the sector representation of the metric. Odd sectors reverse the sign convention of the quadratic form and even sectors preserve it. Because multiplication by a nonzero constant leaves the null set unchanged, all four representations describe the same local light boundary. The full orientation still resides in the coframe: the metric alone cannot distinguish sectors 0 and 2, or sectors 1 and 3.

Only after the geometry has supplied $\vartheta^A_{(g)}$ does particle X apply its own extended Lorentz transformation

$$\mathcal{G}_{(X)}{}^A{}_B = \mathrm{i}^{s_X}\Lambda_{(X)}{}^A{}_B, \qquad \mathrm{d}X^A_{(X)} = \mathcal{G}_{(X)}{}^A{}_B\vartheta^B_{(g)} = \mathrm{i}^{n_g+s_X}\Lambda_{(X)}{}^A{}_B\xi^B{}_a\,\mathrm{d}x^a. \tag{2.7}$$

This is the central composition law. The factor i^{s_X} belongs to the particle transformation, not to a second geometric coframe. The ordinary matrix $\Lambda_{(X)}$ describes motion inside that particle sector. The total label

$$N_X = n_g + s_X \pmod 4 \tag{2.8}$$

therefore records the final phase relative to the reference coframe, but it does not replace the two independent sources of that phase.

The four relative classes are

s_X	relative orientation	name used here
0	ordinary	ordinary particle
1	first side	side particle I
2	opposite	anti-sector particle
3	reflected side	side particle II.

(2.9)

The word *tachyon* will be used as shorthand for the two side classes $s_X = 1, 3$. It does not mean ordinary matter accelerated through c. Likewise, $s_X = 2$ denotes the opposite transformation orientation. Identifying this class with observed antimatter is an additional physical interpretation, not a result of standard quantum field theory.

When the geometry changes chart by k sectors, its coframe and every final particle readout acquire the same additional phase. The particle transformation itself is unchanged:

$$n_g \longmapsto n_g + k, \qquad N_X \longmapsto N_X + k, \qquad s_X = N_X - n_g \longmapsto s_X, \qquad \Lambda_{(X)} \longmapsto \Lambda_{(X)}. \tag{2.10}$$

By contrast, ordinary acceleration changes $\Lambda_{(X)}$ while leaving s_X fixed. Ordinary matter, a side particle and an anti-sector particle therefore retain their class when the geometry changes sector.

Equation (2.7) may be used to form the local quadratic readout $\eta_{AB}\mathrm{d}X^A_{(X)}\mathrm{d}X^B_{(X)}$. It must not be promoted to a new dynamic tensor $g^{(X)}_{ab}$. The only gravitational metric is the geometry field represented in Eq. (2.6); particles differ by the transformations through which they resolve that common field.

2.5 Particle rest and geometric flatness

Particle rest is defined from the readout in Eq. (2.7), not from the value of a coordinate slope. If U^a_X is its tangent, let

$$U^A_{(X)} = \mathcal{G}_{(X)}{}^A{}_B\, \vartheta^B_{(g)a} U^a_X.$$

The particle is at rest relative to the local geometry inside its own sector when its three spatial components vanish. In the adapted representative this is equivalently

$$U^I_{(X)} = 0 \qquad \Longleftrightarrow \qquad \Lambda_{(X)} = \mathbf{1}. \tag{2.11}$$

For $s_X = 0$ this is ordinary rest relative to the geometric coframe. For $s_X = 1$ or 3 the clock axis is side-oriented, so an inherited ordinary chart can assign it an infinite or superluminal-looking slope even though $\Lambda_{(X)}$ is the identity. For $s_X = 2$ the particle transformation has the opposite orientation. There is no absolute rest common to all four readouts.

Geometric sector rest is a different statement. Let u^a_g be the clock of the active geometry chart and $h_g{}^a{}_b = \delta^a{}_b + u^a_g u_{gb}$ its rest projector. A reference congruence is aligned with that chart when $U^a = c\,u^a_g$. It defines a flat sector centre only when the alignment extends through a neighbourhood without tidal deviation:

$$\begin{aligned} &\text{geometric sector rest:} && U^a = c\,u^a_g \quad \Longleftrightarrow \quad h_g{}^a{}_b U^b = 0, \\ &\text{flat geometric centre:} && \nabla_b u^a_g = 0 \text{ throughout a neighbourhood}, \qquad R^a{}_{bcd} = 0. \end{aligned} \tag{2.12}$$

The first line is pointwise and can hold in curved spacetime. The second is a property of the one geometry. Hence a tachyon can be at particle-sector rest in a curved region, and an ordinary particle can move through a flat geometric centre. Neither fact changes the Riemann tensor. Regularisation below relies on the second line, not on setting $\Lambda_{(X)} = \mathbf{1}$ for one particle.

The particle phase must be removed when its components are compared with the geometry norm. From Eq. (2.7),

$$\mathrm{i}^{-2s_X}\eta_{AB}U^A_{(X)}U^B_{(X)} = -c^2, \qquad \eta_{AB}k^A_{(X)}k^B_{(X)} = 0. \tag{2.13}$$

The first expression is the geometry norm written through the particle transformation; it is not a particle metric. The null equation needs no compensating factor because zero is unchanged. The numerical value c and the null boundary are identical in all four orientations. A sector-0 observer may assign a side particle a formal coordinate slope greater than c, but that quotient is not the velocity $\beta_X c$ appearing in its own $\Lambda_{(X)}$.

2.6 Cone orientation and the uncrossed null boundary

The four sector domains share their null boundary. Once a particle sector is chosen, $\Lambda_{(X)}$ tilts its clock and ruler axes inside that domain and Eq. (2.13) places light on its boundary. The boost can approach the boundary but cannot cross it. Hence ordinary matter cannot become a side particle by acceleration, a side particle cannot be slowed into ordinary matter, and an anti-sector particle cannot be converted into ordinary matter by changing observer.

This statement is local and independent of n_g. In flat geometry with $n_g = 0$, particles with $s_X = 0, 1, 2, 3$ may coexist at the same event and use the same null set. A side particle is already centred on $+n^a$ or $-n^a$ before any gravitational sector transition occurs. Its apparent ordinary-sector quantity $\mathrm{d}x/\mathrm{d}t$ can be infinite or formally exceed c, but this is a projection on a clock it does not use. Its measured velocity is the finite $\beta_X c$ obtained from its own sector-transformed components.

A geometry-sector transition is different. When the active field chart moves from n_g to $n_g + 1$, the geometry coframe acquires the next phase before each unchanged particle transformation acts on it. Equation (2.10) shifts every N_X together and preserves both s_X and $\Lambda_{(X)}$. Thus an ordinary particle inside side-oriented black-hole geometry has a side-looking total phase $N_X = 1$, but it is still ordinary relative to the local field because $s_X = 0$.

The global speed limit is therefore the conjunction of two facts. Every ordinary Lorentz motion remains inside its own sector domain, and all four domains possess the same null set. No physical process described here moves a particle through that set. What can change is the active geometry chart or the observer used to project the tangent; neither is a measurement of a particle outrunning light in its own sector.

2.7 The sector continuation and the role of i^n

The extended Lorentz transformation is fixed by

$$\mathcal{G}_{(n)}{}^A{}_B(\Lambda) = \mathrm{i}^n \Lambda^A{}_B, \qquad n = 0, 1, 2, 3. \tag{2.14}$$

The phase chooses the orientation component and Λ is the ordinary Lorentz transformation acting inside it. The group law keeps these roles visible:

$$\mathcal{G}_{(m)}(\Lambda_2)\mathcal{G}_{(n)}(\Lambda_1) = \mathcal{G}_{(m+n)}(\Lambda_2\Lambda_1), \qquad \mathcal{G}_{(n)}(\Lambda)^{-1} = \mathcal{G}_{(-n)}(\Lambda^{-1}), \tag{2.15}$$

with sector addition modulo four. Moreover, $\mathcal{G}_{(n)}^T \eta \mathcal{G}_{(n)} = \mathrm{i}^{2n}\eta$: the quadratic sign alternates, but the null equation is unchanged.

For a particle, Eq. (2.14) is used with $n = s_X$ and $\Lambda = \Lambda_{(X)}$. For the geometry, the sector factor appears separately in its coframe as in Eq. (2.6). These must not be collapsed into one unexplained i^n. Only their ordered composition in Eq. (2.7) has the total phase i^{N_X}.

Once a sector is fixed, the derivation in Section 2.2 applies without modification: $\Lambda^T \eta \Lambda = \eta$ preserves the null rays and the measured relative speed remains below c. The four sectors therefore do not carry four different Lorentz laws. They are four orientation components on which the same Lorentz law acts.

The continuous orbit and the discrete operator label must also be distinguished. An oriented axis can approach a null boundary in one chart and recede from it in the next, so its geometric trajectory is continuous. The algebraic factors $1, i, -1, -i$ only label the four normalisation branches used along that trajectory; the operator i^n is not itself a continuous rotation parameter. Section 3.4 gives the single angle and the projective coordinates that cover this orbit without identifying them with a particle speed.

2.8 The interval cycle and its physical meaning

For a generic orientation label $m \in \mathbb{Z}_4$, successive multiplication by the sector phase gives $\mathrm{d}x^\mu$, $\mathrm{id}x^\mu$, $-\mathrm{d}x^\mu$, $-\mathrm{id}x^\mu$, and again $\mathrm{d}x^\mu$. The corresponding interval is

$$\mathrm{d}s_{(m)}^2 = (-1)^m \mathrm{d}s_{(0)}^2. \tag{2.16}$$

For the geometry coframe one sets $m = n_g$. Applying a particle transformation adds the independent phase s_X, so its final local quadratic readout has $m = N_X$. This does not define a particle-dependent metric. The four steps have a direct reading. Sector 0 uses the original differential and the clock $+t$. The first factor i selects the side normalisation and makes the old positive spatial direction the active clock. The second turn gives $-\mathrm{d}x^\mu$ and reverses both inherited axes, producing the $-t$ orientation. The third gives the reflected side clock $-n^a$; the fourth returns the orientation and the differential to their starting values. The phase is therefore a compact account of the complete axis cycle already listed in Eq. (2.5).

In one selected plane, $\mathrm{d}s_{(0)}^2 = -c^2\mathrm{d}t^2 + \mathrm{d}x^2$, while the inherited sector-1 components give $\mathrm{d}s_{(1)}^2 = -\mathrm{d}x^2 + c^2\mathrm{d}t^2$. The sign change is the algebraic record that the old spatial direction occupies the active time slot and the old time direction belongs to the active spatial decomposition. The same exchange with reversed orientation occurs in sector 3.

Equation (2.16) does not prescribe an independent metric field. In the real reading of every active sector the physical line element has one clock direction and three ruler directions. The factor $(-1)^m$ records the sign convention seen in inherited components. The manifold, common null set and scalar curvature invariants still belong to the one geometry.

2.9 What flat spacetime means in every sector

A cone does not make a region curved merely because it appears tilted or sideways in an inherited coordinate diagram. Flatness means that one active clock–ruler tetrad can be extended throughout a neighbourhood without path-dependent rotation and that the corresponding reference geodesics have no tidal acceleration. Equivalently, the second line of Eq. (2.12) holds there. A constant Lorentz tilt applied to every cone changes the observer, but it does not change this property and therefore does not create curvature.

For geometry sector $n_g = 0$, the flat reference congruence follows the $+t$ axes of a family of parallel cones. Geometry sector $n_g = 1$ uses the same flat construction after the time–space roles have been exchanged, so a selected old spatial direction $+n^a$ supplies its reference clock. Curvature vanishes in both descriptions because the coframes remain parallel throughout the neighbourhood.

Particle sectors are superposed on this geometric choice. Even when $n_g = 0$ everywhere, a particle with $s_X = 1$ can have a side-oriented transformation and be at rest with $\Lambda_{(X)} = \mathbf{1}$. A sector-0 chart may draw its clock horizontally and assign it an infinite slope, but this is neither a divergent physical speed nor a change of the background geometry. Particles with all four values of s_X can therefore coexist in one flat region.

Geometry sectors 2 and 3 repeat the two flat references with opposite orientation. The reversal changes which direction is called future, not the vanishing of curvature. Thus all four geometric sector centres and all four constant particle-sector transformations are compatible with one Minkowski field and the same null boundary c.

The sign cycle in Eq. (2.16) is consistent with this statement. Multiplying an interval by -1 leaves its zero set unchanged, so the null rays that separate the sectors are common. Each active geometry coframe places its selected clock in the $A = 0$ slot. The sign change records the exchange of inherited temporal and spatial roles; it does not introduce a second metric field or a second spacetime.

This also clarifies why particle rest is not a curvature criterion. $\Lambda_{(X)} = \mathbf{1}$ concerns one particle transformation, whereas a flat geometric centre requires a whole neighbouring congruence with no relative turning and no geodesic deviation. The latter is a property of an extended region and cannot be inferred from the sector or velocity of one body.

2.10 Null boundaries and continuity

Neighbouring orientation charts meet only at a null direction. Ordinary Lorentz motion remains inside the particle's chosen sector; only light lies on the common boundary. The geometry can reach that boundary and require a neighbouring chart, but this shifts n_g and N_X together rather than changing the relative label s_X.

The geometry orientation itself is nevertheless continuous. Starting at one sector centre, the clock–ruler plane tilts toward a common null ray. The old unit clock ceases to exist exactly at that ray; the neighbouring chart begins at the same oriented limit and follows the plane away from it toward its own centre. The discrete value n_g therefore records which finite chart is being used, not an impulse delivered to an object and not a discontinuity of the underlying trajectory.

No single projective coordinate covers the complete cycle. The explicit relation between the continuous angle, the local geometry coordinate β_g, and the reciprocal rule is derived in Section 3.4. The important distinction at this stage is that β_g locates the field orientation, whereas β_X is the ordinary within-sector velocity contained in $\Lambda_{(X)}$. A side particle can exist without traversing the geometry orbit, and ordinary matter can traverse that orbit without becoming a side particle.

2.11 PT and CPT closure in the same spacetime

After two geometry turns $\mathrm{i}^2 = -1$, so $\mathrm{d}x^\mu_{(2)} = -\mathrm{d}x^\mu_{(0)}$ and the inherited coordinates are read as $(t, \boldsymbol{x}) \mapsto (-t, -\boldsymbol{x})$. Sector 2 is therefore the PT reflection of sector 0, while sector 3 is the corresponding reflection of the side family. A particle with $s_X = 2$ applies the analogous opposite Lorentz-sector transformation even when $n_g = 0$. Charge conjugation, if imposed on the matter fields, acts at the same events, so the construction is compatible with a closed CPT assignment in one spacetime. The sector phase by itself supplies the PT orientation and does not prove that physical antiparticles are identical to the $s_X = 2$ class.

2.12 From a uniform cone field to curvature

Up to this point every statement concerns one flat geometry. The cone axis may be boosted, exchanged with an inherited spatial direction, or reflected, yet the same choice can be repeated uniformly at neighbouring events. All these operations change the projection of the cone field without changing the fact that its axes can be made parallel throughout the region. Sector orientation alone therefore cannot be a source of gravity.

The passage to curved spacetime begins only when the local construction can no longer be extended uniformly. Each individual event still admits the four orientations and the same null boundary, but the active axes selected at neighbouring events need not remain parallel under transport. If their variation can be removed by one coordinate change or one Lorentz transformation over the whole neighbourhood, the geometry is still flat. If transport along different paths gives different relative orientations and neighbouring geodesics acquire tidal acceleration, the cone field is curved.

This is the precise bridge to general relativity. The sector construction supplies the possible local clock axes and the flat rest configuration against which their variation is compared. The connection describes how the axes change from event to event, and the Riemann tensor decides whether that change is merely a frame choice or genuine curvature. The next section therefore does not add gravity to four separate spaces. It projects one curved metric and one curvature tensor onto the four local orientations already derived in flat spacetime. Particles with different s_X perform different tetrad projections of that same tensor; none of them creates a separate curvature field.

The same bridge explains why regularity is required rather than merely hoped for. If the active curvature diverged, neighbouring timelike geodesics could acquire unbounded relative acceleration and no regular sector-rest neighbourhood would remain in which the universal null limit could be applied to arbitrary objects. A sector continuation is therefore physically complete only when its active metric and invariant curvature stay finite. The causal construction identifies the necessary chart; the tensor calculation in the next sections decides whether that chart actually satisfies the requirement.

3 Curved spacetime: active scales and sector charts

The flat orbit determines which local clock orientations must remain available. Curvature adds a new question: which proper spatial mode is being focused toward a null chart boundary? A coordinate matrix cannot answer this invariantly. The construction must live on the observer's spatial rest bundle and must then be embedded back into the full tangent bundle.

This section concerns the sector n_g of the geometry. A particle's s_X does not choose the metric domain and does not alter the tensors used to regularise it. Once the active geometric coframe has been found, particle readouts are obtained from Eq. (2.7); all of them resolve the same curvature.

The order of the construction is fixed. A congruence determines its proper deformation scales; the solution determines the corresponding critical scales; a tensorial eigenproblem selects the active direction; and that direction is paired with the local clock. Only after this geometric identification is the reciprocal chart of the same GR solution selected. No operator introduced below is an extra metric deformation.

3.1 The rest bundle and the tensor of proper scales

Let u^a be the unit clock field of the active geometry sector. Its local rest space and projector are

$$u^a u_a = -1, \qquad h^a{}_b = \delta^a{}_b + u^a u_b, \qquad h_{ab} = g_{ab} + u_a u_b. \tag{3.1}$$

All scale tensors below are endomorphisms of $u^\perp$: contraction with u^a on either index vanishes. This makes their eigenvalues proper spatial scales rather than coordinate coefficients.

Let ξ_0^a and ξ^a be spatial separation vectors between neighbouring worldlines of a reference congruence. A Jacobi map $\mathcal{D}^a{}_b$ relates them, and its polar part defines the positive scale tensor

$$\xi^a = \mathcal{D}^a{}_b \xi_0^b, \qquad C^a{}_b = (\mathcal{D}^\dagger)^a{}_c \mathcal{D}^c{}_b, \qquad L^a{}_b = (C^{1/2})^a{}_b. \tag{3.2}$$

The adjoint is taken with respect to h_{ab}. Consequently $L^a{}_b$ is self-adjoint and positive on $u^\perp$. Its orthonormal eigenvectors $n^a_{(i)}$ and eigenvalues $L_i > 0$ satisfy $u_a n^a_{(i)} = 0$ and $h_{ab} n^a_{(i)} n^b_{(j)} = \delta_{ij}$. Rigid rotation carried by $\mathcal{D}^a{}_b$ is separated from the physical linear scales.

3.2 Critical scales and the active tensor mode

The deformation tensor alone does not say where a sector chart ends. The given GR solution must also provide a positive spatial tensor $G^a{}_b$ with the same dimensions as $L^a{}_b$. It contains the critical scale in each principal direction. The active modes are defined by the covariant generalised eigenproblem

$$G^a{}_b n^b_{(i)} = q_i L^a{}_b n^b_{(i)}. \tag{3.3}$$

When the two tensors share eigenvectors, $q_i = G_i / L_i$. In the orthonormal tetrad formed by u^a and the three principal vectors $n^a_{(i)}$, their embedding in the full tangent space is the explicit 4×4 form

$$[L^A{}_B] = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & L_1 & 0 & 0 \\ 0 & 0 & L_2 & 0 \\ 0 & 0 & 0 & L_3 \end{pmatrix}, \qquad [G^A{}_B] = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & G_1 & 0 & 0 \\ 0 & 0 & G_2 & 0 \\ 0 & 0 & 0 & G_3 \end{pmatrix}. \tag{3.4}$$

The zero time row and column do not mean that time disappears. They state that $L^a{}_b$ and $G^a{}_b$ measure proper spatial scales on $u^\perp$. Their three spatial eigenvectors tell us *which* ruler direction becomes critical. The next subsection supplies the genuinely four-dimensional operator that pairs this direction with the clock.

The number q_i is a geometric scale ratio, not a material velocity. The active chart contains $q_i < 1$; $q_i = 1$ is its null boundary; and a formal value $q_i > 1$ must be represented from the neighbouring sector centre. Schwarzschild reduces this construction to one areal scale, LTB separates longitudinal and transverse modes, FLRW makes all three modes degenerate, and Kerr selects a position-dependent principal plane.

3.3 Covariant embedding in the active time–space plane

Let n^a be a unit eigenvector of the mode that reaches $q = 1$. The projector onto the active plane span$\{u, n\}$ is

$$P^a{}_b = -u^a u_b + n^a n_b. \tag{3.5}$$

Its complement $\delta^a{}_b - P^a{}_b$ is the transverse two-dimensional screen. The projector makes explicit which old ruler is paired with the clock and which two directions remain spectators.

The covariant quarter-turn generator on that plane is

$$J^a{}_b = u^a n_b + n^a u_b, \qquad J^a{}_c u^c = -n^a, \quad J^a{}_c n^c = u^a, \quad J^a{}_c J^c{}_b = -P^a{}_b. \tag{3.6}$$

It vanishes on the transverse screen. Its exponential therefore has the closed tensorial form

$$\mathcal{R}(\alpha)^a{}_b = [\exp(\alpha J)]^a{}_b = \delta^a{}_b - P^a{}_b + \cos\alpha\, P^a{}_b + \sin\alpha\, J^a{}_b. \tag{3.7}$$

At $\alpha = \pi/2$, u^a is carried to $-n^a$ and n^a to u^a; the local clock and active ruler exchange roles while the transverse screen is unchanged.

The reason for using a 4×4 operator is now explicit. If $\boldsymbol{n} = (n^1, n^2, n^3)^T$ contains the spatial components of the selected unit eigenvector, then

$$[J^A{}_B] = \begin{pmatrix} 0 & \boldsymbol{n}^T \\ -\boldsymbol{n} & 0_{3\times 3} \end{pmatrix}. \tag{3.8}$$

For a basis chosen so that $\boldsymbol{n} = (1, 0, 0)^T$, the continuous orientation operator is

$$[\mathcal{R}^A{}_B(\alpha)] = \begin{pmatrix} \cos\alpha & \sin\alpha & 0 & 0 \\ -\sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{3.9}$$

The spatial matrices in Eq. (3.4) select the critical direction and its proper scale. Equation (3.9) says how that particular ruler direction exchanges roles with time while the two transverse directions remain spectators.

The tensor $\mathcal{R}(\alpha)^a{}_b$ records orientation and the selected plane. It is not an ordinary Lorentz boost and is not multiplied into a completed GR coframe to create a new metric. Such an operation would alter the Einstein tensor and manufacture an effective source even in a vacuum solution. Ordinary particle motion inside each active sector remains described by $\Lambda_{(X)}$ from Section 2.2.

3.4 One continuous angle and four finite projective charts

For one principal mode, the two proper scales determine the inclination in the first oriented quarter by

$$\tan\alpha_i = \frac{G_i}{L_i} = q_i, \qquad \alpha_i = \operatorname{atan2}(G_i, L_i). \tag{3.10}$$

The use of atan2 fixes the quadrant of the ordered pair (L_i, G_i); it is not a rapidity and is not a particle velocity. Equation (3.10) gives the angle from the ordinary centre to its first side centre. The full α_i is obtained by continuing the same oriented clock–ruler plane through successive quarters. Its sector centres are

$$\alpha_i = \frac{n_g \pi}{2},$$

and the null boundaries halfway between them are $\alpha_i = (2n_g + 1)\pi/4$.

Each sector uses one finite coordinate centred on its own clock:

$$\beta_{g,i}^{(n_g)} = \tan\left(\alpha_i - \frac{n_g\pi}{2}\right), \qquad \beta_{g,i}^{(n_g+1)} = -\frac{1}{\beta_{g,i}^{(n_g)}}. \tag{3.11}$$

The first formula says exactly what the symbols mean. The global angle α_i specifies one point on the continuous orbit; n_g chooses the nearest sector centre; and $\beta_{g,i}^{(n_g)}$ is the local projective inclination measured from that centre. At the centre it is zero. At either null edge it is $+1$ or -1. The neighbouring chart assigns the opposite boundary value to the same oriented ray and then evolves toward its own zero.

The reciprocal relation in the second formula follows immediately from $\tan(z-\pi/2) = -1/\tan z$. It also explains the apparent infinity of an inactive chart. If the old coordinate is formally extended from its null edge toward the next centre, its value diverges; the active neighbouring coordinate simultaneously tends to zero. Projective infinity is therefore the zero-inclination centre of the adjacent chart, not an infinite physical velocity.

Two beta symbols are kept because they answer different questions. $\beta_{g,i}^{(n_g)}$ locates the geometry on its sector orbit. β_X belongs to the ordinary matrix $\Lambda_{(X)}$ and is the particle's velocity relative to the local geometry inside its fixed sector. A geometry transition changes the first reading and n_g, while preserving s_X, $\Lambda_{(X)}$, and hence the particle's local state. No active chart carries a normalised massive clock through $|\beta_{g,i}^{(n_g)}| = 1$; the chart is replaced at that null boundary.

3.5 Reciprocal normalisation of the active solution

At $q_i = 1$ two adjacent charts meet at the same critical scale. Beyond that boundary the same mode must be measured from the rest centre of the new sector. The tensorial reciprocal rule fixes the boundary and exchanges a large old ratio with a small new one:

$$L_{(n_g+1)}{}^{a}{}_{b} = G^{a}{}_{c}(L^{-1}_{(n_g)})^{c}{}_{d}G^{d}{}_{b}. \tag{3.12}$$

For a principal scalar mode it reads $L_i^{\star} = G_i^2/L_i$. Applying the rule twice returns the original scale, while the orientation continues through the four-step cycle. Moreover, Eq. (3.3) shows that the same eigenvector has the reciprocal ratio $q_i^{\star} = q_i^{-1}$. Thus the old formal limit $q_i \to \infty$ is the zero-ratio rest centre of the neighbouring chart.

The rule acts on the variables with which a known GR solution is evaluated, not as a conformal or matrix correction to its finished metric. Schematically,

$$g_{ab}^{(n_g)} = \mathrm{i}^{2n_g}\,\mathcal{I}^{*}_{n_g} g_{ab}^{\mathrm{GR}}[L_{(n_g)}{}^{a}{}_{b}]. \tag{3.13}$$

The pullback means that the same functional GR solution is written in the active proper scales. The constant factor $\mathrm{i}^{2n_g} = (-1)^{n_g}$ records the sector sign of its tetrad representation; it leaves the Levi–Civita connection and null set unchanged and does not create another metric field on another spacetime. At the critical surface $L_i = G_i$ the unsigned proper scale agrees in both charts. For a single scale $X^{\star} = X_h^2/X$, the Jacobian equals -1 at $X = X_h$; the radial coordinate orientation therefore reverses while invariant curvature remains continuous.

This sign reversal is the differential expression of the clock–ruler exchange. Along a smooth worldline, $\mathrm{d}X^{\star}/\mathrm{d}\tau = -(X_h^2/X^2)\mathrm{d}X/\mathrm{d}\tau$. The sign of one coordinate component changes at the transition, but the tangent itself is carried by the Jacobian of the chart map and remains timelike. There is no reflection of the body and no physical crossing of its null cone.

3.6 One geometry coframe and the particle readout

Let $\mathcal{I}_{n_g} : U_{n_g} \to M$ be the active geometry chart and $\vartheta^A_{\mathrm{GR}}[L_{(n_g)}]$ a coframe of the same GR solution evaluated in its active scale tensor. The geometric sector coframe is

$$\vartheta^A_{(g)} = \mathrm{i}^{n_g}\, \mathcal{I}^*_{n_g} \vartheta^A_{\mathrm{GR}}[L_{(n_g)}]. \tag{3.14}$$

The pullback selects a regular chart of the field, i^{n_g} selects its orientation component, and the coframe of the GR solution already contains the local geometric deformation. None of these operations creates another geometry.

The particle operation is applied only afterward. Combining Eqs. (2.7) and (3.14) gives

$$\mathrm{d}X^A_{(X)} = \mathrm{i}^{N_X} \Lambda_{(X)}{}^A{}_B\, \mathcal{I}^*_{n_g} \vartheta^B_{\mathrm{GR}}[L_{(n_g)}], \qquad N_X = n_g + s_X \pmod 4. \tag{3.15}$$

This is a transformed differential, not a second coframe field assigned to the manifold. Changing n_g first changes the geometry coframe; changing $\Lambda_{(X)}$ changes one particle's motion; and the conserved s_X fixes which extended Lorentz transformation that particle uses.

The metric belongs only to the geometry:

$$g^{(n_g)} = \eta_{AB}\, \vartheta^A_{(g)} \otimes \vartheta^B_{(g)} = \mathrm{i}^{2n_g} \mathcal{I}^*_{n_g} g^{\mathrm{GR}}. \tag{3.16}$$

The sign alternation is the quadratic trace of the sector orientation. It distinguishes even from odd sectors but not the two opposite orientations within either parity class; the missing information remains in $\vartheta^A_{(g)}$. No analogous $g^{(X)}_{ab}$ is introduced.

Before the orientation sign is attached, every physical or geometric field has one value on M and only chart pullbacks

$$\mathcal{F}_{(n_g)} = \mathcal{I}^*_{n_g} \mathcal{F}, \qquad \mathcal{F} \in \{g_{ab}, T_{ab}, R^a{}_{bcd}, \ldots\}. \tag{3.17}$$

The metric readout $g^{(n_g)}$ is obtained from its pullback by the single orientation factor displayed in Eq. (3.16); it is not a second freely specifiable tensor. The global matter configuration may contain several values of s_X, but there is no signal between separate sector worlds because no such worlds exist. Every particle transformation acts on fields at the same events.

3.7 Connection, curvature, and unchanged field equations

Let $\mathcal{I}$ denote an adjacent reciprocal chart map and let $H^A{}_B = \mathrm{i}\Lambda^A{}_B$ be the corresponding coframe transition. The Cartan connection is not a tensor, whereas its curvature two-form is. Accordingly,

$$\begin{aligned} \omega_{(n_g+1)}{}^A{}_B &= H^A{}_C\, \mathcal{I}^* \omega_{(n_g)}{}^C{}_D (H^{-1})^D{}_B - \mathrm{d}H^A{}_C (H^{-1})^C{}_B, \\ \Omega_{(n_g+1)}{}^A{}_B &= H^A{}_C\, \mathcal{I}^* \Omega_{(n_g)}{}^C{}_D (H^{-1})^D{}_B. \end{aligned} \tag{3.18}$$

The derivative term in the first line is required by the change of coframe; its absence from the second line is precisely tensorial covariance [2, 3].

Scalar contractions are therefore pullbacks of one curvature field. For example,

$$K_{(n_g)} = \mathcal{I}^*_{n_g} (R_{abcd} R^{abcd}). \tag{3.19}$$

Adjacent sectors give the same value at the same boundary event. A normal derivative may reverse sign because the reciprocal Jacobian reverses the normal orientation; this is not a jump in the curvature tensor.

Particle X resolves the same Riemann tensor by acting with its $\mathcal{G}_{(X)}$ on the tetrad indices of the geometric components:

$$R_{(X)}{}^{A}{}_{BCD} = \mathcal{G}_{(X)}{}^{A}{}_{E}(\mathcal{G}^{-1}_{(X)})^{F}{}_{B}(\mathcal{G}^{-1}_{(X)})^{G}{}_{C}(\mathcal{G}^{-1}_{(X)})^{H}{}_{D}R_{(g)}{}^{E}{}_{FGH}. \tag{3.20}$$

This is a change of local components, not a particle-dependent curvature field. Fully contracted scalar invariants are unchanged.

The dynamical equations are also single. With U^a_X the tangent of a freely propagating particle and $U^A_{(X)}, k^A_{(X)}$ its sector-transformed local components,

$$U^b_X\nabla_b U^a_X = 0, \qquad \mathrm{i}^{-2s_X}\eta_{AB}U^A_{(X)}U^B_{(X)} = -c^2, \qquad \eta_{AB}k^A_{(X)}k^B_{(X)} = 0, \qquad G_{ab} = \frac{8\pi G_N}{c^4}T_{ab}. \tag{3.21}$$

There is no sector-dependent Einstein tensor or independently adjustable stress tensor. Different s_X values change the measured tetrad components of $R^a{}_{bcd}$ and their split into temporal and spatial parts, but not its scalar invariants. In particular, inserting $\mathcal{R}(\alpha)^a{}_b$ into a completed coframe while keeping η_{AB} fixed would create a different metric. The orientation tensor selects the active plane; the pullback and reciprocal scale select the regular representative of the original solution.

3.8 Worldlines, conserved particle sector, and invariant regularity

A physical worldline is one map $\gamma_X : \tau_X \mapsto M$. Its coordinate representatives in adjacent sectors satisfy

$$\gamma_{X(n_g+1)} = \mathcal{I}\circ\gamma_{X(n_g)}, \qquad U^a_{X(n_g+1)} = \mathcal{I}_* U^a_{X(n_g)}, \qquad s_X^{(n_g+1)} = s_X^{(n_g)}. \tag{3.22}$$

For $X^\star = X_h^2/X$, the Jacobian equals -1 at the common boundary. Hence one displayed radial component can reverse sign while the geometric tangent and the particle's proper-time orientation remain continuous. A pure geometry chart transition also leaves $\Lambda_{(X)}$ unchanged, so a particle at rest in its own sector remains at rest and a moving particle retains the same local boost.

Geometric flatness still requires a statement about a neighbouring congruence, not about s_X or $\Lambda_{(X)}$. Its obstruction is the geodesic deviation equation

$$\frac{D^2\xi^a}{D\tau^2} = -R^a{}_{bcd}U^b_X\xi^c U^d_X. \tag{3.23}$$

Different particle sectors resolve different components of this one equation because their tetrads and tangents differ. A flat geometric centre requires $R^a{}_{bcd} \to 0$ and is therefore flat for every particle-sector projection at once.

The invariant light limit explains why a chart must change at $q_i = 1$: no geometry clock is continued through its null edge. Reciprocal normalisation measures the same mode from the neighbouring geometric centre. It does not change any s_X and does not, by itself, prove regular curvature. Regularity is established only when Eq. (3.19), or an equivalent complete set of invariants, stays finite on the new domain; flatness requires their tensorial source $R^a{}_{bcd}$ to vanish at the centre.

3.9 Several active modes

When the principal directions of $L^a{}_b$ and $G^a{}_b$ commute, the construction reduces locally to independent two-planes and Eq. (3.7) can be applied mode by mode. If two active planes rotate into one another, their generators need not commute. The orientation is then transported along the path as

$$\mathcal{P}\exp\left[\int\sum_i J_{(i)}{}^a{}_b\,\mathrm{d}\alpha_i\right], \tag{3.24}$$

where $\mathcal{P}$ denotes path ordering. This is the natural generalisation of the same map, but a global existence and uniqueness analysis for arbitrary noncommuting tidal modes lies outside the present examples. Schwarzschild, the radial and angular modes of LTB, isotropic FLRW, and the principal plane of Kerr are sufficient to display the geometric content without claiming that the most general case has already been solved.

4 Sector-normalised general-relativistic examples

The examples now use one procedure. The Jacobi or principal structure fixes $L^a{}_b$; the solution fixes $G^a{}_b$; a mode reaches its null boundary at $q_i = 1$; and Eq. (3.12) supplies the neighbouring chart. Only then are the invariants evaluated. No example obtains a new Einstein equation or a metric correction from the orientation matrix. The examples track geometry sectors n_g. Their reference matter is taken to have $s_X = 0$ unless stated otherwise; particles with other conserved s_X values can be transported through the same charts without changing the curvature calculation.

4.1 Schwarzschild: from the ordinary exterior to side rest

Begin with the standard exterior Schwarzschild line element [4]

$$\mathrm{d}s^2 = -f(R)c^2\mathrm{d}t^2 + \frac{\mathrm{d}R^2}{f(R)} + R^2\mathrm{d}\Omega^2, \qquad f(R) = 1 - \frac{r_s}{R}, \quad r_s = \frac{2G_N M}{c^2}, \tag{4.1}$$

where R is the areal radius and $\mathrm{d}\Omega^2 = \mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2$. The coordinate coefficients in Eq. (4.1) are singular at $R = r_s$, but the horizon itself is regular. The sector transition must therefore be performed in a horizon-regular coframe, such as an ingoing Eddington–Finkelstein frame, rather than inferred from the singular diagonal coordinates [5, 6].

Spherical symmetry reduces the sector test to the areal scale. On the observer's rest space its degenerate tensor representative may be written as $L^a{}_b = Rh^a{}_b$, while the Schwarzschild solution supplies $G^a{}_b = r_s h^a{}_b$. Hence every unit direction on the old spatial sphere has the same eigenvalue. In an orthonormal tetrad whose first vector is the local clock, their full embedding is

$$[L^A{}_B]_{\mathrm{Sch}} = \mathrm{diag}(0, R, R, R), \qquad [G^A{}_B]_{\mathrm{Sch}} = \mathrm{diag}(0, r_s, r_s, r_s). \tag{4.2}$$

The zero entry is the time direction; the three equal spatial entries express spherical degeneracy. These matrices do not rotate or deform the completed metric. They only say that any selected spatial ray reaches the same critical areal scale and may then be paired with the clock by Eq. (3.8). The generalised eigenproblem reduces to

$$q(R) = \frac{r_s}{R}. \tag{4.3}$$

This is a geometric compactness ratio, not the measured velocity of a falling body. The ordinary exterior has $q < 1$, its asymptotic rest centre is $q = 0$, and the horizon is exactly the null chart boundary $q = 1$.

The threefold degeneracy is physically important. It says that the old spatial three-space supplies a sphere of admissible side-sector clocks; it does not create three times on one observer's worldline. A radial observer chooses a unit vector s^a in that family. The tensor $J^a{}_b = u^a s_b + s^a u_b$ then rotates the plane selected by $P^a{}_b = -u^a u_b + s^a s_b$. At the quarter-turn the old clock is read as the new ruler and the chosen old ruler as the new clock, while the transverse screen is unchanged. This statement is made in a horizon-regular frame. It does not add a term to the Schwarzschild metric and does not manufacture a second vacuum solution.

At the first horizon it is n_g, not s_X, that changes from 0 to 1. Ordinary infalling matter has $s_X = 0$ on both sides and therefore changes its total label from $N_X = 0$ to $N_X = 1$. To the exterior chart its motion is side-looking; relative to the local field it is still ordinary matter with the same $\Lambda_{(X)}$. A pre-existing side particle would instead retain $s_X = 1$ while its total label changed from 1 to 2.

Axis exchange and the inverse sphere. The axis exchange must be interpreted before introducing another formula. In sector 0, t is the time direction and the old (x, y, z) space is spatial. In sector 1, every direction in that old three-space is an admissible time direction, while the old time direction becomes the spatial coordinate $X = ct$. Spherical symmetry labels the family of side-time rays by (r, θ, φ): the angles select a ray and r parametrises evolution along it. Sector 0 therefore uses $+t$ as its clock direction and treats (x, y, z) as space. Sector 1 reverses these roles at the level of the projection: the possible clock directions form the family $n^a \in u^\perp$ with $h_{ab} n^a n^b = 1$, while $X = ct$ is spatial. The essential point is that not only one distinguished r direction becomes temporal. The whole old spatial $\mathbb{R}^3$ supplies the possible time axes. A particular observer fixes $(\theta, \varphi) = (\theta_0, \varphi_0)$ and follows one of them, so its single proper time is parametrised by r. The observer does not evolve simultaneously along every ray of the family.

The quantity $R(r)$ measures the areal radius of the sphere formed by all side-time directions at the same sector value r. It is not another name for the selected ray parameter. After the exchange, r labels evolution along a chosen old spatial ray, while $X = ct$ is spatial. Consequently $r = 0$ is a value of the new time coordinate, not the origin of the old spatial radius. This change of roles is what makes an infinite-radius sphere at $r = 0$ geometrically meaningful rather than contradictory.

It remains to determine the size $R(r)$ of that sphere. Restricting Eq. (3.12) to any one of the degenerate eigenvectors gives $L^\star = r_s^2/L$. In the ordinary chart $L = r$, so the neighbouring areal scale is fixed, rather than postulated, as

$$\boxed{R(r) = \frac{r_s^2}{r}}, \qquad 0 < r \le r_s, \tag{4.4}$$

which satisfies $R(r_s) = r_s$ and $\mathrm{d}R/\mathrm{d}r|_{r_s} = -1$. The first equality identifies the same horizon event in both charts; the second records their opposite radial orientations. Two operations must not be conflated. The axis exchange explains why $r = 0$ is a temporal sector centre and why the old time coordinate is now spatial. The reciprocal chart relation then gives the value of the simultaneous areal scale there. Together, and only together, they imply that $r = 0$ is a time limit at which the sphere of side-time directions has infinite areal radius. This is not the identification $0 = \infty$: r and R are coordinates with different roles in the two projections. It would be incorrect to read $r = 0$ as the centre of ordinary spherical coordinates after the exchange.

To keep one spacetime manifest, first perform only the radial relabelling in the underlying

Schwarzschild metric. Since $\mathrm{d}R = -r_s^2\mathrm{d}r/r^2$, Eq. (4.1) becomes

$$\mathrm{d}s^2 = -(1 - r/r_s)c^2\mathrm{d}t^2 + \frac{r_s^4\,\mathrm{d}r^2}{r^4(1 - r/r_s)} + \frac{r_s^4}{r^2}\mathrm{d}\Omega^2. \tag{4.5}$$

Equation (4.5) is the pullback of the same metric under $R = r_s^2/r$, not a new sign-exchanged metric. It therefore remains a vacuum solution wherever the chart is regular: $G_{ab}[\mathcal{I}_1^* g] = \mathcal{I}_1^* G_{ab}[g] = 0$. The sector rule separately says that the selected r direction is read as the side time orientation and the old t direction as spatial. That change belongs to the projection of the axes; it must not be implemented by altering g_{ab}, because doing so would change scalar curvature and create a different geometry.

The flat side centre. The angular coefficient in Eq. (4.5) has the invariant meaning of an area: at fixed side time it is $A_{S^2} = 4\pi r_s^4/r^2$. Consequently the side centre contains an infinite sphere of admissible time directions. This follows from the axis assignments just described together with the reciprocal map Eq. (4.4); neither statement is sufficient alone. A curve with fixed X, θ, φ selects one point on each successive sphere. It does not travel around the entire sphere, and its coordinate stationarity is not the definition of geometric sector rest.

Regularity follows from the curvature of the one Schwarzschild metric, not from the axis exchange itself. There are two useful checks. The Newman–Penrose amplitude Ψ_2 measures its Weyl curvature, while K contracts the complete Riemann tensor. Substituting the radius $R = r_s^2/r$ into their ordinary expressions gives

$$\begin{aligned} \Psi_2(R) &= -\frac{r_s}{2R^3}, & \Psi_{2(1)}(r) &= -\frac{r^3}{2r_s^5}, \\ K(R) &= \frac{48G_N^2M^2}{c^4R^6} = \frac{12r_s^2}{R^6}, & K_{(1)}(r) &= \frac{12r^6}{r_s^{10}}. \end{aligned} \tag{4.6}$$

The powers of r are produced solely by this substitution: R^{-3} becomes r^3/r_s^6, and R^{-6} becomes r^6/r_s^{12}. At the common horizon both charts give $K = 12/r_s^4$. Their first radial derivatives have equal magnitude and opposite sign, $\mathrm{d}K/\mathrm{d}R|_{r_s} = -72/r_s^5$ and $\mathrm{d}K_{(1)}/\mathrm{d}r\Big|_{r_s} = +72/r_s^5$, exactly as required by the Jacobian -1. This is a smooth invariant matching with oppositely oriented chart coordinates, not a curvature cusp. The complete side domain $0 \le r \le r_s$ is finite. At its zero-tilt centre $r = 0$, R is infinite and $K_{(1)}$ vanishes. Hence the side projection approaches its flat reference in the sense of Eq. (2.12). The factor r^6 is not a new curvature law: it is the ordinary Schwarzschild invariant evaluated at the sector radius. The regularisation comes from respecting the side domain and its rest centre, which replaces the extrapolation toward $R = 0$ by $R \to \infty$. The tetrad is not involved in this conclusion.

The reflected side branch. The non-negative scalar $r = \sqrt{x^2 + y^2 + z^2}$ cannot record which side of its centre has been reached. The full oriented coordinates do. The continuation from the first to the second branch is the reflection $x_+^\mu = -x_-^\mu$, with $\mathrm{d}x_+^\mu = -\mathrm{d}x_-^\mu$. The coframe changes orientation with the coordinates, so the geometric tangent is continuous and the quadratic metric is unchanged. The apparent turning point of r is therefore not a bounce of the body.

On the first branch r decreases from r_s to 0, while the simultaneous direction sphere grows from $R = r_s$ to $R = \infty$. On the reflected branch r increases from 0 to r_s, while that sphere contracts from infinity to the second null horizon. The centre does not change the geometry sector: both branches have $n_g = 1$ and share the same three-dimensional family of time directions. A chosen radial geodesic uses the representative $+r$ ray. Only at the second null boundary does the active geometry frame acquire $n_g = 2$, whose time orientation is $-t$. In the original exterior language this part of the same continuation is read as white-hole egress. The transition changes the active chart; it does not make the massive tangent cross a light cone or alter its conserved s_X.

4.2 LTB geometry: longitudinal and transverse sector modes

The Lemaître–Tolman–Bondi dust geometry is the simplest example in which a single spherical radius does not describe the full deformation [7, 8]. Using the mass coordinate F, write $R = R(t, F)$, $R_F = \partial R/\partial F$, and $\Gamma(F) = \sqrt{1 + 2E(F)}$. The comoving line element is

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + \frac{R_F^2}{\Gamma^2}\mathrm{d}F^2 + R^2\mathrm{d}\Omega^2. \tag{4.7}$$

The source dust is taken to have $s_X = 0$ relative to its local geometric coframe. This fixes its particle class but does not enter the directional scale tensors below. The proper separation of neighbouring shells is $\mathrm{d}\ell_\parallel = (R_F/\Gamma)\mathrm{d}F$, whereas both transverse scales are set by the areal radius R. Let $u^a = U^a/c$ be the unit dust clock field, s^a the unit radial vector in its rest space, and $S^a{}_b = h^a{}_b - s^a s_b$ the projector on the spherical screen. Because F is used as the mass coordinate, the radial entry below is the scale coefficient multiplying $\mathrm{d}F$, while the two transverse entries are areal scales. The two tensors required by the sector construction are

$$\begin{aligned} L^a{}_b &= \frac{R_F}{\Gamma} s^a s_b + R\, S^a{}_b, \\ G^a{}_b &= s^a s_b + F\, S^a{}_b. \end{aligned} \tag{4.8}$$

Equation (4.8) is basis independent even though its two eigenspaces coincide with the familiar radial and angular directions.

In the comoving orthonormal ordering $(u, s, e_{\hat\theta}, e_{\hat\varphi})$, the corresponding 4×4 matrices are

$$[L^A{}_B]_{\mathrm{LTB}} = \mathrm{diag}\left(0, \frac{R_F}{\Gamma}, R, R\right), \qquad [G^A{}_B]_{\mathrm{LTB}} = \mathrm{diag}(0, 1, F, F). \tag{4.9}$$

The first spatial entry controls the separation of neighbouring shells; the two repeated entries control the spherical screen. The matrices therefore retain two independent degenerations instead of compressing them into one number. Equation (3.3) produces

$$q_\parallel = \frac{\Gamma}{R_F}, \qquad q_\perp = \frac{F}{R}. \tag{4.10}$$

Thus $R_F \to 0$ drives only the longitudinal eigenvalue beyond the ordinary chart and identifies shell crossing, whereas $R \to 0$ drives the two degenerate transverse eigenvalues and identifies shell focusing. The tensor therefore does not disguise two different degenerations under one scalar. For the longitudinal mode the active plane is generated covariantly by $J_\parallel{}^a{}_b = u^a s_b + s^a u_b$; for a transverse mode the same formula uses a unit vector in the screen.

Applying Eq. (3.12) separately on these eigenspaces gives

$$L_\parallel^\star = \frac{\Gamma}{R_F}, \qquad \mathrm{d}\ell_\parallel^\star = \frac{\Gamma}{R_F}\mathrm{d}F, \qquad L_\perp^\star = R^\star = \frac{F^2}{R}. \tag{4.11}$$

The vanishing old shell separation is thereby represented as an unbounded proper scale measured from the adjacent rest centre, while a transverse collapse is continued by the reciprocal areal scale. These are candidate active-scale data for a sector chart of the same LTB metric and matter fields, not rescalings of a completed coframe. Because the two modes need not become critical together, their reciprocal eigenvalues are first local tensor data. Reconstructing them as one global function $R^\star(t,F)$ and one admissible profile $\Gamma^\star(F)$ imposes the usual differential integrability and Einstein–dust constraints. Whether a chosen pair of profiles $E(F)$ and F gives finite density and curvature throughout the reciprocal domain must still be checked from that specific Einstein–dust solution. The tensor formalism separates the two candidate modes; it does not assert a profile-independent LTB regularity theorem.

4.3 Kerr geometry: the principal sector plane with frame dragging

Rotation changes the active plane, but not the reciprocal construction. In this subsection only, set $G_N = c = 1$. Kerr has no globally preferred radial direction supplied by spherical symmetry. The active plane is instead selected locally by the repeated principal null directions, equivalently by the principal eigenspace of the Weyl operator on bivectors. If Π_{ab} denotes its real principal bivector, the corresponding self-dual object is $\mathcal{U}_{ab} = \Pi_{ab} + \mathrm{i}({}^\star\Pi)_{ab}$. Its real time–radial plane determines a projector $P^a{}_b = -u^a u_b + n^a n_b$ and the same generator $J^a{}_b = u^a n_b + n^a u_b$ as in Eq. (3.6). The new ingredient is that both tensors now vary with position.

In a principal orthonormal tetrad $(u, n, e_{\hat{2}}, e_{\hat{3}})$, this geometric content has the local 4×4 representation

$$[P^A{}_B]_{\mathrm{K}} = \mathrm{diag}(1,1,0,0), \qquad [J^A{}_B]_{\mathrm{K}} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{4.12}$$

Unlike a Schwarzschild coordinate basis, this principal tetrad rotates from point to point. Equation (4.12) is therefore a local tensor matrix, not one constant global array. It identifies the time–principal plane on which the sector orientation acts while leaving the transverse screen to carry the remaining Kerr geometry and frame dragging.

A useful scalar detector of a horizon crossing is

$$q_H(R) = \frac{2MR}{R^2 + a^2}, \qquad q_H = 1 \quad \Longleftrightarrow \quad \Delta := R^2 - 2MR + a^2 = 0. \tag{4.13}$$

This detector identifies both Kerr horizons; it is not used as a material speed and is not assumed to be a generalised eigenvalue of one globally defined pair $L^a{}_b, G^a{}_b$. It only locates the null boundary selected by the principal geometry. For the continuation considered here, the reciprocal radial eigenvalue is anchored at the outer root. With $r_+ = M + \sqrt{M^2 - a^2}$, Eq. (3.12) gives

$$R(r) = \frac{r_+^2}{r}, \qquad 0 < r \le r_+. \tag{4.14}$$

The cross term $g_{t\varphi}$ means that the active clock is the local horizon generator in a corotating, horizon-regular frame, not the coordinate one-form $\mathrm{d}t$ by itself [9]. The tensor $\mathcal{R}(\alpha)^a{}_b$ rotates that clock with the principal vector n^a; the transverse screen and all frame-dragging data remain part of the same Kerr metric. Equation (4.14) changes the radial argument and domain of its sector representative, not its scalar geometry. Thus Kerr does not require an additional tetrad swap rule: its local quarter-turn is already the covariant map of Eq. (3.7), with the principal plane supplying the active direction.

Unlike the Schwarzschild radial projector, the Kerr principal projector $P^a{}_b(x)$ varies across spacetime. Its covariant derivative separates as

$$\nabla_c(\alpha P^a{}_b) = (\nabla_c\alpha)P^a{}_b + \alpha\nabla_c P^a{}_b. \tag{4.15}$$

The first term advances the sector angle; the second rotates the active plane itself and contains the geometric twist associated with frame dragging. It is already part of the Kerr connection and requires neither a new metric nor an extra sector field.

The principal Weyl amplitude makes the substitution especially transparent [10]. In the ordinary principal tetrad it is $-M/(R-\mathrm{i}a\cos\theta)^3$. Replacing R by r_+^2/r and clearing the factor $1/r$ from the denominator gives

$$\Psi_2(R,\theta) = -\frac{M}{(R-\mathrm{i}a\cos\theta)^3}, \qquad \Psi_{2(1)}(r,\theta) = -\frac{Mr^3}{(r_+^2-\mathrm{i}ar\cos\theta)^3}. \tag{4.16}$$

The denominator remains finite at $r=0$, while the numerator contains r^3; the complete Weyl amplitude therefore vanishes at the side centre.

The same conclusion must also be checked without relying on one tetrad component. The ordinary Kerr Kretschmann scalar is

$$K_{\mathrm{K}}(R,\theta) = 48M^2\frac{R^6-15a^2R^4\cos^2\theta+15a^4R^2\cos^4\theta-a^6\cos^6\theta}{(R^2+a^2\cos^2\theta)^6}. \tag{4.17}$$

Substitution of $R=r_+^2/r$ gives the ordinary Kerr scalar along the domain sampled by the reciprocal chart,

$$K_{\mathrm{K}}\left(\frac{r_+^2}{r},\theta\right) = 48M^2r^6\frac{r_+^{12}-15a^2r_+^8r^2\cos^2\theta+15a^4r_+^4r^4\cos^4\theta-a^6r^6\cos^6\theta}{(r_+^4+a^2r^2\cos^2\theta)^6}. \tag{4.18}$$

To obtain this expression, the numerator of Eq. (4.17) is written with a common factor r^{-6} and its denominator with r^{-12}. Their ratio leaves the displayed overall factor r^6; no extra curvature law has been introduced. For a Kerr black hole $r_+>0$, the denominator is nonzero on $0\le r\le r_+$ and the overall factor r^6 makes the scalar vanish at $r=0$. The usual Kerr ring requires $R=0$ and $\theta=\pi/2$, whereas the sector domain has $R\ge r_+$ and approaches $R=\infty$. It therefore does not sample the ring. Because K_{K} is a scalar of the one metric, Eq. (4.18) is already its complete sector representation; no second "actively exchanged" invariant is required.

The non-rotating limit is immediate: $a=0$ gives $r_+=2M=r_s$ and $K_{\mathrm{K}}=48M^2r^6/r_s^{12}=12r^6/r_s^{10}$, precisely the Schwarzschild result in Eq. (4.6).

The continuation after the Kerr centre uses the same oriented reflection as the Schwarzschild case. The tetrad, spin, and frame-dragging data are transported together; there is no independent replacement $a\to-a$. Frame dragging decays at the flat centre and grows again toward the second horizon. There the geometry chart with $n_g=2$ and clock orientation $-t$ takes over, just as in the non-rotating case. A particle transported through this cycle retains its own s_X.

4.4 Spatially flat FLRW: reciprocal scale and the meaning of $t=0$

The FLRW example uses exactly the same sector mechanism as the black-hole examples. The only difference is the geometric quantity that is inverted. For a black hole it is the areal radius; for a homogeneous universe it is the scale factor.

In the ordinary $+t$ projection, write the spatially flat metric as

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + a^2(t)\left(\mathrm{d}\chi^2 + \chi^2\mathrm{d}\Omega^2\right). \tag{4.19}$$

Here $a(t)$ is the ordinary scale factor [11]. Let a_h and ρ_h denote the finite scale factor and density at the null boundary where the ordinary projection ends and the side projection becomes active. The comoving fluid is assigned $s_X = 0$ relative to the active geometric coframe. A change of n_g therefore reorients the fluid and the field together without changing the fluid's particle class.

Homogeneity and isotropy make all three principal modes identical. With u^a the comoving clock and $h^a{}_b$ its rest projector, the complete tensor statement is

$$L^a{}_b = a(t)h^a{}_b, \qquad G^a{}_b = a_h h^a{}_b. \tag{4.20}$$

In a comoving orthonormal tetrad this is

$$[L^A{}_B]_{\mathrm{FLRW}} = \mathrm{diag}(0, a, a, a), \qquad [G^A{}_B]_{\mathrm{FLRW}} = \mathrm{diag}(0, a_h, a_h, a_h). \tag{4.21}$$

All three spatial generalised eigenvalues are $q_i = a_h/a(t)$. Their degeneracy means that every spatial direction reaches the sector boundary together. The 4×4 embedding displays the common clock and the isotropic spatial block without inventing a preferred cosmological axis.

Equation (3.12) gives $L_{(1)}{}^a{}_b = [a_h^2/a(t)]h^a{}_b$, so the neighbouring scale is $a_{(1)} = a_h^2/a_{(0)}$. At the boundary $a(t) = a_h$, both descriptions therefore give the same scale. If τ is proper time along the side-sector clock and $t(\tau)$ labels the same events in the inactive ordinary chart, the pullback representative is

$$\mathcal{I}_1^* g_{\mathrm{FLRW}} = -c^2\mathrm{d}\tau^2 + \left(\frac{a_h^2}{a(t(\tau))}\right)^2 \left(\mathrm{d}\chi^2 + \chi^2\mathrm{d}\Omega^2\right). \tag{4.22}$$

This is one metric written in the active chart, not an independently chosen side metric. No complex time coordinate is introduced. Toward the event labelled $a(t) = 0$ by the inactive ordinary chart, its active scale grows rather than shrinks.

For a fluid with $p = w\rho c^2$, covariant energy conservation gives $\mathrm{d}\ln\rho/\mathrm{d}\ln a = -3(1+w)$. Integrating from the common boundary (a_h, ρ_h) and then inserting the ordinary and reciprocal scales gives

$$\rho_{\mathrm{ord}}(t) = \rho_h\left(\frac{a_h}{a(t)}\right)^{3(1+w)}, \qquad \rho_{\mathrm{side}}(t) = \rho_h\left(\frac{a(t)}{a_h}\right)^{3(1+w)} = \frac{\rho_h^2}{\rho_{\mathrm{ord}}(t)}. \tag{4.23}$$

These are geometry-chart representatives of one scalar field: $\rho_{(n_g)} = \mathcal{I}_{n_g}^*\rho$. The two displayed expressions evaluate the same conservation law at the ordinary scale $a(t)$ and its reciprocal $a_h^2/a(t)$; they are not independently specifiable densities.

For the usual big-bang fluids, $w > -1$. As the inactive ordinary label tends to $t = 0^+$ and $a(t)$ tends to zero, its density expression diverges, whereas the reciprocal side scale becomes infinite and the side density vanishes. Thus the divergent ordinary expression and the empty side-sector centre are two projections of the same geometric limit. Nothing jumps from infinite density to zero density inside one active chart.

The curvature calculation now uses the same matter field, evaluated on the active scale. For a spatially flat FLRW solution, the two Friedmann equations needed below are

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3}\rho, \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}(1+3w)\rho. \tag{4.24}$$

With a denoting the scale factor of whichever projection is active, its Kretschmann scalar is obtained by substituting these two expressions:

$$K = \frac{12}{c^4}\left[\left(\frac{\ddot{a}}{a}\right)^2 + \left(\frac{\dot{a}}{a}\right)^4\right] = \frac{64\pi^2 G_N^2}{3c^4}\left[(1+3w)^2+4\right]\rho^2. \tag{4.25}$$

The squared acceleration term produces the factor $(1+3w)^2$; the squared Hubble term produces the remaining 4. In the side projection, inserting $\rho_{\rm side}$ from Eq. (4.23) therefore gives

$$K_{\rm side} = \frac{64\pi^2 G_N^2}{3c^4}\left[(1+3w)^2+4\right]\rho_h^2\left(\frac{a(t)}{a_h}\right)^{6(1+w)}. \tag{4.26}$$

For $w > -1$, the factor involving $a(t)$ makes $K_{\rm side}$ vanish at the side centre labelled by $t = 0$ in the inactive ordinary chart. Spatially flat FLRW has vanishing Weyl tensor, so all of its curvature is Ricci curvature. The vanishing of $K_{\rm side}$ therefore means that the complete Riemann tensor tends to zero at the side-sector centre, $R^a{}_{bcd} \to 0$.

The phrase "passing through $t = 0$" does not mean continuing the singular ordinary formula from positive to negative t. The $+t$ projection first enters a side branch, reaches its flat centre, and leaves through the reflected branch before the $-t$ projection becomes active. The point labelled $t = 0$ by the inactive ordinary formula is therefore the flat centre between two branches of the same side sector. The opposite $-t$ sector begins only after the reflected side branch reaches the next null boundary.

The same continuation can be read in the physical direction that produces the expanding $+t$ branch. The relevant part of the complete geometry cycle is $n_g = 2 \to 3 \to 0$. The $n_g = 2$ chart has clock orientation $-t$, so a future-directed contraction there lies in what the ordinary $+t$ projection calls its past. After continuation through the flat centre of the $n_g = 3$ chart, the active clock returns to $+t$ and the same evolution is read as expansion toward the ordinary future. The sequence $n_g = 2, 3, 0$ is therefore read as contraction, flat side-sector centre, and expansion. In this sense, the big-bang limit of the ordinary projection is generated by the continuation of a contracting branch lying in its past. Reversing the active time orientation reverses the physical reading of contraction into expansion. The conserved s_X of every comoving particle is unchanged. No second universe or independently created object is introduced: the contracting configuration, the side-sector continuation, and the expanding universe are three projections of one geometric history and the same tensor fields.

4.5 Flat geometric centres and conserved particle sectors

The black-hole and FLRW examples use different coordinates, but the same three-stage distinction applies: a geometry chart changes at a null boundary, the chart reaches its flat centre, and it continues along the reflected branch before the next null boundary is reached. None of these steps changes a particle's relative sector.

For Schwarzschild and Kerr, the side-geometry parameter r decreases from the first horizon to $r = 0$, while the active areal radius grows to $R = \infty$. The value $r = 0$ is therefore the time centre of the $n_g = 1$ chart, not the ordinary spatial point $R = 0$. Curvature vanishes there because the one metric is evaluated at $R \to \infty$. The reflected branch still has $n_g = 1$ and returns to a second horizon; only that null boundary activates $n_g = 2$.

For FLRW, the inactive ordinary chart labels the same kind of limit by $t = 0$. The active reciprocal scale tends to infinity, while density and curvature tend to zero. The reflected side branch returns from that flat centre to the next null boundary. Again, $t = 0$ is not a direct jump from $n_g = 1$ to $n_g = 2$ and is not a particle conversion event.

What the particle carries through the centre. At every point of either branch, Eq. (2.7) remains valid. If the particle was at rest in its own sector, $\Lambda_{(X)} = \mathbf{1}$ remains true. If it had a finite boost $\Lambda_{(X)}$, the same boost relates it to the new local geometric coframe. Its relative label s_X is unchanged at the horizon, at the flat centre, and at the second horizon. The full orientation cycle is therefore

$$
\begin{array}{c|ccccc}
\text{geometry sector } n_g & 0 & 1 & 2 & 3 & 0 \\
\hline
N_X \ (s_X = 0) & 0 & 1 & 2 & 3 & 0 \\
N_X \ (s_X = 1) & 1 & 2 & 3 & 0 & 1 \\
N_X \ (s_X = 2) & 2 & 3 & 0 & 1 & 2 \\
N_X \ (s_X = 3) & 3 & 0 & 1 & 2 & 3.
\end{array}
\tag{4.27}
$$

Each row is shifted when n_g changes, but its fixed offset from the first row is s_X. Ordinary matter passing through a side geometry therefore has a side total phase without becoming a tachyon. A tachyon remains a tachyon even when its total phase is temporarily 2 or 0, and an anti-sector particle retains $s_X = 2$ throughout. The table records the ordered product $\mathrm{i}^{s_X} \Lambda_{(X)} \circ \mathrm{i}^{n_g}$ acting on a geometric differential. It does not merge the geometry phase and particle phase into a single physical sector and does not assign a particle its own metric.

Completion of the geometry orientation cycle. The geometry itself follows the orientation sequence $+t, +n^a, -t, -n^a, +t$, where n^a is the active spatial eigenvector selected by the local curvature. The first and third changes occur at null boundaries; the flat centre lies inside the corresponding side chart and joins its two branches. After four chart orientations the geometry returns to $n_g = 0$. Equation (4.27) shows that every particle simultaneously returns to its original total orientation while retaining its relative class at every intermediate stage.

The completed cycle therefore closes representations of one spacetime and one matter configuration. It does not create four universes, it does not require all particles to have the same s_X, and it does not force a material worldline to traverse the entire geometry orbit in finite proper time.

5 One global field: black holes, cosmology, and closed time

5.1 Four readings do not create four histories

The pullback relation in Eq. (3.17) is not merely a notational convenience. It says that the metric, matter, records and worldlines form one global configuration. If two sector charts contain the same event, they contain the same field value at that event. One description cannot retain a traveller or a decision that is absent from the other.

There is consequently no signal transmitted "between sectors." A change of the physical configuration changes all of its representatives for the same reason that changing a tensor changes its components in every chart. The sector construction is deeper than an ordinary coordinate change because it also changes which local direction is read as time, but it still introduces neither another manifold nor another independently adjustable copy of matter.

This one-field condition does not identify particles with different s_X. They are distinct members of the same matter configuration and may coexist at one event. What must agree across geometry charts is the identity, state, s_X, and $\Lambda_{(X)}$ of each particle. A chart cannot describe the same traveller with $s_X = 0$ while another assigns it $s_X = 1$.

5.2 Proper time and the black–white reading

Every material observer satisfies the local normalisation in Eq. (2.13) and experiences increasing proper time along its own sector-transformed clock. In a geometry chart with $n_g = 2$, however, a future-directed tangent can have a negative component along the inactive ordinary coordinate t. The observer then moves toward what sector 0 calls its past without reversing its own clock, memory, or local causal order. The reversal concerns the projection on another sector's axis, not the sign of proper time.

The same distinction applies to compact objects. A black hole in the $+t$ reading and the corresponding white hole in the $-t$ reading are opposite geometry-sector orientations of one object. This must not be confused with an individual particle carrying $s_X = 2$. The reversal is global: when n_g changes by two, every black-hole entrance in that representation is read as a white-hole exit in the opposite one. It is therefore inconsistent to reverse one chosen hole while retaining all other holes as independent entrances. A traveller cannot assemble a time machine by freely chaining objects that the one global orientation transforms simultaneously.

The complete black-hole continuation derived in Section 4 is accordingly one worldline: ordinary infall, the first side branch, its flat rest centre, the reflected side branch, and the $-t$ reading. Its tangent is carried from chart to chart by Eq. (3.22). The words "infall" and "egress" change with the active time orientation; the event and material state do not. Ordinary infalling matter remains $s_X = 0$ throughout, even while its total phase becomes side or opposite.

5.3 Open histories and exact closed timelike curves

The geometry-frame orbit $n_g = 0 \to 1 \to 2 \to 3 \to 0$ closes its orientation, and Eq. (4.27) closes every total particle phase. Neither fact identifies two events on a material worldline. A complete history has one of two global types. An open timelike curve never returns to precisely the same event and state. An exact closed timelike curve is periodic as a complete Einstein–matter configuration: after one circuit the event, tangent, s_X, $\Lambda_{(X)}$, surrounding fields and geometry all agree.

These types cannot be spliced locally. An open traveller does not become a CTC merely because the geometry enters a side chart, and $s_X = 1$ does not make a side particle a CTC. Conversely, an exact CTC cannot have a first entrance, a last circuit, or an escape tail; any such tail would assign two different continuations to the same repeated state. Its material history is the whole loop and has no external causal segment [12, 13]. A mismatch of position, momentum, memory, or environment does not produce an "almost CTC" but an open curve.

This also shows why the construction does not establish the existence of a CTC. A periodic trajectory would have to arise as a globally matched solution of the coupled field equations, not from local frame closure or from a sequence of independently selectable particle sectors, black holes and white holes. Chronology horizons, stability and quantum backreaction remain additional dynamical questions [14].

5.4 Changing the past without a second history

The phrase "changing the past" must not introduce an unobservable meta-time in which a complete history A exists first and is later overwritten by a complete history B. Within the one-field construction there is only one globally compatible solution at a time. A local decision made while moving toward decreasing ordinary t is a physical event, but its consequences must be solved together with the rest of the metric and matter data.

If that decision removes the conditions required for the journey, the compatible history is an open solution without the journey. It contains no surviving traveller, memory, or record imported from a discarded copy of the future. If the history is an exact CTC, the decision and all physical states on the loop must reproduce themselves and cannot eliminate the loop on one circuit. A hybrid in which the same state both remains periodic and escapes is not a paradoxical solution; it is inconsistent boundary data.

The familiar grandfather contradiction is therefore reformulated as a global existence question. Either the proposed data belong to a self-consistent open solution, or they belong to a self-consistent periodic solution, or no solution satisfies them. The construction excludes coexisting old and new histories because its sectors are representatives of one field. It does not prove uniqueness of the corresponding Einstein–matter boundary-value problem.

5.5 Big Crunch and Big Bang as one transition surface

The FLRW calculation has the same global interpretation. A contracting configuration read with one time orientation and an expanding configuration read with the opposite orientation are not two universes joined by a creation event. Big Crunch and Big Bang are the two temporal readings of one sector transition surface. Scalar fields such as density and curvature agree under the pullback, while quantities odd under time orientation, such as the signed expansion, reverse their reading.

Matter is therefore neither destroyed on the contracting side nor recreated on the expanding side. It is one stress–energy field whose flux is resolved against opposite geometry clocks. The relative sectors of its constituent particles are carried through unchanged. Asking what happened "before" the Big Bang by continuing the $+t$ clock through that surface presupposes the very clock that has ceased to be active. The invariant question is instead which branch of the one spacetime the sector map associates with the expanding branch. In the symmetric FLRW continuation considered here, it is the time-reversed reading of collapse. This is an interpretation of the constructed global atlas; establishing a realistic cosmological solution still requires matching the full matter model across its sector charts.

6 Possible quantum-field-theory consequence

The classical construction allows a particle to carry the discrete relative label s_X in addition to its ordinary Lorentz data. It does not yet provide a quantum operator whose eigenvalues are those labels, an interaction law that conserves them, or a spectrum showing that side-sector asymptotic particles exist. In particular, the word "tachyon" used here denotes a sector orientation and not the standard analytic continuation to a negative mass squared.

The same caution applies to antiparticles. The orientation $s_X = 2$ supplies the PT-reversed transformation class, whereas a physical positron is defined by charge conjugation and the quantum field's representation. Equating these objects would require a derivation of charge, positive frequency and CPT; it does not follow from the phase $\mathrm{i}^2 = -1$ alone.

Virtual internal lines remain a possible place where the sector description could become useful. Their momenta need not obey the mass-shell condition of external states, and a fermion loop admits oppositely oriented descriptions [15, 16]. A future sector theory might organise such segments by s_X, but this is an interpretation rather than a result of the classical geometry. It would not make an internal line a detectable superluminal particle or a controllable channel to the ordinary past.

A quantum completion would need sector-resolved propagators, interaction selection rules, a consistent mass shell and positive-frequency definition, charge conservation, spin–statistics and unitarity. The curvature regularisation and the one-field causal conclusions of the present paper do not depend on that programme.

The complex phases also require an explicit reality condition in any such completion. Here i^n labels orientation: the active metric representation has only the real sign i^{2n}, the null equation remains zero, and the curvature scalars evaluated in the examples are real. A field theory would additionally have to guarantee real proper times, energies, densities, charge fluxes and probabilities. The phase notation alone is not that guarantee.

7 Physical meaning and scope

The construction separates three questions that are often conflated. Particle kinematics specifies the conserved relative class s_X and the ordinary boost $\Lambda_{(X)}$ within it. Geometry continuation specifies the active chart n_g when an inherited clock reaches a null orientation. Dynamical regularity asks whether the one metric has finite curvature on the corresponding domain. A side particle can exist without a geometry-sector transition, and a geometry-sector transition does not make ordinary matter a side particle. Only the third question can establish regularisation.

The dependence of the argument is consequently fixed. The invariant null set supplies four orientation components. A particle at rest has $\Lambda_{(X)} = \mathbf{1}$ in one of them, whereas a flat geometric centre requires $R^a{}_{bcd} = 0$ throughout the limiting neighbourhood. The tensor calculation, not the particle label, decides whether a curved solution reaches that centre. This order prevents reciprocal maps from being chosen merely to remove a singularity.

Curvature has two operational projections. Changing n_g can select a different regular domain of the same solution, while changing s_X or $\Lambda_{(X)}$ changes the components measured by a particle at a fixed event. Neither changes the scalar invariant at that event. Disagreement of scalars on a chart overlap would therefore signal two geometries rather than two representations; disagreement between particle-sector components alone would not.

The reciprocal maps used in the examples should consequently be judged as parts of an atlas, not as replacements for Einstein's equation. A proposed sector continuation is admissible only if its overlap at the null boundary is smooth, its metric and matter data represent one solution, its geodesics can be continued in the active chart, and its invariants remain finite throughout that chart. Approaching zero curvature at a sector centre is a stronger and physically transparent result, but it does not by itself prove global geodesic completeness or stability.

The directional formulation makes this criterion local and calculable. The scale tensor $L^a{}_b$ identifies which proper separation is becoming critical, $G^a{}_b$ fixes the solution-dependent null boundary, and $J^a{}_b$ identifies the clock–ruler plane that changes role. Only the reciprocal scale is used to select the regular representative of the known GR solution. The orientation tensor is never used to manufacture a new metric, which prevents a frame convention from being mistaken for a new stress tensor or for curvature regularisation.

The same one-spacetime requirement controls the later causal interpretation. Opposite orientations cannot carry independently edited histories, and closure of the frame cycle is not closure of a worldline. A macroscopic CTC would require a periodic solution for the complete traveller–geometry state; the microscopic loop is only a possible QFT interpretation. In addition, the present classical work treats s_X as conserved kinematic data; a theory of interactions that can create, annihilate or convert particle sectors has not been supplied.

8 Conclusion

The four-sector construction is most naturally understood as a completion of one Lorentzian orientation, not as the addition of four spacetimes. Once this is imposed, the formal order becomes unambiguous. The geometry first supplies its sector coframe. A particle then acts on that coframe with its own sector-labelled Lorentz transformation. Only the final component readout contains the sum of the two phases. A change of geometry chart can therefore carry the whole local field through the orientation cycle without changing the particle's class or its ordinary motion.

The common null boundary has two complementary consequences. Within a fixed particle sector it is an uncrossed speed limit: ordinary acceleration never turns matter into a side-sector particle. For the geometry it is the edge of one clock chart and the entrance to the neighbouring description of the same field. The 4×4 orientation operator identifies which ruler direction is paired with time, but it does not alter the metric. The reciprocal scale map performs the distinct job of selecting the active variables in which the known GR solution is continued.

Regularity is consequently decided after, not by, the sector rotation. Schwarzschild and Kerr approach their asymptotically flat curvature limit when the inactive zero-radius label is replaced by an infinite reciprocal areal scale. FLRW similarly replaces a vanishing inactive scale by an expanding side scale, while LTB shows why longitudinal and transverse degenerations must be kept separate. These examples support the construction because their invariants can be evaluated explicitly. If an invariant diverged inside an active reciprocal domain, the sector kinematics would not conceal or repair that failure.

The insistence on one spacetime also controls the global interpretation. Opposite time readings do not create editable copies of an event, and different particle sectors are different members of one matter configuration, not different histories. A worldline is either open or a complete periodic solution; closure of the orientation cycle alone cannot make it a CTC. The same one-field requirement links the black–white and collapse–expansion readings without introducing a second universe or a meta-time in which one completed past is overwritten by another.

The remaining work is dynamical. Reciprocal charts must be embedded in full Einstein–matter solutions, their null overlaps and geodesic completeness must be established, and the reality, interaction, stability and quantum conditions for particle sectors must be derived. The present result is the coherent classical starting point for that programme: one causal geometry, two explicitly separated sector operations, and invariant curvature as the criterion that no change of frame can evade.

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