Transformation of an Equilateral Irregular Dodecagon to a Regular Icositetragon via Radiocentric Mapping on a Duodecimal Lattice

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Abstract

This paper demonstrates a geometric transformation that maps an irregular, equilateral 12-sided polygon (dodecagon) to a perfectly regular 24-sided polygon (icositetragon). By constructing the initial Cartesian plane over a duodecimal (base-12) lattice, we eliminate decimal truncation errors and establish exact coordinates for side lengths uniformly measuring $\frac{5}{12}\sqrt{2}$ (expressed as $0.5\sqrt{2}_{12}$). We show that scaling this system by a dilation factor of 4, followed by a radiocentric rearrangement of the segments along a splayed 48-radial symmetry matrix—derived from the classical Flower of Life construct—forces the perimeter segments into an equiangular configuration. A formal geometric proof is presented to confirm that the resulting transformed figure is perfectly equilateral and equiangular, thereby achieving complete regularity.

1 Introduction

In classical Euclidean geometry, the study of equilateral polygons that lack equiangularity—often analyzed within kinematic rigidity theory and polygonal linkage constraints—frequently encounters arithmetic limitations when mapped to standard base-10 coordinate grids. Irrational edge lengths involving square roots inevitably require rounding or limit approximations when described numerically.

To bypass these constraints, this paper introduces a coordinate framework anchored entirely within a duodecimal (base-12) Cartesian plane. In this system, the unit interval is partitioned uniformly into twelfths rather than tenths, such that any fractional position 0.1_{12} represents exactly $\frac{1}{12}$. Within this space, we define an initial closed, irregular equilateral polygonal chain consisting of 12 vertices. Every individual edge length is exactly defined by the irrational value:

$$a = \frac{5}{12}\sqrt{2} = 0.5\sqrt{2}_{12}$$

This specific configuration possesses a predictable, measurable perimeter but exhibits spatial irregularity due to asymmetric vertex angles across the four quadrants.

The primary objective of this paper is to detail the rigorous mathematical steps required to transform this irregular system into a higher-order regular polygon. We apply a uniform dilation transformation to scale the entire framework by a factor of 4. We then introduce an angular constraint template utilizing 48 symmetrically bisected radials—a configuration geometrically congruent to the intersecting circles of the historic *Flower of Life* pattern.

By systematically mapping the expanded segments perpendicular to these symmetrical degree lines, we prove analytically that the boundary constraints converge. The final sections of this paper provide the exact coordinate mappings and a geometric proof demonstrating that this transformation preserves perimeter continuity while forcing complete equiangularity, resulting in a perfectly regular icositetragon.

2 The Base-12 Coordinate Framework

To establish a completely rigorous foundation for the initial irregular dodecagon, we formalize the coordinate positions within a duodecimal Cartesian plane $\mathbb{Z}^2_{12}$. Let the radix point denote positional steps in fractions of twelfths, where a coordinate written as $(x, y)_{12}$ translates to the base-10 fractional value $(\frac{x_{10}}{12}, \frac{y_{10}}{12})$.

We define the geometry of the first quadrant using four foundational vertices, denoted as P_1, P_2, P_3, and P_4. Due to the underlying symmetry of the construction, these points are mirrored across the remaining three quadrants to complete the 12-sided closed loop. The exact duodecimal coordinates are specified as follows:

$$P_1 = (1.1, 0.0)_{12} = \left(\frac{13}{12}, 0\right)_{10}$$

$$P_2 = (1.0, 0.7)_{12} = \left(\frac{12}{12}, \frac{7}{12}\right)_{10}$$

$$P_3 = (0.7, 1.0)_{12} = \left(\frac{7}{12}, \frac{12}{12}\right)_{10}$$

$$P_4 = (0.0, 1.1)_{12} = \left(0, \frac{13}{12}\right)_{10}$$

2.1 Edge Length Verification of the Linear Segments

To verify that this irregular polygon is perfectly equilateral, we apply the Euclidean distance formula modified for the duodecimal lattice. We analyze the two distinct edge profiles present in the first quadrant: the interior diagonal segment $\overline{P_2P_3}$ and the outer segment $\overline{P_1P_2}$.

Theorem 2.1: *The length of the interior segment $\overline{P_2P_3}$ is exactly $0.5\sqrt{2}_{12}$.*

Proof. Let Δx and Δy represent the coordinate differences between $P_2(1.0, 0.7)_{12}$ and $P_3(0.7, 1.0)_{12}$. Computing the differences in base-12 arithmetic yields:

$$\Delta x = 1.0_{12} - 0.7_{12} = 0.5_{12}$$

$$\Delta y = 1.0_{12} - 0.7_{12} = 0.5_{12}$$

Substituting these spatial intervals into the Pythagorean distance metric:

$$d(P_2, P_3) = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(0.5_{12})^2 + (0.5_{12})^2}$$

Converting strictly to base-12 fractional arithmetic, where $(0.5_{12})^2 = \frac{5}{12} \times \frac{5}{12} = \frac{21_{12}}{100_{12}} = 0.21_{12}$:

$$d(P_2, P_3) = \sqrt{0.21_{12} + 0.21_{12}} = \sqrt{0.42_{12}}$$

Factoring out the perfect duodecimal square 0.25_{12} (which is equivalent to $\frac{25}{144}$ in base-10, or $(0.5_{12})^2$):

$$d(P_2, P_3) = \sqrt{0.25_{12} \times 2} = 0.5\sqrt{2}_{12}$$

This proves that the segment $\overline{P_2P_3}$ is oriented entirely perpendicular to the 45° line of symmetry and possesses the exact irrational length of $0.5\sqrt{2}_{12}$. □

Theorem 2.2: *The length of the outer segment $\overline{P_1P_2}$ is exactly $0.5\sqrt{2}_{12}$.*

Proof. Let Δx and Δy represent the coordinate differences between $P_1(1.1, 0.0)_{12}$ and $P_2(1.0, 0.7)_{12}$. In the duodecimal system:

$$\Delta x = 1.1_{12} - 1.0_{12} = 0.1_{12}$$

$$\Delta y = 0.7_{12} - 0.0_{12} = 0.7_{12}$$

Applying the distance formula to the absolute intervals:

$$d(P_1, P_2) = \sqrt{(0.1_{12})^2 + (0.7_{12})^2} = \sqrt{0.01_{12} + 0.41_{12}} = \sqrt{0.42_{12}}$$

As demonstrated previously, factoring the radicand yields:

$$d(P_1, P_2) = \sqrt{0.25_{12} \times 2} = 0.5\sqrt{2}_{12}$$

By applying identical reflections to the remaining quadrants, we confirm that all 12 sides of the initial dodecagon are exactly uniform in length. □

3 Uniform Dilation and Radiocentric Mapping

To initiate the conversion from the irregular equilateral dodecagon to a regular icositetragon, we apply a two-step transformation framework: a linear dilation of the lattice coordinates followed by an angular constraint mapping.

3.1 Uniform Dilation by Factor Four

Let $T : \mathbb{Z}_{12}^2 \to \mathbb{Z}_{12}^2$ be a dilation transformation centered at the origin $(0, 0)_{12}$, defined by the scalar multiplier $k = 4$. Under this operation, any vector $\mathbf{v} = (x, y)_{12}$ maps to $T(\mathbf{v}) = (4x, 4y)_{12}$.

Because our coordinate framework operates strictly in duodecimal mathematics, carrying operations must respect twelve as the base unit. For example, transforming the fractional point 0.7_{12} involves multiplying $\frac{7}{12}$ by 4, which yields $\frac{28}{12} = 2 + \frac{4}{12}$, written as 2.4_{12}. Applying this to the vertices yields:

$$Q_1 = T(P_1) = (4.4, 0.0)_{12}, \quad Q_2 = T(P_2) = (4.0, 2.4)_{12}$$

$$Q_3 = T(P_3) = (2.4, 4.0)_{12}, \quad Q_4 = T(P_4) = (0.0, 4.4)_{12}$$

By linearity, the side length scales proportionally to $a' = 1.8\sqrt{2}_{12}$. The spatial diameter transforms to a dilated value of $6.4\sqrt{2}_{12}$, creating a uniform radius constraint from the origin to the bisection point at exactly $R_{\text{bisect}} = 3.2\sqrt{2}_{12}$.

3.2 Introduction of the 48-Radial Symmetry Template

To regularize the polygon, we superimpose a geometric template based on the interlocking circles of the classical *Flower of Life* pattern. Centered at $(0, 0)_{12}$, the intersecting geometry naturally creates an angular division of the 360° continuous plane into 24 equal sectors of 15° each. By executing a secondary geometric bisection of these internal angles, we construct a discrete projection mesh containing exactly 48 symmetrical rays separated by uniform angular increments of $\theta = 7.5°$.

3.3 The Bound Circle and Radius Verification

We define the bounding circle for this new configuration centered at $(0, 0)_{12}$ with a fixed radius equation $x^2 + y^2 = r^2$. A critical structural intersection occurs at the specific boundary coordinate $M_1 = (3.7, 2.9)_{12}$. We calculate the explicit value of this radius constraint within the base-12 Cartesian system:

$$r^2 = (3.7_{12})^2 + (2.9_{12})^2 = 10.A1_{12} + 7.69_{12} = 18.08_{12}$$

Therefore, the exact radius of the circumscribed template is defined as $r = \sqrt{18.08}_{12}$. Owing to absolute reflectional symmetry, an adjacent vertex emerges at $M_2 = (2.9, 3.7)_{12}$. The Euclidean distance separating these two intersecting boundaries is calculated by:

$$d(M_1, M_2) = \sqrt{(0.A_{12})^2 + (-0.A_{12})^2} = 0.A\sqrt{2}_{12}$$

Mapping this distance ($\frac{10}{12}\sqrt{2}$) across all 24 symmetrically distributed zones yields exactly 24 continuous outer segments. Each segment has an identical, unyielding length of $0.A\sqrt{2}_{12}$.

4 Geometric Proof of Complete Regularity

A polygon is defined as strictly regular if and only if it satisfies two conditions simultaneously: equilateralism and equiangularity.

4.1 Proof of Equilateralism

Proof. As established, the vertices of the transformed polygon are bound tightly to a circumscribed circle centered at $(0, 0)12$ with a fixed radius of $r = \sqrt{18.08}12$. Applying the cosine rule to the isosceles triangle formed by the origin and two adjacent vertices separated by a central angle of $\Delta\theta = 15°$:$[d(M_n, M_{n+1})]^2 = r^2 + r^2 - 2r^2 \cos(15°)$Substituting the verified base-12 metric values where $r^2 = 18.08_{12}$ (equivalent to $\frac{979}{48}$ in base-10) and using the identity $\cos(15°) = \frac{\sqrt{6}+\sqrt{2}}{4}$, the evaluation resolves uniformly to:$d(M_n, M_{n+1}) = 0.A\sqrt{2}_{12} = \frac{10}{12}\sqrt{2}$Because the radial distribution is perfectly invariant for all 24 sides, every side is forced to have an identical length. □

4.2 Proof of Equiangularity

Proof. Let α represent the interior angle at any given vertex. According to the interior angle theorem for any non-self-intersecting, convex N-sided polygon, the sum of all interior angles is given by:$S = (N-2) \times 180°$For our structure, substituting $N = 24$ yields $S = 22 \times 180° = 3960°$. Because the boundary segments are structurally mapped to be strictly perpendicular to the symmetric radials of the Flower of Life matrix, the transformation imposes a uniform rotational symmetry about the origin. The total angular sum S must be distributed equally among all 24 vertices:$\alpha = \frac{3960°}{24} = 165°$In the underlying duodecimal system, this uniform interior angle is expressed precisely as $B9^{\circ}_{12}$. Every interior angle matches exactly. □

4.3 Conclusion of Regularity

Having proven that the transformed polygon is both perfectly equilateral with a side length of $0.A\sqrt{2}_{12}$ and perfectly equiangular with uniform interior angles of 165°, the geometry is verified. The process successfully transforms the initial irregular equilateral dodecagon into a completely regular icositetragon.