The Volume-Uniform Poincaré Walls: Machine-Checked Obstructions for Flat and Fluctuation-Sector Block-Poincaré Routes to Combes–Thomas Coercivity in Lattice Yang–Mills

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Abstract

Inside a Lean 4 formalization programme for four-dimensional $\mathrm{SU}(N_c)$ lattice Yang–Mills, we report two machine-checked *negative* results and the machine-checked infrastructure that makes them meaningful. The positive substrate is: (i) a fixed-volume Combes–Thomas chain for self-adjoint coercive finite-range lattice operators, instantiated on the flat gauge-fixed covariance of the physical shell, with coercivity constant $c = \min(1, a)/C_{\mathrm{P}}$ fed by a proved fixed-volume flat Hodge/block-Poincaré inequality; and (ii) the concrete adjoint model of $\mathrm{SU}(n)$ — $\mathfrak{su}(n)$ with the trace inner product, $\dim_{\mathbb{R}} \mathfrak{su}(n) = n^2-1$, and the isometric transport to Euclidean coordinates — so that the abstract adjoint-model interface has a concrete nontrivial inhabitant and the flat-lane results can be instantiated with the genuine matricial adjoint model. The first wall states that, under the block normalization actually used by the formalized chain, every flat Hodge/block-Poincaré constant obeys $L^d/L^2 \le C_{\mathrm{P}}$ on the fine torus of side LN', hence the volume-uniform Poincaré gate is *provably false* for $d \ge 3$, $N_c \ge 2$, and no positive coercivity constant survives all volumes through this route. The route consumed by the fixed-volume endpoint is therefore closed, by theorem rather than by impression. A second wall stands in the fluctuation sector. For $d \ge 3$ and a transported half-period square-wave mode on the exact fine side $(2M)N'$, the formalization proves $\|QA\|^2 \le (2M)^{-1}\|A\|^2$, the exact identity $\langle A,\, K_0 A\rangle = 8((2M)N')^{-1}\|A\|^2$, and therefore a Rayleigh numerator at most $9(2M)^{-1}\|A\|^2$. Every quotient Poincaré constant is thus at least $2M/9$, so the volume-uniform fluctuation-sector gate is also provably false for every positive N', $d \ge 3$, $N_c \ge 2$, and every adjoint model. Everything stated here is checked by Lean 4 against a pinned Mathlib, with zero `sorry`, zero project axioms, and a committed axiom-oracle transcript. A dependency record, theorem–artifact map, and reproduction instructions expose the complete proof chain. Both walls concern the current unscaled line-integral block map with the current unweighted coarse norm; neither gate is claimed to be necessary, equivalent, or exhaustive for Yang–Mills theory. No claim toward a continuum construction or a mass-gap theorem is made.

1 Introduction

Constructive approaches to the four-dimensional Yang–Mills mass gap in the Balaban tradition [1, 2] route spectral information through renormalization-group covariances whose kernels must decay *uniformly in the volume.* A standard engine for such decay is the Combes–Thomas argument [3]: a self-adjoint coercive finite-range operator has an inverse whose kernel entries decay exponentially in operator norm, at a rate governed by the coercivity constant. Whoever supplies the coercivity therefore controls the physics; and in a gauge-fixed lattice formalization the natural supplier is a Poincaré-type inequality tying the field norm to a Hodge term plus a block-averaging constraint.

This paper reports what happened when that plan was executed *formally*, in Lean 4 [6] against Mathlib [7], inside a long-running formalization programme whose earlier machine-checked artifacts include a volume-uniform Wilson-loop area law via a formalized cluster expansion [8], rooted-tree majorants for polymer expansions with holes [9], a mechanized Bessel-ratio calculus for lattice gauge expansions [10, 11, 12], an exact two-dimensional SU(2) theory [13], and the audited programme map [15].

The outcome has four layers, and we state them with the labels they deserve.

Positive (fixed volume). A complete Combes–Thomas chain for coercive finite-range operators on finite metric graphs, instantiated at the flat gauge-fixed covariance of the physical shell: for every fixed volume there exist a positive rate θ and amplitude $2/c$ with $\left\|(K^{-1})_{q,p}\right\|_{\mathrm{op}} \leq (2/c)\, e^{-\theta\, d(q,p)}$ for all bond pairs (q,p), where $c = \min(1,a)/C_{\mathrm{P}}$ and C_{P} is a *proved* fixed-volume flat Hodge/block-Poincaré constant (Section 3).

Positive (non-vacuity). The abstract adjoint-model interface consumed by the chain is inhabited by the *true* matricial adjoint model of $\mathrm{SU}(n)$: $\mathfrak{su}(n)$ as the real inner-product space of traceless skew-Hermitian matrices under the trace form, its real dimension n^2-1, and the isometric transport to $\mathbb{R}^{n^2-1}$ carrying $\mathrm{Ad}(g)X = gXg^{\dagger}$ (Section 4). The flat-lane results also admit the trivial inhabitant, since only $\mathrm{Ad}(1)$ is consumed there; the matricial construction removes the intended-model vacuity and prepares the nontrivial-background lane.

Negative (the constant-sector wall). Under the block normalization actually used by the formalized chain — the unscaled line-integral block map, which sends constants to L times constants — *every* flat Hodge/block-Poincaré constant obeys

$$\frac{L^d}{L^2} \;\leq\; C_{\mathrm{P}}$$

on the fine torus of side $N = LN'$. Consequently (Section 5): the volume-uniform Poincaré gate is *false* for $d \geq 3$, $N_c \geq 2$; the Combes–Thomas coercivity fed by this route obeys $c \leq \min(1,a)\, L^2/L^d \to 0$; and no single $c_0 > 0$ is dominated at every volume by the route's coercivity. The per-volume constants *do* exist, so the obstruction kills a real route, not an empty one.

Negative (the fluctuation-sector wall). The first witness is excluded by restricting to cochains orthogonal to every constant generator (Section 6); this non-transfer alone was result-neutral. The registered one-sided falsifier is then evaluated completely (Section 7). Its Rayleigh numerator decays like $(2M)^{-1}$ along even block scales, forcing $2M/9 \leq C_{\mathrm{P}}$ and refuting the volume-uniform quotient gate for every positive fixed coarse side N'. Thus removing the constant sector does not rescue the current unscaled block normalization. The argument still respects the falsifier's asymmetry: divergence refutes the gate, whereas a bounded quotient on this mode would not have proved it.

Honest scope

This paper proves no mass gap, no volume-uniform Combes–Thomas bound, and nothing about a thermodynamic or continuum limit. In particular, it makes no claim toward the continuum Yang–Mills construction or a mass-gap theorem [5]. Neither Poincaré gate used here is proved necessary, equivalent, or exhaustive for that problem.

Both walls are theorems about *one specific normalization*: the current unscaled line-integral block map with the current unweighted coarse norm. The second wall is uniform in every positive fixed coarse side N', but only for that normalization; it shows that restriction to the fluctuation sector alone does not rescue the route. The results do not exclude a differently rescaled or weighted block operator, whose gate must be formulated and audited anew, or coercivity extracted from an interacting Wilson Hessian. Their significance is navigational: two concrete routes are closed by machine-checked theorems. No numerical or percentage interpretation as progress toward the Clay problem is asserted.

2 The formal setting

All objects below are Lean definitions in the repository accompanying this paper (Section 9); we give their mathematical content and their Lean names.

Cochains. Fix $d, N, N_c \in \mathbb{N}$, all positive. Sites form the discrete torus $\Lambda_N = (\mathbb{Z}/N)^d$ (`FinBox d N`); positive bonds are pairs $b = (x, k)$ of a site and a direction (`PhysicalBond d N`). A *physical one-cochain* is a map A from positive bonds to the internal coordinate space $\mathbb{R}^{N_c^2-1}$ (`SUNLieCoord Nc`), with the ℓ^2 inner product in both the bond and internal indices (`PhysicalGaugeOneCochain d N Nc`).

The block map. For $N = LN'$, the flat block constraint $Q\colon$ fine cochains (side LN') $\to$ coarse cochains (side N') assigns to a coarse bond the *unscaled* line integral (sum) of the L fine bond values along the corresponding block line (`flatBlockConstraintQCLM`, built on `linAvg`). The normalization fact that drives everything below is exact (`flatBlockConstraintQCLM_constant`):

$$Q\,(A_v) \;=\; L \cdot A_v^{\text{coarse}}, \tag{1}$$

where A_v denotes the direction-wise constant cochain $A_v(x, k) = v_k$ for a fixed family $v\colon \{1, \ldots, d\} \to \mathbb{R}^{N_c^2-1}$ (`constantPhysicalGaugeOneCochain`).

The flat Hodge operator. At the trivial gauge background, $K_0 = D_1^\dagger D_1 + D_0 D_0^\dagger$ (`flatGaugeHodgeK0CLM`), so that $\langle A,\, K_0 A\rangle = \|\operatorname{curl} A\|^2 + \|\operatorname{div} A\|^2$ with the standard lattice curl on plaquettes and backward divergence on sites (`flatGaugeHodgeK0_inner_right`). Constants are flat-harmonic: $K_0 A_v = 0$ (`flatGaugeHodgeK0CLM_constantPhysicalGaugeOneCochain`).

The Poincaré predicate. For an adjoint model ρ (Section 4) and $C_{\mathrm{P}} \in \mathbb{R}$, `FlatGaugeHodgePoincare d L N' Nc` ρ `CP` is the statement

$$0 < C_{\mathrm{P}} \quad\wedge\quad \forall A:\; \|A\|^2 \;\le\; C_{\mathrm{P}}\left(\langle A,\, K_0 A\rangle + \|QA\|^2\right), \tag{2}$$

over fine cochains at side LN'. The repository proves, at every fixed volume, that such constants exist (`exists_flatGaugeHodgePoincare`); this non-vacuity is what makes the negative results of Section 5 statements about a live route.

3 The fixed-volume Combes–Thomas endpoint

The Combes–Thomas chain is formalized for abstract self-adjoint coercive finite-range operators on finite metric graphs: the tilt algebra and tilted entry bounds, the block Schur bound, coercivity survival under tilting at the budget $M(e^{\theta R} - 1)N_R \le c/2$, the tilted inverse with $\|K_\theta^{-1}\| \le 2/c$, and the kernel extraction at a root (`PhysicalCoerciveCombesThomas*.lean`). Instantiated at the flat gauge-fixed covariance of the physical shell (stencils and locality proved, not assumed), the endpoint reads:

Theorem 3.1 (fixed-volume Combes–Thomas; `flatGaugeFixedCovariance_CT_fixedVolume`, `exists_flatGaugeFixedCovariance_CT_fixedVolume`). *Fix the volume parameters (d, L, N'), a mass $a > 0$, and a flat Hodge/block-Poincaré constant C_P as in (2). Let $c = \min(1, a)/C_\mathrm{P}$. Then there exists $\theta > 0$ meeting the explicit tilt budget such that for all bond pairs (q, p) and every internal vector v,*

$$\left\|(K^{-1})_{q,p}\, v\right\| \;\le\; \frac{2}{c}\, e^{-\theta\, d(q,p)}\, \|v\|,$$

equivalently $\|(K^{-1})_{q,p}\|_{\mathrm{op}} \le (2/c)\, e^{-\theta\, d(q,p)}$ — the form quantified literally by `PhysicalCovarianceExponentialKernelBound`.

Fixed volume means fixed volume: C_P, hence c and θ, may depend on (d, L, N', a). The question the rest of this paper answers is whether *this* c can be made volume-uniform. It cannot.

4 The true adjoint model of $\mathrm{SU}(n)$

The chain consumes an abstract interface `SUNAdjointModel n`: a family of isometries $\mathrm{Ad}(g)$ of $\mathbb{R}^{n^2-1}$ indexed by $g \in \mathrm{SU}(n)$ with $\mathrm{Ad}(1) = \mathrm{id}$, $\mathrm{Ad}(gh) = \mathrm{Ad}(g)\mathrm{Ad}(h)$, and inner-product invariance. A trivial inhabitant ($\mathrm{Ad}(g) = \mathrm{id}$) exists and suffices for the flat lane, where only $\mathrm{Ad}(1)$ is consumed; leaving matters there would invite a vacuity objection the moment a non-trivial background appears. The repository therefore constructs the genuine object:

Theorem 4.1 (the matricial adjoint model; `matrixSUNAdjointModel`). *Let $\mathfrak{su}(n)$ be the real vector space of traceless skew-Hermitian $n \times n$ complex matrices with the trace form $\langle X, Y\rangle = \mathrm{Re}\,\mathrm{tr}(X^\dagger Y)$ (`SUNAdjointMatrixSubstrate`, `SUNAdjointInnerSpace`). Then $\dim_\mathbb{R} \mathfrak{su}(n) = n^2 - 1$ (`finrank_suLie`), the reindexed orthonormal-basis representation gives a linear isometric equivalence $\mathfrak{su}(n) \simeq \mathbb{R}^{n^2-1}$ (`suLieCoordIso`), and conjugating the adjoint action $X \mapsto gXg^\dagger$ by this isometry yields a `SUNAdjointModel n` whose action laws and inner-product invariance are proved, not hypothesized.*

The dimension count is itself formalized (Hermitian/skew decomposition, multiplication-by-i equivalence, ambient real dimension $2n^2$, and rank–nullity through the imaginary-trace functional with explicit surjectivity witness $i \cdot \mathbf{1}$). The coordinate choice is the non-canonical standard orthonormal basis; any other choice conjugates the model by a fixed orthogonal map, and nothing downstream depends on it.

5 The wall

Everything in this section is proved in `PhysicalGaugeFlatPoincare.lean` (the fixed-volume audit) and `PhysicalPoincareWall.lean` (the volume-uniform obstruction).

5.1 The constant sector forces the constant

Lemma 5.1 (exact sector normalization; `flatBlockConstraint_controls_constantSector`). *For every v,*

$$\|A_v\|^2 \;=\; \frac{L^d}{L^2}\,\|QA_v\|^2.$$

Proof. $\|A_v\|^2 = N^d \sum_k \|v_k\|^2$ with $N = LN'$ (`norm_sq_constantPhysicalGaugeOneCochain`), while by (1), $\|QA_v\|^2 = L^2 N'^d \sum_k \|v_k\|^2$ (`flatBlockConstraintQCLM_constant_norm_sq`); divide. □

Theorem 5.2 (the wall, fixed volume; `flatPoincare_constantSector_lower_bound`). *Let $N_c \geq 2$ and let C_{P} satisfy (2). Then*

$$\frac{L^d}{L^2} \;\leq\; C_{\mathrm{P}}.$$

Proof. For $N_c \geq 2$ the internal space $\mathbb{R}^{N_c^2-1}$ is nontrivial, so there is v with $\sum_k \|v_k\|^2 > 0$ (`exists_pos_constantSector`; for $N_c = 1$, $\mathfrak{su}(1) = 0$ and the sector is empty — the hypothesis is stated, not hidden). Apply (2) to A_v: the Hodge term vanishes ($K_0 A_v = 0$), so $\|A_v\|^2 \leq C_{\mathrm{P}}\|QA_v\|^2$, and Lemma 5.1 gives the claim after cancelling $\|QA_v\|^2 > 0$. □

Corollary 5.3 (linear form; `flatPoincare_linear_lower_bound`). *If moreover $d \geq 3$, then $L \leq C_{\mathrm{P}}$. (For $d = 4$ the forced growth is L^2; the linear form is all the divergence the obstruction needs. $d = 2$ is the scale-invariant exemption $L^d/L^2 = 1$ and is deliberately not claimed.)*

5.2 The route's coercivity dies with the volume

Theorem 5.4 (coercivity upper bound; `flatPoincare_coercivity_upper_bound`). *For $a > 0$, $N_c \geq 2$, and any C_{P} as in (2),*

$$\frac{\min(1,a)}{C_{\mathrm{P}}} \;\leq\; \min(1,a)\,\frac{L^2}{L^d}.$$

Theorem 5.5 (no volume-uniform coercivity through flat Poincaré). Lean: *`no_volumeUniform_coercivity_via_flatPoincare`.*

Let $d \geq 3$, $N_c \geq 2$, $a > 0$. Suppose $c_0 \in \mathbb{R}$ is dominated at every volume: for every $L \in \mathbb{N}$ there exists $C_{\mathrm{P}}(L)$ satisfying (2) at side $(L+1)N'$ with $c_0 \leq \min(1,a)/C_{\mathrm{P}}(L)$. Then $c_0 \leq 0$.

Proof. If $c_0 > 0$, choose (Archimedes) n with $n > \min(1,a)/c_0$. By Corollary 5.3, $C_{\mathrm{P}}(n) \geq n + 1 > \min(1,a)/c_0$, whence $\min(1,a)/C_{\mathrm{P}}(n) < c_0$, contradicting domination at volume $n+1$. □

Theorem 5.6 (the gate is false; `volumeUniformFlatPoincareGate_false`). *For $d \geq 3$, $N_c \geq 2$, there is no single $C_{\mathrm{P}} > 0$ satisfying (2) at every volume $L+1$: the proposition `VolumeUniformFlatPoincareGate` — with the constant quantified* before *the volume — is provably false.*

Remark 5.7 (non-vacuity of the killed route; `perVolume_flatPoincare_family`). *The hypothesis family of Theorem 5.5 is inhabited: per-volume constants exist at every volume. The obstruction closes a route that is genuinely open at each fixed volume.*

Remark 5.8 (quantifier discipline). *The gate places its constant before the volume quantifier. The reversed order — for every volume there exists a constant — is exactly the (true) per-volume statement, and conflating the two is the standard way such obstructions get lost in prose. The formalization makes the distinction type-level.*

6 The constant-sector quotient interface

The wall witness lives in one sector. The natural continuation restricts the Poincaré question to its orthogonal complement — the physical fluctuation space. The repository formalizes the *interface* (`PhysicalPoincareSectorQuotient.lean`); we use the word "interface" deliberately: no quotient type or packaged orthogonal-complement submodule is constructed, and the predicate is a restriction to fluctuation cochains.

Definition 6.1 (fluctuation cochains; `IsFluctuationCochain`)**.** A is a fluctuation cochain if $\langle A_v, A\rangle = 0$ for every constant generator A_v — equivalently, A is orthogonal to the span of the constant sector. (The generator-wise form is chosen for formal robustness.)

Proposition 6.2 (interface facts)**.** (i) *The constant sector is flat-harmonic (`constantSectorLin_harmonic`) — which is why only the block term sees it, with the normalization of Lemma 5.1.* (ii) *The full predicate (2) implies its restriction to fluctuation cochains at the same constant (`quotientFlatPoincare_of_flatPoincare`); the restricted statement is strictly weaker, so Theorem 5.6 does* not *refute the quotient gate.* (iii) *Restricted constants exist at every fixed volume (`exists_quotientFlatPoincare`).* (iv) (Non-transfer.) *A nonzero constant cochain is not a fluctuation cochain (`constant_not_fluctuation`): the wall* witness *is excluded from the fluctuation space by construction.*

Remark 6.3 (what the interface alone establishes)**.** *At the interface stage, the volume-uniform quotient gate (`VolumeUniformQuotientPoincareGate`) was neither proved nor refuted: non-transfer removes the W-1 witness, not the possibility of a second wall. The next section supplies that wall with a different, genuine fluctuation witness. This chronology matters logically: the W-1 obstruction is not being reused outside its domain.*

7 The low-mode falsifier and the second wall

A *one-sided falsifier* for the quotient gate needs an explicit fluctuation family on which the quadratic form of (2) can be evaluated. The repository constructs the family (`PhysicalPoincareLowModeFalsifier.lean`), evaluates its Hodge term exactly (`PhysicalPoincareLowModeHodge.lean`), transports it to the precise fine side consumed by the block map, and closes the Rayleigh estimate and endpoint (`PhysicalPoincareLowModeBlock.lean`).

Definition 7.1 (the half-period (two-interface) square-wave mode; `squareModeCochain`)**.** On even side lengths $N = 2M$ ($M \geq 1$), with oscillation direction j, bond direction i, and internal vector w:

$$A(x,k) \;=\; \begin{cases} \varepsilon(x_j)\, w, & k = i, \\ 0, & k \neq i, \end{cases} \qquad \varepsilon(m) = \begin{cases} +1, & m < M, \\ -1, & m \geq M, \end{cases}$$

(`squareSign`). The square profile is chosen over the cosine so that every identity below is an exact finite ± 1 computation; odd side lengths are out of scope by declared parameterization (a falsifier needs only *some* volume sequence).

Proposition 7.2 (exact identities)**.** (i) $\sum_m \varepsilon(m) = 0$ *(`sum_squareSign`), hence A is a fluctuation cochain (`squareModeCochain_isFluctuation`) — exact orthogonality, no approximation.* (ii) $\|A\|^2 = (2M)^d\, \|w\|^2$ *exactly (`norm_sq_squareModeCochain`).* (iii) *For*

$w \neq 0$ the mode equals no constant cochain (`squareModeCochain_ne_constant`), and the fluctuation space is nontrivial for $N_c \geq 2$ (`exists_nonzero_fluctuation`).

For arbitrary positive coarse side N', the base witness with half-period MN' lives on the side $(MN')+(MN')$, whereas the physical block predicate expects $(M+M)N'$. These sides are propositionally equal but their cochain types carry size-indexed instances. The formalization therefore uses canonical coordinate, bond, site, and plaquette equivalences, lifted to linear isometries, rather than an opaque dependent cast. The resulting `blockScaleSquareModeCochain` preserves fluctuation orthogonality, norm, shifts, flat curl, flat divergence, and Hodge energy.

Theorem 7.3 (exact transported Hodge ratio)**.** Lean: *`flatGaugeHodgeK0_inner_blockScaleSquareModeCochain_eq_div`. Let A be the transported mode on fine side $(2M)N'$, with $M, N' > 0$. For every adjoint model,*

$$\langle A,\, K_0 A\rangle = \frac{8}{(2M)N'}\,\|A\|^2. \tag{3}$$

The constant 8 is the exact cost of the two jump interfaces. For the longitudinal mode it lies in the divergence term; for a transverse mode it lies in the curl term.

Theorem 7.4 (physical block contraction)**.** Lean: *`norm_sq_flatBlockConstraintQCLM_le_inv_mul`. For every physical cochain A and $d \geq 3$,*

$$\|QA\|^2 \leq \frac{1}{L}\,\|A\|^2. \tag{4}$$

This is the current unscaled line-integral map measured in the current unweighted coarse norm; no field rescaling or hidden volume weight is inserted.

Taking $L = 2M$ in Theorem 7.4, combining with (3), and using $N' \geq 1$ yields the certified Rayleigh numerator bound

$$\langle A,\, K_0 A\rangle + \|QA\|^2 \leq \frac{9}{2M}\,\|A\|^2 \qquad (\texttt{blockScaleSquareMode\_rayleigh\_numerator\_le}). \tag{5}$$

The Hodge contribution is exact; the block contribution is the general upper bound (4), and no pointwise equality for it is claimed.

Theorem 7.5 (linear lower bound in the fluctuation sector)**.** Lean: *`quotientPoincare_squareMode_linear_lower_bound`. Let $M, N' > 0$, $d \geq 3$, $N_c \geq 2$, and suppose the quotient Poincaré predicate holds at block scale $2M$ with constant C_{P}. Then*

$$\frac{2M}{9} \leq C_{\mathrm{P}}.$$

Proof. Choose a unit internal vector (available because $N_c \geq 2$) and the longitudinal transported mode. It is a fluctuation cochain and has strictly positive norm. Apply the quotient predicate, use (5), and cancel $\|A\|^2 > 0$. □

Theorem 7.6 (the quotient gate is false)**.** Lean: *`volumeUniformQuotientPoincareGate_false`. For every positive N', every $d \geq 3$, every $N_c \geq 2$, and every `SUNAdjointModel Nc`,*

$$\neg\ \texttt{VolumeUniformQuotientPoincareGate}\ d\ N'\ N_c\ \rho.$$

Proof. A putative gate supplies one constant C_{P} before the block-scale quantifier. Theorem 7.5 forces $2M/9 \leq C_{\mathrm{P}}$ for every positive M. Choosing M by the Archimedean property gives a contradiction. The even subsequence suffices against the universal block-scale quantifier. □

Dependency record for the second wall

The endpoint above is not obtained by assuming the quotient gate and then repackaging it. The following table records the analytic chain in the order in which the final contradiction consumes it.

Certified step	Statement, hypotheses, and role
Block contraction	For every physical cochain and $d \geq 3$, $\|QA\|^2 \leq L^{-1}\|A\|^2$. This uses the concrete unscaled block map and unweighted coarse norm; it is the only block estimate entering the falsifier.
Exact Hodge ratio	For the canonically transported square-wave mode on fine side $(2M)N'$, $M, N' > 0$, $\langle A, K_0 A\rangle = 8((2M)N')^{-1}\|A\|^2$. The transport preserves the size-indexed structures and the exact energy.
Rayleigh numerator	With $L = 2M$ and $N' \geq 1$, the two previous rows give $\langle A, K_0 A\rangle + \|QA\|^2 \leq 9(2M)^{-1}\|A\|^2$. No equality for the block term is asserted.
Quotient lower bound	If the quotient Poincaré predicate holds at scale $2M$ with constant C_{P}, then the nonzero fluctuation witness and the Rayleigh bound force $2M/9 \leq C_{\mathrm{P}}$.
Gate endpoint	A volume-uniform gate chooses C_{P} before the scale. The Archimedean choice of M contradicts the preceding lower bound, yielding $\neg$`VolumeUniformQuotientPoincareGate` $d\ N'\ N_c\ \rho$ for every $N' > 0$, $d \geq 3$, $N_c \geq 2$, and every adjoint model.

Remark 7.7 (one-sided discipline and surviving alternatives)**.** *The falsifier has fired in its conclusive direction: the diverging lower bound refutes the gate, rather than merely eliminating a candidate. Had the quotient remained bounded on this mode, the gate would still not have followed without an all-modes or spectral-minimality argument. The theorem is specific to the current unscaled Q and current unweighted coarse norm. A rescaled or weighted block operator defines a different gate and remains open, as does coercivity from the interacting Hessian.*

8 Theorem–artifact map

Statement	Lean artifact and module under `YangMills/RG/`
CT endpoint, fixed volume	`flatGaugeFixedCovariance_CT_fixedVolume` `PhysicalShellCombesThomasEndpoint`
CT positive-rate witness	`exists_flatGaugeFixedCovariance_CT_fixedVolume` `PhysicalShellCombesThomasEndpoint`
$\dim_{\mathbb{R}} \mathfrak{su}(n) = n^2 - 1$	`finrank_suLie` `SUNAdjointDimension`
Isometric transport	`suLieCoordIso` `SUNAdjointModelInstance`
True adjoint model	`matrixSUNAdjointModel` `SUNAdjointModelInstance`
Sector witness ($N_c \geq 2$)	`exists_pos_constantSector` `PhysicalPoincareWall`
Wall, fixed volume	`flatPoincare_constantSector_lower_bound` `PhysicalPoincareWall`
Wall, linear form ($d \geq 3$)	`flatPoincare_linear_lower_bound` `PhysicalPoincareWall`

Statement	Lean artifact and module under `YangMills/RG/`
Coercivity upper bound	`flatPoincare_coercivity_upper_bound` `PhysicalPoincareWall`
No volume-uniform c_0	`no_volumeUniform_coercivity_via_flatPoincare` `PhysicalPoincareWall`
Gate false ($d \geq 3$, $N_c \geq 2$)	`volumeUniformFlatPoincareGate_false` `PhysicalPoincareWall`
Per-volume non-vacuity	`perVolume_flatPoincare_family` `PhysicalPoincareWall`
Fluctuation predicate	`IsFluctuationCochain` `PhysicalPoincareSectorQuotient`
Full $\Rightarrow$ quotient	`quotientFlatPoincare_of_flatPoincare` `PhysicalPoincareSectorQuotient`
Non-transfer of the witness	`constant_not_fluctuation` `PhysicalPoincareSectorQuotient`
Quotient gate predicate	`VolumeUniformQuotientPoincareGate` `PhysicalPoincareSectorQuotient`
Square profile, zero sum	`sum_squareSign` `PhysicalPoincareLowModeFalsifier`
Mode is fluctuation	`squareModeCochain_isFluctuation` `PhysicalPoincareLowModeFalsifier`
Mode norm, exact	`norm_sq_squareModeCochain` `PhysicalPoincareLowModeFalsifier`
Exact transported Hodge ratio	`flatGaugeHodgeK0_inner_blockScaleSquareModeCochain_eq_div` `PhysicalPoincareLowModeBlock`
Physical block contraction	`norm_sq_flatBlockConstraintQCLM_le_inv_mul` `PhysicalPoincareLowModeBlock`
Rayleigh numerator bound	`blockScaleSquareMode_rayleigh_numerator_le` `PhysicalPoincareLowModeBlock`
Quotient linear lower bound	`quotientPoincare_squareMode_linear_lower_bound` `PhysicalPoincareLowModeBlock`
Quotient gate false	`volumeUniformQuotientPoincareGate_false` `PhysicalPoincareLowModeBlock`

Every listed name is exercised by the committed axiom-oracle script. The headline wall theorems, including all five new W-3c rows above, print exactly `[propext, Classical.choice, Quot.sound]` on the frozen tree.

9 Reproducibility

Repository. https://github.com/lluiseriksson/THE-ERIKSSON-PROGRAMME (branch `main`). The Lean tree described here is that of source checkpoint a17d7816, sealed together with its transcript and live documentation by commit 33072662; this manuscript revision is added in a strictly editorial later commit that changes no Lean file. The unified release is anchored by the immutable tag `poincare-wall-v1.3-2026-07-14`; its manifest records the PDF and source hashes without inserting a self-referential commit identifier into the PDF.

Toolchain. Lean `leanprover/lean4:v4.29.0-rc6`; Mathlib pinned to commit `0764272048`... (lakefile and manifest agree). Build: `lake build YangMillsCore` — expected `Build completed successfully (8412 jobs)`.

Axiom oracle. `lake env lean oracle_check.lean` executes 2250 `#print axioms` invocations (2247 distinct output names); the committed transcript `ORACLE-20260714-a17d7816.txt` records the full raw output with provenance header: 2228 results on a nonempty subset of `[propext, Classical.choice, Quot.sound]` (finely 2145 + 65 + 18) and 22 axiom-free results; zero `sorryAx`, zero nonstandard axiom sets. SHA-256 hashes (git blob form, LF): script `CBEFB14D…`, raw output `4176F9E9…`.

Verification ledger. The append-only verification ledger documents every checkpoint of the results here (Addenda 467–509), including the measurement-discipline rules (measure the pushed tree; blob-hash artifacts; positive-assertion counter sweeps) and the corrections history.

Reconciliation provenance. Version 1.3 supersedes two editorial candidates without rewriting either of their seals: the fixed-$N' = 2$ review candidate contributed its strict scope and dependency-record discipline, while the later W-3c tree contributed the proved endpoint for every positive N'. The latter is the theorem stated here. No generalization is inferred from the former, and no Lean source differs from checkpoint `a17d7816`.

Continuation from this submission

The Hodge evaluation, block estimate, Rayleigh bound, and endpoint decision are now Theorems 7.3–7.6, not future work. The surviving registered routes are: a rescaled or weighted block operator with its normalization re-audited from first principles, and coercivity extracted from the interacting Wilson Hessian. No claim that either route succeeds is made. Any continuation will retain the same discipline — exact identities, declared parameterizations, and one-sided claims only where only one side is proved.

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