Exact Two-Dimensional $SU(2)$ Yang–Mills in Lean: Weyl Integration, Heat-Kernel Convolution, Migdal Invariance, and the Exact Simple-Loop Area Law

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Abstract

We give a kernel-checked Lean 4/Mathlib formalization of the exact heat-kernel solution of two-dimensional $SU(2)$ Yang–Mills theory along the chain from Haar measure to a concrete Migdal subdivision move. The development identifies the transported Haar probability on $SU(2) \simeq S^3$ with the canonical spherical probability, proves the required all-order orbital integration law, derives translated character convolution for every pair of irreducible labels, passes from finite character sums to the infinite heat-kernel semigroup by dominated convergence, and integrates an actual edge shared by two faces. We then define finite oriented disk cellulations independently of any reduction tree, using vertices, paired half-edges, cyclic face words, incidence, Euler characteristic, and positive face areas. Every connected dual graph admits a certified elimination schedule, every valid schedule reduces to the heat kernel at total area, and any two choices give the same amplitude. For the first cyclic unreduced lattice, a three-face disk, we additionally construct a triangular gauge-fixing equivalence that preserves the original triple product Haar measure and prove equality with the reduced amplitude. We then close the global edge model: for every primal-connected, facially well-formed finite disk cellulation, a rooted spanning tree selects $V-1$ distinct physical edges and induces an explicit measurable equivalence $SU(2)^E \simeq SU(2)^{V\setminus\{r\}} \times SU(2)^{E\setminus T}$ preserving product Haar. All facial holonomies transform simultaneously by basepoint conjugation, the heat-kernel density loses every tree/gauge coordinate, and the unreduced edge integral equals the gauge-fixed chord integral exactly. Finally, a tree–cotree theorem constructs compatible primal gauge and dual elimination trees for every certified physical disk cellulation. The boundary-conditioned original-edge amplitude is therefore defined without an auxiliary chart and is proved equal to the heat kernel at total face area and retained exterior holonomy. The schedule is hidden from the public loop object, which supplies a nontrivial exact-area-law package. Combined with all-label coefficient extraction, this yields the exact normalized simple-loop identity

$$\mathbb{E}[W_n(C)] = \exp\left[-\frac{n(n+2)}{4}\,\mathrm{Area}(C)\right].$$

All displayed endpoints compile without `sorry`, `admit`, or project-local axioms. The mathematical identities are classical; the claimed contribution is their concrete end-to-end machine-checked composition, not a new analytic solution of two-dimensional Yang–Mills theory.

1 Statement and claim boundary

Let dg be normalized Haar probability on $SU(2)$, let χ_n be the character of the irreducible representation of dimension $d_n = n + 1$, and put

$$c_n = \frac{n(n+2)}{4}, \qquad K_t(g) = \sum_{n \geq 0} d_n e^{-tc_n} \chi_n(g), \qquad t > 0.$$

With convolution

$$(f * h)(g) = \int_{SU(2)} f(x)h(x^{-1}g)\, dx,$$

the formalized chain is

$$\begin{aligned} \text{Haar on } S^3 &\Longrightarrow \text{all-order orbital law} \Longrightarrow \chi_n * \chi_m \Longrightarrow K_s * K_t = K_{s+t} \\ &\Longrightarrow \text{Migdal} \Longrightarrow \text{conditioned edge model} \\ &\Longrightarrow \text{simple-loop coefficient.} \end{aligned}$$

This result is distinct from the programme's earlier strong-coupling area-law theorem [6]. The earlier theorem is a volume-uniform *inequality* for finite lattices and $SU(N_c)$ in an explicit convergence window, obtained from a Kotecký–Preiss cluster expansion. The present theorem is an *exact identity* in the two-dimensional $SU(2)$ heat-kernel model, obtained by Haar integration and subdivision. The former is cited context; it is not used to discharge any exact two-dimensional statement here.

No claim is made here about four-dimensional Yang–Mills, a continuum construction in four dimensions, or a new proof of the classical exact solution. A priority claim about formalization should remain conditional on a systematic literature and repository search.

2 From spherical Haar measure to the orbital law

Write an element of $SU(2)$ in its standard form

$$g = \begin{pmatrix} a & b \\ -\bar{b} & \bar{a} \end{pmatrix}, \qquad |a|^2 + |b|^2 = 1.$$

Earlier modules construct a homeomorphism $SU(2) \simeq S^3$, realize left multiplication as an ambient real-linear isometry, prove preservation of the canonical `Measure.toSphere` measure, and use uniqueness of normalized Haar measure to obtain literal equality of the two probability measures.

The new all-order step studies $u = |a|^2$. In dimension four, the squared mass in one complex rail is uniform on $[0, 1]$. Rather than assume this distribution, the formal proof establishes its moment recurrence, proves

$$\int_{SU(2)} u^k\, dg = \frac{1}{k+1} \qquad (k \geq 0),$$

and then identifies measures on the compact interval using polynomial density. The resulting literal endpoint is:

```
theorem su2FirstRailMassMeasure_eq_uniform :
  su2FirstRailMassMeasure = su2UnitIntervalMeasure
```

For $x, y \in [-1, 1]$, choose representatives with half-traces x, y and average their product over conjugation. The preceding uniform law reduces the orbital integral to a one-variable

Chebyshev integral. For the Chebyshev polynomial U_m of the second kind, the formalized formula is

$$\int_{SU(2)} U_m\left(\tfrac{1}{2}\operatorname{tr}(kak^{-1}b)\right) dk = \frac{U_m(\frac{1}{2}\operatorname{tr} a)U_m(\frac{1}{2}\operatorname{tr} b)}{m+1}.$$

The proof includes the endpoint cases $x = \pm 1$; no division by a vanishing sine is hidden in the statement.

3 All-order translated character convolution

Characters are implemented concretely as

$$\chi_n(g) = U_n\left(\tfrac{1}{2}\operatorname{tr} g\right).$$

Conjugacy invariance, left and right Haar invariance, Fubini, and the orbital law give the translated orthogonality relation

$$\int_{SU(2)} \chi_n(x)\chi_m(x^{-1}g)\, dx = \delta_{nm}\frac{\chi_n(g)}{n+1}. \tag{1}$$

This is stronger than orthogonality at the identity and is exactly the consumer needed by the heat-kernel product.

Theorem 3.1 (Machine-checked character convolution)**.** *For every $n, m \in \mathbb{N}$ and $g \in SU(2)$,*

$$(\chi_n * \chi_m)(g) = \begin{cases} \chi_n(g)/(n+1), & n = m, \\ 0, & n \neq m. \end{cases}$$

The public Lean endpoints are listed below. They are theorems about the actual normalized Haar integral, not fields projected from an abstract character package.

```
integral_su2CharacterReal_mul_translate
su2CharacterChebyshev_convolution
```

4 Infinite heat-kernel semigroup

For a cutoff N, expand both finite kernels, interchange the finite sums and the Haar integral, and apply (1). The Kronecker selector leaves a single sum and the Casimir weights multiply:

$$e^{-sc_n}e^{-tc_n} = e^{-(s+t)c_n}.$$

This proves the finite convolution identity. The infinite theorem then uses the already established uniform Casimir majorant for positive times. Partial kernels converge pointwise and uniformly; the product is bounded by the product of two summable majorants, so dominated convergence passes the Haar integral to the limit.

Theorem 4.1 (Concrete heat semigroup)**.** *For $s, t > 0$ and every $g \in SU(2)$,*

$$\int_{SU(2)} K_s(x)K_t(x^{-1}g)\, dx = K_{s+t}(g).$$

Equivalently, as functions, $K_s * K_t = K_{s+t}$.

The corresponding Lean declarations, including the function-level consumer, are:

```
su2HeatKernelPartial_convolution
su2HeatKernel_convolution
su2HeatKernel_convolution_consumer
```

5 A concrete Migdal move

Consider two adjacent faces of areas s and t. Let a, b encode the external boundary holonomies and integrate their shared oriented edge x. The actual two-face density in the development is

$$D_{s,t}^{a,b}(x) = K_s(ax)K_t(x^{-1}b).$$

Left invariance changes variables $y = ax$ and the heat semigroup gives

$$\int_{SU(2)} D_{s,t}^{a,b}(x)\, dx = K_{s+t}(ab). \tag{2}$$

Theorem 5.1 (Two-face Migdal subdivision)**.** *Equation* (2) *holds for all $a, b \in SU(2)$ and all $s, t > 0$. It is proved by a genuine Haar integration over the shared edge and invokes the concrete infinite heat-kernel semigroup.*

This closes the first nontrivial subdivision move: two holonomy variables, two faces, and one integrated shared edge. The endpoints below contain no package hypothesis.

```
su2Migdal_twoFace_merge
su2Migdal_subdivision_invariant
```

6 A cyclic product-Haar gauge fixing

The two-face move alone does not identify a reduced evaluator with the original lattice integral when the face-dual graph has cycles. We therefore formalize the first cyclic test explicitly. Consider a disk split into three bounded faces by three spokes meeting at one interior vertex. Its face dual is K_3, equivalently the three-cycle. Write $x, y, z \in SU(2)$ for the original spoke-edge variables and $a, b, c \in SU(2)$ for the fixed boundary arcs. The three face holonomies are

$$xay^{-1}, \qquad ybz^{-1}, \qquad zcx^{-1}.$$

Thus the unreduced density and measure are

$$K_{t_1}(xay^{-1})K_{t_2}(ybz^{-1})K_{t_3}(zcx^{-1})\, dx\, dy\, dz. \tag{3}$$

Gauge transformations at the interior vertex act by simultaneous left multiplication. Lean constructs the measurable equivalence

$$\Phi(x, y, z) = (x, x^{-1}y, x^{-1}z), \qquad \Phi^{-1}(u, v, w) = (u, uv, uw).$$

The skew-product theorem and left invariance of normalized Haar prove the literal measure identity

$$\Phi_*(dx\, dy\, dz) = dx\, dy\, dz.$$

After substitution, conjugation invariance of the heat kernel removes the first coordinate and gives

$$K_{t_1}(av^{-1})K_{t_2}(vbw^{-1})K_{t_3}(wc).$$

The pure-gauge integral has mass one. Applying the reversed two-face Migdal identity first to w and then to v yields the fully unreduced result

$$\int_{SU(2)^3} K_{t_1}(xay^{-1})K_{t_2}(ybz^{-1})K_{t_3}(zcx^{-1})\, dx\, dy\, dz = K_{t_1+t_2+t_3}(abc). \tag{4}$$

The proof does not store a Jacobian certificate or assume an abstract gauge package. The map, its inverse, measurability, product-Haar preservation, density factorization, Fubini integrability, and both edge integrations are all kernel checked. The principal endpoints are:

```
su2ThreeSpokeFaceDual_connected
su2ThreeSpokeGaugeFix_measurePreserving
su2ThreeSpoke_density_gaugeFactorization
su2ThreeSpoke_unreducedIntegral_eq_gaugeFixedIntegral
su2ThreeSpoke_unreducedIntegral_eq_heatKernel
```

Equation (4) is the first concrete product-Haar instance. It is now subsumed by the uniform original-edge theorem below, whose spanning-tree gauge fixing is constructed directly from the incidence data of the independent disk object.

6.1 Uniform finite-valence theorem

The preceding three-spoke map is not intrinsically three-dimensional. Let I be any finite type and let

$$d\mu_I = \bigotimes_{i \in I} dg_i$$

be the normalized product Haar probability on $SU(2)^I$. The formalization constructs, without choosing an enumeration of I, the measurable equivalence

$$\begin{aligned} \Phi_I \colon SU(2) \times SU(2)^I &\to SU(2) \times SU(2)^I, \\ \Phi_I(x, y) &= (x, (x^{-1} y_i)_{i \in I}). \end{aligned}$$

with inverse $(u, v) \mapsto (u, (uv_i)_i)$. The product measure is Mathlib's `Measure.pi`. Using finite-product Haar invariance and the skew-product theorem, Lean proves

$$(\Phi_I)_*(dg \otimes d\mu_I) = dg \otimes d\mu_I \tag{5}$$

for every finite I. It also proves that the diagonal action $(x, y_i) \mapsto (ux, uy_i)$ preserves the same measure.

The result is consumed at the integral level. If $F : SU(2) \times SU(2)^I \to \mathbb{C}$ is invariant under that diagonal action, then

$$\int F(x, y)\, dg(x)\, d\mu_I(y) = \int F(1, y)\, d\mu_I(y). \tag{6}$$

More generally, the source contains an explicit factorization theorem for any density whose pullback through Φ_I^{-1} loses its first coordinate. The principal uniform endpoints are:

```
su2FiniteStarDiagonalLeft_measurePreserving
su2FiniteStarGaugeFix_measurePreserving
su2FiniteStar_integral_eq_gaugeFixed
su2FiniteStar_integral_eq_identitySlice
```

Equations (5)–(6) remove the fixed-valence limitation of the local gauge step. The next two subsections compose them with an ordered spanning tree, first for vertex coordinates and then for all original edge variables and all facial holonomies simultaneously.

6.2 Global rooted-tree vertex coordinates

The development now performs the global composition at the level of vertex coordinates. A value of `SU2RootedTreeOrder n` records a rooted tree on $n + 1$ vertices by adjoining one leaf at a time, together with the already present parent of each new leaf. Its parent–child list is embedded in the final index type. For every finite connected simple graph, Lean constructs an equivalence

$$e : \mathrm{Fin}(n + 1) \simeq V$$

and such an order for which every recorded parent–child pair is an actual graph edge. The proof removes a vertex whose complement remains connected, recurses on the induced graph, and reattaches the vertex across a witnessed edge.

For vertex variables $x_0, \dots, x_n \in SU(2)$, the associated single global map retains the root and replaces each later variable by its tree increment:

$$\Psi_T(x)_0 = x_0, \qquad \Psi_T(x)_k = x_{p(k)}^{-1} x_k \quad (k > 0). \tag{7}$$

It is implemented recursively by splitting the last coordinate, applying the triangular Haar shear, transforming the older coordinates, and reassembling the finite product. The inverse is part of the measurable equivalence, not an existence assertion. Induction and finite-product Haar invariance prove

$$(\Psi_T)_* \left(\bigotimes_{v \in V} dg_v \right) = \bigotimes_{v \in V} dg_v. \tag{8}$$

For a `SU2FiniteDiskCellulation`, primal adjacency is derived from an actual oriented half-edge and made into a simple graph. Under the explicit hypothesis that this primal graph is connected, Lean chooses the certified tree internally, transports (7) through the vertex enumeration, and returns a global measure-preserving equivalence on `C.Vertex -> SU2`. The principal endpoints are:

```
SU2RootedTreeOrder.exists_validForGraph_of_connected
SU2RootedTreeOrder.coordinateEquiv_measurePreserving
SU2FiniteDiskCellulation.exists_rootedSpanningTree_of_primal_connected
SU2FiniteDiskCellulation.RootedSpanningTree.vertexCoordinateEquiv_measurePreserving
SU2FiniteDiskCellulation.exists_rootedTreeVertexCoordinateEquiv_measurePreserving
```

The edge module consumes this tree and proves the stronger decomposition

$$SU(2)^E \simeq SU(2)^{V \setminus \{r\}} \times SU(2)^{E \setminus T},$$

as a literal measurable equivalence. Each construction step chooses an oriented parent–child half-edge; strict ordering of parents before children proves that the resulting physical tree edges are injective. The complement $E \setminus T$ is therefore an actual finite subtype, not a cardinality-only placeholder. After correcting the stored orientation of each tree edge, the map reconstructs potentials $g_r = 1$ and $g_k = g_{p(k)} a_k$, and sends each chord to

$$X_e = g_{s(e)} U_e g_{t(e)}^{-1}.$$

Finite-product relabelling, coordinatewise inversion and a Haar-preserving skew product prove preservation of normalized product Haar.

The physical edge record `SU2EdgeConnectedDiskCellulation` additionally requires an explicit once-around, duplicate-free boundary enumeration for each bounded face. This condition is stated because the older combinatorial record did not exclude a formally declared face with no incident half-edge. For every well-formed face f, Lean proves simultaneously

$$H_f(g \cdot U) = g_{v_f} H_f(U) g_{v_f}^{-1}.$$

Consequently the complete heat-kernel density is gauge invariant. Under the inverse edge equivalence it factors only through the chord block, and the normalized gauge coordinates integrate to one:

$$\int_{SU(2)^E} \prod_f K_{t_f}(H_f(U))\, dU = \int_{SU(2)^{E \setminus T}} \prod_f K_{t_f}(H_f(U_T = 1, X))\, dX. \tag{9}$$

Both sides of (9) integrate every physical edge, including the exterior boundary edges, and are therefore scalars. They are not identified with the boundary-dependent amplitude. Instead, the formalization separately exposes one exterior holonomy g as a Haar coordinate and defines the conditioned original-edge integral $Z_C(g)$.

The remaining compatibility problem is solved internally. A physical dual spanning tree selects $F-1$ distinct internal edges. In construction order, the rest of the newly attached facial cycle bypasses the selected edge; hence removing the first k selected edges preserves primal connectedness. After all $F-1$ removals, Lean extracts a primal spanning tree from the connected complement while retaining the physical dart witnesses, so parallel edges cannot be confused by classical choice. An exterior edge outside that tree becomes the boundary anchor. This gives a compatible primal tree, dual tree, and conditioned coordinate chart for every certified physical disk.

The terminal theorem is therefore unconditional on auxiliary chart data:

$$Z_C(g) = K_{\sum_f t_f}(g). \tag{10}$$

The proof composes the measure-preserving conditioned edge–chord equivalence, the physical Migdal fold, identification of the residual word with the exterior holonomy, and heat-kernel invariance under inversion. The principal new endpoints are:

```
SU2FiniteDiskCellulation.RootedSpanningTree.treeEdge_injective
SU2FiniteDiskCellulation.exists_rootedSpanningTree_avoiding_of_connected
SU2FiniteDiskCellulation.RootedSpanningTree.globalEdgeGaugeEquiv_measurePreserving
SU2FiniteDiskCellulation.RootedSpanningTree.gaugeTransform_eq_gaugeFixedEdgeConfiguration
SU2EdgeConnectedDiskCellulation.allFaceHolonomies_gaugeTransform
SU2EdgeConnectedDiskCellulation.edgeHeatKernelDensity_globalEdgeFactorization
SU2EdgeConnectedDiskCellulation.unreducedEdgeIntegral_eq_chordGaugeFixedIntegral
SU2DualRootedEliminationTree.primalGraphAvoiding_selectedEdgeRange_connected
SU2BoundaryDiskCellulation.nonempty_physicalBoundaryEliminationChart
SU2BoundaryDiskCellulation.conditionedEdgeModelAmplitude_eq_heatKernel
```

7 Independent finite cellulations and elimination

The analytic evaluator used by the development is a labelled binary schedule. If a schedule node has left and right subproblems L, R, its amplitude is defined recursively by

$$A_C(a,b) = \int_{SU(2)} A_L(a,x)A_R(x^{-1},b)\,dx.$$

The earlier tree module proves by structural induction that

$$A_C(a,b) = K_{T(C)}(ab).$$

At each node the induction hypotheses expose two heat kernels and the proof invokes the concrete two-face theorem.

The new point is that the geometric input is no longer this evaluator syntax. The new independent cellulation structure stores finite types of vertices, edges, half-edges, and bounded faces; a fixed-point-free involution pairs the two half-edges of every edge; source and target incidence is linked to a cyclic face-successor permutation; `none` denotes the exterior face; and the disk Euler relation $V-E+F=1$ is part of the certificate. The dual face graph is derived from the two sides of internal edges.

A labelled elimination schedule is valid when its leaves list every bounded face exactly once and each binary merge is witnessed by a genuine adjacency in the derived dual graph. Lean proves the following existence theorem for an arbitrary finite connected dual graph. If the graph has one vertex, the schedule is a leaf. Otherwise, a vertex whose deletion leaves a connected induced subgraph is removed, the proof recurses on the smaller finite type, and the vertex is reattached across an actual adjacent edge. Thus no acyclicity assumption is used.

For any valid schedule S, permutation of the face list gives $T(S) = \sum_f t_f$. The tree reduction then proves

$$A_{C,S}(a,b) = K_{\sum_f t_f}(ab). \tag{11}$$

Consequently, for all valid S_1, S_2,

$$A_{C,S_1}(a,b) = A_{C,S_2}(a,b).$$

The public structure `SU2ConnectedDiskCellulation` stores only the cellulation and connectedness of its dual. A schedule is chosen internally from the existence theorem; the preceding equality proves that every other valid choice is observationally identical. Its principal declarations are:

```
SU2FaceEliminationSchedule.exists_schedule_of_connected_graph
SU2FaceEliminationSchedule.exists_valid_schedule_of_dual_connected
SU2FaceEliminationSchedule.amplitude_eq_of_valid_schedules
SU2ConnectedDiskCellulation.amplitude_eq_heatKernel
SU2ConnectedDiskCellulation.amplitude_eq_any_valid_schedule
```

For each representation label n, the formalization defines a plane-loop theory whose loops are connected independently specified cellulations, whose area is $\sum_f t_f$, whose string tension is $n(n+2)/4$, and whose Wilson functional integrates the schedule-independent amplitude. Equation (11) followed by coefficient extraction gives a nontrivial concrete instance `su2ConnectedDiskExactAreaLawPackage`. The generic public interface is then consumed by `su2ConnectedDisk_simpleLoop_areaLaw_exact`; no field of that package remains assumed. The trivial-label case also proves the normalization

$$\int_{SU(2)} A_C(1,g)\, dg = 1.$$

For a general dual graph with cycles, the physical construction now identifies the schedule evaluator with the conditioned original-edge model rather than merely comparing two reduced expressions. Equation (10) holds for every facially well-formed, primal- and dual-connected certified disk cellulation. The earlier three-cycle computation remains a concrete diagnostic instance of this general theorem.

8 Exact simple-loop coefficient

The same repository proves, for every label n and positive area t, that the normalized Wilson character $W_n = \chi_n/(n+1)$ extracts the corresponding heat coefficient:

$$\int_{SU(2)} W_n(g) K_t(g)\, dg = e^{-tc_n} = \exp\left[-t\frac{n(n+2)}{4}\right].$$

The proof first establishes the identity for partial character sums and then passes to the full kernel by dominated convergence. Its final consumer is:

```
su2_exact_simpleLoop_areaLaw
```

Together with the iterated Migdal reduction, this is the exact two-dimensional route from the original conditioned edge integral for every certified finite combinatorial disk: chart and schedule choices do not change the amplitude, and the effective face has the Casimir coefficient prescribed by its total area.

9 Audit and reproducibility

The checked source is built with Lean 4 and Mathlib. The public root module imports the concrete orbit, character-convolution, heat-semigroup, and exact area-law modules. A repository-wide placeholder audit finds no occurrence of `sorry`, `admit`, or a project-local `axiom` declaration in the Lean development. Printing axioms for the 167 audited public endpoints returns only the standard kernel dependencies

`propext`, `Classical.choice`, `Quot.sound`.

The toolchain and Mathlib revision are pinned in the repository. GitHub Actions builds the public root target from a clean checkout and separately rejects placeholders.

Authoritative proof artifact. The complete machine-checked proofs are contained in the public repository `lluiseriksson/lean-2d-yang-mills` [7]. The immutable mathematical snapshot for this manuscript is commit `a1fbea97cbe673d383dbb4bc5e2a2fb70dbf190a`. In particular, `SU2RootedTreeGaugeFixing.lean` contains the connected-graph spanning-tree construction and global product-Haar theorem; the preceding modules contain the Haar–sphere identification, orbital integration, character convolution, heat semigroup, Migdal move, finite-cellulation reduction, tree–cotree connectivity, conditioned original-edge evaluation, and exact coefficient extraction. `AuditMigdalClosure.lean` prints the kernel dependencies of all 167 public endpoints. Thus the repository is not supplementary numerics or pseudocode: it is the compilable proof object to which the theorem claims refer.

Principal new declarations.

```
su2FirstRailMoment_recurrence
su2FirstRailMoment_eq
su2FirstRailMassMeasure_eq_uniform
intervalIntegral_chebyshevU_product_formula
integral_su2FirstEntryMass_eq_unitInterval
integral_su2Haar_orbit_character_general
integral_su2CharacterReal_mul_translate
su2CharacterChebyshev_convolution
su2HeatKernelPartial_convolution
su2HeatKernel_convolution
su2Migdal_twoFace_merge
su2Migdal_subdivision_invariant
su2_exact_simpleLoop_areaLaw
SU2PlanarFaceTree.internalEdgeCount_eq_faceCount_sub_one
su2PlanarCellulationAmplitude_eq_heatKernel
su2PlanarCellulationAmplitude_eq_effectiveFace
su2TreePlanar_wilsonExpectation_eq_casimir
su2TreePlanarExactAreaLawPackage
su2TreePlanar_simpleLoop_areaLaw_exact
integral_su2PlanarCellulationAmplitude_eq_one
SU2FaceEliminationSchedule.exists_schedule_of_connected_graph
SU2FaceEliminationSchedule.exists_valid_schedule_of_dual_connected
SU2FaceEliminationSchedule.amplitude_eq_of_valid_schedules
SU2ConnectedDiskCellulation.amplitude_eq_heatKernel
SU2ConnectedDiskCellulation.amplitude_eq_any_valid_schedule
su2ConnectedDisk_wilsonExpectation_eq_casimir
su2ConnectedDiskExactAreaLawPackage
su2ConnectedDisk_simpleLoop_areaLaw_exact
integral_su2ConnectedDiskAmplitude_eq_one
su2ThreeSpokeGaugeFix_measurePreserving
su2ThreeSpoke_density_gaugeFactorization
su2ThreeSpoke_gaugeFixedIntegral_eq_heatKernel
```

```
su2ThreeSpoke_unreducedIntegral_eq_gaugeFixedIntegral
su2ThreeSpoke_unreducedIntegral_eq_heatKernel
su2FiniteStarDiagonalLeft_measurePreserving
su2FiniteStarGaugeFix_measurePreserving
su2FiniteStar_integral_eq_gaugeFixed
su2FiniteStar_integral_eq_identitySlice
SU2RootedTreeOrder.exists_validForGraph_of_connected
SU2RootedTreeOrder.coordinateEquiv_measurePreserving
SU2FiniteDiskCellulation.exists_rootedSpanningTree_of_primal_connected
SU2FiniteDiskCellulation.RootedSpanningTree.vertexCoordinateEquiv_measurePreserving
SU2FiniteDiskCellulation.exists_rootedTreeVertexCoordinateEquiv_measurePreserving
SU2FiniteDiskCellulation.RootedSpanningTree.globalEdgeGaugeEquiv_measurePreserving
SU2FiniteDiskCellulation.RootedSpanningTree.gaugeTransform_eq_gaugeFixedEdgeConfiguration
SU2EdgeConnectedDiskCellulation.allFaceHolonomies_gaugeTransform
SU2EdgeConnectedDiskCellulation.unreducedEdgeIntegral_eq_chordGaugeFixedIntegral
SU2FiniteDiskCellulation.exists_rootedSpanningTree_avoiding_of_connected
SU2DualRootedEliminationTree.primalGraphAvoiding_selectedEdgeRange_connected
SU2BoundaryDiskCellulation.nonempty_physicalBoundaryEliminationChart
SU2PhysicalBoundaryEliminationChart.conditionedEdgeIntegral_eq_heatKernel
SU2BoundaryDiskCellulation.conditionedEdgeModelAmplitude_eq_heatKernel
```

10 Scope and next frontier

The present artifact closes the compact-group analytic chain, introduces an independent finite oriented disk-cellulation object, constructs elimination schedules for every connected dual graph, and proves independence of the schedule. Dual cycles are allowed. The full boundary-conditioned original-edge model is linked through a universally constructed tree–cotree chart to the physical Migdal fold and hence to the heat kernel at total area. The construction is uniform in the certified finite disk cellulation, proves simultaneous facial covariance and removes all $V-1$ gauge coordinates exactly.

The artifact does not identify the abstract combinatorial maps with planar isotopy classes, treat nonsimple or intersecting loops, construct a continuum measure, or evaluate positive-area higher-genus partition functions. The remaining frontier is therefore geometric rather than a missing edge-model bridge: comparison with planar isotopy classes, nonsimple and intersecting Wilson loops, surfaces of higher topology, and continuum constructions.

References

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