

Octonionic Associator-Driven Baryogenesis

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Abstract

We propose a compact effective mechanism for baryogenesis in which the CP-odd source is the oriented phase of the octonionic associator. The signed imaginary projection of the associator defines a dimensionless field Θ_A that couples derivatively to the baryon current with strength c_Θ . In a homogeneous radiation-dominated Universe this coupling gives an effective baryonic chemical potential, and baryon-number-violating reactions freeze a nonzero asymmetry when their rate falls below the Hubble rate. The observed baryon-to-photon ratio fixes the model-independent target $c_\Theta \Theta_A / T_D \simeq 2.4 \times 10^{-8}$ at freeze-out. For Hubble-driven relaxation this corresponds to perturbative associator orientations at high decoupling temperatures, $T_D \sim 10^{12} - 10^{15}$ GeV. We give an explicit baryonic projector, a representative dimension-six baryon-violating operator, the effective equation of motion for Θ_A , and the conditions required to avoid sphaleron washout and low-energy nucleon-decay constraints.

Keywords: octonions, baryogenesis, nonassociativity, CP, early Universe

1. Introduction

The observed Universe contains a small but robust excess of matter over antimatter. This is usually expressed as

$$\eta_B \equiv \frac{n_B - n_{\bar{B}}}{n_\gamma} \simeq 6.1 \times 10^{-10}, \quad (1)$$

or equivalently

$$Y_B \equiv \frac{n_B - n_{\bar{B}}}{s} \simeq 8.7 \times 10^{-11}, \quad (2)$$

where $s/n_\gamma \simeq 7.04$. Precision cosmology fixes the baryon abundance accurately; in the base Λ CDM model, Planck 2018 gives $\Omega_b h^2 = 0.02237 \pm 0.00015$ [1]. A microscopic explanation must satisfy the Sakharov conditions: baryon-number violation, C and CP violation, and departure from thermal equilibrium [2]. Although the Standard Model contains anomalous baryon-plus-lepton violation and CKM CP violation, it does not naturally yield the observed asymmetry without additional dynamics [3, 4, 5, 6, 7, 8].

This Letter develops an effective mechanism in which the required CP-odd source is not an independent axion-like or ad hoc scalar field, but the oriented nonassociative phase of an octonionic field triplet. Octonions form the largest normed division algebra and are nonassociative but alternative [9]. Their automorphism group is G_2 , and their imaginary sector carries a canonical oriented three-form [10]. The associator is the algebraic obstruction to replacing the octonionic product by an associative product. Equivalently, it measures the failure of three local octonionic amplitudes to lie in a common quaternionic subalgebra. Since every quaternionic subalgebra is associative,

a nonzero associator is precisely a signed departure from the associative sectors that can support ordinary low-energy complex or quaternionic quantum mechanics.

The central observation of this Letter is that this departure is orientation-sensitive. The norm of the associator is CP even, while a signed imaginary projection can be CP odd. We show that a slowly varying CP-odd associator phase acts as an effective baryonic chemical potential, in close analogy with spontaneous baryogenesis [15, 16], while its microscopic origin is tied to nonassociative geometry.

2. Octonionic structure and associator tensor

Let $\{1, e_i\}$, $i = 1, \dots, 7$, be an orthonormal basis of $\mathbb{O}$ with multiplication

$$e_i e_j = -\delta_{ij} + f_{ijk} e_k, \quad f_{ijk} = f_{[ijk]}, \quad (3)$$

where the nonzero f_{ijk} are fixed by a chosen Fano-plane orientation. For $x = x^0 + x^i e_i$ and $y = y^0 + y^i e_i$, use

$$\langle x, y \rangle = \text{Re}(\bar{x}y) = x^0 y^0 + x^i y^i. \quad (4)$$

For three octonionic fields $\Psi_I(x) \in \mathbb{O}$, define

$$A(\Psi_1, \Psi_2, \Psi_3) = (\Psi_1 \Psi_2) \Psi_3 - \Psi_1 (\Psi_2 \Psi_3). \quad (5)$$

The triplet structure is minimal because alternativity gives $A(x, x, y) = A(y, x, x) = 0$. In components,

$$[e_b, e_c, e_d] = C^a_{bcd} e_a, \quad C_{abcd} = \langle e_a, [e_b, e_c, e_d] \rangle. \quad (6)$$

After lowering the first index with the Euclidean octonionic metric, C_{abcd} is a totally antisymmetric four-tensor determined by f_{ijk} . Equivalently, for the convention in Eq. (3), one may write $C_{abcd} = 2\psi_{abcd}$, where $\psi = *_\varphi \varphi$ is the four-form dual to

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the canonical G_2 three-form $\varphi_{abc} = f_{abc}$. The factor of two is conventional and may be absorbed into the normalization of M_* below. The tensor vanishes on every quaternionic subalgebra and changes sign under reversal of the Fano-plane orientation. Thus the associator is a covariant order parameter for nonassociative orientation, not merely an additional interaction. A CP-even stabilizing term is

$$\mathcal{L}_A = -\frac{\lambda}{2}\|A\|^2, \quad \|A\|^2 = \langle A, A \rangle, \quad \lambda \geq 0. \quad (7)$$

This term favors an approximately associative low-energy projection but is insensitive to the sign of the associator orientation.

3. Baryonic projector and CP-odd associator phase

We now make the baryon projector explicit. Work in the complexified algebra $\mathbb{C} \otimes \mathbb{O}$ and choose a preferred imaginary unit $u = e_7$. With the central complex unit i , define

$$\varepsilon_{\pm} = \frac{1}{2}(1 \pm iu), \quad \varepsilon_{\pm}^2 = \varepsilon_{\pm}, \quad \varepsilon_+ \varepsilon_- = 0. \quad (8)$$

The stabilizer of u inside G_2 is $SU(3)$, so the remaining six imaginary units split into three complex color directions. Choose creation-annihilation operators $\alpha_r^\dagger, \alpha_r$, $r = 1, 2, 3$, satisfying $\{\alpha_r, \alpha_s^\dagger\} = \delta_{rs}$ on the minimal left ideal generated by ε_+ , as in division-algebra ideal constructions of Standard-Model multiplets [11]. The color-number operator and baryon generator are

$$N_c = \sum_{r=1}^3 \alpha_r^\dagger \alpha_r, \quad Q_B = \frac{1}{3} N_c. \quad (9)$$

The explicit baryonic projector used below is the spectral projector onto states with nonzero Q_B ,

$$\mathcal{P}_B = 1 - \prod_{r=1}^3 (1 - \alpha_r^\dagger \alpha_r), \quad \mathcal{P}_B^2 = \mathcal{P}_B, \quad [\mathcal{P}_B, Q_B] = 0. \quad (10)$$

This realizes the lepton-quark split algebraically: the vacuum ideal state has $Q_B = 0$, while one-color excitations carry $B = 1/3$. To avoid an additional undefined field, define the baryonic reference projection as

$$\Psi_B \equiv \mathcal{P}_B \Psi_0, \quad (11)$$

where Ψ_0 is a fixed local octonionic reference amplitude, or equivalently the slowly varying baryonic component of the same microscopic multiplet. It is normalized such that $[\Psi_B] = 1$.

The CP-odd associator phase is then

$$\Theta_A(x) = \frac{1}{M_*^4} \text{Im} \langle A(\Psi_1, \Psi_2, \Psi_3), \Psi_B \rangle. \quad (12)$$

With canonical scalar dimension $[\Psi_I] = [\Psi_B] = 1$, one has $[A] = 3$, hence Θ_A is dimensionless. The relevant microscopic

operation is an octonionic CP transformation: ordinary space-time parity and complex conjugation, accompanied by reversal of the oriented G_2 structure,

$$\begin{aligned} CP : \quad i &\mapsto -i, & f_{ijk} &\mapsto -f_{ijk}, \\ \Psi(t, \mathbf{x}) &\mapsto \Psi^{CP}(t, -\mathbf{x}). \end{aligned} \quad (13)$$

Since C_{abcd} is linear in the oriented Fano structure, Eq. (12) obeys

$$CP_{\text{oct}} : \quad \Theta_A(t, \mathbf{x}) \mapsto -\Theta_A(t, -\mathbf{x}). \quad (14)$$

Thus Θ_A is fixed as the CP-odd orientation of the microscopic associator rather than introduced as an independent scalar. The notation CP_{oct} emphasizes that the sign change of f_{ijk} is not an independent four-dimensional field transformation, but a reversal of the internal nonassociative orientation.

4. Derivative coupling and baryon-number violation

The leading shift-symmetric interaction between the associator phase and baryon number is the dimension-four operator

$$\mathcal{L}_{\text{int}} = c_{\Theta} \partial_{\mu} \Theta_A J_B^{\mu}, \quad [J_B^{\mu}] = 3, \quad [c_{\Theta}] = 0. \quad (15)$$

Up to a boundary term, $\mathcal{L}_{\text{int}} = -c_{\Theta} \Theta_A \nabla_{\mu} J_B^{\mu}$, so it contributes only if baryon number is violated. In the numerical estimates below $c_{\Theta} = 1$ corresponds to the minimal normalization. A concrete effective choice is the standard dimension-six baryon-violating operator [12, 13]

$$\mathcal{L}_B = \frac{c_B}{\Lambda_B^2} \epsilon_{abc} (q_L^a q_L^b) (q_L^c \ell_L) + \text{h.c.}, \quad \Delta B = 1, \quad (16)$$

where color and weak contractions are suppressed. This representative operator has $\Delta B = 1$ and $\Delta L = 1$, hence $\Delta(B - L) = 0$. If it acts above the electroweak sphaleron epoch, the generated asymmetry must therefore either be stored partly in a sector not equilibrated by sphalerons, freeze in after sphaleron decoupling, or be generalized to a $B - L$ -violating completion. The effective calculation below only assumes that a baryon-number-violating channel remains out of equilibrium after T_D .

In the effective theory,

$$\begin{aligned} \nabla_{\mu} J_B^{\mu} &= i\Delta B (O_B - O_B^{\dagger}), \\ O_B &= \frac{c_B}{\Lambda_B^2} \epsilon_{abc} (q_L^a q_L^b) (q_L^c \ell_L). \end{aligned} \quad (17)$$

Thermal averaging gives the parametric baryon-violating relaxation rate

$$\Gamma_B(T) = \kappa_B |c_B|^2 \frac{T^5}{\Lambda_B^4}, \quad (18)$$

with κ_B an order-one phase-space and multiplicity coefficient. Freeze-out is defined by

$$\Gamma_B(T_D) = H(T_D), \quad T_D \simeq \left[\frac{1.66 \sqrt{g_*} \Lambda_B^4}{\kappa_B |c_B|^2 M_{\text{Pl}}} \right]^{1/3}. \quad (19)$$

5. Freeze-out calculation of the baryon asymmetry

In a homogeneous early Universe Eq. (15) becomes

$$c_\Theta \partial_\mu \Theta_A J_B^\mu = c_\Theta \dot{\Theta}_A n_B, \quad \mu_B^{\text{eff}} = c_\Theta \dot{\Theta}_A. \quad (20)$$

For relativistic degrees of freedom and $|\mu_B| \ll T$, expansion of the thermal distributions gives

$$n_B - n_{\bar{B}} = \frac{g_B}{6} \mu_B^{\text{eff}} T^2 + \mathcal{O}\left(\frac{\mu_B^3}{T}\right). \quad (21)$$

With $s = 2\pi^2 g_* T^3/45$, the instantaneous equilibrium value is

$$Y_B^{\text{eq}} = \frac{15g_B}{4\pi^2 g_*} c_\Theta \frac{\dot{\Theta}_A}{T}. \quad (22)$$

The one-mode Boltzmann equation

$$\dot{Y}_B = -\Gamma_B(T) [Y_B - Y_B^{\text{eq}}(T)] \quad (23)$$

then gives, for slowly varying $\dot{\Theta}_A/T$,

$$Y_B(T < T_D) \simeq \frac{15g_B}{4\pi^2 g_*} c_\Theta \frac{\dot{\Theta}_A}{T} \Big|_{T_D}. \quad (24)$$

Converting to the baryon-to-photon ratio,

$$\eta_B \simeq 7.04 \frac{15g_B}{4\pi^2 g_*} c_\Theta \frac{\dot{\Theta}_A}{T} \Big|_{T_D}. \quad (25)$$

For $g_* = 106.75$ and $g_B = 1$,

$$\eta_B \simeq 2.51 \times 10^{-2} c_\Theta \frac{\dot{\Theta}_A}{T} \Big|_{T_D}. \quad (26)$$

Matching Eq. (1) gives

$$\boxed{c_\Theta \frac{\dot{\Theta}_A}{T} \Big|_{T_D} \simeq 2.4 \times 10^{-8}} \quad (27)$$

and therefore

$$\boxed{\frac{\Delta E_{B\bar{B}}}{T_D} = 2 c_\Theta \frac{\dot{\Theta}_A}{T} \Big|_{T_D} \simeq 4.8 \times 10^{-8}.} \quad (28)$$

Only a part-in- 10^8 nonassociative energy bias at freeze-out is needed.

6. Effective dynamics of the associator phase

The previous result is model independent once the effective coupling (15) exists. To make the source dynamical, introduce the homogeneous effective action

$$S_\Theta = \int d^4x \sqrt{-g} \left[\frac{f_A^2}{2} (\partial \Theta_A)^2 - V_A(\Theta_A) + c_\Theta \partial_\mu \Theta_A J_B^\mu \right], \quad (29)$$

where f_A is the stiffness of the associator orientation. Variation gives

$$f_A^2 (\ddot{\Theta}_A + 3H\dot{\Theta}_A) + V_A'(\Theta_A) = -c_\Theta \nabla_\mu J_B^\mu. \quad (30)$$

After baryon violation freezes out, the source term vanishes and Hubble friction damps $\dot{\Theta}_A$, suppressing late-time CP violation. In the slow-roll regime, Eq. (30) reduces to the logarithmic parametrization

$$\dot{\Theta}_A = q_A H \Theta_A, \quad (31)$$

with $q_A \simeq -V_A'/(3f_A^2 H^2 \Theta_A)$. During radiation domination,

$$H(T) = 1.66 \sqrt{g_*} \frac{T^2}{M_{\text{Pl}}}, \quad (32)$$

with $M_{\text{Pl}} = 1.22 \times 10^{19}$ GeV. Combining Eqs. (27) and (32) yields

$$c_\Theta q_A \Theta_A \simeq \frac{2.4 \times 10^{-8}}{1.66 \sqrt{g_*}} \frac{M_{\text{Pl}}}{T_D}. \quad (33)$$

For $g_* = 106.75$,

$$c_\Theta q_A \Theta_A \simeq 1.4 \times 10^{-9} \frac{M_{\text{Pl}}}{T_D}. \quad (34)$$

Hence

$$T_D = 10^{15} \text{ GeV} : \quad c_\Theta q_A \Theta_A \simeq 1.7 \times 10^{-5}, \quad (35)$$

$$T_D = 10^{12} \text{ GeV} : \quad c_\Theta q_A \Theta_A \simeq 1.7 \times 10^{-2}, \quad (36)$$

$$T_D = 10^{10} \text{ GeV} : \quad c_\Theta q_A \Theta_A \simeq 1.7. \quad (37)$$

Thus the mechanism is naturally high scale. For $T_D \sim 10^{12} - 10^{15}$ GeV, the required effective orientation $c_\Theta q_A \Theta_A$ is perturbatively small. At electroweak temperatures, by contrast, Hubble-driven relaxation would be far too slow unless Θ_A changes nonadiabatically during a phase transition.

7. Consistency with washout and proton decay

The same operator that generates baryon violation in the early Universe can mediate nucleon decay at low energy. A conservative dimensional estimate is

$$\tau_p \sim \frac{\Lambda_B^4}{|c_B(0)|^2 m_p^5}. \quad (38)$$

Current water-Cherenkov searches require partial lifetimes of order 10^{34} yr in leading channels [14], so a viable model must either take a sufficiently large Λ_B , a small low-energy coefficient, or a temperature-dependent projection factor. The last option is natural here because baryon violation is tied to nonassociative orientation. We write

$$c_B(T) = c_{B,0} F_A(T), \quad F_A(T \ll T_D) \rightarrow 0, \quad (39)$$

where F_A measures the overlap between the nonassociative sector and the baryonic ideal. A concrete parametrization is, for example, $F_A(T) \propto \|A(T)\|/M_*^3$ or equivalently a small-angle form $F_A(T) \propto |\sin \Theta_A(T)|$. Then the low-temperature suppression is not inserted independently; it follows from the decay of

the nonassociative orientation after the associative low-energy projection. Washout is avoided if

$$\mathcal{W} \equiv \int_{t_D}^{\infty} \Gamma_B(t) dt = \int_0^{T_D} \frac{\Gamma_B(T)}{H(T)} \frac{dT}{T} < 1. \quad (40)$$

For $F_A(T)$ decreasing below T_D , this condition is stronger than $\Gamma_B(T_D) = H(T_D)$ only by an order-one factor. Combining Eq. (19) with $T_D = 10^{12}$ GeV gives $\Lambda_B \sim 3 \times 10^{13}$ GeV for $\kappa_B |c_{B,0}|^2 \sim 1$; proton decay then requires the low-temperature suppression $|F_A(0)| \ll 1$, whereas $T_D \sim 10^{15}$ GeV is compatible with a GUT-scale Λ_B .

8. Sakharov conditions and falsifiability

The three Sakharov requirements are implemented as follows. Baryon-number violation is encoded explicitly by Eqs. (16)–(17), or by a sphaleron analogue with the same current nonconservation. When the representative B – L -conserving operator in Eq. (16) is used above the electroweak scale, sphaleron washout must be avoided by one of the mechanisms stated above or by replacing it with a B – L -violating completion. CP violation is supplied by the oriented associator phase Θ_A , which changes sign under Eq. (14). Departure from equilibrium occurs when Γ_B/H crosses unity at T_D , with residual washout bounded by Eq. (40).

The mechanism is falsifiable because it predicts a sharp relation between the freeze-out scale, the baryon-violating scale, and the associator velocity:

$$c_{\Theta} \dot{\Theta}_A|_{T_D} \simeq 2.4 \times 10^{-8} T_D. \quad (41)$$

Together with Eq. (19), this fixes the correlated target

$$\Lambda_B \simeq \left[\frac{\kappa_B |c_{B,0}|^2 F_A^2(T_D) T_D^3 M_{\text{Pl}}}{1.66 \sqrt{g_*}} \right]^{1/4}. \quad (42)$$

Equivalently, for Hubble-driven relaxation it predicts Eq. (34). A microscopic octonionic theory that fixes $\Theta_A(t)$, q_A , $F_A(T)$, and M_* therefore also fixes T_D and Λ_B . Conversely, if independent particle-physics input fixes the baryon-violating scale, the required associator orientation is no longer adjustable.

A remaining cosmological constraint is isocurvature: fluctuations of Θ_A must satisfy schematically $|\delta Y_B/Y_B| \simeq |\delta \Theta_A/\dot{\Theta}_A|$ below the observed baryon-isocurvature limit. If Θ_A is light during inflation, the conservative estimate $\delta \Theta_A \simeq H_{\text{inf}}/(2\pi f_A)$ gives

$$\frac{H_{\text{inf}}}{2\pi f_A |\Theta_A|} \lesssim \mathcal{O}(10^{-5}). \quad (43)$$

This bound can be avoided or weakened if the relevant roll occurs after inflation, if Θ_A is heavy during inflation, or if the associator stiffness f_A is sufficiently large.

9. Discussion

The essential result is Eq. (27). The observed baryon asymmetry does not require a large matter-antimatter preference. It requires a small effective CP-odd nonassociative velocity,

$$c_{\Theta} \dot{\Theta}_A/T_D \simeq 2.4 \times 10^{-8}. \quad (44)$$

This is naturally compatible with an early high-energy epoch in which the octonionic sector is not yet fully projected onto an associative low-energy subalgebra. As the Universe cools, the associator phase relaxes and the CP-odd bias disappears, while the baryon asymmetry remains frozen.

The proposal differs from standard spontaneous baryogenesis in the origin of the phase. In conventional models the derivative coupling is usually introduced for a scalar or pseudo-scalar field. Here the phase is the oriented projection of the octonionic associator itself. This makes the CP-odd source geometrical: it measures the time evolution of nonassociative orientation in the microscopic algebraic structure.

The construction fixes the effective projector, a representative baryon-violating operator, the dynamical equation for the associator phase, and the consistency conditions for washout and nucleon decay. A complete ultraviolet theory must still calculate the coefficients f_A , V_A , c_B , Λ_B , κ_B and the projection function $F_A(T)$ from the full octonionic master action. Nevertheless, the low-energy baryogenesis prediction is fixed by Eqs. (25)–(42): an octonionic theory of the early Universe must produce the small dimensionless ratio in Eq. (27) while satisfying Eq. (40).

10. Conclusion

We have shown that, within an effective spontaneous-baryogenesis description, an oriented octonionic associator phase can reproduce the observed matter-antimatter asymmetry if it couples derivatively to the baryon current. The observed value $\eta_B \simeq 6.1 \times 10^{-10}$ follows from a small freeze-out bias $c_{\Theta} \dot{\Theta}_A/T_D \simeq 2.4 \times 10^{-8}$. For Hubble-driven relaxation this corresponds to $c_{\Theta} q_A \Theta_A \simeq 1.7 \times 10^{-5}$ at $T_D = 10^{15}$ GeV or $c_{\Theta} q_A \Theta_A \simeq 1.7 \times 10^{-2}$ at $T_D = 10^{12}$ GeV. Viability further requires washout suppression and consistency with nucleon-decay bounds, naturally implemented if the baryon-violating overlap $F_A(T)$ switches off after the associative low-energy projection.

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