The Increase in Center of Mass Momentum of a Three Body System as Predicted by a General Invariant Impulse Equation

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Abstract

We conducted an experiment where a rotator rotated in a horizontal plane with respect to the earth. The rotator at one end was connected to a vertical shaft that was embedded in a horizontal bearing assembly. The bearing assembly was attached to a sliding platform that was constrained to move with one degree of freedom along a linear track. The main point of the experiment was to determine if the orbital angular speed of the rotator increased due to fictitious forces. The accelerometer attached to the rotator showed a local increase in angular speed of the rotator when it passed the 90 degrees point and the 270 degrees point of its angular displacement.

Based on this invariant increase in angular speed, we derived a general invariant impulse equation that remarkably predicted this increase leads to an increase in the center of mass momentum of a system.

1. Introduction

In May of 1921 Albert Einstein gave four lectures at Princeton University. He had already completed his General Theory of Relativity at this time and his stress-energy tensor, reduced to the weak field limit, yielded linearized, gravitoelectromagnetic equations that hinted at Machian effects. In lecture four he referred to this by stating three important principles:

"What is to be expected along the line of Mach's thought?

1. The inertia of a body must increase when ponderable masses are piled up in its neighbourhood.

2. A body must experience an accelerating force when neighbouring masses are accelerated, and, in fact, the force must be in the same direction as the acceleration.

3. A rotating hollow body must generate inside of itself a "Coriolis field," which deflects moving bodies in the sense of the rotation, and a radial centrifugal field as well" [1].

Einstein acknowledged the magnitude of this accelerating force under normal laboratory conditions would be so tiny it would be immeasurable because the mathematics showed this force would be proportional to the inverse of the speed of light squared.

In 1953 Dennis Sciama expanded on this concept in his paper "On the Origin of Inertia" to show the relative accelerated motion of the total mass of the universe with respect to a linearly accelerated reference frame was immense enough that it would induce a significant gravitoelectric field with respect to the accelerated frame. This linearized gravity field would give rise to fictitious forces on bodies that conform exactly with Newton's second law $f = ma$ [2].

These theoretical insights of Einstein and Sciama were profound. But these linear fictitious forces arise with respect to a linearly accelerated reference frame, not with respect to an inertial laboratory frame. However, any increase in orbital angular velocity measured by an accelerometer attached to a rotating body is a frame-invariant, proper increase in speed since all reference frames would agree with the readings of the sensor. Because of this observable fact, we can derive a general invariant impulse equation based on this increase in angular speed that predicts an increase in the linear center of mass momentum of a system with respect to an inertial, laboratory reference frame.

We note that an important and relevant property of this linear fictitious force is that it does not comply with Newton's third law. There are no equal and opposite contact forces acting on a system when fictitious forces manifest in a linearly accelerated system. If this were true, then by Euler's first law, an increase in the center of mass momentum of a system predicted by the impulse equation would be impossible.

2. Materials and Methods

The experiment reported in this paper as far as we know is unique and never has been previously published in the existing physics literature.

A rotating body (rotator) with a vertical shaft at one end was inserted into a bearing assembly that was embedded in a platform. This platform could slide with one degree of freedom with respect to the y-axis. One Pasco accelerometer was attached to the platform and another accelerometer to the rotator. The platform accelerometer measured the proper linear acceleration of the platform; the rotator accelerometer measured the proper angular speed of the rotator. The platform was initially set against a barrier so that it could not move in the negative y-direction.

A counter-clockwise rotation was imparted to the rotator when it was at rest at the 180 degrees position. When the rotator passed the 0 degrees point, the platform began accelerating in the positive y-direction. The proper linear acceleration of the platform and the proper angular speed of the rotator were transferred via Blue Tooth to a Pasco program on a computer that simultaneously plotted the graphs of the data. The data generated from within the accelerometers was with respect to the proper time of the accelerometers.

Results

The data graphed a sinusoidal wave with zero acceleration for the platform at the zero degrees point, a maximum acceleration at the 90 degrees point, followed by zero acceleration at the 180 degrees point. The crucial data showed the angular speed of the rotator locally increased when it passed the 90 degrees and 270 degrees point of its angular displacement.

The results of this rotator-sliding frame experiment motivated us to perform a second experiment where instead of a single rotator, we used two spheres that transversed quarter-circle barriers after entering the platform along the x-axis simultaneously with the same initial velocity. As the spheres rounded the curves, centrifugal reactive forces accelerated the system in the negative y-direction, inducing fiction forces acting on the spheres in the positive y-direction. (The induced fictitious forces are always opposite the acceleration or deacceleration). The platform was attached to a Pasco smart cart and as the cart rolled backwards, wheel friction impulses acted on the wheels of the cart in the positive y-direction.

After rounding the curves, the two spheres made inelastic collisions with two barriers that contained magnets and after 30 test trials the mean ratio of the friction impulse to the post collision momentum was ~ 8.2 per cent. Although this was a low value and strongly suggested there were other external impulses that contributed to the final momentum, we could not stand on this experiment because there were too many unaccounted, speed-dependent friction effects between the wheels and the track that could have contributed to the final momentum. Below is a figure representing the experiment.

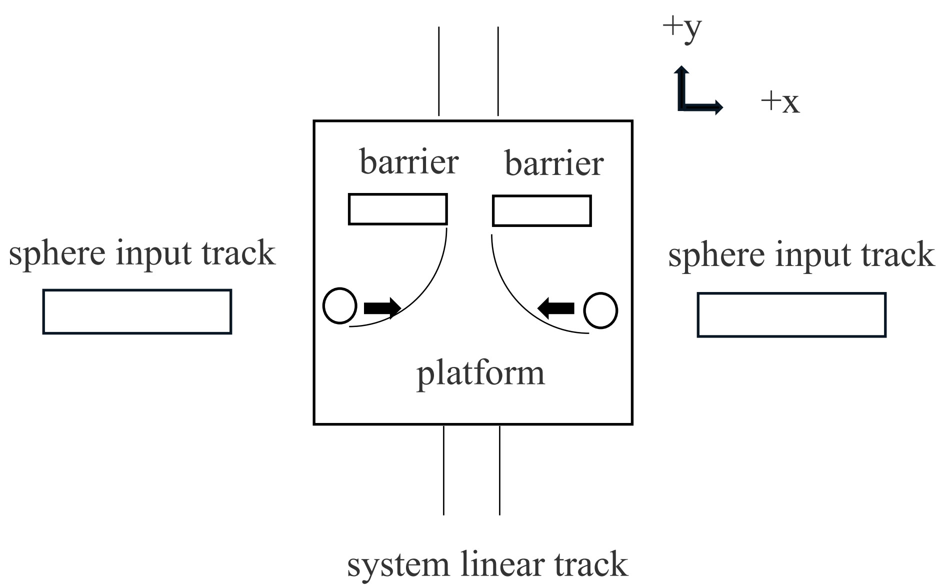

Figure 1. Top-down view of the experiment.

We concluded the ideal environment for the experiment would be in a free fall, micro-gravity environment where these friction issues would be totally eliminated.

4.1 Discussion – The General Invariant Impulse Equation

The derivation of the equation uses a system similar to the one shown in Fig. 1 above, except there will be no cart or linear track, only the platform and the spheres. This system is within the environment of a flat Minkowski, inertial space. In this derivation we are limited to non-relativistic speeds. This means we can invoke Galilean relativity where proper time is sufficiently equal to coordinate time in a first approximation. So, we treat these times as frame invariant. The centrifugal reactive forces acting on the sides of the curved barriers as the spheres round the curves are also nearly frame invariant. This is because the centrifugal reactive force associated with the strain of the inner walls of the curves as the spheres press against the curves follows Hooke's law. In this law force equals the frame invariant constant K times Δl, the amount of compression of the walls. At non-relativistic speeds, the difference in the measurement of Δl across different frames would be insignificant. Hence, we treat the force as nearly frame invariant. Multiplying these two invariants, centrifugal reactive force and time, gives us a frame invariant centrifugal reactive impulse that acts on the inner walls of the curves.

The centrifugal reactive force has a magnitude of $nkm_1r\omega^2$. In this expression n is the number of spheres, $k = \frac{m_2}{nm_1+m_2}$, m_1 is the mass of each sphere, m_2 is the mass of the platform, r is the radius of curvature of the path of the center of mass of each sphere, and ω is the orbital angular velocity of each sphere. In this derivation the orbital angular speed ω of each sphere is constant as they round their curves and we assume no friction between the spheres and the inner walls of the curved barriers.

In a real world setting these friction forces would slow the speed of the spheres, reduce the linear acceleration of the system, and diminish the magnitude of the effects discussed in this derivation. However, since these friction forces are internal force pairs, by Euler's first law they in of themselves will have no impact on the center of mass momentum of the system.

Finally, we are only interested in the y-component of the impulse J, so we include a sine factor in front of the force equation. In the experiment shown in Fig. 1 the platform is constrained to move with one degree of freedom on a track with respect to the y-axis. This constraint also applies in this derivation because even though the system is in free fall, the spheres enter the curves simultaneously from opposite directions and at the same speed. This cancels the centrifugal reactive force components along the x-axis so that the platform follows a straight-line path along the y-axis.

The final form of the y-component of the reactive centrifugal force equation is given by:

$$f_y = \ sin(\omega t)\, nkm_1 r\omega^2 \qquad (1)$$

The k factor arises because we assume the spheres and m_2 are near-rigid bodies and have the same instantaneous y-component of linear acceleration.

We integrate the force with respect to time to determine the impulse J on m_2 with respect to the y-axis. This integral is expressed by:

$$J_y = \int_{\frac{\theta_i}{\omega}}^{\frac{\theta_f}{\omega}} nsin(\omega t) km_1 r\omega^2\, dt \qquad (2)$$

Recognizing the indefinite integral of $\int \sin(\omega t)\, dt$ is equal to $-\frac{1}{\omega}\cos(\omega t) + C$ and applying this to Eq. (2) and after moving the constants out of the expression, we evaluate the above integral, giving us the general invariant impulse equation:

$$J_y = n\, k\, m_1 r\omega\left[cos(\theta_i) - \cos(\theta_f)\right] \qquad (3)$$

This equation expresses the y-component of the impulse that acts on the platform by the action of the centrifugal reactive force on the inner walls of the curves as the spheres round their curves from Θ_i to Θ_f.

Alternatively, we can use the tangential velocity of the sphere in the equation by setting $r\omega = v_i$, which gives us the expression:

$$J_y = n\,k\,m_1 v_i\left[cos(\Theta_i) - \cos(\Theta_f)\right] \tag{3a}$$

For the special case where the initial angles are 0 degrees, 90 degrees, 180 degrees, or 270 degrees and where there is either a 90 degrees counterclockwise or clockwise angular displacement, then the y-impulse on the platform simplifies to:

$$J_y = n\,k\,m_1 v_i \tag{3b}$$

The sign of the above special case impulse, Eq. (3b), must be adjusted by multiplying the y-impulse by one for a counterclockwise 90 degrees angular displacement and by negative one for a clockwise 90 degrees angular displacement to obtain the correct y-direction sign of the impulse. The correct sign for the y-impulse using equation (3a) is obtained by the same convention regardless of the initial and final angles.

We now demonstrate two cases using both Eq. (3b) and Eq. (3a) to determine the final y-impulse on the platform. We begin with a constant speed case. The initial speed of each sphere is 2.7 m/s. We use Eq. (3b) to determine the y-impulse on the platform. Given the initial values, the impulse on the platform equates to -1.80 N s, which gives a momentum value of -1.80 kg · m/s. Dividing this by the mass of the platform gives a velocity of -1.80 m/s. This is the velocity of the platform after the spheres round their curves immediately before they make their inelastic collisions.

Next, we apply Galilean velocity transformation. We add the final velocity of each sphere 2.7 m/s with respect to the platform frame to the final velocity of the platform -1.80 m/s which gives us .9 m/s with respect to the laboratory frame. Finally, we multiply the mass of each sphere by their laboratory velocities to obtain a total lab momentum of 1.80kg · m/s. We add this to the final momentum of the platform -1.80 kg · m/s which equals 0 kg · m/s. This was the initial momentum of the system with respect to the y-axis which means momentum is conserved as expected.

In the second case the initial speed of each sphere is 1 m/s but increases by .1 m/s for every 5 degrees angular displacement from 270 degrees to 360 degrees by the action of fictitious forces. The final speed of each sphere after they round the curves is 2.7 m/s. We apply the general impulse Eq. (3a) to determine an impulse for each angular increment then sum the incremental impulses to get the total y-impulse on the platform. Table 1 below tabulates the results.

Table 1. Increasing Speed Case

Sphere Initial Speed M/S	Sphere Final Speed M/S	Initial Angle Degrees	Final Angle Degrees	Incremental Impulse N · S
1	1	270	275	-0.058103828
1.1	1.1	275	280	-0.063427786
1.2	1.2	280	285	-0.068136694
1.3	1.3	285	290	-0.072107618

1.4	1.4	290	295	-0.075224911
1.5	1.5	295	300	-0.077381738
1.6	1.6	300	305	-0.078481532
1.7	1.7	305	310	-0.07843933
1.8	1.8	310	315	-0.077183006
1.9	1.9	315	320	-0.074654372
2	2	320	325	-0.070810135
2.1	2.1	325	330	-0.065622703
2.2	2.2	330	335	-0.059080829
2.3	2.3	335	340	-0.051190078
2.4	2.4	340	345	-0.041973129
2.5	2.5	345	350	-0.031469878
2.6	2.6	350	355	-0.019737371
2.7	2.7	355	360	-0.006849543

The sum of the incremental y-impulses acting on the platform equals ~ - 1.07 N s. Note, this is significantly less in magnitude than in the constant speed case where the y-impulse on the platform equaled -1.80 N s. Yet, the final speed of the sphere is 2.7 m/s with respect to the platform frame which was the same as the speed in the constant speed case. This implies there is a gain in momentum. Using the same analysis that we used above in the first case, we determine the final center of mass momentum of the system equals ~2.2 kg · m/s in the positive y-direction. The system has gained momentum with respect to an inertial laboratory frame by the action of a frame invariant increase in the orbital angular speeds or tangential speeds of the spheres due to fictional forces.

4.2 Discussion – The Energy Implications

We now address a final point in this discussion dealing with the energy implications. If a system acquires momentum as predicted by the general impulse equation, it follows that the system acquires kinetic energy. Since kinetic energy and change in kinetic energy are a frame-dependent phenomena, an interesting energy dynamic occurs when we add an initial velocity to the system before the spheres around their curves. We show a general result of this mathematically.

We begin with the impulse-momentum theorem. Here, we define the increase in center of mass momentum is due a virtual impulse denoted by $J_{virtual}$. By the impulse-momentum theorem this impulse is implied if there is a change in momentum as shown below:

$$J_{virtual} = mv_{f\ system} - mv_{i\ system} \tag{4}$$

In the above m is the total mass of the system which includes the spheres and the platform.

Solving for $v_{f\ system}$, we obtain:

$$v_{f\ system} = \frac{(J_{virtual} + mv_{i\ system})}{m} \tag{5}$$

Substituting the above for $v_{f\ system}$ in the expression for ΔKE, gives us :

$$\Delta KE = \frac{1}{2}\, m[\frac{(J_{virtual} + mv_{i\ system})}{m}]^2 - \frac{1}{2} mv_{i\ system}^2 \tag{6}$$

Simplifying the above, we arrive at:

$$\Delta KE = \frac{J_{virtual}^2}{2m} + J_{virtual}\, v_{i\,system} \qquad (7)$$

The second term on the right of the above equation shows that the change in kinetic energy of a system while keeping the impulse $J_{virtual}$ and mass the same is impacted by the initial velocity of the system. This can be easily confirmed by noting that with respect to a boosted inertial reference frame the initial velocity of the system will always impact the change in kinetic energy of the system. This is an observable fact. And by the first postulate of special relativity this phenomenon observed by a boosted inertial frame must also be possible with respect to an inertial laboratory frame.

An even more interesting consequence has to do with the energy that is responsible for the initial kinetic energy of the spheres. For example, suppose the spheres are propelled by compressed springs that are attached to the platform. The elastic potential energy of each spring (epe) is a *frame invariant scalar*. This implies when we apply Eq. (7) above, there always exists a boosted inertial frame where respect to this frame the initial velocity of the system will yield a change in kinetic energy of the system that is *greater than the elastic potential energy (epe) of the springs.* And again, by the first postulate of special relativity this phenomenon observed by a boosted inertial frame must also be possible with respect to an inertial laboratory frame.

We elaborate further on this point by referencing a more complex system. First, we need to define what a cycle N means. In this system the spheres are propelled by springs, enter and round the quarter-circle curves, then have elastic collisions with barriers of high elasticity, bounce back, then round the curves again in the opposite direction. They return to their initial positions, resting against their two recompressed springs. This is defined as one cycle N. Recall the virtual impulse J_{virtual} discussed earlier. We can find the value for this impulse empirically by measuring the change in momentum of the system after one cycle, denoting it by $J_{\text{one cycle}}$.

Denoting the total number of springs as n and the energy per compressed spring as epe, the cumulative energy applied to the spheres after N cycles is:

$$E_{\text{cumulative}} = N\, n\, epe \tag{8}$$

The cumulative final velocity of the system after N cycles is equal to:

$$v_{f\ \text{system}} = \frac{N\, J_{\text{one cycle}}}{m} \tag{9}$$

Substituting Eq. (9) above into the general formula for kinetic energy gives us:

$$KE_{\text{final}} = \frac{(N\, J_{\text{one cycle}})^2}{2m} \tag{10}$$

We note that Eq. (8) is a linear expression and Eq. (10) is quadratic. This means when Eq. (8) and Eq. (10) are plotted on a graph where the x-axis is N and the y-axis is energy, there will be a cycle N where the change in kinetic energy of the system will cross over the cumulative elastic potential energy. This gap will widen as N increases.

5. Conclusion

The results of the rotator-sliding platform experiment showed an increase in angular speed of the rotator. This frame invariant increase in speed was the consequence of a fictitious force acting on it while the platform was linearly accelerating. Derivation of a general invariant impulse equation showed that the center of mass of a system would increase with respect to an inertial reference frame due to the increase in orbital angular speed of the spheres.

We recommend an experiment using synchronized sphere-propelling devices attached to the platform, similar to what is shown in Figure 1, be performed at one of the two NASA drop towers located at Glenn Research Center in Cleveland, Ohio. In this micro-gravity environment, the friction issues relating to the cart wheels and the track would be totally eliminated and would yield conclusive results. After the spheres collide, if the post collision speed of the platform is zero, then the experiment would confirm no net gain in the center of mass momentum of the system. If a post collision speed is greater than zero, then the center of mass momentum has increased as predicted by the general invariant impulse equation.

Declaration of AI Use

During the preparation of this work, the author used generative AI to assist with formatting the paper according to submission standards. The author reviewed and edited the content and takes full responsibility for the publication.

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