The Tensional Structure Principle:
Mathematical Foundations and Lagrangian Formulation

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Abstract

This paper develops the full Lagrangian formulation of the Tensional Structure Principle (TSP). In TSP, the fundamental substrate of reality is the entanglement domain **E**, composed of qubit-scale flexible mats described by a stiffness field with a tension field as the primary dynamical quantity.

Mass induces distortions in the mat field, producing gravity as a tension gradient. We construct the action, derive the Euler–Lagrange equations, identify the stress–tension tensor, and show how dark energy arises from a uniform residual tension. This paper provides the mathematical backbone for the conceptual TSP framework.

Keywords

Entanglement; tension network; mat field; Planck-scale structure; dark energy; phase transition; gravity; black holes; structural cosmology; spacetime; cosmic acceleration; Lagrange.

Preface

The Tensional Structure Principle (TSP) emerged from my attempt to identify a simple, physically meaningful foundation for gravity and spacetime. The TSP is a structural theory: it describes what spacetime is made of, how it forms, and why gravity behaves as it does.

Instead of beginning with geometry or fields, the TSP begins with a single primitive: tension. The universe is modeled as a nonlocal network of Planck-scale elements—mats—connected through tension. Spacetime is the equilibrium configuration of this network. Gravity is the gradient of tension between adjacent mats.

My earlier work defined a two-domain model [1, 2, 3, 4, 5], consisting of an **S** domain and an **E** domain. We identify **S** matter to follow Einstein relativity. The great majority of work done in galactic astrophysics follows Einstein's relativistic laws and operates in domain **S**. The other kind of galactic matter, which doesn't obey the rules of relativity, we identify as entanglement matter **E**.

My subsequent work [6] used the two-domain model and proposed a conceptual framework in which both the gravitational constant **G** and the phenomenon commonly referred to as dark energy arise from a single physical mechanism of extended mats — formed during an early-universe phase transition.

1. Introduction

The Tensional Structure Principle proposes that spacetime and gravitation emerge from the mechanical behavior of mats in domain **E**. My companion paper [7] introduced the conceptual structure; this paper provides the formal mathematical foundation. Although the **E** domain is a place where fields do not reside, we nonetheless apply the power of Lagrangian analysis in the context of this paper and obtain successful results.

The goals of this paper are to:

1. Define the mat field $M(x)$and stiffness field $m(x)$.
2. Construct the tension field T_μ.
3. Build the full Lagrangian density.
4. Derive the field equations.
5. Identify the gravitational and dark-energy terms.
6. Show how spacetime geometry in **S** emerges from tension dynamics in **E**.

2. Field Content of the Theory

2.1 Mat Field

Each mat is described by a scalar field:

$$M(x)$$

representing local mat configuration, stiffness, and deformation.

2.2 Stiffness Field

The stiffness field $m(x)$ determines the local mechanical resistance of the mat.

2.3 Tension Field

The fundamental dynamical quantity is the tension field:

$$T_\mu(x) = \partial_\mu m(x)$$

This replaces curvature as the generator of gravitational effects.

3. The Action of the Tensional Structure Principle

The total action [8] is:

$$S = \int d^4x \, \mathcal{L}_{\text{TSP}}$$

with Lagrangian density:

$$\mathcal{L}_{\text{TSP}} = \mathcal{L}_M + \mathcal{L}_T + \mathcal{L}_{\text{int}} + \mathcal{L}_\Lambda$$

We now define each term.

4. Mat Dynamics

4.1 Mat Kinetic Term

The kinetic term for the mat field is:

$$\mathcal{L}_M = \frac{1}{2}(\partial_\mu M)(\partial^\mu M)$$

4.2 Mat Potential

The potential encodes the liquid-to-frozen phase transition [9]:

$$V(M) = \alpha M^2 + \beta M^4 + \gamma$$

The sign of αdetermines the phase.

5. Tension Field Dynamics

The tension field is derived from the stiffness field:

$$T_\mu = \partial_\mu m$$

The kinetic term is:

$$\mathcal{L}_T = \frac{1}{2}(\partial_\mu T_\nu)(\partial^\mu T^\nu)$$

This term governs how tension propagates through the mats.

6. Interaction Term: Mass and Distortion

Mass modifies the mat configuration through a mass-defect term:

$$\mathcal{D}(x) = \rho(x)\,\delta M(x)$$

The interaction Lagrangian is:

$$\mathcal{L}_{\text{int}} = -\mathcal{D}(x) M(x)$$

Variation of the action yields the distortion equation:

$$\nabla^2 M = \mathcal{D}(x)$$

This is the TSP analog of the Poisson equation.

7. Gravity as a Tension Gradient

The gravitational field is not curvature but:

$$g_\mu = \partial_\mu T$$

where:

$$T = \sqrt{T_\mu T^\mu}$$

Gravity arises from spatial variation in tension:

$$g_\mu \propto \partial_\mu T$$

This is the central dynamical equation of TSP gravity.

8. Dark Energy from Residual Tension

The freeze-out of the early universe leaves a uniform tension:

$$T_0 = \text{constant}$$

This contributes a cosmological-constant-like term:

$$\mathcal{L}_\Lambda = -T_0$$

Thus:

$$\Lambda_{\text{TSP}} = T_0$$

Dark energy is simply **tension without gradient** [10].

9. Euler–Lagrange Equations

9.1 Mat Field Equation

Variation with respect to M gives:

$$\partial_\mu \partial^\mu M + V'(M) = \mathcal{D}(x)$$

9.2 Stiffness Field Equation

Variation with respect to m yields:

$$\partial_\mu \partial^\mu T^\mu = J(x)$$

where $J(x)$is the tension source term.

10. Stress–Tension Tensor

The stress–tension tensor is [11]:

$$\Theta_{\mu\nu} = (\partial_\mu M)(\partial_\nu M) + (\partial_\mu T_\alpha)(\partial_\nu T^\alpha) - g_{\mu\nu}\mathcal{L}_{\text{TSP}}$$

This tensor determines how tension in **E** projects into geometry in **S**.

11. Emergent Spacetime Geometry

Spacetime curvature in **S** is not fundamental. It is defined by the projection:

$$G_{\mu\nu}^{(S)} = f(\Theta_{\mu\nu})$$

for some projection map f. This is the TSP analog of the Einstein equation.

12. Predictions

TSP predicts that:

- Gravity varies with mat stiffness $m(x)$ [12].
- Dark energy density decreases as mats relax.
- Black holes correspond to mat-folding singularities.
- Gravitational waves correspond to tension-wave propagation.
- Entanglement entropy scales with $| T |$.

13. Conclusion

This paper provides the full Lagrangian formulation of the Tensional Structure Principle. The theory replaces curvature with tension, mass with distortion, and dark energy with uniform residual tension. Together with my companion paper [7], this establishes TSP as a unified mechanical foundation for spacetime, gravity, and cosmic acceleration.

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