Electromagnetic-Gravitational Geometric Unification via Cylindrical Helical Spacetime Flow and the Frenet-Serret Frame
— Dimensional Self-Consistency and Geometric Conservation Laws under Velocity-Field Representation

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Abstract

Within the framework of Topological Residual Theory (TRT), this paper rigorously establishes the first-principles physical picture that “space itself flows at the speed of light along curves in a right-handed cylindrical helical form.” In this picture, length and time originate from different aspects of the same spacetime flow motion, thereby providing a geometric explanation for the origin of the constancy of the speed of light.

To fundamentally resolve the dimensional mismatch among different physical fields that arises in classical field theory from differing measurement protocols, we propose a **Velocity-Field Representation** scheme: the electric, gravitational, and magnetic fields are directly defined as the three local velocity components of the light-speed spacetime flow along the orthogonal basis of the Frenet-Serret (FS) frame:

$$\mathbf{E} = c\mathbf{t}, \quad \mathbf{A} = c\mathbf{n}, \quad \mathbf{B} = c\mathbf{b}.$$

This achieves a complete unification of all underlying field dimensions to velocity at the kinematic level. Since the flow speed of spacetime is fixed at the speed of light c, different physical interactions essentially correspond to the allocation of this motion budget among the three orthogonal directions: tangential propagation, normal centripetal constraint, and binormal torsion. The allocation mechanism is precisely controlled by the helical pitch angle α.

By introducing the time definition $ds = c\,dt$, we derive the first-order time-evolution equations of the three fields under the characteristic rotation frequencies of the FS frame (bending frequency $\omega_\kappa = c\kappa$, twisting frequency $\omega_\tau = c\tau$). Under the rigid rotation of the frame, the total kinetic energy density $U = \frac{1}{2}(\mathbf{E}\cdot\mathbf{E} + \mathbf{A}\cdot\mathbf{A} + \mathbf{B}\cdot\mathbf{B}) = \frac{3}{2}c^2$ remains strictly constant. Utilizing the rigid rotational dynamics of the frame and the vector and scalar triple-product identities, we elucidate the physical nature by which the “self-work” terms of the rigid triad automatically vanish during rotation, thereby revealing the deep geometric mechanism of dissipation-free energy flow.

The macroscopic observable fields — centripetal gravitational acceleration $g = c^2\kappa\mathbf{n}$, tangential electric field strength $e = c^2\kappa\mathbf{t}$, and magnetic rotational angular frequency $B_{\text{macro}} = c\tau\mathbf{b}$ — emerge naturally as the time derivatives or rotational frequencies of the underlying velocity fields

after being modulated by the motion-budget allocation. This provides a self-consistent first-principles framework for the geometric unification of gravity and electromagnetism.

Keywords: Frenet-Serret geometry; right-handed cylindrical helical spacetime flow; velocity-field representation; dimensional self-consistency; energy conservation; motion-budget allocation

1. Introduction

1.1 Physical Picture Revision and the Core Idea of TRT

Traditional physics usually regards particles as entities moving in a static background space. However, within the TRT framework, this picture is revised: it is not that particle entities move independently in space, but rather that the spatial background surrounding objects (or the space fluid, vacuum background) itself flows at the speed of light c along curves in a cylindrical helical form. Particles or objects can be regarded as topological singularities or wave packets in this flow field, and their physical properties (such as mass, spin, charge, gravity, and electromagnetic effects) completely emerge from the geometric deformation of the space flow field (curvature κ, torsion τ) through the topological residual mechanism.

The space flow lines exhibit right-handed cylindrical helical characteristics (right-handed chirality, $\tau > 0$), which determines the directionality of the time arrow (irreversible phase accumulation) and the breaking of chiral symmetry. The Frenet-Serret moving frame $\{\mathbf{t}, \mathbf{n}, \mathbf{b}\}$ can precisely describe the local geometric properties of this “flowing space conduit”: the tangential basis $\mathbf{t}$ carries the energy propagation of light-speed flow, the principal normal basis $\mathbf{n}$ points toward the central axis of the helix (centripetal constraint, corresponding to gravity), and the binormal basis $\mathbf{b}$ carries circumferential torsion (corresponding to electromagnetic oscillation).

1.2 Light-Speed Invariance, the Dimensional Problem, and Motion-Budget Allocation

The fundamental starting point of this model is that the physical world is essentially a description of the motion of space itself. Space is not a static background container but flows continuously at the speed of light along curves in a right-handed cylindrical helical form. In this picture, length and time are not two independent primitive dimensions but two different parametrizations of the same underlying motion. When the arc length of the motion changes, the corresponding intrinsic temporal ordering changes synchronously so as to maintain the fundamental ratio of the speed of light. This is the geometric origin of the invariance of the speed of light — distance (numerator) and time (denominator) are not independent but arise as two aspects of the same motion.

On this basis, the dimensional problem is resolved at the most fundamental level. The root cause of the different dimensions of the electric, magnetic, and gravitational fields in classical physics lies in the adoption of different macroscopic measurement protocols. In the present model, however, the spacetime background itself flows at the speed of light; therefore, the only fundamental physical quantity at the deepest level is the velocity of the flow. We directly define the electric, gravitational, and magnetic fields as the velocity components of the light-speed spacetime flow along the three orthogonal directions of the Frenet-Serret frame:

$$\mathbf{E} = c\mathbf{t}, \quad \mathbf{A} = c\mathbf{n}, \quad \mathbf{B} = c\mathbf{b}.$$

All three fields thus share the same dimension $[L/T]$ at the kinematic level. In the evolution equations, the motion-type fields are transformed into one another through the frame rotation frequencies ($\omega_\kappa = c\kappa$, $\omega_\tau = c\tau$). The frequency factors naturally provide the dimensional transition from velocity to acceleration, so that both sides of the equations consistently carry the dimension $[L/T^2]$, without the need to introduce any artificial scaling constants or conversion factors. This approach eliminates the dimensional conflict of classical field theory from the most fundamental kinematic level.

Since the underlying flow speed is fixed at the speed of light c, different physical interactions correspond to different allocations of this motion budget among the three orthogonal directions of the frame. The helical pitch angle α is precisely the control parameter of this allocation (see Section 2.5 for the mathematical formulation of motion-budget allocation).

1.3 Current Positioning of the Model

The present model is currently positioned at the stage of geometric kinematics + field evolution. Its core lies in reconstructing the rotational and projection relations among the classical electric, magnetic, and gravitational direction vectors through the intrinsic geometric properties of spacetime flow (curvature and torsion), and in solving the dimensional self-consistency of physical quantities from the underlying level. The current stage focuses on the construction of kinematic laws and the geometric covariant framework, and does not involve the complete derivation of externally added dynamical source terms or the microscopic quantum dynamics inside particles. The construction of the micro-dynamics framework is the core subsequent work (see Section 7.2).

1.4 Orthogonality of the Three Fields and the Motivation for Geometric Unification

The three fields strictly correspond to the three orthogonal directions of the moving frame and are therefore mutually perpendicular. This orthogonality guarantees the disappearance of energy cross terms, pure orthogonal channels for geometric coupling, and the directional transmission of Poynting-like energy flow along the flow direction $\mathbf{t}$.

This mapping realizes the following natural unification of concepts at the geometric level:

- Electric field = propagation velocity component of spacetime flow itself along the tangential direction.
- Gravitational field = contraction velocity component of spacetime flow lines along the principal normal direction (centripetal constraint).

- Magnetic field = circumferential torsional velocity component of spacetime flow lines along the binormal direction (circumferential torsion).

Within this framework, gravity is interpreted as the "residual of residuals": microscopic electromagnetic residuals, after statistical averaging over a macroscopic number of particles $N \approx 10^{80}$, are scaled by $1/\sqrt{N}$ fluctuations to produce extremely weak macroscopic gravity. This is intrinsically consistent with Sakharov's idea of induced gravity.

2. Frenet-Serret Geometric Modeling of Spacetime Flow Lines and Definition of Physical Parameters

2.1 Right-Handed Cylindrical Helical Parametrization (Local Approximation Note)

Let the cylindrical radius be r (characterizing the radial scale of the flow-line conduit), θ the rotation angle parameter, and the axial pitch be P. Define the dimensionless topological parameter $\beta = \tan\alpha = P/(2\pi r)$, where α is the helical pitch angle.

Note: The right-handed cylindrical helical parametrization adopted in this paper is a local approximation along the flow line. When the curvature and torsion vary slowly along the flow line, this approximation is valid; the global structure in real spacetime can be obtained by integrating the FS frame evolution along the flow line.

The position vector of the spacetime flow line is parametrized in the right-handed helical convention (θ increasing corresponds to positive advancement in the z-direction) as:

$$\mathbf{r}(\theta) = (r\cos\theta,\ r\sin\theta,\ r\beta\theta).$$

2.2 Arc-Length Element and Unit Tangent Vector

Differentiating the position vector with respect to the parameter θ:

$$\frac{d\mathbf{r}}{d\theta} = (-r\sin\theta,\ r\cos\theta,\ r\beta).$$

Its magnitude is:

$$\left|\frac{d\mathbf{r}}{d\theta}\right| = r\sqrt{1+\beta^2}.$$

Since the arc-length element satisfies $ds = |\, d\mathbf{r}/d\theta \,|\ d\theta$, we obtain:

$$ds = r\sqrt{1+\beta^2}\, d\theta \quad \Rightarrow \quad \frac{ds}{d\theta} = r\sqrt{1+\beta^2}.$$

The unit tangent vector $\mathbf{t}$ (indicating the direction of light-speed flow and carrying energy propagation) is defined as the derivative of the position vector with respect to arc length s:

$$\mathbf{t} = \frac{d\mathbf{r}}{ds} = \frac{dr/d\theta}{ds/d\theta} = \frac{1}{\sqrt{1+\beta^2}}(-\sin\theta,\ \cos\theta,\ \beta).$$

2.3 Curvature κ and Unit Principal Normal Vector $\mathbf{n}$

According to the Frenet-Serret formula:

$$\frac{d\mathbf{t}}{ds} = \kappa\mathbf{n}.$$

Using the chain rule to compute $d\mathbf{t}/ds$:

$$\frac{d\mathbf{t}}{ds} = \frac{dt/d\theta}{ds/d\theta} = \frac{1}{r(1+\beta^2)}(-\cos\theta,\ -\sin\theta,\ 0).$$

Since $\mathbf{n}$ is a unit vector (magnitude 1), its coefficient is the curvature κ:

$$\kappa = \left|\frac{d\mathbf{t}}{ds}\right| = \frac{1}{r(1+\beta^2)}.$$

The corresponding unit principal normal vector $\mathbf{n}$ (always pointing toward the central axis of the cylinder, representing the inward centripetal constraint of the spacetime flow field and corresponding to the direction of gravitational acceleration) is:

$$\mathbf{n} = (-\cos\theta,\ -\sin\theta,\ 0).$$

Physical meaning: The larger the curvature κ, the stronger the bending and the stronger the gravitational effect. The direction of $\mathbf{n}$ is the direction in which the gravitational field points toward the center acceleration.

2.4 Torsion τ and Unit Binormal Vector $\mathbf{b}$

The unit binormal vector $\mathbf{b}$ is perpendicular to both $\mathbf{t}$ and $\mathbf{n}$ and carries circumferential torsion (corresponding to electromagnetic oscillation). It is defined as their cross product:

$$\mathbf{b} = \mathbf{t} \times \mathbf{n} = \frac{1}{\sqrt{1+\beta^2}}(\beta \sin\theta,\ -\beta\cos\theta,\ 1).$$

According to the Frenet-Serret formula:

$$\frac{d\mathbf{b}}{ds} = -\tau\mathbf{n}.$$

The torsion τ is calculated as:

$$\tau = \frac{\beta}{r(1+\beta^2)}.$$

2.5 Angle Parametrization and Topological Essence (Mathematical Formulation of Motion-Budget Allocation)

Using the trigonometric identity $1+\beta^2 = \sec^2\alpha = 1/\cos^2\alpha$, the curvature and torsion can be written precisely as:

$$\kappa = \frac{\cos^2\alpha}{r}, \quad \tau = \frac{\sin\alpha\cos\alpha}{r}.$$

The ratio of the two eliminates the absolute spatial scale r and is determined solely by the topological property of the spacetime flow line (the helical pitch angle α):

$$\frac{\tau}{\kappa} = \tan\alpha.$$

Core Insight and Motion-Budget Allocation:

$\tan\alpha$ is a purely topological parameter that precisely regulates the coupling strength ratio between curvature (gravity) and torsion (magnetism). Under the right-handed helical convention, we define $\tau > 0$ (positive torsion), ensuring consistency between the handedness of the flow field and the direction of the time arrow.

Mathematical formulation of motion-budget allocation: Since the underlying motion speed is fixed at the speed of light c, the helical pitch angle α precisely determines the relative weights (allocation proportions) of the total motion "budget" in the three orthogonal directions through $\kappa \propto \cos^2 \alpha$ and $\tau \propto \sin \alpha \cos \alpha$:

- When $\alpha \to 0$, the motion budget is mainly allocated to circumferential torsion, and the magnetic (motion-type) effect is relatively enhanced.
- When $\alpha \to \pi/2$, the motion budget is mainly allocated to tangential straight-line propagation, and the electric (motion-type) propagation effect dominates.
- Near intermediate values, centripetal constraint (acceleration-type) and torsion reach dynamic balance.

This allocation directly determines the relative strengths and coupling characteristics of the three fields in physics. The dimensionless parameter $\tan \alpha = \tau/\kappa$ is the precise mathematical expression of this allocation ratio, thereby unifying the geometric origin of the three fields at the first-principles level.

3. Orthogonal Moving Frame and Velocity-Field Representation (Containing Dimensional Self-Consistency)

3.1 Properties of the Orthogonal Moving Frame

The FS frame $\{\mathbf{t}, \mathbf{n}, \mathbf{b}\}$ constitutes a right-handed orthogonal unit moving frame at any point on the flow line, satisfying:

- $\mathbf{t} \cdot \mathbf{n} = \mathbf{t} \cdot \mathbf{b} = \mathbf{n} \cdot \mathbf{b} = 0$
- $|\mathbf{t}| = |\mathbf{n}| = |\mathbf{b}| = 1$
- $\mathbf{b} = \mathbf{t} \times \mathbf{n}$ (right-handed orientation)

This directly guarantees that the three physical field directions defined by this frame are strictly mutually perpendicular.

3.2 Precise Correspondence of the Three Fields and the Velocity-Field Representation (Core of Dimensional Unification)

To eliminate the dimensional inconsistency of classical theory from its root, we define the mapping between the physical field vectors and the geometric frame as follows (Velocity-Field Representation):

- **Electric velocity field $\mathbf{E}$**: $\mathbf{E} = c\mathbf{t}$ (dimension $[L/T]$)
  Represents the propagation velocity component of the spacetime background flow along the tangential direction, carrying energy/perturbations along the light-speed direction (primarily motion-type).
- **Gravitational velocity field $\mathbf{A}$**: $\mathbf{A} = c\mathbf{n}$ (dimension $[L/T]$)
  Mathematically a velocity component representing the contraction velocity component of the spacetime background flow along the principal normal direction (corresponding to centripetal

constraint); **physically, it represents the strength of the acceleration-type centripetal constraint**. Its observable effect emerges as an acceleration-type gravitational field after frequency modulation.

- **Magnetic velocity field $\mathbf{B}$**: $\mathbf{B} = c\mathbf{b}$ (dimension $[L/T]$)
  Represents the torsional velocity component of the spacetime background flow along the binormal direction, corresponding to circumferential torsion (primarily motion-type, angular character).

This definition completely resolves the dimensional problem at the most fundamental level: all fields possess the same dimension $[L/T]$, without introducing any arbitrary scaling constant E_0 or artificial unit conversion. The subsequent macroscopic observable fields (accelerations and frequencies) emerge naturally as time derivatives or frequency modulations of these velocity fields. The electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$ primarily participate in motion-type propagation and torsion, while the gravitational constraint strength $\mathbf{A}$ undergoes the transition to acceleration-type through the bending frequency.

3.3 Dot-Product Relations of the Three Fields (Orthogonality)

Due to the orthogonality and normalization of the moving frame, the dot products of the three fields strictly satisfy:

$$\mathbf{E} \cdot \mathbf{A} = 0, \quad \mathbf{E} \cdot \mathbf{B} = 0, \quad \mathbf{A} \cdot \mathbf{B} = 0.$$

This orthogonality property has the following direct physical consequences:

1. No direct work cross terms: There is no direct conductive work cross term between the three fields; energy is transmitted in mutually perpendicular directions (orthogonal transmission).
2. Additivity of energy density: The total energy density of the system can be simply added without cross-interference terms.
3. Directional energy flow: Poynting-like energy flow is strictly transmitted along the tangential direction $\mathbf{t}$ (spacetime flow direction).

4. Three-Field Coupling Dynamics and Time Evolution

4.1 Basic Setting and Unified Frame Evolution

Since the spacetime background flows along the flow line at the speed of light, we introduce the time definition:

$$ds = c\,dt \quad \Rightarrow \quad \frac{d}{dt} = c\frac{d}{ds}.$$

Combining with the characteristic self-rotation frequencies of the frame $\omega_\kappa = c\kappa$ and $\omega_\tau = c\tau$, the standard Frenet-Serret spatial evolution equations can be uniformly written in time-evolution form:

$$\frac{d\mathbf{t}}{dt} = \omega_\kappa \mathbf{n}, \quad \frac{d\mathbf{n}}{dt} = -\omega_\kappa \mathbf{t} + \omega_\tau \mathbf{b}, \quad \frac{d\mathbf{b}}{dt} = -\omega_\tau \mathbf{n}.$$

4.2 Time-Derivative Relations of the Directions of the Three Fields (Velocity-Field Version)

Using the characteristic frequency form, we directly derive three mutually coupled velocity-field evolution relations (completely self-consistent, no dimensional problem):

Relation 1: Time variation of the electric field direction (influenced by the gravitational field)

$$\frac{d\mathbf{E}}{dt} = \omega_\kappa \mathbf{A}.$$

Physical picture: The propagation velocity of the electric field $\mathbf{E}$ is deflected because of the self-rotation frequency ω_κ of the spacetime flow line, and the deflection change points toward the gravitational velocity field direction $\mathbf{n}$. This geometrically corresponds precisely to the first-principles picture of “gravitational bending of light rays”; the magnitude of the deflection angular velocity is determined by the bending angular frequency ω_κ.

Relation 2: Time variation of the gravitational field direction (jointly influenced by the electromagnetic double fields)

$$\frac{d\mathbf{A}}{dt} = -\omega_\kappa \mathbf{E} + \omega_\tau \mathbf{B}.$$

Physical picture: The gravitational velocity field $\mathbf{A}$ is simultaneously subjected to two aspects of geometric feedback:

- $-\omega_\kappa \mathbf{E}$: The geometric reaction of electric field flow on gravity (the “push-back” caused by curvature).
- $\omega_\tau \mathbf{B}$: The driving of the gravitational direction by magnetic torsion (planar rotation caused by torsion).

This shows that the gravitational field is not locally isolated; it changes direction because of electric field propagation and magnetic torsion.

Relation 3: Time variation of the magnetic field direction (influenced by the gravitational field)

$$\frac{d\mathbf{B}}{dt} = -\omega_\tau \mathbf{A}.$$

Physical picture: The magnetic velocity field $\mathbf{B}$ is mainly influenced by the torsion frequency ω_τ of the gravitational direction $\mathbf{n}$, reflecting the geometric torsion coupling between the magnetic field and the gravitational field.

4.3 Summary Table of Mutual Derivative Relations of the Three Fields

Coupling Relation	Mathematical Evolution Form	Physical Picture Explanation	Geometric Origin
Electric field influenced by gravity	$d\mathbf{E}/dt = \omega_\kappa \mathbf{A}$	Electric field propagation direction is “bent” by gravity	Bending frequency ω_κ
Gravity jointly influenced by electromagnetic feedback	$d\mathbf{A}/dt = -\omega_\kappa \mathbf{E} + \omega_\tau \mathbf{B}$	Gravitational direction is driven by both electric field reaction and magnetic torsion	Bending frequency + twisting frequency
Magnetic field influenced by gravity	$d\mathbf{B}/dt = -\omega_\tau \mathbf{A}$	Magnetic field direction is “twisted” by gravity	Twisting frequency ω_τ

4.4 Geometric Emergence of Macroscopic Observable Fields

The gravitational acceleration, electric field strength, and magnetic field strength that we observe macroscopically are essentially the time derivatives or rotational frequencies of the underlying motion-type velocity fields after being modulated by the frame rotation frequencies. The helical pitch angle α, acting as the control parameter for the total motion budget of the light-speed spacetime flow, has already determined the allocation proportions of this budget among the three directions through the expressions of curvature and torsion:

$$\kappa = \frac{\cos^2\alpha}{r}, \quad \tau = \frac{\sin\alpha\cos\alpha}{r}.$$

Consequently, the forms of the macroscopic fields already incorporate this allocation effect.

1. Emergence of the Macroscopic Gravitational Field g (Core Transition from Motion-Type to Acceleration-Type)

Although the gravitational velocity field $\mathbf{A}$ exists mathematically as a velocity component, it physically represents the strength of the centripetal constraint. This strength is modulated by the bending frequency $\omega_\kappa = c\kappa$ and is thereby transformed into the observable acceleration-type gravitational field:

$$g = \omega_\kappa \mathbf{A} = c^2 \kappa \mathbf{n} \quad (\text{dimension: } [L/T^2])$$

Physical picture: The motion-type centripetal constraint strength is transformed into an acceleration-type field through the bending frequency. Because $\kappa \propto \cos^2 \alpha$, when α varies, the proportion of the motion budget allocated to the normal direction changes accordingly, directly affecting the magnitude of the macroscopic gravitational strength. This is precisely the geometric mechanism by which light-speed spacetime flow generates centripetal acceleration due to curvature, consistent with Sakharov's idea of induced gravity as a statistical residual.

2. Emergence of the Macroscopic Electric Field e

The electric field (motion-type) undergoes a time variation through the same bending frequency and thereby emerges as an acceleration-type electric field strength:

$$e = \omega_\kappa \mathbf{E} = c^2 \kappa \mathbf{t} \quad (\text{dimension: } [L/T^2])$$

Physical picture: The motion-type electric field $\mathbf{E}$ is transformed into the observable electric field e under the action of the bending frequency. Because $\kappa \propto \cos^2 \alpha$, variations in α alter the allocation proportion of the motion budget in the tangential direction, thereby influencing the macroscopic electric field strength. It shares the same origin (curvature) with gravity but is projected tangentially, reflecting the difference in motion-budget allocation along different directions.

3. Emergence of the Macroscopic Magnetic Field B_{macro}

As a motion-type torsional quantity, the magnetic field has an observable effect that directly manifests as an angular frequency:

$$B_{\text{macro}} = \omega_\tau \mathbf{B} = c\tau \mathbf{b} \quad (\text{dimension: } [T^{-1}])$$

Physical picture: The torsional effect of the motion-type magnetic field $\mathbf{B}$ (or its basis vector $\mathbf{b}$) directly appears as the macroscopic magnetic field frequency. Because $\tau \propto \sin \alpha \cos \alpha$, variations in α change the allocation proportion of the motion budget in the binormal direction, thereby regulating the macroscopic magnetic rotational angular velocity.

5. Energy Conservation: Geometric Proof Based on Orthogonality and Cross-Product Properties of the Three Fields

The three fields $\mathbf{E} = c\mathbf{t}$, $\mathbf{A} = c\mathbf{n}$, $\mathbf{B} = c\mathbf{b}$ are strictly orthogonal and constitute a local right-handed triad. The evolution equations describe the rigid rotation of this frame in space (bending frequency $\omega_\kappa = c\kappa$, twisting frequency $\omega_\tau = c\tau$). Under this geometric motion, the total energy density of the system remains constant locally.

Below we first give a direct substitution proof, and then reveal its geometric essence through the vector triple-product identity.

5.1 Direct Proof of the Rate of Change of Energy

Define the total energy density of the three-field system as the direct non-interfering superposition of the energy densities of the three orthogonal components:

$$U = \frac{1}{2}(\mathbf{E} \cdot \mathbf{E} + \mathbf{A} \cdot \mathbf{A} + \mathbf{B} \cdot \mathbf{B}) = \frac{3}{2}c^2.$$

Since the magnitudes $|\mathbf{E}| = |\mathbf{A}| = |\mathbf{B}| = c$ are constant, the local total energy density is constant. Taking the time derivative with respect to t:

$$\frac{dU}{dt} = \mathbf{E} \cdot \frac{d\mathbf{E}}{dt} + \mathbf{A} \cdot \frac{d\mathbf{A}}{dt} + \mathbf{B} \cdot \frac{d\mathbf{B}}{dt}.$$

Substituting the first-order time-coupled evolution equations of the three fields:

$$\frac{dU}{dt} = \mathbf{E} \cdot (\omega_\kappa \mathbf{A}) + \mathbf{A} \cdot (-\omega_\kappa \mathbf{E} + \omega_\tau \mathbf{B}) + \mathbf{B} \cdot (-\omega_\tau \mathbf{A}).$$

Expanding each term and extracting common factors:

$$\frac{dU}{dt} = \omega_\kappa(\mathbf{E} \cdot \mathbf{A} - \mathbf{A} \cdot \mathbf{E}) + \omega_\tau(\mathbf{A} \cdot \mathbf{B} - \mathbf{B} \cdot \mathbf{A}).$$

Due to the symmetry of the dot product and the mutual orthogonality of the three fields ($\mathbf{E} \cdot \mathbf{A} = \mathbf{A} \cdot \mathbf{B} = \mathbf{E} \cdot \mathbf{B} = 0$), all terms on the right-hand side cancel to zero:

$$\frac{dU}{dt} = 0.$$

The local total energy density U does not change with time; the system energy is strictly conserved. At this point, the frame triad undergoes rigid rotation, each component velocity magnitude remains constant at c, and orthogonality guarantees no self-work dissipation.

5.2 Rigidity of the Three Fields and the Vector Triple-Product Identity

To reveal the geometric root of the above conservation, we utilize the vector triple-product identity (BAC-CAB rule):

$$\mathbf{E} \times (\mathbf{A} \times \mathbf{B}) = \mathbf{A}(\mathbf{E} \cdot \mathbf{B}) - \mathbf{B}(\mathbf{E} \cdot \mathbf{A}).$$

Since the three fields are strictly orthogonal ($\mathbf{E} \cdot \mathbf{A} = 0$, $\mathbf{E} \cdot \mathbf{B} = 0$), both terms on the right-hand side are zero; hence:

$$\mathbf{E} \times (\mathbf{A} \times \mathbf{B}) = 0.$$

Similarly, we obtain:

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{E}) = 0, \quad \mathbf{B} \times (\mathbf{E} \times \mathbf{A}) = 0.$$

This shows that the cross product of any two fields is always perpendicular to the third field. The three fields remain linearly dependent and mutually orthogonal at any instant, forming an undeformable rigid triad. The bending and twisting of the frame only cause overall rigid rotation and do not destroy the orthogonality or magnitude relations among the three fields. This rigid structure is the geometric prerequisite for energy conservation.

5.3 Energy Conservation Proof Based on the Triple-Product Identity

On the basis of the rigid triad of the three fields, energy conservation can be obtained more directly from the vector identity. For any field $\mathbf{F}$, its time rate of change is always geometrically perpendicular to itself; therefore it satisfies:

$$\mathbf{F} \cdot \frac{d\mathbf{F}}{dt} = 0.$$

This property is equivalent to the scalar triple-product identity $\mathbf{F} \cdot (\mathbf{F} \times \mathbf{G}) \equiv 0$ holding for any vector $\mathbf{G}$. Applying this to the three-field system, the dot product of each component's rate of change with itself is as follows:

- $\mathbf{E} \cdot \frac{d\mathbf{E}}{dt} = \mathbf{E} \cdot (\omega_\kappa \mathbf{A}) = \omega_\kappa (\mathbf{E} \cdot \mathbf{A}) = 0$
  (Because the electric velocity field $\mathbf{E}$ is along $\mathbf{t}$ and the gravitational velocity field $\mathbf{A}$ is along $\mathbf{n}$, the two bases satisfy strict orthogonality $\mathbf{t} \cdot \mathbf{n} = 0$.)
- $\mathbf{B} \cdot \frac{d\mathbf{B}}{dt} = \mathbf{B} \cdot (-\omega_\tau \mathbf{A}) = -\omega_\tau (\mathbf{B} \cdot \mathbf{A}) = 0$
  (Because the magnetic velocity field $\mathbf{B}$ is along $\mathbf{b}$ and the gravitational velocity field $\mathbf{A}$ is along

$\mathbf{n}$, the two bases satisfy strict orthogonality $\mathbf{b} \cdot \mathbf{n} = 0$.)

- $\mathbf{A} \cdot \frac{d\mathbf{A}}{dt} = \mathbf{A} \cdot (-\omega_\kappa \mathbf{E} + \omega_\tau \mathbf{B}) = -\omega_\kappa(\mathbf{A} \cdot \mathbf{E}) + \omega_\tau(\mathbf{A} \cdot \mathbf{B}) = 0$

Therefore, the time derivative of the total energy density is directly zero:

$$\frac{dU}{dt} = 0.$$

Physical essence: During the rigid rotation of the three fields, energy flows without loss only among the three orthogonal directions (tangential velocity, normal velocity, binormal velocity) in a closed cycle. Any “self-work” term automatically disappears due to the triple-product identity, forming a closed cycle without net dissipation. This is geometrically identical to the mechanism in classical electromagnetic waves where the Poynting vector $\mathbf{E} \times \mathbf{B}$ satisfies $\mathbf{E} \cdot (\mathbf{E} \times \mathbf{B}) = 0$, except that it is here extended to a unified three-field system that includes the gravitational velocity field $\mathbf{A}$.

6. Three-Layer Topological Structure and Topological Invariants

6.1 Three-Layer Topological Expansion

Within the TRT framework, the geometric structure of spacetime flow lines naturally expands into three topological layers, each producing specific geometric residuals and corresponding to different interactions:

1. **First layer: Strong core (naked magnetic tube)**
   Located in the central axis region of the helical flow line, dominated by pure torsion (torsion τ). Corresponds to the “naked” magnetic flux tube structure, appearing topologically as a tubular singularity. This layer has no curvature dominance and exhibits extremely strong chirality.
2. **Second layer: Weak/spin layer (chiral symmetry breaking)**
   Located in the outer envelope layer dominated by helical torsion. The existence of τ leads to parity and chiral symmetry breaking, naturally giving rise to particle spin and weak interaction residuals.
3. **Third layer: Electromagnetic boundary layer (Gaussian sphere S^2)**
   Located in the outermost layer, dominated by curvature κ projection. It encloses the entire flow conduit with a Gaussian sphere S^2 (boundary conditions induced by the FS frame). The electromagnetic field “freezes” and projects outward in this layer, producing standard Maxwell electromagnetic interactions. The Gauss-Bonnet theorem plays a constraining role in this layer:

$$\int_{S^2} K \, dA = 2\pi\chi(S^2) = 4\pi \quad \text{(Euler characteristic } \chi = 2\text{)}.$$

The three layers are coupled through the projection residual mechanism: geometric deformation residuals from inner layers project outward, producing effective potentials and physical coupling constants.

6.2 Topological Invariants and the Physical Meaning of $\tan\alpha$

- **Euler characteristic** $\chi = 2$: The topological invariant of the Gaussian sphere boundary layer, constraining the total curvature integral over the cross-section of the spacetime conduit.
- **Gauss-Bonnet theorem and imperfect closure**: When a one-dimensional helical fiber winds in three-dimensional space, it cannot achieve perfect topological closure. This “imperfection” produces geometric residuals that are precisely the geometric source of all physical charges (mass, charge) in TRT.
- **Dimensionless topological invariant** $\tan\alpha = \tau/\kappa$: This is the most core topological invariant. Because it eliminates the scale parameter r, its value is completely determined by the ratio of the axial pitch to the circumference of the helical flow line. In the renormalization group flow, $\tan\alpha$ flows to a specific geometric fixed point, precisely controlling the coupling strength ratio among gravity, magnetism, and electricity.

6.3 Residual Mechanism and Gravitational Emergence

Each layer projection produces residuals: microscopic electromagnetic residuals, after statistical averaging over a macroscopic number of particles $N \approx 10^{80}$, through $1/\sqrt{N}$ statistical fluctuation scaling, emerge as macroscopic gravitational residuals at extremely weak scales. This level of emergence naturally realizes the geometric unification from the quantum (chiral breaking, spin) to classical gravity (quantitative relations such as $G \approx \mu_0 \alpha^2$ are given in previous TRT work).

7. Physical Meaning and Outlook

7.1 Unified Picture and Advantages

This model clearly shows that the electric, magnetic, and gravitational fields are not independent existences, but rather projections and time deflections of the same spacetime flow geometry in different directions:

- Spacetime flow itself (tangential velocity) → electric field $\mathbf{E}$
- Spacetime centripetal deflection (normal acceleration) → gravitational field g
- Spacetime circumferential torsion (binormal frequency) → magnetic field B_{macro}

When spacetime flow lines possess both curvature and torsion, the three fields are naturally coupled through the FS connection, with coupling coefficients determined by the characteristic rotation frequencies. The model is fully compatible with classical mathematical tools (Frenet-Serret formulas, Gauss-Bonnet theorem, renormalization group flow, and holographic principle), and binds the origin of all interactions self-consistently to the local motion of spacetime geometry.

Summary of First-Principles Advantages (Integrated):

1. **Unified architecture**: Single motion entity (light-speed helical flow) + single geometric tool (FS frame) + single topological parameter ($\tan\alpha$).
2. **Underlying self-consistency**: The dimensional problem is completely resolved at the most fundamental level (velocity field), with no artificial factors or redundant constants.
3. **Natural emergence**: Evolution equations, energy conservation, and macroscopic field emergence are all geometrically self-consistent and closed.
4. **Conceptual overlap**: Naturally connects with Sakharov's induced gravity, Zitterbewegung inner clock, chiral symmetry breaking, and other frontier concepts.

7.2 Subsequent Research Directions and Core Missing Link

1. **Construction of the micro-dynamics framework (core missing link)**:
The present model is currently mainly at the geometric kinematics stage and does not yet contain self-consistent dynamical feedback. The construction of micro-dynamics is the most important missing link in the current model. Subsequent research needs to establish a mechanism for transitioning from continuous velocity fields and acceleration flows to localized "particle wave packets." This requires introducing nonlinear geometric self-interaction, treating particles as dynamically stable solutions of topological singularities in flowing space (similar to soliton solutions), and giving the micro-dynamical evolution equations of singularities under external field deflection, thereby achieving a complete derivation at the micro-dynamics level.
2. **Derivation of fundamental constants**:
Deepen the geometric derivation and renormalization factor calculation of more fundamental constants (such as the reduced Planck constant $\hbar$, Newtonian gravitational constant G, Planck mass, etc.).
3. **Higher-order topological residuals**:
Perform quantitative calculations of higher-order residuals (around the 21st order) as dark matter candidates.
4. **Experimental exploration**:
Carry out tabletop topological residual tests and plasma helical flow simulation experiments.

8. Conclusion

In this paper, within the strict Frenet-Serret geometric framework, we have established a complete three-field coupling model of right-handed cylindrical helical light-speed spacetime flow. By proposing the Velocity-Field Representation scheme — in which the electric field is directly represented as the velocity field $\mathbf{E} = c\mathbf{t}$, the gravitational field is represented (mathematically as the velocity component $\mathbf{A} = c\mathbf{n}$, physically as acceleration-type centripetal constraint strength) as $g = c^2\kappa\mathbf{n}$, and the magnetic field is represented as the rotational angular frequency field $B_{\text{macro}} = c\tau\mathbf{b}$ — we have resolved the problem of dimensional mismatch among physical fields from the most fundamental kinematic level and achieved a highly self-consistent first-principles description. At the same time, we explicitly point out that the present model is currently at the stage of "geometric kinematics + field evolution."

We have defined the curvature κ and torsion τ, and clarified the correspondence relations and orthogonal dot-product properties among the electric, gravitational, and magnetic directions. All underlying fields are unified in dimension $[L/T]$, and the left- and right-hand sides of the dynamical evolution equations match perfectly in dimension without any additional artificial conversion. By introducing the parametrization of flow time $ds = c\,dt$, we derived the first-order mutual evolution equations for the directions of the three fields and elucidated that the rotation frequencies are the essential coupling strengths of the frame evolution.

Based on the mutual orthogonality of the three fields and the rigid-triad constraint, we provided a direct geometric proof of strict local energy conservation ($dU/dt = 0$) and, from the geometric nature of the vanishing of scalar triple products, demonstrated the self-consistent physical constraint of dissipation-free energy flow. Finally, we revealed the three-layer topological invariant structure containing the Euler characteristic and $\tan\alpha$, indicating the decisive role of dimensionless invariants in determining the strengths of the three fields, and thereby provided a self-consistent geometric first-principles framework for the unification of gravity and electromagnetism.

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