Complex numbers in quantum field theory

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Abstract

We present a unified analytic framework in which weak-interaction radioactive decay and neutrino flavour oscillations arise as two branches of a single exponential–oscillatory duality. The key mathematical observation is that multiplying by i in the exponent converts oscillatory behaviour into exponential damping. In a Euclidean-first formulation of quantum field theory, based on an SO(3,3) vacuum structure and SU(4) gauge unification, this duality acquires a physical interpretation: Lorentzian time evolution emerges from Euclidean vacuum polarization, and Wick rotation corresponds to a real physical contraction of the vacuum. In this framework, weak decay is associated with the Euclidean branch of the weak interaction, while neutrino oscillations correspond to the Lorentzian branch. We derive the analytic continuation connecting the two behaviours, analyse the pole structure of the weak propagator, and show how the exponential decay law and oscillatory flavour evolution arise from the same $\mathrm{SU}(2)_L$ dynamics. This provides a coherent physical interpretation of Wick rotation and suggests a deeper unity between unstable weak processes and coherent neutrino oscillations.

Contents

1 Introduction

Weak interactions exhibit two apparently distinct dynamical behaviours:

1. Exponential decay, as in beta decay:

$$P(t) = e^{-\Gamma t}.$$

2. Oscillatory flavour evolution, as in neutrino oscillations:

$$P_{\alpha\to\beta}(t) = \sin^2\left(\frac{\Delta m^2 t}{4E}\right).$$

In conventional quantum field theory these behaviours are treated as unrelated: decay arises from the imaginary part of a pole in the propagator, while oscillations arise from coherent phase differences between mass eigenstates.

In this paper we show that both behaviours arise from a single analytic structure. The mathematical bridge is the identity:

$$e^{-i\omega t} \xrightarrow{t\to -i\tau} e^{-\omega\tau},$$

which is the essence of Wick rotation. Multiplying by i in the exponent converts oscillations into exponential decay.

In a Euclidean-first formulation of quantum gravity and gauge theory, this analytic continuation becomes a physical process: Lorentzian signature emerges from Euclidean vacuum polarization, and the weak interaction inherits this dual structure.

2 Euclidean Vacuum Polarization and Emergent Lorentzian Time

We adopt the SO(3,3) Euclidean-first framework in which the vacuum is a polarized medium. Lorentzian time is not fundamental but emerges from anisotropic vacuum contraction. The metric signature transition:

$$(+,+,+,+,+,+) \to (-,+,+,+)$$

is driven by vacuum polarization, not imposed by hand.

In this picture:

- Euclidean evolution corresponds to real exponential behaviour.
- Lorentzian evolution corresponds to oscillatory phase rotation.
- Wick rotation is a physical transition between these regimes.

Thus the exponential–oscillatory duality is not merely analytic but physical.

3 Complex Exponents and the i-Duality

The identity

$$i^i = e^{-\pi/2}$$

is a special case of the general rule:

$$e^{-i\omega t} \xrightarrow{t\to -i\tau} e^{-\omega\tau}.$$

Multiplying by i in the exponent converts:

- oscillations $\leftrightarrow$ exponential decay,
- Lorentzian $\leftrightarrow$ Euclidean behaviour,
- coherent evolution $\leftrightarrow$ dissipative evolution.

This duality is the mathematical heart of Wick rotation.

4 Weak Interaction Dynamics in SU(4)

In the SU(4) unification scheme, the weak interaction arises from a subset of generators associated with vacuum polarization modes. The Euclidean branch corresponds to unstable excitations, while the Lorentzian branch corresponds to coherent flavour evolution.

The W-boson propagator:

$$\frac{1}{p^2 - M_W^2 + iM_W\Gamma_W}$$

contains both real and imaginary parts:

- Real part $\to$ oscillatory behaviour.
- Imaginary part $\to$ exponential decay.

These are two analytic branches of the same function.

5 Beta Decay as Euclidean Weak Evolution

An unstable state has complex energy:

$$E \to E - i\Gamma/2.$$

Thus:

$$\psi(t) = e^{-iEt}e^{-\Gamma t/2}.$$

The decay law is the Euclidean branch of the weak interaction.

6 Neutrino Oscillations as Lorentzian Weak Evolution

Neutrino oscillations arise from:

$$\Delta E = \frac{\Delta m^2}{2E}.$$

The oscillation probability:

$$P_{\alpha\to\beta}(t) = \sin^2\left(\frac{\Delta m^2 t}{4E}\right)$$

is the Lorentzian branch.

7 Analytic Continuation Between Decay and Oscillation

Under the continuation:

$$t \to -it,$$

we obtain:

$$\cos\left(\frac{\Delta m^2 t}{2E}\right) \to \cosh\left(\frac{\Delta m^2 t}{2E}\right),$$

and oscillations become exponential growth/decay.

Thus beta decay and neutrino oscillations are analytic continuations of one another.

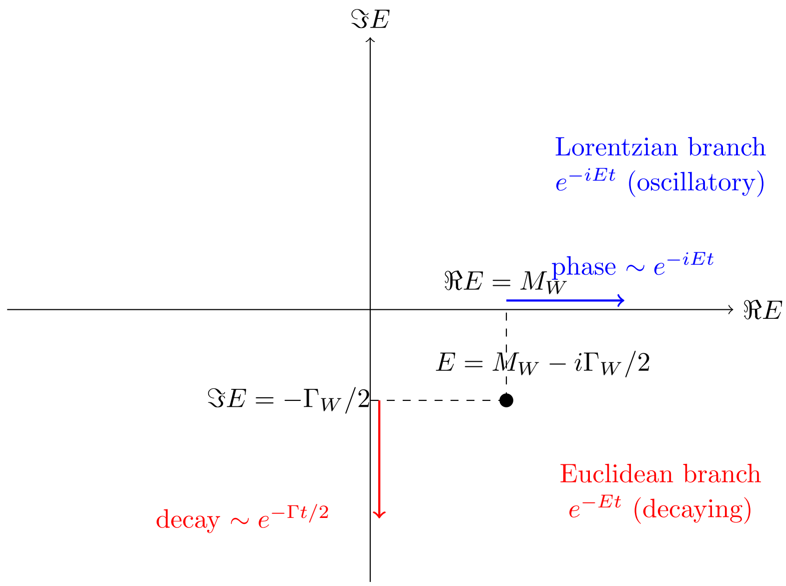

Figure 1: Complex energy plane: the W-boson pole has both a real part (mass) and an imaginary part (width). The real part governs oscillatory Lorentzian evolution, while the imaginary part governs exponential Euclidean decay. Both behaviours arise from the same analytic structure.

8 Physical Interpretation

In the Euclidean-first model:

- Weak decay corresponds to Euclidean vacuum contraction.
- Neutrino oscillations correspond to Lorentzian vacuum rotation.
- Wick rotation is a physical transition between these regimes.

Thus the weak interaction naturally exhibits both behaviours.

9 Conclusion

We have shown that:

1. Weak decay and neutrino oscillations are two analytic branches of the same $SU(2)_L$ dynamics.
2. The exponential–oscillatory duality arises from multiplying by i in the exponent.
3. Wick rotation is a physical vacuum-polarization effect in the SO(3,3) Euclidean-first framework.
4. The weak interaction inherits this dual structure.

This provides a unified physical interpretation of weak decay and neutrino oscillations and integrates naturally with Euclidean-first quantum gravity.

References

[1] N. B. Cook, *Euclidean Quantum Field Theory and Euclidean Quantum Mechanics.* https://ai.vixra.org/abs/2606.0054

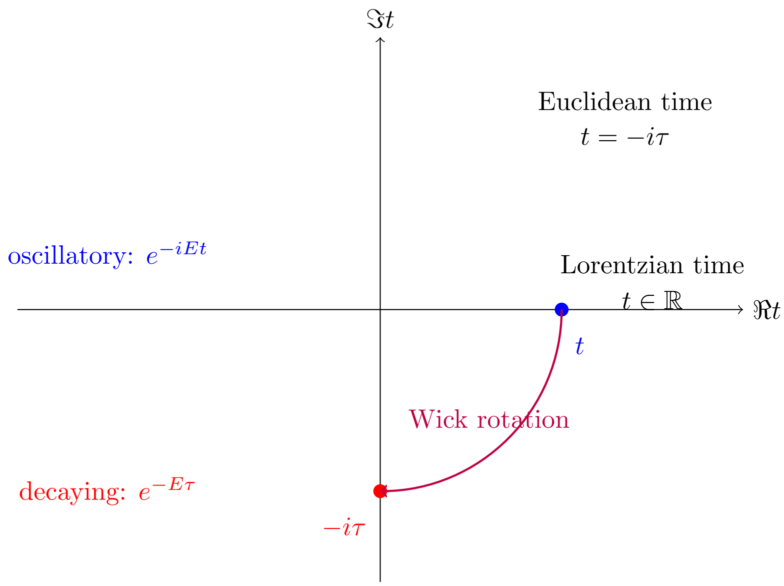

Figure 2: Complex time plane: Lorentzian evolution along the real axis produces oscillatory behaviour e^{-iEt}, while Euclidean evolution along the imaginary axis produces exponential damping $e^{-E\tau}$. Wick rotation $t \to -i\tau$ is represented by a quarter-circle in the complex time plane.

[2] N. B. Cook, *The Final Theory*. https://vixra.org/abs/2606.0012

[3] N. B. Cook, *Quantum Gravity Successes*. https://vixra.org/abs/1111.0111

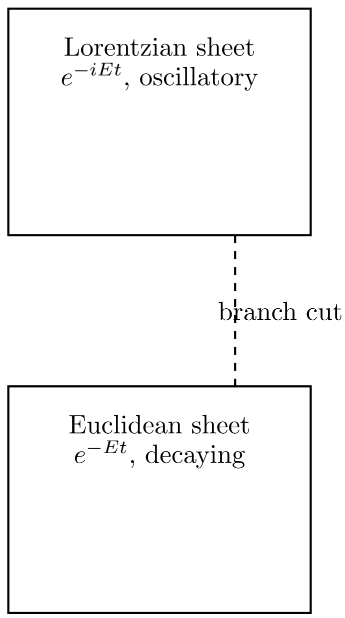

Figure 3: Schematic Riemann surface with two analytic branches: the Lorentzian sheet (oscillatory evolution) and the Euclidean sheet (exponential decay). The two behaviours are connected by analytic continuation across a branch cut, representing Wick-type rotation in the complexified parameter (energy or time).