The Increase in the Center of Mass Momentum of a Three-Body System due to Linear Acceleration through Minkowski Spacetime

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Abstract

Euler's first law requires that changes in the center of mass momentum of a system of bodies can only occur when external impulses act on a system. We report the results of an experiment where after 30 test trials the mean value of known external friction impulses acting on a three-body system accounted for only ~ 8.2 per cent (standard deviation .037 and standard error .0068) of the increase in the final momentum of the system, leaving a ~ 91.8 per cent discrepancy. The three-body system consisted of two spheres, constrained to roll around quarter-circle barriers attached to a third body. Since the spheres were constrained from following their natural straight-line geodesic paths in the metric of flat spacetime, centrifugal real forces emerged radially outward from the center of mass of the spheres, pushing on the inner walls of the curved barriers. This caused the system to accelerate, inducing torque inertial forces on the spheres which increased their orbital angular speed. This increase in speed accounted for the ~ 91.8

per cent impulse discrepancy of the experiment, and thus, when added to the friction impulse, accounted for the total impulse that caused the increase in the center of mass momentum of the system per Euler's first law.

1. Introduction

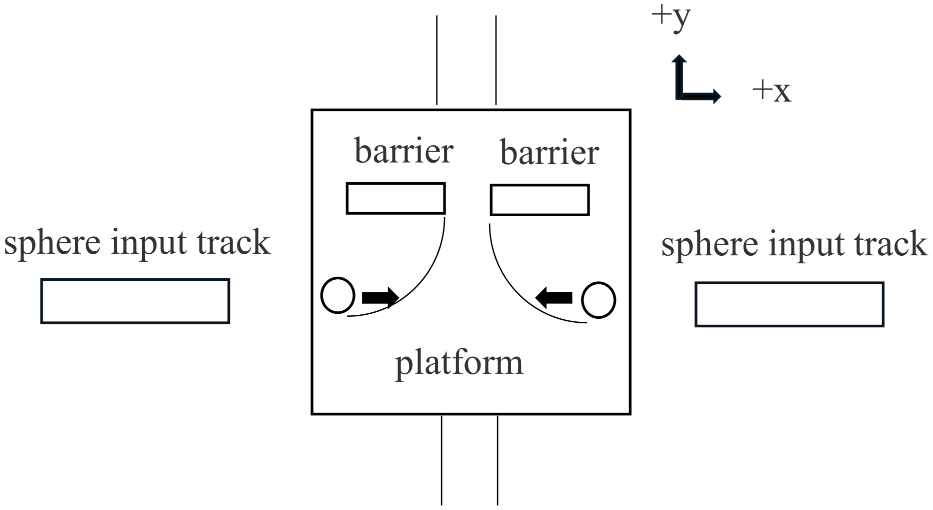
The experiment reported in this paper as far as we know is unique and never has been previously published in physics journals. The initial interest which led to the reported experiment in this paper was due to the results of a prior experiment we performed. In this preliminary experiment a rotating body (rotator) with a vertical shaft at one end was inserted into a bearing assembly that was embedded in a platform. This platform could slide with one degree of freedom with respect to the y-axis. One Pasco accelerometer was attached to the platform and another accelerometer to the rotator. The platform accelerometer measured the proper linear acceleration of the platform; the rotator accelerometer via its internal 3-axis gyroscope measured the proper angular speed of the rotator. The platform was initially butted against a barrier so that it could not move in the negative y-direction. A counter-clockwise rotation was imparted to the rotator when it was at rest at the 180 degrees position. When the rotator passed the 0 degrees point, the platform accelerated in the positive y-direction. The acceleration data of the platform and the angular velocity of the rotator from their accelerometers was transferred via Blue Tooth to a Pasco program on a computer that simultaneously plotted the graph of the data.

The data graphed a sinusoidal wave with zero acceleration for the platform at the zero degrees point, a maximum acceleration at the 90 degrees point, followed by zero acceleration at the 180 degrees point. When the rotator passed the 90 degrees point, the accelerometer on the rotator showed an increase in the rotator's orbital angular speed.

Since the increase in the proper orbital angular speed of the rotator was frame-invariant, we asked could there be an increase in the center of mass momentum of a similar linearly-accelerated system with respect to our inertial lab frame due to this frame-variant increase in orbital angular speed?   This was the motivation for the experiment reported in this paper

2. Materials and Methods

Figure 1



system linear track

In this experiment we attached a platform to the top of a Pasco Smart Cart which was free to travel on top of an aluminum linear track horizontal to the earth with one degree of freedom as shown above in the top-down view of Figure 1. Attached to the surface of the platform were two quarter-circle curves and two barriers. The platform, curves, barriers, and Pasco Smart Cart taken together constituted the mass designated as m_2. Two spheres of equal mass, designated by m_1, entered the surface of the platform simultaneously, initially rolling along the x-axis with equal velocities, designated by v_i.

Each sphere had a m_1 mass of .5330 kg ± .0001 kg. The m_2 mass was .4492 kg ± .0001 kg. A digital level with a resolution of .1 degrees was used to check if the linear track was level with respect to the horizontal x and y-axes. Each barriers the spheres collided into contained neodymium magnets to prevent bounce back after the collision of the spheres. The diameter of each sphere was ~ .051 meters and the radius of each curve was ~ .042 meters.

As the spheres rounded their curves, radially outward centrifugal real forces emerging from the spheres, acted on the curves and caused the spheres-platform-cart assembly to acquire a rectilinear y-component of acceleration in the negative y-direction. Immediately, after rounding and exiting the curves, both spheres made inelastic collisions with the barriers in the positive y-direction, causing the system to acquire a post collision momentum in the positive y-direction. The time and velocity values of the system were communicated via Blue Tooth from the Pasco Smart Cart to a laptop computer. The velocity and time data from the Pasco Smart Cart were recorded at a 50 HZ sampling rate.

The core of this experiment was to measure the impact of friction impulses on the system to see if they accounted for all of the final momentum of the system. When the platform rolled backwards in the negative y-direction as the spheres rounded their curves, external friction impulses acted on the cart's wheels in the positive y-direction. The friction force for each trial was determined by dividing the post collision momentum of the system by the time it took the system to come to a stop. The friction impulse was calculated by multiplying the time of the back movement of the cart in the negative y-direction times this friction force.

It was not necessary to determine the rolling friction between the spheres and platform's surface and curved barriers since these were internal force pairs that would have no impact on the center of mass of the system. Air friction effects were also ignored since the speeds of the cart and spheres were small enough that any air friction drag effects would have negligible impact on the outcome.

3. Results

Table 1. Data summary of 30 Test Trials

Initial Sphere Velocity m/s	Back Time s	Friction Force N	Friction Impulse N · s	Post Collision Velocity m/s	Post Collision Momentum Kg · m/s	Impulse to Momentum Ratio
0.65	0.154	0.0249	0.0038	0.047	0.071	0.05
0.65	0.116	0.0704	0.0082	0.053	0.080	0.10
0.65	0.171	0.0131	0.0022	0.025	0.038	0.06
0.66	0.148	0.0144	0.0021	0.028	0.042	0.05
0.65	0.141	0.0281	0.0040	0.033	0.050	0.08
0.65	0.192	0.0309	0.0059	0.051	0.077	0.08
0.66	0.264	0.0360	0.0095	0.034	0.052	0.18
0.65	0.137	0.0492	0.0067	0.077	0.12	0.06
0.65	0.186	0.0370	0.0069	0.044	0.067	0.10
0.65	0.132	0.0292	0.0039	0.032	0.048	0.08
0.65	0.135	0.0206	0.0028	0.035	0.053	0.05
0.66	0.155	0.0498	0.0077	0.047	0.071	0.11

0.66	0.164	0.0489	0.0080	0.06	0.091	0.088
0.66	0.2	0.0323	0.0065	0.046	0.070	0.09
0.67	0.212	0.0463	0.0098	0.063	0.095	0.10
0.65	0.136	0.0374	0.0051	0.039	0.059	0.09
0.66	0.195	0.0543	0.0106	0.033	0.050	0.21
0.67	0.195	0.0150	0.0029	0.023	0.035	0.08
0.67	0.132	0.0349	0.0046	0.052	0.08	0.06
0.65	0.186	0.0410	0.0076	0.059	0.089	0.09
0.66	0.133	0.0172	0.0023	0.036	0.055	0.04
0.66	0.19	0.0175	0.0033	0.045	0.068	0.05
0.65	0.149	0.0379	0.0056	0.037	0.06	0.10
0.65	0.198	0.0147	0.0029	0.028	0.042	0.07
0.65	0.152	0.0388	0.0059	0.042	0.06	0.09
0.66	0.127	0.0147	0.0019	0.032	0.048	0.04
0.66	0.2	0.0219	0.0044	0.028	0.04	0.10
0.66	0.19	0.0172	0.0033	0.037	0.056	0.06
0.66	0.21	0.0124	0.0026	0.027	0.041	0.06
0.73	0.137	0.0249	0.0034	0.051	0.08	0.04

Statistical summary of impulse to momentum ratio for 30 test trials

Mean	.082
Std Dev	.037
Std Error	.0068

The back time was the time the platform moved backwards from its initial at rest state to the time it attained its maximum value. Post collision velocity was the maximum velocity of the platform right after the collision of the two spheres with their barriers.

It was crucial in this experiment that the two spheres entered the platform with the same speed and that they entered the curves simultaneously. This was necessary in order to cancel out any forces acting on the platform along the x-axis. We were only interested in forces that acted along the y-axis. If the two spheres did not enter the platform simultaneously at the same speed, the cart would twist completely off the track and for these extreme cases the test data run was rejected.

Because the experiment was built of low-density PLA fiber, the system was susceptible to high frequency mechanical vibrations and noise which clouded the interpretation of the final post collision velocity of the spheres and time increment readings and often required a conservative interpretation of the data. This interpretation always filtered the results of the experiment to maximize the impact of the friction impulses on the system so as not to underestimate the contribution of the friction impulses.

In future replications of the experiment, it is recommended that higher density materials be used to dampen the mechanical vibrations while holding the ratio of the total mass of the spheres to the mass of the platform to be at least 2:1. Initial velocity of the spheres should be as high as possible as the acceleration of the system is proportional to this speed.

4. Discussion

Einstein initially hoped that Mach's interpretation of Newton's famous bucket experiment [1,2] would endure as a foundational principle for his general theory of relativity. Mach proposed inertial forces arose from relative acceleration with respect to the cosmic mass of the universe. Newton asserted that inertial forces arose from acceleration with respect to absolute space. Einstein previously found that the Minkowski metric was one of the possible mathematical vacuum solutions in his field equations but he doubted that they implied any physical realities.

However, Einstein eventually rejected Mach's relativism when it was pointed out to him in Willem de Sitter's paper [3] that inertial forces could still emerge in a universe devoid of matter.

Technically, this experiment was performed on the surface of the earth which means the Schwarzschild metric applies. However, since the experiment was performed horizontal to the surface of the earth, we note that an approximate Minkowski metric can be assumed in the analysis of the experiment. There were two space inertial force effects that we suggest were responsible for the ~ 91.8 per cent impulse discrepancy measured in the experiment. The first space inertial force effect was due to the curved barriers that forced the spheres to change their direction from their straight-line geodesic path in Minkowski spacetime. This caused a reaction from space itself, causing real centrifugal forces to act radially outward from the center of mass of the spheres that pushed on the inner walls of the curves. We acknowledge this is a chicken or egg question. Which comes first? Real centrifugal forces acting radially outward through the center of mass of each sphere or centripetal forces acting on the spheres from the constraints of the curved barriers? We suggest they both occur precisely simultaneously. The physical, measurable reality, shown in the preliminary experiment, and this sphere experiment is there was a centrifugal real force that pushed the spheres against the inner walls of the curve that caused both the system and the spheres to have an instantaneous component of linear acceleration in the negative y-direction.

Because the spheres now had an instantaneous rectilinear y-component acceleration in the negative y-direction, this meant the instantaneous linear speed of the spheres increased in the negative y-direction, and therefore, to deviate from their natural geodesic *constant speed* path in Minkowski spacetime. This is when the second space inertial force effect occurred. Again, space itself opposed the increase in linear speed of the spheres off their natural constant speed geodesic path, causing a real inertial force reaction on the spheres in the positive y-direction.

Because this was an interaction between the spheres and space itself, there were no equal and opposite forces that acted on the system in the negative y-direction. And Euler's first law predicts that any increase in the center of mass momentum of a body within a multi-body system due to an external impulse affects an increase in the center of mass momentum of the whole system. Thus, we suggest that it was the linear acceleration of the system through Minkowski space that accounted for the ~ 91.8 per cent impulse discrepancy measured in the experiment and when added to the friction impulse accounts for the total increase in the center of mass momentum of the system.

We now discuss a second approach to account for the increase in the center of mass momentum of the system. But first, we must derive what we denote as the general impulse equation. We begin its derivation by examining the centrifugal reactive contact force acting on the inside of the curved barriers. This force has a magnitude of $nkm_1r\omega^2$. In this expression n is the number of spheres, $k = \frac{m_2}{nm_1+m_2}$, m_1 is the mass of each sphere, m_2 is the mass of the platform, r is the radius of curvature of the path of the center of mass of each sphere, and ω is the orbital angular velocity of each sphere. Note that $m_1r\omega^2$ is the real centrifugal radially outward force acting through the center of mass of the spheres. We will assume the orbital angular speed ω of each sphere remains constant as they round their curves in this derivation and we ignore friction between the sphere and the curves and platform surface since these internal force pairs will have no impact on the center of mass of the system.

Finally, since we are only interested in the y-component of the impulse J, we include a sine expression factor in front of the force equation. (Recall the platform is constrained to move with one degree of freedom along the y-axis.) The final form of the y-component of the reactive centrifugal force equation is given by:

$$f_y = \ sin(\omega t) \, nkm_1 r\omega^2 \qquad (1)$$

The k arises because we assume the spheres and m_2 are rigid bodies, and therefore, have the same instantaneous y-component of linear acceleration.

We integrate the force with respect to time to determine the impulse J on m_2 with respect to the y-axis. This integral is expressed by:

$$J_y = \int_{\frac{\theta_i}{\omega}}^{\frac{\theta_f}{\omega}} nsin(\omega t) km_1 r\omega^2 \, dt \qquad (2)$$

Recognizing the indefinite integral of $\int \sin(\omega t)\, dt$ is equal to $-\frac{1}{\omega}\cos(\omega t) + C$ and applying this to equation (2) and after moving the constants out of the expression, we evaluate the above integral, giving us the general impulse equation:

$$J_y = n\, k\, m_1 r\omega\left[cos(\theta_i) - \cos(\theta_f)\right] \qquad (3)$$

Alternatively, we can set $r\omega = v_i$, which gives us:

$$J_y = n\, k\, m_1 v_i \left[cos(\theta_i) - \cos(\theta_f)\right] \qquad (3a)$$

For the special case where the initial angle is 90 degrees or 270 degrees and where there is a 90 degrees angular displacement in either the clockwise or counter-clockwise direction, the above equation reduces to:

$$J_y = n\, k\, m_1 v_i \tag{3b}$$

The sign of the calculated impulse depends on the initial angle of the spheres.

We now apply the above impulse equations in hypothetical examples that focuses on the impact the speed of a sphere has on the center of mass momentum of a system. We begin with a constant sphere velocity case to show that the momentum of the center of mass of the system does not change. To simplify, we use one sphere. The masses of m_1 and m_2 each is 1 kg. The platform is constrained to move with one degree of freedom along the y-axis.

The initial speed of the sphere is 1 m/s and the angular counter-clockwise displacement is from 270 degrees to 0 degrees. We simplify further by invoking Equation (3b) which for the initial values give us a final momentum of the platform of -.5 N s and a final velocity of -.5 m/s after the sphere exits the curve in the positive y-direction.

The final momentum of the sphere with respect to our lab frame in the positive y-direction is .5 kg · m/s. This is determined by applying the Galilean transformation equation, by adding the final velocity of m_2 to the velocity of m_1 with respect to the m_2 frame, which gives us the velocity of the sphere with respect to our laboratory frame. Multiplying this velocity by the mass of the sphere gives us the final momentum with respect to our lab frame. Hence, the total momentum of the center of mass of the system is equal to the final momentum of the platform which is -.5 kg · m/s plus the final momentum of the sphere which is .5 kg · m/s. This equals 0 kg · m/s which was the initial momentum of the system along the y-axis, and hence, momentum of the sphere-platform system is conserved as expected.

We now show the case where a hypothetical external impulse acts on the sphere which increases the orbital angular speed of the sphere as it rounds the curve. We keep everything the same as above except the speed of the sphere increases by .1 m/s for every 5 degrees angular displacement from 270 degrees to 0 degrees. We apply the general impulse equation (3a) for each angular increment, then sum the incremental impulses. Table 2 below summarizes the results.

Table 2. Increasing Orbital Angular Speed Result

Sphere Initial Speed M/S	Sphere Final Speed M/S	Initial Angle Degrees	Final Angle Degrees	Incremental Impulse N · S
1	1	270	275	-0.04358
1.1	1.1	275	280	-0.04757
1.2	1.2	280	285	-0.0511
1.3	1.3	285	290	-0.05408
1.4	1.4	290	295	-0.05642
1.5	1.5	295	300	-0.05804
1.6	1.6	300	305	-0.05886
1.7	1.7	305	310	-0.05883
1.8	1.8	310	315	-0.05789
1.9	1.9	315	320	-0.05599
2	2	320	325	-0.05311
2.1	2.1	325	330	-0.04922
2.2	2.2	330	335	-0.04431

2.3	2.3	335	340	-0.03839
2.4	2.4	340	345	-0.03148
2.5	2.5	345	350	-0.0236
2.6	2.6	350	355	-0.0148
2.7	2.7	355	0	-0.00514

This time the sum of the incremental impulses from the last column equals ~ -0.7464 N s. From this as before we determine the momentum of m_2 in the negative y-direction to be ~ -.746 kg m/s. The momentum of the sphere with respect to our lab frame is ~1.95 kg m/s. The sum of the two gives us a net increase in momentum of the center of mass of the sphere-platform equal to ~1.2 kg m/s in the positive y-direction. Hence, there was an increase in the center of mass momentum of the sphere-platform system.

We pointed out in the introduction that the orbital angular speed of the rotator increased as it passed the 90 degrees point. The same dynamic occurred in the spheres-platform experiment. The increase in speeds of the spheres due to the *external* physical effects of spacetime itself on the spheres, even when we include the reduction in speed due to the *internal* friction forces between the spheres, platform, and the walls, increased the center of mass momentum of the system as demonstrated above when we applied the general impulse equation.

We now address a final point in this discussion dealing with the energy implications of the experiment. If a system acquires a net momentum by the action of an external force multiplied by the time the force acts, it follows that the system acquires kinetic energy by the same force multiplied by the displacement of the force as predicted by the work-energy theorem. Since kinetic energy and change in kinetic energy is a frame dependent phenomenon, an interesting dynamic occurs when we factor in the initial velocity of the system used in this experiment on its impact on the final kinetic energy of the system. We show a general result of this mathematically as follows.

We begin with the impulse-momentum theorem using the external "space" impulse J_{space} that acts on the spheres as they round their quarter-circle barriers. The total mass of the system is given by m. Thus, we have the following expression, where $v_{f\ system}$ is the final velocity of the system and $v_{i\ system}$ is the initial velocity of the system:

$$J_{space} = mv_{f\ system} - mv_{i\ system} \tag{4}$$

Solving for $v_{f\ system}$, we obtain:

$$v_{f\ system} = \frac{(J_{space} + mv_{i\ system})}{m} \tag{5}$$

Substituting the above for $v_{f\ system}$ in the expression for ΔKE,

gives us :

$$\Delta KE = \frac{1}{2}\, m[\frac{(J_{space} + mv_{i\ system})}{m}]^2 - \frac{1}{2} m v_{i\ system}^2 \tag{6}$$

Simplifying the above, we arrive at:

$$\Delta KE = \frac{J_{space}^2}{2m} + J_{space}\ v_{i\ system} \tag{7}$$

The second term on the right of the above equation shows that the change in kinetic energy of a system while keeping the impulse J_{space} and mass the same is impacted by the initial velocity of the system. This is not just a mathematical, theoretical result. This can be confirmed by noting that with respect to a boosted inertial reference frame the initial velocity of the system will always impact the change in kinetic energy of the system. This is an observable, physical fact. And by the first postulate of special relativity this phenomenon observed by a boosted inertial frame must also be possible with respect to an inertial laboratory frame.

An even more interesting consequence has to do with the energy that is responsible for the initial kinetic energy of the spheres. For example, suppose the spheres are propelled by compressed springs that are attached to the platform. The elastic potential energy of the springs (epe) is a frame invariant scalar. When the springs expand, they do work on the spheres by displacing the spheres along the x-axis, imparting an initial kinetic energy to the springs before they enter the curves. By the same reasoning above, there always exists a boosted inertial reference frame for which with respect to this frame there is an initial velocity of the system along the y-axis such that the change in kinetic energy of the system will be *greater* than the total frame invariant elastic potential energy (epe) of the springs. And again, by the first postulate of special relativity this phenomenon observed by a boosted inertial frame must also be possible with respect to an inertial laboratory frame.

5. Conclusion

The ~ 91.8 per cent discrepancy measured in this experiment demonstrated that friction impulse was not sufficient to account for the total external impulses that acted on the system. It was acceleration of the system through Minkowski space that highly suggests that the discrepancy was a direct consequence of the spheres moving contrary to the natural paths defined by the Minkowski geodesics and the consequential real inertial force reaction of space itself on the spheres.

It is recommended that further experiments be conducted in a near zero friction environment on an air track or within a plane in a zero-g dive or by a CubeSat in space where the effects of external friction forces would be reduced significantly. Appendix A contains a set of equations that can be used for further testing and confirmation of the effect in near frictionless environments.

Appendix A

This appendix presents a set of five equations for predicting the motions of a more complicated system. These equations apply in idealized flat Minkowski spacetime where external friction impulses or other external impulses are nearly eliminated such as in the environment of inter-stellar space. In this more elaborate system, the total mass m is the sum of the masses of the spheres, the platform, and all attachments to the platform.

We need to define what a cycle N means. In this system two spheres are propelled by the release of two compressed springs, afterwards the spheres enter and round quarter-circle curves, then have elastic collisions with elastic barriers in the positive y-direction after existing the curves, bounce back, and then round the curves again in the opposite direction, returning to their initial positions, and finally resting against the two recompressed springs. This is defined as one cycle N. We denote a spacetime impulse $J_{\text{spacetime}}$, defined by measuring the change in momentum of the system after one cycle.

The compressed springs which propel the spheres store elastic potential energy (epe) which is a scalar or invariant quantity that does not depend on the velocity of the system. Denoting the total number of springs as n and the energy per compressed spring as epe, the cumulative energy applied to the spheres after N cycles is:

$$E_{\text{cumulative}} = N\,n\,epe \qquad (A1)$$

The cumulative final velocity of the system, with external impulse $J_{\text{spacetime}}$, after N cycles equates to:

$$v_{f\text{ system}} = \frac{N\,J_{\text{spacetime}}}{m} \qquad (A2)$$

Substituting $v_{f\text{ system}}$ into the formula for kinetic energy yields:

$$KE_{\text{final}} = \frac{(N\,J_{\text{spacetime}})^2}{2m} \qquad (A3)$$

The cycle N at which KE_{final} equals $E_{\text{cumulative}}$ is determined by setting Equation (A1) equal to Equation (A3), then solving for N, giving us:

$$N = \frac{2\,m\,epe}{J_{spacetime}^2} \qquad \text{(A4)}$$

For other engineering applications and insight, it is useful to graph both Equation (A1) and Equation (A3) to visually see their relationship with N on the x-axis and energy on the y-axis.

The value for m can easily be predetermined by direct measurement. The value for epe can be calculated knowing the spring constant and the degree of compression. The spacetime impulse $J_{\text{spacetime}}$ is determined experimentally by activating a single cycle when the system is at rest and measuring the system's final velocity v_f, giving us the spacetime impulse as:

$$J_{\text{spacetime}} = m\, v_f \tag{A5}$$

Declaration of AI Use

During the preparation of this work, the author used generative AI to assist with spelling, academic formatting, and citation protocols. The author reviewed and edited the content and takes full responsibility for the publication.

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The author identifies as an independent researcher and received no institutional support for this study. All experimental trials were conducted privately.

Data Availability

Pasco data relating to the experiments is available for review upon request.

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