Octonionic non-associativity and the emergence of particle physics

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Abstract

We formulate a particle-physics sector in an octonionic projection framework in which the Standard Model is interpreted as the low-energy associative projection of a deeper non-associative field dynamics. The fundamental objects are three octonion-valued fields $\Psi_I \in \mathbb{O}$, $I = 1, 2, 3$, whose non-associative content is measured by the octonionic associator $[\Psi_1, \Psi_2, \Psi_3] = (\Psi_1\Psi_2)\Psi_3 - \Psi_1(\Psi_2\Psi_3)$. A positive semidefinite associator potential dynamically suppresses non-associative excitations at low energies and selects local associative subalgebras. We show how the chain $\mathbb{O} \to \mathbb{H}_{\text{phys}} \to \mathbb{C}$ motivates the emergence of complex quantum fields and Standard-Model-like residual gauge sectors related to $SU(3)_c$, $SU(2)_L$, and $U(1)_Y$. We emphasize that the exact chiral hypercharge spectrum is not derived here; it requires an explicit projector representation. In this construction the associator is not merely an algebraic obstruction, but a dynamical source for projection, symmetry breaking, generation structure, mass hierarchy corrections, neutrino mixing corrections, Higgs-sector deformations, and possible fifth-force phenomenology. We briefly distinguish this proposal from division-algebraic Standard-Model reconstructions, where octonions primarily encode particle representations and charge assignments; here the new ingredient is the promotion of the associator to a field-theoretic quantity. The framework provides a constrained octonionic projection mechanism leading to Standard-Model-like particle sectors, while the complete derivation of the observed chiral charge spectrum remains an open task.

Keywords: octonions, non-associativity, particle physics, Standard Model, associator dynamics, effective field theory

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1. Introduction

The Standard Model of particle physics is extraordinarily successful, yet several of its structural features remain unexplained from within the model itself: the origin of its gauge group, the number of fermion generations, the hierarchy of Yukawa couplings, the pattern of neutrino mixing, and the relation between internal quantum numbers and spacetime geometry. The octonions provide a natural mathematical arena in which several of these questions can be reformulated. They form the largest normed division algebra, possess a canonical G_2 automorphism group, and contain associative complex and quaternionic subalgebras as special sectors [1–3, 8, 9].

The central idea pursued here is that observed particle physics is not the most fundamental description, but the low-energy associative projection of an underlying octonion-valued field theory. The key algebraic measure of departure from ordinary associative physics is the associator

$$[x, y, z] = (xy)z - x(yz), \qquad x, y, z \in \mathbb{O}. \tag{1}$$

In conventional quantum field theory, products of fields, operators, and matrices are associative. The appearance of a nonzero octonionic associator therefore signals a sector outside standard complex Hilbert-space quantum mechanics and outside ordinary Lie-algebraic gauge theory.

The distinctive assumption of this work is that the associator is promoted to a dynamical field-theoretic quantity. Its norm enters the action through a positive semidefinite potential, and the observed Standard Model emerges when this potential dynamically selects approximately associative local projections. In the exact low-energy limit,

$$[\Psi_1, \Psi_2, \Psi_3] \simeq 0, \tag{2}$$

one recovers ordinary complex spinor and gauge-field dynamics. At higher energies or in regions of strong projection curvature, small deviations become possible.

This approach is related in spirit to earlier division-algebraic approaches to particle physics, including the octonionic quark model of Gunaydin and Gursey and the later programs of Dixon and Furey [2, 4–7]. The focus here, however, is different. Furey's program reconstructs Standard-Model representations, charges, and symmetry-breaking patterns from division algebras and associated Clifford structures. The present model instead aims to describe a dynamical mechanism by which those associative particle-physics structures emerge as projections of a non-associative octonionic sector. Throughout the paper, statements about the emergence of the Standard-Model gauge and matter content should therefore be understood as a dynamical embedding and motivation, not as a completed derivation of all hypercharges, chiralities, and Yukawa parameters.

2. Octonionic algebra and associator dynamics

Let

$$\mathbb{O} = \mathrm{span}_{\mathbb{R}}\{1, e_1, \ldots, e_7\} \tag{3}$$

be the algebra of real octonions. The imaginary units satisfy

$$e_i e_j = -\delta_{ij} + f_{ijk} e_k, \tag{4}$$

where f_{ijk} are the completely antisymmetric structure constants of the chosen Fano-plane convention. The octonions are alternative but not associative. Hence the associator is completely antisymmetric in its three arguments and vanishes whenever two arguments are equal.

This fact has an important consequence for model building. A single octonion-valued field cannot generate a nontrivial cubic associator by itself because

$$[\Psi, \Psi, \Psi] = 0. \tag{5}$$

The minimal non-associative matter sector therefore contains three independent octonionic fields,

$$\Psi_I(x) \in \mathbb{O}, \qquad I = 1, 2, 3. \tag{6}$$

The basic non-associative invariant is

$$A(x) = [\Psi_1(x), \Psi_2(x), \Psi_3(x)]. \tag{7}$$

The proposed effective action follows the octonionic foundation framework developed in Ref. [10].

$$\begin{aligned} S = \int d^4x \sqrt{-g} \Bigg[& \frac{1}{2\kappa}(R - 2\Lambda) - \sum_{I=1}^{3} \frac{\alpha_I}{2} g^{\mu\nu} \langle \nabla_\mu \Psi_I, \nabla_\nu \Psi_I \rangle \\ & - V_{\mathrm{loc}}(\Psi_1, \Psi_2, \Psi_3) - \frac{\lambda}{2} \left\| [\Psi_1, \Psi_2, \Psi_3] \right\|^2 - \xi R \sum_I \langle \Psi_I, \Psi_I \rangle \Bigg]. \end{aligned} \tag{8}$$

Here $\langle \cdot, \cdot \rangle$ is the real octonionic inner product induced by the norm, $\lambda \geq 0$ stabilizes the non-associative sector, and ξ is a dimensionless nonminimal curvature coupling. In natural units, a canonical scalar normalization gives $[\Psi_I] = 1$ and $[\lambda] = -2$, so that one may write

$$\lambda = \frac{\eta_A}{M_*^2}, \tag{9}$$

with η_A dimensionless and M_* the scale at which non-associative effects become relevant.

The associator contribution to the potential is

$$V_A = \frac{\lambda}{2} \|A\|^2 . \tag{10}$$

It favors dynamically selected associative configurations. Thus the Standard Model is identified with the sector in which V_A is minimized or strongly suppressed.

Variation of Eq. (8) with respect to $\overline{\Psi}_I$ gives schematically

$$\alpha_I \Box \Psi_I - \frac{\partial V_{\mathrm{loc}}}{\partial \overline{\Psi}_I} - \lambda\, \mathrm{D}_I^{\dagger} A - 2\xi R \Psi_I = 0, \tag{11}$$

where D_I is the linear map obtained by varying the associator with respect to Ψ_I. For example,

$$\delta_I A = [\delta\Psi_1, \Psi_2, \Psi_3] \tag{12}$$

for $I = 1$, with analogous expressions for $I = 2, 3$. Equation (11) shows explicitly that non-associativity backreacts on the effective matter dynamics.

3. Associative projection and the origin of quantum fields

The observed particle-physics sector is assumed to arise by local projection onto an associative subalgebra. The fundamental chain is

$$\mathbb{O} \longrightarrow \mathbb{H}_{\mathrm{phys}} \longrightarrow \mathbb{C}. \tag{13}$$

The quaternionic subalgebra $\mathbb{H}_{\mathrm{phys}} \subset \mathbb{O}$ is associative and can support spinorial structures, while the final complex subalgebra supplies the ordinary complex phase structure of quantum mechanics.

Let $\mathcal{P}_{\mathbb{H}}$ and $\mathcal{P}_{\mathbb{C}}$ denote local projectors. The observed low-energy field is

$$\psi_f(x) = \mathcal{P}_{\mathbb{C}} \mathcal{P}_{\mathbb{H}} \Psi_I(x), \tag{14}$$

where f labels the projected fermion species. The complementary components are heavy, confined, or dynamically suppressed by V_A:

$$\Psi_I = \Psi_I^{\mathrm{assoc}} + \Psi_I^{\mathrm{nonassoc}}, \qquad \|\Psi_I^{\mathrm{nonassoc}}\| \ll \|\Psi_I^{\mathrm{assoc}}\| \tag{15}$$

in the low-energy regime.

The physical interpretation is that electrons, quarks, neutrinos, and other observed matter fields are stable projection modes rather than primitive degrees of freedom. The ordinary complex Hilbert-space structure appears when the non-associative sector is dynamically frozen:

$$\epsilon_A(E) \equiv \eta_A \left(\frac{E}{M_*}\right)^2 \ll 1. \tag{16}$$

4. Gauge sectors from residual projection symmetry

The automorphism group of the octonions is G_2. Selecting a preferred imaginary unit or complex direction reduces this symmetry. The stabilizer of a chosen imaginary unit in G_2 is isomorphic to $SU(3)$. Therefore the emergence of a color-like symmetry is naturally associated with the projection

$$G_2 \longrightarrow SU(3)_c. \tag{17}$$

This motivates the identification of quark color with residual rotations of the octonionic imaginary sector orthogonal to a selected complex direction.

A further choice of quaternionic subalgebra supplies an $SU(2)$-like structure, and the selected complex subalgebra supplies a $U(1)$ phase. The qualitative chain is

$$G_2 \quad \to \quad SU(3)_c \quad \to \quad SU(2)_L \times U(1)_Y, \tag{18}$$

with the target low-energy gauge structure represented by

$$G_{\mathrm{SM}} = SU(3)_c \times SU(2)_L \times U(1)_Y. \tag{19}$$

The model does not assume that Eq. (18) is a literal subgroup chain in every sector, nor does it by itself derive the complete chiral hypercharge spectrum. Rather, it states that Standard-Model-like gauge generators can arise as connection components associated with local variations of the selected associative projectors. A full Standard-Model embedding must reproduce, in a fixed projector representation,

$$Y(q_L)=\frac{1}{6},\quad Y(u_R)=\frac{2}{3},\quad Y(d_R)=-\frac{1}{3},\quad Y(\ell_L)=-\frac{1}{2},\quad Y(e_R)=-1, \tag{20}$$

and must also show why the weak $SU(2)$ sector acts only on left-handed fields.

If the projector $\mathcal{P}(x)$ varies over spacetime, ordinary derivatives are replaced by covariant derivatives,

$$\partial_\mu \longrightarrow D_\mu = \partial_\mu + \mathcal{A}_\mu, \tag{21}$$

with

$$\mathcal{A}_\mu = G^a_\mu T_a + W^i_\mu \tau_i + B_\mu Y. \tag{22}$$

Here T_a, τ_i, and Y are the effective generators of $SU(3)_c$, $SU(2)_L$, and $U(1)_Y$ after projection. Thus gluons, weak bosons, and the hypercharge gauge boson are interpreted as connection fields of the associative projection geometry. The chiral nature of the weak interaction requires an additional compatibility condition between the internal projector and the spacetime spin projector,

$$P_L=\frac{1-\gamma^5}{2},\qquad P_R=\frac{1+\gamma^5}{2}, \tag{23}$$

namely

$$D_\mu\psi_L = (\partial_\mu + igW^a_\mu\tau_a + ig'YB_\mu)\psi_L, \tag{24}$$

$$D_\mu\psi_R = (\partial_\mu + ig'YB_\mu)\psi_R, \tag{25}$$

up to color terms where appropriate. Equation (25) is imposed as a required consistency condition of the embedding; deriving it from a unique octonionic projector remains an open representation-theoretic step.

5. Quarks, leptons, and charge assignments

After choosing a complex direction in $\mathbb{O}$, the complexified imaginary sector decomposes qualitatively as

$$\mathbb{O}_\mathbb{C} \sim \mathbb{C}\oplus\mathbb{C}^3. \tag{26}$$

The singlet component is naturally associated with leptonic states, while the triplet is associated with colored quark states:

$$\mathbb{C} \longleftrightarrow \text{lepton singlet sector}, \tag{27}$$

$$\mathbb{C}^3 \longleftrightarrow \text{quark color triplet sector}. \tag{28}$$

Thus leptons are color singlets and quarks transform as color triplets:

$$\ell \sim \mathbf{1}_{SU(3)}, \qquad q \sim \mathbf{3}_{SU(3)}. \tag{29}$$

The electric charge operator is then the projected combination

$$Q = T_3 + \frac{Y}{2}, \tag{30}$$

where T_3 and Y are generated by the quaternionic and complex projection sectors. In this framework the electric charge of a fermion is not assigned independently; it is an eigenvalue of the projected generator acting on the corresponding octonionic projection state:

$$Q_f \psi_f = Q\Big(\mathcal{P}_{\mathbb{C}} \mathcal{P}_{\mathbb{H}} \Psi_I\Big). \tag{31}$$

A complete derivation of all Standard-Model charge eigenvalues requires specifying the explicit representation of the projector algebra and verifying Eq. (20) together with anomaly cancellation. The present paper gives the dynamical framework in which such representation-theoretic constructions can be embedded, but it does not claim a completed charge-spectrum derivation.

6. Generation structure from associator eigenmodes

The three generations do not follow as a proven theorem from alternativity alone. Rather, the model proposes a natural dynamical hypothesis: the observed family triplication is associated with the lowest stable eigenmodes of the fundamental associator sector. This distinction is important. The mathematical fact is that a nonzero octonionic associator requires three independent arguments; the physical identification with the three fermion families requires an explicit spectral calculation.

The associator functional is

$$\mathcal{V}_A[\Psi_1, \Psi_2, \Psi_3] = \left\| [\Psi_1, \Psi_2, \Psi_3] \right\|^2. \tag{32}$$

Its variational derivative defines the associator operator acting on the three-field configuration,

$$(\mathcal{A}_A)_I(\Psi) \equiv \frac{\delta}{\delta \overline{\Psi}_I} \left\| [\Psi_1, \Psi_2, \Psi_3] \right\|^2 , \qquad I = 1, 2, 3. \tag{33}$$

A projected eigenmode problem may then be written schematically as

$$\mathcal{A}_A \Psi^{(n)} = a_n \Psi^{(n)}, \qquad n = 1, 2, 3, \ldots . \tag{34}$$

The working hypothesis is that the three lowest stable projected eigenmodes correspond to the observed fermion families,

$$\begin{aligned} \Psi^{(1)} &\longleftrightarrow \text{first generation}, \\ \Psi^{(2)} &\longleftrightarrow \text{second generation}, \\ \Psi^{(3)} &\longleftrightarrow \text{third generation}. \end{aligned} \tag{35}$$

In particle notation this gives the schematic correspondence

$$\begin{aligned} (e, \nu_e, u, d) &\longleftrightarrow \Psi^{(1)}, \\ (\mu, \nu_\mu, c, s) &\longleftrightarrow \Psi^{(2)}, \\ (\tau, \nu_\tau, t, b) &\longleftrightarrow \Psi^{(3)}. \end{aligned} \tag{36}$$

Equations (33)–(36) are therefore not introduced as an already completed derivation of the family spectrum, but as the precise spectral problem that the model must solve. The exact fermion masses, mixing angles, and CP phases require the explicit diagonalization of $\mathcal{A}_A$ or, equivalently, of the associator Hessian around the chosen vacuum.

For perturbative analysis one expands around a vacuum configuration,

$$\Psi_I = \Psi_{I,0} + \delta\Psi_I, \tag{37}$$

so that

$$V_A = V_A(\Psi_0) + \frac{1}{2}\, \delta\Psi_I \, \mathcal{M}_A^{IJ} \, \delta\Psi_J + \cdots . \tag{38}$$

The corresponding Hessian eigenvalue problem is

$$\mathcal{M}_A^{IJ} u_J^{(n)} = a_n u_I^{(n)}. \tag{39}$$

The hierarchy of eigenvalues a_n can then feed into the hierarchy of effective Yukawa couplings through the projection coefficients in Eq. (41).

7. Masses, Yukawa couplings, and Higgs-sector corrections

At low energies the usual Higgs mechanism is recovered. Fermion masses take the form

$$m_f^{\mathrm{SM}} = y_f v. \tag{40}$$

The associator potential has mass dimension four. Since $[\Psi_I] = 1$, $[A] = 3$, and $[\lambda] = -2$, the combination $\lambda \|A\|^2$ is an energy density, not directly a fermion mass. Fermion mass corrections must therefore enter through projected dimensionless Yukawa corrections or through a canonically normalized mediator. A dimensionally consistent parametrization is

$$m_f = v \left[y_f^{(0)} + c_f^{(A)} \eta_A \frac{\Phi_0^2}{M_*^2} + \mathcal{O}(M_*^{-4}) \right], \tag{41}$$

where Φ_0 denotes the characteristic vacuum amplitude of the projected octonionic fields and $c_f^{(A)}$ is a dimensionless projection coefficient. Equivalently,

$$y_f^{\mathrm{eff}} = y_f^{(0)} + c_f^{(A)} \eta_A \frac{\Phi_0^2}{M_*^2} + \cdots . \tag{42}$$

For scalar associator fluctuations, the Hessian of V_A instead produces mass-squared corrections of the schematic form

$$\Delta m_{\mathrm{scalar}}^2 \sim \eta_A \frac{\Phi_0^4}{M_*^2}. \tag{43}$$

This gives a possible dynamical origin for the observed mass hierarchy: different fermions occupy different projection modes and therefore receive different dimensionless coefficients $c_f^{(A)}$.

The Higgs potential is likewise interpreted as the associative projection of a more general octonionic potential,

$$V(\Psi) = V_{\mathrm{loc}}(\Psi) + \frac{\lambda}{2} \|[\Psi_1, \Psi_2, \Psi_3]\|^2 . \tag{44}$$

After projection,

$$V_H(H) = -\mu^2 H^\dagger H + \rho_H (H^\dagger H)^2 + \Delta V_A(H), \tag{45}$$

where ρ_H denotes the Standard-Model Higgs quartic coupling. The correction ΔV_A shifts Higgs self-interactions. For a leading dimension-six projection,

$$\Delta V_A(H) = \frac{c_H \eta_A}{M_*^2} (H^\dagger H)^3 + \cdots , \tag{46}$$

one obtains schematically

$$\frac{\Delta\lambda_{hhh}}{\lambda_{hhh}^{\mathrm{SM}}} = C_{hhh} c_H \eta_A \frac{v^2}{M_*^2} + \cdots , \tag{47}$$

where C_{hhh} depends on the precise normalization of the operator basis and on possible field redefinitions. Such effects are expected to be small when $E \ll M_*$, but they provide a concrete target for future precision measurements.

8. Neutrino mixing

Neutrino oscillations are a natural place to search for projection-induced corrections. In the Standard Model extended by neutrino masses,

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle, \tag{48}$$

where U is the PMNS matrix. In the octonionic projection framework,

$$U_{\alpha i} = U_{\alpha i}^{(0)} + \delta U_{\alpha i}^{(A)}, \tag{49}$$

where the correction must be dimensionless. A consistent parametrization is

$$\delta U_{\alpha i}^{(A)} = c_{\alpha i}^{(A)} \eta_A \frac{\Phi_0^2}{M_*^2} + \cdots . \tag{50}$$

Equivalently, an oscillation Hamiltonian correction may be written as

$$\Delta H_{\alpha\beta}^{(A)} = \eta_A \frac{E^2}{M_*^2} \mu_A (\Theta_A)_{\alpha\beta}, \tag{51}$$

where μ_A has dimension of energy and Θ_A is dimensionless. Therefore

$$P_{\nu_\alpha \to \nu_\beta} = P_{\mathrm{SM}} + \Delta P_A(E, L), \tag{52}$$

where E is the neutrino energy and L is the baseline. Since the effect depends on both projection geometry and energy, it need not mimic a simple shift of the standard mass-squared differences.

9. High-energy deviations and contact operators

Integrating out heavy non-associative modes generates higher-dimensional effective operators. The leading correction can be parameterized by

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \frac{\eta_A}{M_*^2}\mathcal{O}_A + \mathcal{O}(M_*^{-4}), \tag{53}$$

where $\mathcal{O}_A$ is a dimension-six operator encoding projected associator exchange or associator-induced contact interactions. Examples include four-fermion operators $(\bar{f}\gamma_\mu f)(\bar{f}\gamma^\mu f)$, Higgs operators $(H^\dagger H)^3$, and current-Higgs operators $(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{f}\gamma^\mu f)$. Accordingly, scattering amplitudes take the form

$$\mathcal{M} = \mathcal{M}_{\text{SM}} + \epsilon_A(E)\mathcal{M}_A, \tag{54}$$

with $\epsilon_A(E)$ defined in Eq. (16). Cross sections become

$$\sigma = \sigma_{\text{SM}}\left[1 + c_A\eta_A\left(\frac{E}{M_*}\right)^2 + \mathcal{O}\left(\frac{E^4}{M_*^4}\right)\right]. \tag{55}$$

Promising channels include high-mass dilepton production, dijet angular distributions, top-pair production, and future high-energy e^+e^- precision observables. The characteristic prediction is not a random violation of the Standard Model, but an energy-growing deviation whose tensor and flavor structure is determined by the associator projection operator.

10. Effective fifth interaction

The scalar magnitude of the associator has canonical dimension three and therefore is not itself a canonically normalized scalar field. A dimension-one collective mode may be defined by

$$\chi_c = \frac{1}{M_*^2}\left\|[\Psi_1, \Psi_2, \Psi_3]\right\|. \tag{56}$$

Its fermion coupling can then be written in the standard form

$$\mathcal{L}_{\text{int}} = g_c\chi_c\bar{\psi}\psi, \tag{57}$$

with dimensionless g_c. Equivalently, without the redefinition in Eq. (56), the coupling must be treated as a higher-dimensional operator suppressed by M_*^2. Exchange of χ_c generates a Yukawa potential

$$V_A(r) = -\frac{g_c^2}{4\pi}\frac{e^{-m_A r}}{r}. \tag{58}$$

For macroscopic bodies this can be written as

$$\begin{aligned} V(r) &= -\frac{Gm_1m_2}{r}\left[1+\alpha_A e^{-r/\ell_A}\right], \\ \ell_A &= m_A^{-1} \quad (\ell_A = \hbar/m_A c \text{ in SI units}). \end{aligned} \tag{59}$$

This interaction is not added by hand; it is the residual force associated with incomplete suppression of non-associativity. Current fifth-force searches constrain α_A and ℓ_A, while future short-distance and precision interferometric experiments could further test this sector.

11. Matter–antimatter conjugation

Antiparticles arise from conjugating the projected octonionic states. For an octonion-valued field

$$\Psi = \psi_0 + \sum_{a=1}^{7} \psi_a e_a, \tag{60}$$

the octonionic conjugate is

$$\overline{\Psi} = \psi_0 - \sum_{a=1}^{7} \psi_a e_a. \tag{61}$$

After projection to the complex associative sector this operation induces the usual charge-conjugated state,

$$\psi_f \longrightarrow \psi_f^c. \tag{62}$$

Thus, at the level of the projected particle interpretation,

$$\begin{aligned} \Psi &\longrightarrow \text{particle}, \\ \overline{\Psi} &\longrightarrow \text{antiparticle}. \end{aligned} \tag{63}$$

The sign reversal of the relevant charges follows because the projected $U(1)$ generator and the non-Abelian color generators act in the conjugate representation. This gives a natural algebraic interpretation of matter and antimatter within the same octonionic field space.

In the octonionic projection framework, the observed matter-antimatter asymmetry may originate from a non-associative CP-odd projection bias during the early-universe transition from the full octonionic phase to the associative four-dimensional sector. The associator acts as an intrinsic source of orientation, chirality, and effective CP violation, thereby allowing baryonic matter and antimatter to be produced with slightly different probabilities.

Table 1: Schematic classification of particle classes in the octonionic projection framework. The entries denote the proposed low-energy interpretation of different algebraic structures after minimization or suppression of the associator potential.

Octonionic or projection structure	Physical particle class
$\mathbb{C}$-singlet projection	leptons
$\mathbb{C}^3$-triplet projection	quarks and color degrees of freedom
local projection connection $\mathcal{P}(x)$	gauge bosons
radial vacuum fluctuation $\delta\Psi_{\text{radial}}$	Higgs-like scalar mode
associator fluctuation δA	sterile or associator-sector excitations
associator eigenmodes $\Psi^{(n)}$	fermion generations
octonionic conjugation $\overline{\Psi}$	antiparticles

12. Classification of particle classes

The particle classes are not created by the associator as by an ordinary creation operator. Instead, the associator potential selects stable projection sectors and suppresses unstable non-associative components. The resulting classification is summarized in Table 1.

This table should be read as a classification map rather than as a completed Standard-Model spectrum calculation. In particular, the lepton–quark split follows from singlet versus triplet projections, gauge bosons arise as local connection fields of the projector, the Higgs mode is associated with radial stabilization of the vacuum, and possible new particles correspond to residual fluctuations of the associator sector.

13. Compact emergence chain

The full construction can be summarized by the following sequence:

$$\boxed{\begin{array}{c} \Psi_I \in \mathbb{O} \\ \Downarrow \\ A(\Psi_1, \Psi_2, \Psi_3) = [\Psi_1, \Psi_2, \Psi_3] \\ \Downarrow \\ V_A = \dfrac{\lambda}{2} \left\| A \right\|^2 \\ \Downarrow \\ \text{stabilization of an associative projection} \\ \Downarrow \\ \mathbb{O} \to \mathbb{H}_{\text{phys}} \to \mathbb{C}_{\text{em}} \\ \Downarrow \\ SU(3)_c \times SU(2)_L \times U(1)_Y \\ \Downarrow \\ \text{quarks, leptons, gauge bosons, Higgs modes, and generations.} \end{array}} \tag{64}$$

The decisive physical operation is the minimization of

$$\left\| [\Psi_1, \Psi_2, \Psi_3] \right\|^2 . \tag{65}$$

The components that remain stable under this minimization constitute the observable associative particle-physics sector.

14. Core interpretation

The fundamental associator therefore classifies particle sectors by their distance from an associative projection. In the infrared limit,

$$A \simeq 0 \quad \Rightarrow \quad \text{known associative particle physics,} \tag{66}$$

whereas incomplete suppression gives

$$A \neq 0 \quad \Rightarrow \quad \text{new non-associative particle modes.} \tag{67}$$

The central dynamical question is consequently

$$\begin{array}{l}\text{Which components of } \Psi_I \text{ remain as stable projections}\\ \text{after minimizing } \ \|[\Psi_1,\Psi_2,\Psi_3]\|^2\,?\end{array} \tag{68}$$

In this sense, leptons, quarks, gauge bosons, Higgs-like modes, antiparticles, generations, and possible sterile associator particles arise as different stable projections and fluctuations of the three fundamental octonion-valued fields under the associator potential.

15. Relation to Furey's division-algebraic program

Furey's work develops an algebraic reconstruction of Standard-Model representations from the tensor product of normed division algebras, especially structures involving $\mathbb{R}\otimes\mathbb{C}\otimes\mathbb{H}\otimes\mathbb{O}$ and associated Clifford-algebraic ideals [4]. Later work with Hughes describes division-algebraic symmetry-breaking cascades such as Spin(10) $\to$ Pati-Salam $\to$ left-right symmetric theory $\to$ Standard Model plus $B-L$ [6].

The present model is complementary but distinct. It does not primarily aim to reconstruct Standard-Model multiplets from ideals. Instead, it promotes the octonionic associator to a dynamical object in an action principle. In this sense,

$$\text{Furey: algebraic encoding of particle representations,} \tag{69}$$

whereas

$$\text{this work: dynamical projection from non-associativity.} \tag{70}$$

This distinction is essential. Here the associator can carry energy, influence masses, drive projection, generate corrections, and potentially mediate new interactions.

16. Experimental predictions

The framework is predictive only after the associator scale M_*, the dimensionless strength η_A, and the projection-dependent coefficients are specified. Nevertheless, several model-independent signatures follow from the assumption that the Standard Model is the infrared associative projection and that non-associativity is encoded by the operator $\mathcal{O}_A$ in Eq. (53). The leading expansion parameter is

$$\epsilon_A(E) = \eta_A \left(\frac{E}{M_*}\right)^2, \tag{71}$$

so deviations are suppressed at low energy but grow in high-energy, high-curvature, or high-coherence experiments.

16.1. Collider signatures

At hadron and lepton colliders the cleanest signal is an energy-growing deformation of high-invariant-mass distributions. For a partonic process $ab \to cd$ one obtains

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{SM}} \left[1 + C_{abcd}(\theta)\, \eta_A \frac{\hat{s}}{M_*^2} + \mathcal{O}\!\left(\frac{\hat{s}^2}{M_*^4}\right)\right], \tag{72}$$

where $\hat{s}$ is the partonic center-of-mass energy and C_{abcd} is fixed by the projection tensor of the participating fermions. The most sensitive channels are expected to be

$$pp \to \ell^+\ell^-, \qquad pp \to jj, \qquad pp \to t\bar{t}, \qquad e^+e^- \to f\bar{f}. \tag{73}$$

The characteristic prediction is not merely a resonance peak. If the associator mode is heavy, the first observable effect is a smooth excess or deficit in high-energy tails and angular distributions. If a light or narrow associator excitation exists, an additional resonance-like structure may appear at

$$\sqrt{\hat{s}} \simeq m_A. \tag{74}$$

A practical collider fit can therefore use the two-parameter template

$$\frac{\Delta\sigma}{\sigma_{\mathrm{SM}}} = c_A \eta_A \left(\frac{E}{M_*}\right)^2, \tag{75}$$

with c_A channel-dependent. A null result yields a lower bound on $M_*/\sqrt{\eta_A}$.

16.2. Fermion-family and flavor observables

If the three generations are associator eigenmodes, flavor violation is controlled by the mismatch between the mass basis and the associator basis. Denoting this mismatch by a small matrix Θ_A, rare processes receive corrections of the schematic form

$$\mathcal{A}(f_i \to f_j X) = \mathcal{A}_{\mathrm{SM}} + \eta_A \frac{E^2}{M_*^2} (\Theta_A)_{ij} \mathcal{A}_A. \tag{76}$$

The strongest qualitative prediction is a hierarchy

$$|\Delta\mathcal{O}_{3\leftrightarrow 2}| \gtrsim |\Delta\mathcal{O}_{2\leftrightarrow 1}| \gtrsim |\Delta\mathcal{O}_{1\leftrightarrow 1}|, \tag{77}$$

whenever the associator eigenvalues increase with fermion generation. Consequently, top, bottom, tau, and heavy-flavor observables are natural search channels.

16.3. Neutrino oscillations

The neutrino sector can test non-associative projection effects because oscillation experiments are directly sensitive to small phase shifts. The leading correction may be written as

$$P_{\nu_\alpha\to\nu_\beta}(E,L) = P^{\mathrm{SM}}_{\alpha\beta}(E,L) + \eta_A\left(\frac{E}{M_*}\right)^2 F_{\alpha\beta}\left(\frac{L}{E},\Theta_A,\varphi_A\right), \tag{78}$$

where $F_{\alpha\beta}$ encodes the associator-induced mixing angles and phases. Unlike a pure shift of Δm^2_{ij}, the correction scales with energy through E^2/M_*^2. Long-baseline and atmospheric neutrino measurements can therefore distinguish the model from standard oscillation-parameter renormalization by testing whether residuals increase with energy.

16.4. Higgs-sector prediction

Projection of the associator potential modifies the effective Higgs potential by $\Delta V_A(H)$. To leading order this may be parameterized as

$$V_H(H) = V_H^{\mathrm{SM}}(H) + \frac{c_H\eta_A}{M_*^2}(H^\dagger H)^3 + \mathcal{O}(M_*^{-4}). \tag{79}$$

The trilinear Higgs self-coupling is shifted by

$$\frac{\Delta\lambda_{hhh}}{\lambda^{\mathrm{SM}}_{hhh}} = C_{hhh}c_H\eta_A\frac{v^2}{M_*^2} + \cdots , \tag{80}$$

where v is the electroweak vacuum expectation value and C_{hhh} is an order-one coefficient fixed by the operator normalization and field basis. Therefore Higgs-pair production provides a direct test of the projected associator potential.

16.5. Fifth-force and precision-interferometry tests

If the canonically normalized associator mode χ_c is light, the model predicts a Yukawa correction to Newtonian gravity,

$$\begin{aligned} V(r) &= -\frac{Gm_1m_2}{r}\left[1+\alpha_A e^{-r/\ell_A}\right], \\ \ell_A &= m_A^{-1} \quad (\ell_A = \hbar/m_A c \text{ in SI units}). \end{aligned} \tag{81}$$

A complementary microscopic test is matter-wave interferometry. A coherent path separation d over an interrogation time T acquires an additional phase

$$\Delta\phi_A \simeq \frac{1}{\hbar}\int_0^T \Delta V_A[d(t)]\,dt. \tag{82}$$

The prediction is strongest for mesoscopic masses, long coherence times, and path separations comparable to ℓ_A.

16.6. Cosmological and black-hole signatures

A homogeneous associator condensate contributes an effective energy density

$$\rho_A = \frac{\lambda}{2}\left\langle \|[\Psi_1,\Psi_2,\Psi_3]\|^2 \right\rangle + \langle V_{\rm loc}\rangle. \tag{83}$$

Its Friedmann contribution can be written phenomenologically as

$$H^2(z) = H^2_{\Lambda\rm CDM}(z) + \frac{8\pi G}{3}\rho_A(z). \tag{84}$$

The characteristic prediction is a late-time or environment-dependent deviation rather than a universal rescaling of Newton's constant. In compact objects the same term may regularize the central region by producing a finite de-Sitter-like core density. Observable consequences include small shifts of photon-ring radii, ringdown frequencies, and possible deviations in the final stages of evaporation.

16.7. Summary of experimental targets

The leading signatures can be summarized as

collider tails: $\Delta\sigma/\sigma_{\rm SM} \sim \eta_A E^2/M_*^2,$ (85)
neutrino oscillations: $\Delta P_{\alpha\beta} \sim \eta_A E^2/M_*^2,$ (86)
Higgs self-coupling: $\Delta\lambda_{hhh}/\lambda_{hhh}^{\rm SM} \sim \eta_A v^2/M_*^2,$ (87)
fifth forces: $\Delta V/V_N = \alpha_A e^{-r/\ell_A},$ (88)
cosmology: $\Delta H^2 = (8\pi G/3)\rho_A(z).$ (89)

These predictions make the model falsifiable: the associative Standard-Model limit is recovered for $M_* \to \infty$, $\eta_A \to 0$, or complete dynamical suppression of the associator.

17. Phenomenological hierarchy

The model is consistent with known low-energy particle physics when the associator sector is heavy or weakly excited:

$$E \ll M_* \quad \Rightarrow \quad \epsilon_A(E) \ll 1. \tag{90}$$

The leading phenomenological effects are ordered as follows:

$$\text{low-energy precision tests:} \qquad \Delta\mathcal{O}/\mathcal{O} \sim \epsilon_A(E), \tag{91}$$

$$\text{collider tails:} \qquad \Delta\sigma/\sigma \sim (E/M_*)^2, \tag{92}$$

$$\text{neutrino mixing:} \qquad \Delta P_A(E, L), \tag{93}$$

$$\text{Higgs self-coupling:} \qquad \Delta\lambda_{hhh}^{(A)}, \tag{94}$$

$$\text{fifth forces:} \qquad \alpha_A e^{-r/\ell_A}. \tag{95}$$

Thus the Standard Model is recovered as a stable infrared fixed projection, while deviations increase with energy, curvature of the projection bundle, or incomplete associator suppression.

18. Discussion

The conceptual novelty of the model can be summarized in four statements. First, the Standard Model is interpreted as an associative infrared projection rather than a fundamental starting point. Second, the three-field octonionic sector is minimal because alternativity forbids a nonzero associator from a single repeated field. Third, the gauge groups arise as residual symmetries and connections of local associative projections. Fourth, mass hierarchies and generation structure are related to eigenmodes of the associator potential.

Several tasks remain for a complete theory. One must construct explicit projector representations that reproduce all Standard-Model hypercharges, define the chiral weak sector without ambiguity, compute the full spectrum of associator modes, and compare the resulting effective operators with collider and precision bounds. The present work should therefore be read as a dynamical framework rather than a completed derivation of all Standard-Model data.

19. Conclusion

We have described how particle physics can emerge from an octonionic model in which non-associativity is dynamical. The fundamental algebraic object is the associator

$$[\Psi_1, \Psi_2, \Psi_3], \tag{96}$$

whose squared norm enters the action as a stabilizing potential. The Standard Model is recovered when the associator is suppressed and the fields project onto associative subalgebras,

$$\mathbb{O} \to \mathbb{H}_{\mathrm{phys}} \to \mathbb{C}. \tag{97}$$

In this limit, complex quantum fields, Standard-Model-like gauge connections, and ordinary low-energy interactions appear, provided that the projector representation reproduces the required chiral hypercharge assignments. Away from this limit, the associator generates new physical effects: generation-mode splittings, mass-hierarchy corrections, neutrino-mixing deviations, Higgs-sector deformations, high-energy contact operators, and possible fifth-force interactions.

The central result is therefore

$$\boxed{\begin{array}{c}\text{particle physics emerges as the stable associative projection}\\ \text{of octonionic non-associative dynamics.}\end{array}} \tag{98}$$

The new element compared with purely algebraic division-algebraic reconstructions is the field-theoretic role of the associator as a source of energy, symmetry breaking, and phenomenology.

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