Unified Framework of Gravity and Electromagnetism and Consistency of Fundamental Constants Based on a Geometric Model of Spatial Point Helical Motion (Integrated Revised Edition)

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Abstract

Within the framework of Topological Residual Theory (TRT), this study explores the unification of gravitational and electromagnetic interactions at the level of spacetime geometry. Utilizing the cylindrical helical motion of spatial points (geometrization of Zitterbewegung) as the dynamical foundation, we derive the wave equation

$\Box \mathbf{L} = 0$

via the variational principle of the displacement field $\mathbf{L}$. Using the geometric structure of the local cylindrical orthonormal frame $\{\hat{\mathbf{e}}_r, \hat{\mathbf{e}}_\phi, \hat{\mathbf{e}}_z\}$ induced by local helical symmetry, the electric, magnetic, and gravitational fields are defined as projections of the time derivatives of $\mathbf{L}$ onto this orthonormal frame. Consequently, the mutual orthogonality of the three fields emerges as a geometric necessity of the frame decomposition rather than an independent postulate. The gravitational field is derived via two independent pathways: (i) the frame kinematics and time evolution pathway ($\mathbf{A} = d(\mathbf{E} \times \mathbf{B})/dt$), and (ii) the second-order derivative pathway of the wave equation ($\mathbf{A} = c^2\mathbf{W}$). The mathematical consistency between these two pathways indicates that gravity can be interpreted as a second-order residual effect of electromagnetic oscillations propagating at the speed of light.

Furthermore, charge is defined as the topological charge (winding number / Hopf invariant) of the helical displacement field on a closed sphere

S^2

(Euler characteristic $\chi = 2$), which manifests as the effective rate of mass change only during the transient state of topological structure formation. Under the compatibility condition of the flux change rate (local topological flux continuity), we naturally derive the vacuum relations $\epsilon_0 \mu_0 = 1/c^2$ and $G = \alpha^2 \mu_0$. Based on this unified topological mechanism, the model provides a self-consistent geometric explanation for both charge quantization and particle spin.

Keywords: Unified field theory; spatial helical motion; variational principle of displacement field; cylindrical orthonormal frame; gravity and electromagnetism; consistency of physical constants; Topological Residual Theory

I. FUNDAMENTAL POSTULATES AND FIRST-PRINCIPLES OF THE UNIFIED FIELD THEORY

The core of Topological Residual Theory (TRT) is geometric priority and topological necessity. The fundamental postulates are as follows:

1. Cylindrical helical motion of spatial points (geometrization of Zitterbewegung)

This theory restricts the cylindrical helical motion of spatial points to the spacetime region surrounding a local source (such as an elementary charge). The

z

-axis is defined as the intrinsic symmetry axis of the local topological source (corresponding to its spin direction). In macroscopic or source-free regions, due to the statistical isotropy of the source orientations, the global space strictly maintains global Lorentz covariance and isotropy.

Any spatial point in the Cartesian coordinate system simultaneously undergoes uniform transverse circular motion (radius

R

, angular frequency ω, linear velocity $\mathbf{V} = R\omega\hat{\mathbf{e}}_\phi$) and uniform axial linear motion at the speed of light (velocity $\mathbf{C} = c\hat{\mathbf{e}}_z$). These superpose to form a cylindrical helical trajectory, with the geometric ratio defined as:

$$\alpha = \frac{V}{c} = \frac{R}{h}$$

where h is the axial pitch (distance traversed per revolution). The helical motion described here represents the dynamical evolution of the geometric phase of the spacetime metric/displacement field. The phase velocity of its transverse modulated wave can exceed the speed of light, analogous to the phase velocity in classical waveguides or de Broglie matter waves, while the group velocity of physical entities and energy remains strictly bounded by the speed of light c, thereby preserving relativistic causality. In subsequent derivations, this geometric ratio is shown to correspond to the fine-structure constant.

2. Minimal geometric definition of the three fields (without intermediate coefficients)

The electric, magnetic, and gravitational fields are constructed directly from the fundamental variables of the helical motion, namely

$\mathbf{C}$

, $\mathbf{V}$, and ω.

3. Principle of orthogonality (equivalence of motion and orthogonality)

The perpendicularity of any two fields mathematically necessitates the generation of a third field; conversely, physical motion is manifested as the mutual orthogonality of the three fields. Mathematically, this is expressed as the equivalence between triple cross products and temporal derivatives.

4. Consistency of gravitational and electromagnetic fluxes

Gravity is interpreted as the effective residual effect of electromagnetic oscillations (resembling Sakharov's induced gravity, but fully geometrized) [1]. The macroscopic coherent force scales as

$\propto N$

(in-phase superposition of a large number of helices), while the residual fluctuating force scales as $\propto \sqrt{N}$ (stochastic phase statistics; for $N \sim 10^{80}$, the ratio of forces is $\sim 10^{-40}$). The two are unified through the topological properties of the same closed sphere S^2 (Euler characteristic $\chi = 2$, solid angle 4π) [2].

5. Wave equation as the dynamical skeleton

The displacement field

$\mathbf{L}$

satisfies the d'Alembert equation $\Box\mathbf{L} = 0$, guaranteeing causality and propagation at the speed of light.

Variational Principle of the Displacement Field L and Cylindrical Orthonormal Frame (New First-Principles Layer)

To establish a higher level of first-principles rigor, we begin with the variational principle of the displacement field

$\mathbf{L}$

. Considering $\mathbf{L}$ to be a Lorentz-invariant vector field, its action over the Minkowski spacetime $\mathrm{R}^{1,3}$ is defined as:

$$S[\mathbf{L}] = \frac{1}{2} \int_{\mathrm{R}^{1,3}} \left(|\partial_t \mathbf{L}|^2 - c^2 |\nabla \mathbf{L}|^2 \right) d^4x$$

Consider a family of parameterized perturbations

$\mathbf{L}_\epsilon = \mathbf{L} + \epsilon\boldsymbol{\eta}$

, where $\boldsymbol{\eta}$ is an arbitrary smooth test field with compact support. Requiring the physical field to extremize the action yields:

$$\left.\frac{d}{d\epsilon}\right|_{\epsilon=0} S[\mathbf{L}_\epsilon] = \int_{\mathbb{R}^{1,3}} (\partial_t \mathbf{L} \cdot \partial_t \boldsymbol{\eta} - c^2 \nabla \mathbf{L} \cdot \nabla \boldsymbol{\eta}) \, d^4x = 0$$

Performing integration by parts with respect to the temporal and spatial variables separately (where the boundary terms vanish at infinity due to the compact support of

$\boldsymbol{\eta}$

), we obtain:

$$\int_{\mathbb{R}^{1,3}} \boldsymbol{\eta} \cdot (-\partial_t^2 \mathbf{L} + c^2 \Delta \mathbf{L}) \, d^4x = 0$$

By the fundamental lemma of the calculus of variations, the arbitrariness of

$\boldsymbol{\eta}$

necessitates:

$$\Box \mathbf{L} = 0$$

which is the d'Alembert wave equation. This equation provides a rigorous dynamical foundation for the helical ansatz.

The helical symmetry of the local source naturally induces a cylindrical coordinate system and its associated orthonormal frame

$\{\hat{\mathbf{e}}_r, \hat{\mathbf{e}}_\phi, \hat{\mathbf{e}}_z\}$

, satisfying $\hat{\mathbf{e}}_i \cdot \hat{\mathbf{e}}_j = \delta_{ij}$. Any vector can be uniquely decomposed orthogonally. This provides a geometric necessity for the orthogonality of the three fields: the fields are defined as projections of the temporal derivatives of the displacement field $\mathbf{L}$ onto this frame, with their perpendicularity directly guaranteed by the orthonormality of the frame rather than by an independent physical postulate.

II. DEFINITION OF THE ELECTRIC FIELD

In the cylindrical orthonormal frame, the component of the first-order temporal derivative of the displacement field (i.e., the velocity field) along the axial direction is defined as the background electric field:

$$\mathbf{E} := (\partial_t \mathbf{L}) \cdot \hat{\mathbf{e}}_z \hat{\mathbf{e}}_z = c\hat{\mathbf{e}}_z$$

which remains strictly constant under the helical ansatz.

Physical Interpretation:

The electric field is physically understood as the intrinsic drift velocity of space itself along the symmetry axis at the speed of light (which is uniform and divergence-free,

$\nabla \cdot \mathbf{E} = 0$

). When space undergoes topological divergence (mass increasing from 0 to m) or convergence (mass decreasing from m to 0), this axial drift manifests as an effective radial charge effect. The background axial field provides the propagation channel, and all electromagnetic phenomena are manifestations of this axial drift modulated by transverse rotation. The effective radial field $\mathbf{E}_{\text{eff}} = E_r\hat{\mathbf{r}}$ is governed by the strength of the topological flow and decoupled from the background field, thereby resolving the divergence conflict inherent in traditional point-source models.

III. DERIVATION OF THE GRAVITATIONAL FIELD VIA THE CYLINDRICAL ORTHONORMAL FRAME AND KINEMATICS (GEOMETRIC NECESSITY OF THREE-FIELD ORTHOGONALITY)

First-Principles Definition of the Magnetic Field

The azimuthal component of the velocity field defines the magnetic field:

$$\mathbf{B} := \frac{1}{c}(\partial_t \mathbf{L}) \cdot \hat{\mathbf{e}}_\phi \hat{\mathbf{e}}_\phi$$

or equivalently represented as $\mathbf{B} = \mathbf{V}'/c$ (where $\mathbf{V}'$ is the transverse projection of the velocity). This definition arises directly from the helical rotation without intermediate coefficients.

Geometric Necessity of Three-Field Orthogonality

- Velocity:

  $\mathbf{v} = \partial_t \mathbf{L} = v_z \hat{\mathbf{e}}_z + v_\phi \hat{\mathbf{e}}_\phi$

  .

- Definition:

  $\mathbf{E} \sim v_z \hat{\mathbf{e}}_z$

  and $\mathbf{B} \sim (v_\phi/c)\hat{\mathbf{e}}_\phi$.

- Since the frame is orthonormal, i.e.,

  $\hat{\mathbf{e}}_z \perp \hat{\mathbf{e}}_\phi \perp \hat{\mathbf{e}}_r$

  , the cross product $\mathbf{E} \times \mathbf{B}$ is strictly directed along the radial direction ($\hat{\mathbf{e}}_z \times \hat{\mathbf{e}}_\phi = -\hat{\mathbf{e}}_r$).

The gravitational field is defined as the radial component of the displacement field's acceleration:

$$\mathbf{A} := -(\partial_t^2 \mathbf{L}) \cdot \hat{\mathbf{e}}_r \hat{\mathbf{e}}_r$$

Under the helical ansatz, $|\partial_t^2 \mathbf{L}|_r = R\omega^2 = V^2/R$, which corresponds to the centripetal acceleration.

The cross product relations naturally emerge as:

$$\mathbf{A} = \frac{d}{dt}(\mathbf{E} \times \mathbf{B}) = \frac{\omega}{c}(\mathbf{C} \times \mathbf{V})$$

where the negative sign (pointing toward the center) originates from the topological tension pointing toward the interior of the helix. Here, the principle of orthogonality is a direct mathematical consequence of the **orthonormal frame and helical symmetry decomposition** rather than an independent physical postulate.

IV. DERIVATION OF THE GRAVITATIONAL FIELD VIA THE WAVE EQUATION AND EMERGENCE OF $\epsilon_0 \mu_0 = 1/c^2$

The wave equation pathway (independent of frame kinematics, yet entirely consistent with it):

For the helical ansatz, which strictly satisfies

$\Box \mathbf{L} = 0$
:

$$L_x = R\cos\psi, \quad L_y = R\sin\psi, \quad L_z = ct, \quad \psi = \omega t - kz, \quad k = \omega/c$$

The gravitational field is defined as the second-order temporal derivative:

$$\mathbf{A} = \frac{\partial^2 \mathbf{L}}{\partial t^2} = -(ck)^2 R(\cos\psi, \sin\psi, 0)$$

Defining the combined electromagnetic field as

$\mathbf{W} = \partial^2 \mathbf{L}/\partial z^2$
, the wave equation directly yields:

$$\mathbf{A} = c^2 \mathbf{W} \iff \frac{\partial^2 \mathbf{L}}{\partial t^2} = c^2 \frac{\partial^2 \mathbf{L}}{\partial z^2}$$

Comparison and Equivalence of the Two Pathways:

- **Frame kinematics pathway:**

  $\mathbf{A} = d(\mathbf{E} \times \mathbf{B})/dt$

  (geometric coupling derived from frame projection and velocity decomposition).

- **Wave equation pathway:**

  $\mathbf{A} = \partial^2 \mathbf{L}/\partial t^2 = c^2 \partial^2 \mathbf{L}/\partial z^2$

  (directly guaranteed by the variational principle).

The exact mathematical equivalence between the two pathways proves that gravity is a second-order residual effect of electromagnetic oscillations propagating at the speed of light. The variation of the gravitational field with spatial position generates electromagnetic fields, and the two are linked via

c^2

.

Emergence of vacuum constant relations:

In this unified framework, the magnetic field is defined as

$\mathbf{B} = \mathbf{V}' \times \mathbf{E}/c^2$

. In classical electromagnetism, it is defined as $\mathbf{B} = \epsilon_0 \mu_0 (\mathbf{V}' \times \mathbf{E})$。 Directly comparing the two expressions for the same physical quantity yields:

$$\epsilon_0 \mu_0 = \frac{1}{c^2}$$

This relation serves as the consistency condition between the geometrically defined magnetic field and its classical representation.

V. TOPOLOGICAL DEFINITIONS OF MASS AND CHARGE (UPGRADED VERSION)

Definition of Mass:

Within the TRT framework, mass is defined as the field energy carried by the helical displacement field

$\mathbf{L}$

within the volume V enclosed by the closed sphere S^2, divided by c^2. Specifically, this energy density is derived directly from the canonical energy density associated with the action $S[\mathbf{L}]$:

$$\mathrm{H} = \frac{1}{2} \left(| \partial_t \mathbf{L} |^2 + c^2 | \nabla \mathbf{L} |^2 \right)$$

Thus, the complete expression for mass is:

$$m = \frac{1}{c^2} \int_V \mathrm{H} dV = \frac{1}{c^2} \int_V \frac{1}{2} \left(|\partial_t \mathbf{L}|^2 + c^2 |\nabla \mathbf{L}|^2 \right) dV$$

Dimensional Analysis:

In the above expression,

$|\partial_t \mathbf{L}|^2$ has the dimension of [energy/volume], and $c^2 |\nabla \mathbf{L}|^2$ shares the same dimension. Consequently, integrating and dividing by c^2 yields the correct dimension of mass for m (kg in the SI system, and energy/c^2 in natural units). In the minimal geometric model, we set the proportionality constant to 1, and subsequently introduce Newton's gravitational constant G by matching this to the classical gravitational flux $\oint_S \mathbf{A} \cdot dS = -4\pi G m$, thereby establishing a dimensional bridge to macroscopic mechanics.

Pure Topological Definition of Charge:

Charge is fundamentally defined as the topological charge (winding number / Hopf invariant) of the helical displacement field on the closed sphere

S^2 [5,6]:

$$q_{\text{top}} = \frac{1}{4\pi} \iint_{S^2} \mathbf{n} \cdot (\partial_\theta \mathbf{n} \times \partial_\phi \mathbf{n}) d\theta d\phi$$

This definition is entirely determined by the Euler characteristic of S^2 ($\chi = 2$), representing an integer or half-integer topological invariant that is time-independent and conserved (with continuity guaranteed by the wave equation).

Only when the topological soliton is in a transient state of formation or dynamic assembly does the energy

m vary with time, during which the effective electromagnetic response manifests as:

$$q_{\text{eff}} \approx \frac{m}{c^2 t_{\text{char}}}$$

where the characteristic time scale t_{char} is entirely determined by geometric parameters ($t_{\text{char}} \sim h/c \sim 2\pi/\omega$).

Combining the wave equation

$\mathbf{A} = c^2\mathbf{W}$

with the flux relations yields the unified mass-topological charge equation:

$$c^2 q_{\mathrm{top}} = \frac{0 - 4\pi m}{t_{\mathrm{char}}} \quad (\mathrm{or}\ \pm q_{\mathrm{top}} = \pm \frac{4\pi m}{c^2 t_{\mathrm{char}}})$$

where positive charges correspond to spherically diverging spatial waves, and negative charges correspond to converging/annihilating waves.

Remark on the Calculability of the Topological Integrals:

It should be emphasized that while the definitions of topological charge

q_{top}

and field mass m outline their fundamental geometric and topological origins, the direct computation of their specific numerical values for a particular elementary particle (e.g., the electron's rest mass of $9.11 \times 10^{-31}\ \mathrm{kg}$) requires solving the exact non-linear soliton equations of the displacement field $\mathbf{L}$ in the core region.

Specifically, the topological charge

q_{top}

yields a strictly quantized integer ($n = 0, \pm 1, \pm 2, \ldots$), which mathematically explains the origin of charge quantization. To map this dimensionless topological index to the SI unit (Coulomb), the dimensional bridging constant ϵ_0 is employed.

For the field mass

m

, the integral represents the canonical self-energy of the spatial deformation. In standard topological soliton physics (such as the Skyrme or Hopfion models), calculating the exact mass-energy of a localized soliton requires solving the full non-linear Euler-Lagrange equations, which typically necessitates numerical computation. In the current framework, the simplified helical ansatz represents the asymptotic field behavior outside the non-linear core. Once the full non-linear soliton core of $\mathbf{L}$ is solved under a given characteristic frequency ω, the absolute mass can be quantitatively evaluated. This approach is conceptually identical to standard Quantum Field Theory (QFT), where particle masses are determined by the coupling of fields and their self-energies.

VI. DERIVATION OF FLUX INTEGRALS AND THE RELATIONS AMONG THE THREE CONSTANTS

Integrating over a Gaussian sphere

$S = 4\pi r^2$

enclosing the origin O, we obtain the gravitational flux using Newton's gravitational field equation:

$$\oiint_S \left(\frac{Gm}{r^2}\right) \cdot dS = -4\pi Gm$$

Using Coulomb's electrostatic field equation, we obtain the flux of the combined electromagnetic field (where the net magnetic flux contribution over the closed sphere is zero, leaving only the electric field component):

$$\oiint_S \left(\frac{q_{\text{top}}}{4\pi\epsilon_0 h^2}\right) \cdot dS = \oiint_S \left(\frac{\alpha^2 q_{\text{top}}}{4\pi\epsilon_0 r^2}\right) \cdot dS = \frac{\alpha^2 q_{\text{top}}}{\epsilon_0}$$

(Note that the fine-structure constant is defined here as the geometric ratio $\alpha = r/h$).

According to this unified framework, the gravitational field is the root cause of the accelerated spatial oscillations in the

xy

-plane (the cause), whereas the electromagnetic field represents the propagation of these accelerated oscillation effects along the z-axis (the effect).

The combined electromagnetic field represents the rate of change of the gravitational field within a unit time

$t_0 = 1$

(within the minimal model of the unified field theory, the unit of time and unit of spatial length can be set to 1) . Therefore, in the minimal model of the unified field theory, and neglecting dimensional differences, the flux of the combined electromagnetic field must be equal to the change in the gravitational flux over the unit time t_0 as it increases from 0 to $4\pi Gm$, or decreases from $4\pi Gm$ to zero, yielding:

$$\frac{0 - 4\pi Gm}{t_0} = \frac{\alpha^2 q_{\text{top}}}{\epsilon_0}$$

When the unit time t_0 is set to 1, this equation reduces to:

$$0 - 4\pi Gm = \frac{\alpha^2 q_{\text{top}}}{\epsilon_0}$$

It must be specifically pointed out that the equivalence relation satisfied by the gravitational and electromagnetic fields in the wave equation is fundamental and does not require a unit-time constraint to hold; however, the electric flux is formed by the change of gravitational flux over a unit time, it must be constrained by the prerequisite of the "unit time." Furthermore, since the net magnetic flux over any closed sphere is zero, the magnetic field yields no net contribution, making the integration of the combined electromagnetic field mathematically identical to the integration of the electric field.

Now, we compare the flux change rate matching equation:

$$\frac{0 - 4\pi G m}{t_0} = \frac{\alpha^2 q_{\text{top}}}{\epsilon_0}$$

with the minimal mass-charge relation derived in Section V:

$$q_{\text{top}} = -\frac{4\pi m}{c^2 t_0} \implies c^2 q_{\text{top}} = \frac{0 - 4\pi m}{t_0}$$

Substituting the expression for

q_{top}
into the flux matching equation and canceling the common factors m and t_0, we immediately obtain:

$$G = \frac{\alpha^2}{\epsilon_0 c^2}$$

Combining this with the previously proven vacuum relation

$\epsilon_0 \mu_0 = 1/c^2$
directly yields:

$$G = \alpha^2 \mu_0$$

This completes the proof.

The relations among the three constants are proven. The entire logical chain is closed: **geometric locking (**

α
) → variational wave connection (c^2**) → flux conservation and temporal change rate matching → classical bridging → self-consistent emergence of constants**. All steps support each other, forming a rigorous closed loop.

VII. GEOMETRIC-TOPOLOGICAL ORIGIN OF CHARGE QUANTIZATION AND SPIN

- **Charge Quantization:** The flux variation must change discretely in integer multiples of 4π (a topological necessity of the closed sphere S^2, where the Euler characteristic $\chi = 2$ strictly constrains the solid angle to 4π). Positive charges correspond to spherically diverging spatial waves, while negative charges correspond to converging/annihilating waves.

- **Geometric-Topological Origin of Spin:** The same closed sphere encloses the helical displacement lines/vortices, where the quantization of the winding number or link number corresponds directly to the particle spin. Mathematically, spin can be associated with the topological charge of the helical orientation field on S^2 (resembling a Skyrmion/Hopf structure):

$$\eta = \frac{1}{4\pi} \iint_{S^2} \mathbf{n} \cdot (\partial_\theta \mathbf{n} \times \partial_\phi \mathbf{n}) d\theta d\phi$$

  Taking integer or half-integer values, this is topologically consistent with spin quantization (including $\hbar/2$). Both charge and spin share the same closed-sphere topological mechanism, providing a unified geometric-topological origin.

VIII. CONCLUDING REMARKS

The entire proof chain is coherent and profound: beginning with first-principles geometric postulates, the field definitions and three-field orthogonality are derived from the variational principle of the displacement field

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combined with the cylindrical orthonormal frame (geometric necessity). The mathematical equivalence of the two independent pathways yields the gravitational field, which upon comparison directly produces $\epsilon_0 \mu_0 = 1/c^2$. Integrating the pure topological charge definition via Gaussian flux and comparing with classical forms naturally reveals $G = \alpha^2 \mu_0$. Concurrently, charge quantization and spin are naturally explained within the same topological framework. The introduction of the variational principle and the orthonormal frame adds depth and first-principles consistency to the theoretical structure. All steps mutually support each other, establishing a rigorous closed loop that fully embodies the first-principles philosophy of TRT in "understanding the universe."

Unified Relations of the Three Constants (Core Results):

$$\epsilon_0 \mu_0 = \frac{1}{c^2}, \quad G = \frac{\alpha^2}{\epsilon_0 c^2} = \alpha^2 \mu_0$$

These relations are obtained under the geometric flux framework by matching the minimal model of TRT with classical forms, highlighting the central roles of helical geometry (

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) and propagation at the speed of light (c^2) in a unified description. They also serve as a reminder that the establishment of relations between constants involves components of dimensional bridging.

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