Dynamical Threshold Theory of Sleep: A Claim-Bounded Integrated Model and Audit Synthesis for Sleep-Like Recovery Transitions

v0.3.1 Integrated Revision

Keiji Yoshimura

Independent Researcher

May 25, 2026

Repository: https://github.com/yokken0907/dynamical-threshold-theory-of-sleep

Abstract

This integrated revision combines the original Dynamical Threshold Theory of Sleep (DTTS) model manuscript with the subsequent v0.2.0–v0.2.8 audit sequence and v0.3.0 synthesis addendum. DTTS is treated here as a claim-bounded network-dynamical prototype for sleep-like recovery transitions in finite adaptive networks. The original model represents a multicellular system as a graph of adaptively coupled oscillatory elements carrying local homeostatic load, local sleep propensity, and selectively protected interactions. Wake-like dynamics are represented as noisy adaptive synchronization under continued perturbation, while sleep-like dynamics emerge when local load accumulation and coherence degradation drive a thresholded recovery gate into a sleep-dominant renormalization regime. The audit sequence adds threshold, hysteresis, perturbation-recovery, topology/noise, failure-boundary, rescue-classification, and frozen holdout tests. Across the tested toy-model envelope, threshold-like gate onset, hysteresis-like persistence, structurally aligned selective protection, and rescue-sensitive regimes were observed. However, expanded audits also identified failure boundaries, and weak-protection conditions remained unstable under harsher holdout. This manuscript does not claim clinical sleep modeling, biological sleep validation, EEG or wearable-device prediction, diagnosis, treatment guidance, or a complete theory of physiological sleep.

Reader guidance and medical non-claim. This manuscript is a mathematical and computational prototype study. It is not medical advice, not a clinical sleep model, not a diagnostic or treatment tool, and not a claim of validation against human or animal sleep data. All results should be interpreted as reduced network-dynamical evidence for sleep-like recovery-transition behavior inside the implemented toy-model and audit settings.

1 Introduction

Sleep is associated with homeostatic recovery, local use-dependent regulation, state-dependent reorganization of network interactions, and temporally structured circadian gating. Major explanatory traditions emphasize synaptic homeostasis and down-selection [1–4], local sleep-like events and use-dependent regional regulation [5–7], and two-process homeostatic/circadian regulation [8, 9]. These traditions are individually productive, but they are often expressed at different levels of description.

DTTS was proposed to express these motifs in a single reduced dynamical language. The aim is not to replace biological sleep science with a toy model. The aim is narrower: to ask whether a finite adaptive network with local load, local coherence, a thresholded sleep-like gate, and protected coupling updates can produce recovery-transition motifs that are mathematically closed and numerically auditable.

Synchronization models, including the Kuramoto family and small-world network synchronization studies, provide a natural mathematical background for treating collective coherence and its degradation [10, 11]. DTTS extends that reduced language by adding local burden variables, sleep-like gate dynamics, and selective coupling renormalization.

2 Model

2.1 Network and phase dynamics

We represent the system as a network of N coupled oscillatory nodes on an undirected graph $G = (V, E)$ with adjacency matrix $A_{ij} \in \{0, 1\}$. Each node has phase $\theta_i(t)$, intrinsic frequency ω_i, local sleep gate $q_i(t)$, and perturbation susceptibility σ_i. The phase dynamics are

$$\frac{d\theta_i}{dt} = \omega_i + \sum_{j=1}^{N} A_{ij} K_{ij} \sin(\theta_j - \theta_i) + \sigma_i (1 - q_i) \xi_i(t), \tag{1}$$

where K_{ij} is the adaptive coupling and $\xi_i(t)$ denotes independent Gaussian noise. Local coherence is

$$R_i(t) = \left| \frac{1}{d_i} \sum_{j=1}^{N} A_{ij} e^{i(\theta_j - \theta_i)} \right|. \tag{2}$$

2.2 Local load and sleep-like gate

Each node carries homeostatic load $s_i(t)$:

$$\frac{ds_i}{dt} = a(1 - R_i) + \chi u_i(t) - b q_i s_i, \tag{3}$$

where $u_i(t)$ is a localized use-dependent drive. The sleep-like gate evolves according to

$$\tau_q \frac{dq_i}{dt} = -q_i + \Sigma \left(\beta_s (s_i - s_c) - \beta_c C(t) - \beta_R (R_i - R_c) \right), \tag{4}$$

where $C(t)$ is a circadian-like permissive term and Σ is a sigmoid. In this reduced setting, “sleep-like” means only a modeled recovery-gate transition. It does not mean physiological sleep.

2.3 Selective protection and coupling updates

Adaptive coupling weights are updated by

$$\frac{dK_{ij}}{dt} = (1 - \bar{q}_{ij}) \left[\eta_W P_{ij} (K_{\max} - K_{ij}) - \delta_W K_{ij} \right] - \bar{q}_{ij} \eta_K (\mu K_{ij} - \kappa P_{ij}), \tag{5}$$

where P_{ij} is a protection field representing prior structural/coherence history. During a sleep-dominant interval, couplings relax toward the protection-dependent fixed point

$$K^*_{ij} = \frac{\kappa}{\mu} P_{ij}. \tag{6}$$

3 Reduced free-energy closure

A key purpose of the original DTTS formulation was to avoid unbounded adaptive-coupling pathologies. In the sleep-dominant limit, the dynamics can be written as a gradient flow of a reduced free energy

$$\mathcal{F}_s(K, s \mid P) = \sum_{i<j} A_{ij} \left(\frac{\mu}{2} K_{ij}^2 - \kappa P_{ij} K_{ij} \right) + \frac{\nu}{2} \sum_i s_i^2. \tag{7}$$

In the limit $q_i \to 1$, the coupling and load dynamics satisfy $\dot{K}_{ij} \propto -\partial \mathcal{F}_s / \partial K_{ij}$ and $\dot{s}_i \propto -\partial \mathcal{F}_s / \partial s_i$, giving $d\mathcal{F}_s/dt \leq 0$ within the reduced subsystem.

4 Original proof-of-concept diagnostics

The original DTTS numerical prototype used a hierarchical modular small-world network with localized filtered-Poisson drive. Figure 1 summarizes the core behavior: local burden accumulates, mean sleep propensity rises into a sleep-dominant regime, mean coupling strength downscales toward a finite band, and the reduced free energy relaxes after an initial transient.

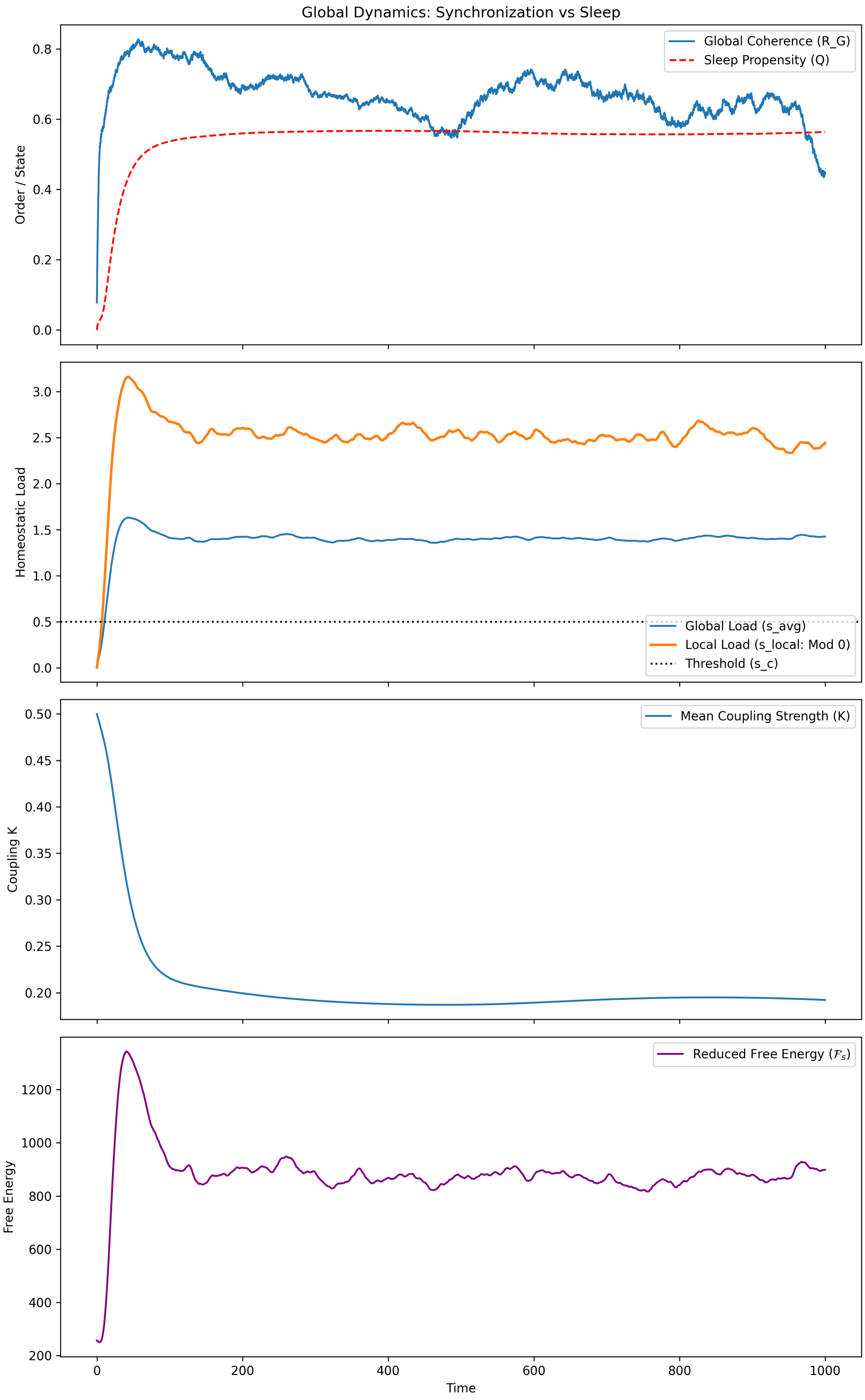

Figure 1: Original DTTS simulation summary. Top to bottom: global coherence R_G and mean sleep propensity Q; homeostatic load; mean coupling strength; and reduced free energy. This is a proof-of-concept network-dynamical diagnostic, not a biological sleep-data fit.

The original selective-protection diagnostic is shown in Figure 2. Edges with higher pre-sleep protection were relatively preserved, while less protected edges were more strongly down-selected. This supports the model-internal idea of selective renormalization rather than uniform weakening.

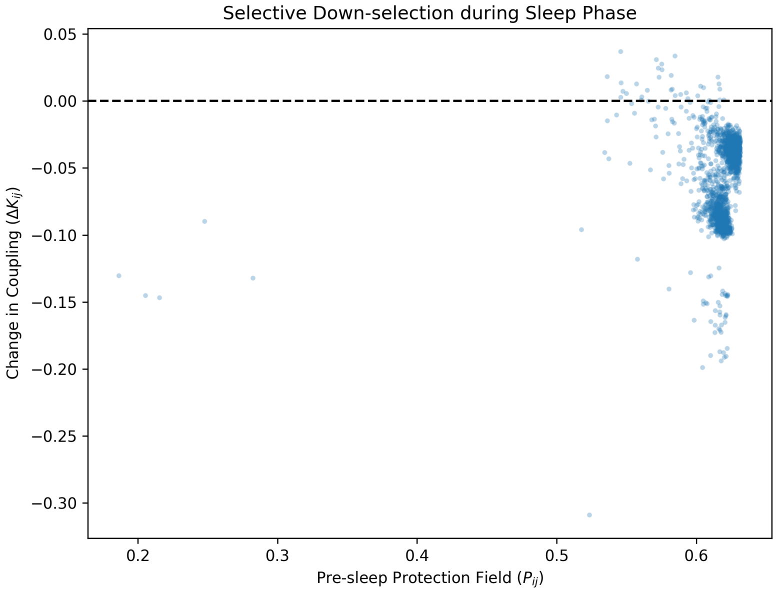

Figure 2: Original selective down-selection diagnostic: coupling change ΔK_{ij} versus pre-sleep protection field P_{ij}. The figure supports only a model-internal selective-protection claim.

5 Extended audit sequence

The v0.2.x audit sequence was designed to test and bound the original mechanism rather than merely add positive examples. Table 1 summarizes the locked phase ledger.

Phase	Role	Locked contribution
v0.2.0	Threshold/protection preflight	Threshold-drive candidate identified; sleep-like gate onset and protection controls established.
v0.2.1	Closed-loop hysteresis	Baseline hysteresis-like gap exceeded memoryless control.
v0.2.2	Circadian/multi-cycle audit	Circadian-like permissive gate modulated onset timing; protection remained active.
v0.2.3.1	Perturbation recovery	Acute load pulses evoked gate response; no-gate control suppressed it.
v0.2.4	Topology/noise robustness	Core gate/protection effects persisted across simplified topology and noise settings.
v0.2.5	Conservative envelope	No failure boundary found inside the conservative envelope; claims restricted to that envelope.
v0.2.6	Expanded failure boundary	Failure cases identified; DTTS behavior shown not to be universal.
v0.2.7	Mechanism rescue	Failure modes separated into gate, protection, and unresolved-load boundaries with distinct rescue responses.
v0.2.8	Frozen rescue holdout	Frozen rescue policies generalized for most rescue-sensitive regimes; weak-protection remained unstable.

Table 1: Claim-bounded DTTS v0.2.0–v0.2.8 audit ledger.

6 Failure boundaries and frozen rescue holdout

The v0.2.8 frozen-rescue holdout used six scenario classes, five simplified topologies, and four noise levels, yielding 240 holdout cells. The mean frozen-policy pass rate was 0.7729. Native pass count was 1; rescue holdout pass count was 4; unstable rescue boundary count was 1; persistent failure boundary count was 0. The remaining unstable boundary was weak protection, indicating that structural protection is not merely decorative but a central condition for the modeled selective-renormalization effect.

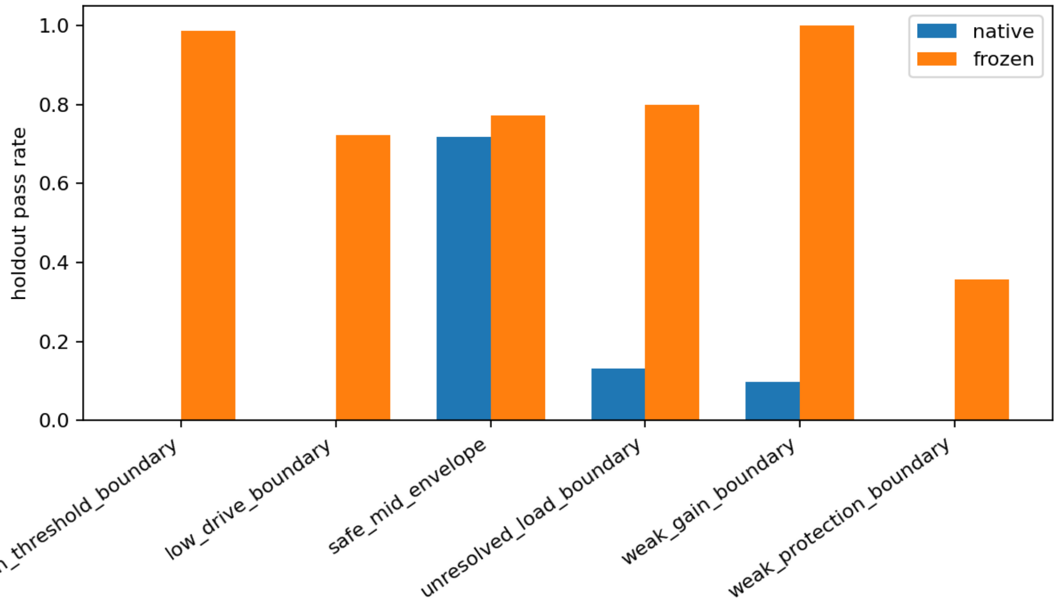

Figure 3: Frozen rescue-policy pass-rate comparison from the v0.2.8 holdout audit. Frozen rescue policies generalized for most rescue-sensitive regimes, while weak-protection remained unstable.

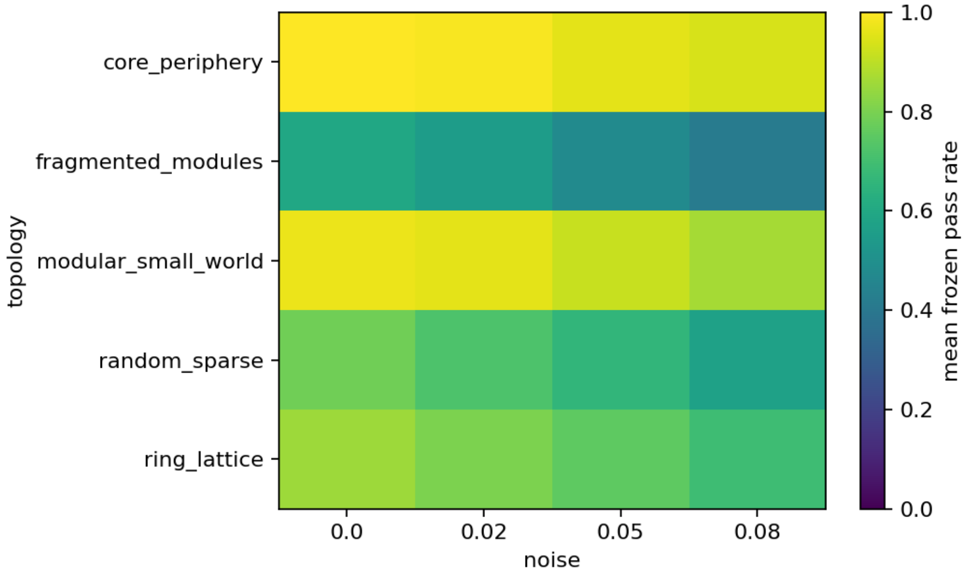

Figure 4: Topology/noise holdout heatmap for frozen rescue-policy generalization. The plot is an internal toy-model robustness diagnostic and should not be interpreted as a biological sleep validation result.

7 Integrated interpretation

The integrated interpretation is narrower than the original title might suggest but stronger as a research object. DTTS should be read as a network-dynamical recovery-transition prototype with four interacting elements: local load accumulation, thresholded gate onset, structurally aligned selective protection, and regime-dependent rescue sensitivity.

The positive evidence is that the implemented network can show threshold-like gate onset, hysteresis-like persistence, perturbation-recovery response, and selective protection. The negative evidence is equally important: expanded sweeps found conditions under which the desired signatures weaken or fail. Low drive, high threshold, weak gate gain, weak protection, and unresolved-load regimes can disrupt the modeled mechanism. This converts DTTS from a broad sleep claim into a bounded regime-classification framework.

8 Limitations

The audits use simplified networks, synthetic load dynamics, synthetic protection fields, and internal toy-model controls. No human data, animal data, polysomnography, EEG, wearable-device data, clinical sleep annotation, pharmacological intervention, or treatment outcome is included. The phrase "sleep-like" refers only to a modeled recovery-gate transition, not to physiological sleep.

This manuscript also does not claim that sleep is reducible to a Kuramoto-like network or that biological sleep is generated by the exact variables used here. The model is a hypothesis-generating scaffold for reasoning about recovery-transition motifs in finite adaptive networks.

9 Data and code availability

The public GitHub repository is available at https://github.com/yokken0907/dynamical-threshold-theory-of-sleep. The repository contains manuscript materials, claim-boundary documentation, source-defined Evaluation-Only license files, and evidence packages for the audit sequence. Repository archives should be treated as repository-release artifacts, not as standalone peer-reviewed article identifiers.

10 Conflict of interest

The author declares no institutional, financial, or commercial conflict of interest related to this study.

AI assistance disclosure

The author used AI assistance for drafting, code generation, numerical-audit scaffolding, and manuscript preparation. The author remains responsible for the final claim boundary, interpretation, and release decisions.

11 Conclusion

DTTS v0.3.1 integrates the original model manuscript and the later audit sequence. The resulting claim is intentionally bounded: in the implemented finite adaptive-network toy model, sleep-like recovery-transition motifs can arise through local load accumulation, thresholded gate onset, and structurally aligned selective protection, while extended audits identify both rescue-sensitive regimes and failure boundaries. This is a claim-bounded network-dynamical prototype, not a biological or clinical theory of sleep.

References

[1] Tononi, G., and Cirelli, C. (2003). Sleep and synaptic homeostasis: a hypothesis. *Brain Research Bulletin*, 62(2), 143–150.

[2] Tononi, G., and Cirelli, C. (2006). Sleep function and synaptic homeostasis. *Sleep Medicine Reviews*, 10(1), 49–62.

[3] Cirelli, C., and Tononi, G. (2015). Sleep and Synaptic Homeostasis. *Sleep*, 38(10), 1615–1624.

[4] Tononi, G., and Cirelli, C. (2014). Sleep and the Price of Plasticity. *Neuron*, 81(1), 12–34.

[5] Vyazovskiy, V. V., et al. (2011). Local sleep in awake rats. *Nature*, 472(7344), 443–447.

[6] Krueger, J. M., et al. (2019). Local Sleep. *Sleep Medicine Reviews*, 43, 14–21.

[7] Siclari, F., et al. (2018). Local aspects of sleep and wakefulness. *Current Opinion in Neurobiology*, 44, 222–227.

[8] Borbely, A. A., et al. (2022). The two-process model of sleep regulation. *Journal of Sleep Research*, 31(6), e13598.

[9] Deboer, T. (2018). Sleep homeostasis and the circadian clock. *Neurobiology of Sleep and Circadian Rhythms*, 5, 68–77.

[10] Acebron, J. A., et al. (2005). The Kuramoto model. *Reviews of Modern Physics*, 77(1), 137–185.

[11] Hong, H., et al. (2002). Synchronization on small-world networks. *Physical Review E*, 65(2), 026139.