Topology Residual Entropy Increase Theory: Constant Closed Loop and Topological Hierarchy
——Proof of the Self-Consistency of the Three-Layer π Expansion and Five Residual Iterations

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Abstract
The Topology Residual Entropy Increase Theory (TRT) takes the "right-handed cylindrical helical motion of space at the speed of light" ($\omega r = c$) as its sole foundational axiom. Through a three-layer π topological expansion ($\Omega = 4\pi + \pi + \pi \approx 137.0363$, $\alpha = 1/\Omega$) and five residual iterations, it achieves a self-consistent closed loop for the fundamental constants and physical quantities G, h, α, e, m_p, q_p, as well as the Hawking temperature T_H and Bekenstein-Hawking entropy S of black holes.

This paper systematically demonstrates, from five independent dimensions—geometric-topological necessity, dimensional self-consistency and residual bridging, the critical balance criterion (μ_0 cancellation when $p + q = 0$), high consistency with experimental values, and minimality of self-consistency—that the "three-layer + five-iteration" structure is the unique minimal structure within the current framework that simultaneously satisfies geometric self-consistency, dimensional closure, critical balance, and experimental agreement. In addition, the theory naturally derives the holographic property of space (with residual networks serving as holographic carriers, and black hole event horizons as well as large-scale cosmic structures acting as effective holographic screens). It also proposes a series of falsifiable predictions, including an upper geometric limit on the Hawking temperature of Planck-mass black holes, the Bekenstein-Hawking entropy of Planck-mass black holes approaching $4\pi k_B$, and the linking number of early-universe residual networks decaying as t^{-1}. These predictions provide clear directions for testing with LISA, gamma-ray telescopes, and CMB observations. This paper serves as a phased summary of the Topology Residual Unified Field Theory series and lays a solid theoretical foundation for subsequent prediction verification, cosmological applications, and experimental design.

Keywords: Topology Residual Theory; fine-structure constant; gravitational constant; Planck constant; holographic principle; black hole thermodynamics; falsifiable predictions; induced gravity

1. Introduction

The Topology Residual Entropy Increase Theory (TRT) originates from long-term reflection on the origin of fundamental constants and the grand unification of forces. Taking "space as a right-handed cylindrical helical fluid moving at the speed of light" ($\omega r = c$) as its sole foundational axiom, and combining it with the topological necessity—derived from the Gauss-Bonnet theorem—that a one-dimensional helix cannot perfectly close into a three-dimensional sphere, the theory constructs an extremely minimal geometric-topological-residual-driven unified framework. Previous works in the series have successfully derived the geometric-observer expressions for $G = \mu_0 \alpha^2$ and h, and through five residual iterations have achieved complete cancellation of μ_0 for multiple Planck-scale quantities and black-hole thermodynamic quantities, revealing a striking self-consistent closed loop.

However, one core question has persistently lingered: why precisely the "three-layer π expansion + five residual iterations"? Does this structure possess uniqueness and necessity? This paper aims to systematically address this question. We will proceed through five independent dimensions (geometric-topological necessity, dimensional self-consistency requirements, critical balance criterion, degree of agreement with experimental values, and minimality of self-consistency) in a progressive argumentation, proving that the "three-layer + five-iteration" structure is the unique minimal structure within the current framework that simultaneously satisfies all self-consistency conditions. At the same time, the paper supplements a rigorous derivation of the holographic property of space and systematically organizes falsifiable predictions, thereby providing a clear pathway for experimental verification of the theory.

The structure of this paper is as follows: Section 2 briefly reviews the theoretical framework and core concepts; Section 3 presents the multi-argument proof; Section 4 displays the constant closed-loop and μ_0-independent formula table; Section 5 demonstrates the holographic property of space; Section 6 presents specific falsifiable predictions; Section 7 discusses the relationship with the standard holographic principle and Sakharov's induced gravity; and the final section provides conclusions and outlook.

2. Theoretical Framework and Core Concepts

2.1 Foundational Axiom and Topological Residual Necessity

The sole foundational axiom of TRT is that space is intrinsically right-handed cylindrical helical motion at the speed of light, i.e., $\omega r = c$. This axiom directly inherits ideas from Zitterbewegung and helical particle models but elevates them to the dynamical nature of space itself rather than a particle property. From the Gauss-Bonnet theorem, the Euler characteristic of a sphere is $\chi = 2$; a one-dimensional helical curve cannot perfectly close into a boundary-free surface in three-dimensional space, and thus necessarily generates a topological residual. This residual constitutes the geometric origin of all fundamental forces and constants.

2.2 Three-Layer π Topological Expansion

To simultaneously accommodate the three known interactions (strong, weak, and electromagnetic) while producing a fine-structure constant in high agreement with experiment, a three-layer topological folding must be performed:

- Layer 1 (strong force / bare magnetic tube): $\Omega_1 = \pi$
- Layer 2 (weak force / spin): $\Omega_2 = \pi$

- Layer 3 (electromagnetic macroscopic boundary): $\Omega_3 = 4\pi$

The total measure is $\Omega = 4\pi + \pi + \pi \approx 137.0363$, so $\alpha = 1/\Omega \approx 1/137.0363$.

If the number of layers is not equal to 3, Ω deviates significantly from the experimental value and geometric self-consistency cannot be achieved. The three-layer structure corresponds to different topological hierarchical projections of the three interactions and simultaneously satisfies the geometric requirement of the Gauss-Bonnet theorem that equipotential surfaces collapse to S^2.

2.3 Five Residual Iterations and the Planck Constant

Starting from the maximum rigidity of the bare magnetic tube μ_0, each multiplication by α represents one topological folding and accumulation of residual. After exactly five iterations (corresponding to the α^5 in the expression for h), the system reaches the Planck limit: continuous topological termination ($\chi \approx 2$ saturation, “nearly complete”), while satisfying dimensional self-consistency in SI units (requiring the introduction of Factor_h to bridge L, I, T, M). If the number of iterations $n \neq 5$, it is impossible to reach the critical point simultaneously and achieve cancellation of μ_0.

Core formulas:

$$G = \mu_0 \alpha^2$$

$$h = \frac{\mu_0 \alpha^5 \ln(10) \cdot \text{Factor}_h}{c^2}$$

Here, Factor_h is the dimensional bridging and higher-order residual correction factor, and $\ln(10)$ represents the cognitive residual arising from the observer’s choice of the decimal base.

2.4 Geometric Definition of Mass and Spacetime Normalization

Combining the spacetime normalization equations and geometric mass definition already established in the series, mass is understood as an emergent quantity from electromagnetic-geometric residuals rather than an independent fundamental quantity. This viewpoint is in the same lineage as Sakharov’s (1967) induced gravity idea, but within the present framework it provides a quantitative realization through the explicit three-layer π topology and five iterations.

3. Multi-Argument Proof: Uniqueness and Necessity of the Three-Layer π Expansion and Five Residual Iterations

This section employs a “progressive + synthetic” mode to argue from five independent dimensions that the “three-layer + five-iteration” structure is the unique minimal self-consistent structure.

3.1 Argument One: Intrinsic Necessity of Geometric Topology

From the foundational axiom ($\omega r = c$) together with the Gauss-Bonnet theorem, a one-dimensional helix cannot perfectly close into a three-dimensional sphere ($\chi = 2$); the necessity of residual generation is beyond doubt. If the number of layers $\neq 3$, the corresponding interactions cannot be fully accommodated, and Ω cannot yield the experimental value $\alpha \approx 1/137.036$. Geometric self-consistency strictly requires the three-layer structure.

3.2 Argument Two: Dimensional Self-Consistency and the Mandatory Requirement of Residual Bridging

The dimensional requirement of Factor_h (L, I, T, M) is introduced. Only five iterations (α^5) can make h and G precisely match this dimensional requirement when $p + q = 0$, thereby achieving closure of the four SI base quantities. If the iteration number $n \neq 5$, dimensional non-closure will result or additional new mechanisms will be required, destroying minimality.

3.3 Argument Three: Critical Balance Criterion and Scaling Consistency (Definition and Origin of $p + q = 0$)

The strict mathematical origin of the $p + q = 0$ criterion is as follows: starting from the core formulas, $h \propto \mu_0^5 \alpha^5 \text{Factor}_h \ln(10) c^{-2}$ (result of five residual iterations) and $G \propto \mu_0 \alpha^2$. For an arbitrary physical quantity X, if its expression can be written in power-law form $X \propto h^p G^q c^r \alpha^s \text{Factor}_h^t \ln(10)^u$ (other geometric factors), then the total exponent of μ_0 is $p + q$. If and only if $p + q = 0$, μ_0 is precisely cancelled and the remaining expression is determined solely by pure geometric-topological residuals (specific powers of α, Factor_h, $\ln(10)$, c, and dimensionless parameters such as Q/q_p, a/M). This is precisely the mathematical fingerprint of the $n = 5$ iteration reaching critical self-consistency.

Multiple independent physical quantities simultaneously satisfy this criterion: m_p ($p = 1/2, q = -1/2$), T_H and e ($p = 1, q = -1$), S ($p = -1, q = 1$), etc. Only $n = 5$ can produce a consistent and reasonable window of α powers ($\alpha^{3/2}$ to α^3 and the dual α^{-3}); this is not a coincidence but the joint result of the iteration number and the geometric requirements of the three-layer π structure. Black-hole thermodynamic quantities and Planck-scale quantities simultaneously satisfy $p + q = 0$, profoundly indicating that electromagnetism and gravity have achieved geometric unification at the $n = 5$ level.

Summary Table of Major Formulas Independent of Vacuum Permeability μ_0

The following table summarizes the core formulas within the TRT framework that become completely independent of μ_0 (and ϵ_0) after substituting $G = \mu_0 \alpha^2$ and $h = (\mu_0 \alpha^5 \ln(10) \cdot \text{Factor}_h)/c^2$. All these formulas satisfy the mathematical criterion $p + q = 0$ (the total exponent of μ_0 in $h^p G^q$ is zero), embodying the critical balance of five residual iterations.

No.	Physical Quantity	Expression (after substitution)	Remarks
1	Planck mass m_p	$m_p = \sqrt{\frac{\alpha^3 \ln(10) \cdot \mathrm{Factor}_h}{2\pi c}}$	Core critical quantity; μ_0 completely cancelled
2	Planck charge q_p	$q_p = \sqrt{\frac{2\alpha^3 \ln(10) \cdot \mathrm{Factor}_h}{c^3}}$	Directly related to charge scale
3	Elementary charge e	$e = \sqrt{\frac{2\alpha^6 \ln(10) \cdot \mathrm{Factor}_h}{c^3}}$	Derived from $e = q_p\sqrt{\alpha}$
4	Hawking temperature (Schwarzschild black hole) T_H	$T_H = \frac{\mathrm{Factor}_h \cdot \alpha^3 \cdot c \cdot \ln(10)}{16\pi^2 M k_B}$	Black-hole thermodynamics; μ_0 cancelled
5	Bekenstein-Hawking entropy (Schwarzschild black hole) S	$S = \frac{8\pi^2 M^2 c k_B}{\mathrm{Factor}_h \cdot \alpha^3 \cdot \ln(10)}$	Structurally complementary to T_H
6	Reissner-Nordström extremal mass M_{ext}	$M_{\mathrm{ext}} = \frac{Qc}{2\sqrt{\pi\alpha}}$	Charge term μ_0 cancelled; can be rewritten as $M_{\mathrm{ext}} = (Q/q_p) \times f(\alpha, \mathrm{Factor}_h, \ln(10), c)$
7	Kerr-Newman extremal condition (after geometric-unit simplification)	$M^2 = a^2 + (\frac{Qc}{2\sqrt{\pi\alpha}})^2$	Balance among mass, spin and charge; μ_0-independent
8	Peak energy of Hawking radiation from primordial black holes E_{peak}	$E_{\mathrm{peak}} \approx 2.82 k_B T_H$ (where T_H uses the expression in row 4)	Spectral peak precisely determined by $\alpha^3/\mathrm{Factor}_h \cdot \ln(10)$
9	Any quantity X satisfying the criterion	$X \propto h^p G^q$ with $p + q = 0$	General criterion: any combination in which the powers of h and G are exactly opposite is μ_0-independent

Supplementary Notes

- l_p and t_p still retain μ_0 dependence ($l_p \propto \mu_0 \alpha^{7/2}/c^{5/2} \times \cdots$), because they are purely geometric quantities and the square root of the μ_0^2 term in the hG combination cannot be completely cancelled.
- Application of the general criterion: as long as the total power of h and G in a physical expression satisfies $p + q = 0$, the result after substitution is necessarily μ_0-independent. This explains why black-hole thermodynamics, Planck-scale quantities, and the elementary charge simultaneously exhibit this property.

- Role of residual iteration order $n = 5$: precisely because $h \propto \alpha^5$, after cancellation the remaining powers of α (e.g., $m_p \propto \alpha^{3/2}$, $T_H \propto \alpha^3$) can achieve geometric self-consistency with the three-layer π topology.

3.4 Argument Four: High Degree of Agreement with Experimental Values and Non-Accidentality

After substituting experimental values, the residual for G is only 0.261% and that for h is only 0.543%. These small residuals are natural emergent results of the geometric structure rather than parameter tuning. This stands in sharp contrast to the Standard Model, in which constants must be input independently, and underscores the "non-fine-tuned" character. The "net-effect window" of residual iteration order $n = 5$ is extremely narrow and stable; deviation from this window causes quantities such as m_p and T_H to deviate seriously from their actual measured values.

3.5 Argument Five: Comprehensive Demonstration of Minimality and Uniqueness

Synthesizing the preceding four arguments, only $n = 5$ can simultaneously satisfy geometric self-consistency (three-layer π), dimensional closure (Factor_h bridging), critical balance ($p + q = 0$), experimental agreement (residual < 1%), and minimality (no need for higher-order residuals). Any other number of layers or iterations will cause at least one condition to fail. Therefore, the "three-layer + five-iteration" structure possesses strict uniqueness and necessity.

3.6 Deep Significance Revealed by These Powers of α ($3/2$ to 3, and -3)

In the μ_0-independent formulas listed above, the powers of the fine-structure constant α mainly concentrate in the following ranges:

- Positive powers: $\alpha^{3/2}$ (Planck mass m_p), α^3 (Hawking temperature T_H, elementary charge e, etc.)
- Negative powers: α^{-3} (Bekenstein-Hawking entropy S)

These powers do not appear arbitrarily; they are the direct mathematical fingerprints of five residual iterations combined with the critical balance criterion $p + q = 0$. The following explains their profound significance from multiple levels.

1. Origin and Calculation of the Powers (Strict Traceability)

Theoretical basis:

- $h \propto \alpha^5$ (five iterations)
- $G \propto \alpha^2$

Key combinations:

- $h/G \propto \alpha^{5-2} = \alpha^3$

- $G/h \propto \alpha^{2-5} = \alpha^{-3}$
- $(h/G)^{1/2} \propto \alpha^{3/2}$ (square-root structure of Planck mass)

Therefore:

- $T_H \propto h/G \Rightarrow \alpha^3$
- $S \propto G/h \Rightarrow \alpha^{-3}$
- $m_p \propto (h/G)^{1/2} \Rightarrow \alpha^{3/2}$
- $e \propto \sqrt{\alpha} \cdot q_p \Rightarrow \alpha^3$ (effective power)

Precise duality: the α^3 of T_H and the α^{-3} of S are exactly opposite, perfectly corresponding to the conjugate relationship between temperature and entropy in thermodynamics.

2. What Do These Powers Indicate? (Core Interpretation)

(1) The "net-effect" window of residual iteration is narrow and stable

After subtracting the α^2 contribution of G from the five iterations (α^5), the net power falls into the narrow interval $3/2 \sim 3$ (and the dual -3). This shows:

- The theory already reaches sufficient balance at the fifth iteration; higher-order residuals are not required to produce scales consistent with experiment.
- If the iteration number $n \neq 5$, the net power deviates from this window, causing m_p, T_H, etc., to deviate seriously from measured values. Hence $n = 5$ is the "minimal sufficient iteration number" strictly selected by geometric self-consistency.

(2) The square-root structure reflects the "emergent" nature of mass

Only m_p exhibits the half-integer power $3/2$, because the definition of Planck mass contains a square root (geometric average of energy, spacetime, and gravity). This indicates:

- Mass is not a basic input but "emerges" by taking the square root of the critical combination of h and G.
- The half-integer power is the signature of the joint action of geometric topology (requiring square-root dimensional balance) and residual iteration.

(3) Symmetry of positive and negative powers embodies thermodynamic and topological closure

The strict duality $T_H \propto \alpha^3$ and $S \propto \alpha^{-3}$ shows that black-hole thermodynamic quantities lie within the thermodynamic closed loop of the residual network. This is fully consistent with the philosophy of the paper—"measurement is residual, residual is entropy increase"—temperature and entropy are two conjugate aspects of the same residual network.

(4) Low-order powers indicate "low-complexity criticality"

The highest power is only 3 (or -3); higher powers such as α^4 or α^5 do not appear. This shows:

- These critical quantities (Planck scale, black-hole horizon) involve only the lowest-order residual protection (five iterations already suffice).
- Higher-order residuals are strictly confined by the six-degree-of-separation network and appear only at deeper levels (or higher Russian-doll hierarchies). This explains why magnetic monopoles, deviations near the Planck limit, etc., are all well "protected."

(5) Connection with the foundational axiom and observer participation

α itself originates from the three-layer π topology ($\Omega = 4\pi + \pi^2 + \pi$). The powers $3/2 \sim 3$ can be regarded as measures of different "depths of topological folding":

- $3/2$: requires additional geometric averaging (mass emergence)
- 3: direct h/G ratio (temperature, charge)
- -3: inverse ratio of entropy

The appearance of $\ln(10)$ incorporates the observer's cognitive residual into this power structure, making the entire system a triple self-consistency of "geometry + residual + observer."

3. Summary: Theoretical Picture Revealed by These Powers

These powers from $\alpha^{3/2}$ to α^3 (and the precisely dual α^{-3}) jointly indicate:

1. Critical balance is low-order and highly efficient: five residual iterations already suffice to completely internalize the initial electromagnetic rigidity μ_0, producing scales in high agreement with experiment.
2. Mass, temperature, entropy, and charge are governed by the same residual network at the critical point; they merely exhibit the $3/2 \sim 3$ power spectrum through different geometric operations (square root versus direct ratio).
3. Thermodynamic duality and topological closure are highly consistent: the α^3/α^{-3} duality between T_H and S is the direct embodiment of residual entropy increase and observer closure on macroscopic black holes.
4. Falsifiable fingerprint: if future high-precision measurements reveal that the dependence of m_p , black-hole T_H, S, etc., on α deviates from these powers, it will challenge the five-iteration hypothesis.

In short, these powers are the "topological fingerprints" left by five residual iterations at the critical balance point. They both explain why so many different physical quantities simultaneously摆脱 dependence on μ_0 and provide concrete, falsifiable scaling behavior of the theory at the Planck scale and black-hole horizons.

4. Constant Closed Loop and μ_0-Independent Physical Quantities

By substituting $G = \mu_0 \alpha^2$ and $h = (\mu_0 \alpha^5 \ln(10) \cdot \mathrm{Factor}_h)/c^2$, all physical quantities satisfying $p + q = 0$ achieve complete cancellation of μ_0; the remaining expressions depend only on pure geometric-topological residuals (α, Factor$_h$, $\ln(10)$, c). This closed loop profoundly reveals the geometric unified origin of electromagnetism and gravity: gravity is the induced result of electromagnetic vacuum rigidity (bare magnetic tube) through three-layer π topological projection, consistent with Sakharov's induced-gravity idea, but here a clear topological realization is provided.

The "emergent" essence of mass is clearly manifested here: the remaining dimension in the m_p expression is LI (length × current); theoretically, mass M is generated by electromagnetic-geometric residual emergence rather than being an independent base quantity. Factor$_h$ serves as the residual bridge, precisely compensating the electromagnetic dimension introduced by μ_0 while simultaneously bridging mass and spacetime.

The μ_0-independent formula summary table in Section 3.3 together with the deep-meaning analysis of powers in Section 3.6 jointly constitute the core evidence set of "critical balance" in the theory: from the Planck scale to black-hole horizons and then to the elementary charge, all quantities protected by the residual network have摆脱 explicit dependence on the initial electromagnetic rigidity μ_0.

5. Holographic Property of Space

The Topology Residual Theory can naturally derive that space possesses a holographic property. This is not an additional assumption but emerges from its core mechanisms (right-handed helical axiom + residual network + statistical topological phase transition).

5.1 Residual Network as Holographic Carrier

In TRT, space is not an empty background but a dynamic medium composed of right-handed helical fluid. Information and entropy are primarily stored in residual networks: residual networks consist of mutually linked helical vortex tubes, with topological invariants (linking number Lk, writhe, etc.) encoding information. Black-hole event horizons, large-scale cosmic structures, and even microscopic particle scales can all be regarded as residual networks at different levels. This directly leads to information being primarily stored in "boundary/surface-like" structures rather than being uniformly distributed throughout the volume, thereby naturally generating the holographic property.

5.2 Rigorous Derivation of Black-Hole Holography

After substituting the black-hole entropy formula: $S = k_B N_{\text{link}}/C_0$, where N_{link} is the total linking number of the residual network. When a black hole forms, the internal helical fluid is compressed into an extreme residual network; the event horizon can be viewed as the effective boundary (holographic screen) of this network. Internal "volume" information is encoded in the topological structure of the boundary network via the linking number.

Finite entropy of Planck-mass black holes: $S_p \approx 4\pi k_B$

This indicates that at the Planck scale there exists a minimal holographic unit. Its entropy is determined by pure topological invariants rather than diverging infinitely. This provides a natural ultraviolet cutoff for the holographic principle. The replacement formula for Hawking temperature $T_H \propto Q/M$ further supports the image of the event horizon as an information-release boundary.

5.3 Cosmic Holography and General Spatial Holography

From the evolution equation of $N_{\text{res}}(t)$, the total cosmic entropy $S_{\text{total}} \propto N_{\text{res}}(t)$. The early universe (Planck era) consisted of a large number of Planck-mass extreme knots; the residual networks of these knots can be regarded as the initial "holographic screens." As statistical decoherence and structure formation proceed, residual networks continuously expand and become more complex; entropy growth mainly arises from reconstruction of the networks on "surfaces." The early-universe solution $N_{\text{res}}(t) \propto t^{-1}$ suggests that early entropy behavior was dominated by boundary-like residual networks.

At different scales, TRT exhibits holographic features: at microscopic scales, particles act as stable knots whose charge, spin, and mass are encoded by chiral residuals and linking; at macroscopic/cosmic scales, large-scale structure (cosmic web) can be viewed as the "surface" structure after residual networks have been stretched; in statistical phase-transition mechanisms, the Gauss-Bonnet theorem forces equipotential surfaces to collapse to spheres (S^2), which itself is a geometric realization of "surface dominance." General conclusion: because the linking number of residual networks encodes information and statistical decoherence tends to make effective descriptions "surface-ize," space possesses a holographic property at the level of effective description.

5.4 Observer Relativity of Statistical Decoherence and Three-Layer Topology (Unified Perspective)

Three-layer topology and statistical decoherence are not two completely separate processes but two complementary aspects of the same $n = 5$ residual-iteration network. Topology provides the geometric skeleton (three-layer π geometry, network closure, self-consistent matching, dominating charge phase transition $\alpha^{5/2} \to \alpha^3$); statistical decoherence provides dynamics and organization (random-walk residuals $\sim \sqrt{N}$, decoherence, entropy increase, dominating mass phase transition and macroscopic emergence). Factor$_h$ and $\ln(10)$ serve both simultaneously.

Observer relativity is key: from a smaller (more microscopic, more fundamental) perspective, the structure appears more three-layer topological (geometric constraints Ω, three-layer π, helical structure dominant, structural matching obvious); from a larger (more macroscopic, higher-level) perspective, it appears more statistical-decoherence-like (residual statistical averaging, $\sqrt{N}$ emergence, classicalization more pronounced). At the black-hole event horizon—an intermediate critical surface—the two reach their strongest fusion: the horizon geometry is topological (extreme balance, inner and outer horizons merged), while information/entropy S is statistical ($\alpha^{-3} \ln(10)$). In the cascade of phase transitions (charge topological phase transition → mass statistical-decoherence phase transition → black-hole horizon fusion phase transition), the two alternately dominate, but the underlying layer remains the unfolding of the same residual-iteration mechanism at different observer levels. This unifies “topological rigidity” and “statistical flexibility,” giving TRT greater explanatory power.

6. Design of Specific Falsifiable Predictions

Based on the preceding derivations, TRT has designed a series of specific, falsifiable predictions, prioritizing directions that are relatively clear, possess quantitative features, and are in principle testable by current or near-future experiments/observations.

6.1 Black-Hole Physics Direction (Most Directly Testable)

Prediction 1: Hawking temperature of Planck-mass black holes has a geometric upper limit

When black-hole mass approaches the Planck mass m_p, its Hawking temperature approaches a finite value determined by the geometric residual rate α:

$$T_{H,\text{Planck}} \approx \frac{\alpha^3 kc}{16\pi^2 m_p k_B} \quad (\text{where } k = \ln(10) \cdot \text{Factor}_h)$$

Traditional theory predicts that temperature diverges, whereas the present framework predicts the existence of a geometric finite upper limit. Test method: evaporation signals of primordial black holes (PBHs) (γ-rays, gravitational waves). If future experiments (e.g., LISA, gamma-ray telescopes) detect a cutoff or characteristic peak at the low-mass end of the PBH evaporation spectrum rather than unlimited high-temperature behavior, this prediction is supported. If black-hole temperatures near the Planck mass are observed to be significantly higher than this geometric upper limit, or if there is completely no cutoff behavior, the prediction is falsified.

Prediction 2: Black-hole entropy approaches a geometric constant at Planck mass

The Bekenstein-Hawking entropy of Planck-mass black holes approaches $S_p \approx 4\pi k_B$ (the dominant term is a pure geometric constant, almost independent of mass details). Test method: indirect constraints on the entropy/information-storage behavior of low-mass black holes via gravitational-wave observations (future LISA or next-generation detectors), or comparison with theoretical and numerical-relativity simulations.

6.2 Early-Universe and CMB Direction (Highest Potential)

Prediction 3: Linking number of early-universe residual networks decays as $1/t$

In the radiation-dominated early universe, the total linking number of residual networks evolves approximately as $N_{\text{res}}(t) \propto t^{-1}$. This leads to a specific scaling behavior of early entropy density. Test method: specific shape and amplitude of CMB non-Gaussianity (f_{NL} parameter). If future CMB observations (e.g., Simons Observatory, CMB-S4) detect non-Gaussian features consistent with t^{-1} evolution in a specific multipole range, this prediction is supported.

Prediction 4: Minimal holographic unit exists at the Planck scale

The Bekenstein-Hawking entropy of Planck-mass black holes $S_p \approx 4\pi k_B$ implies the existence at the Planck scale of a minimal holographic unit determined by pure topological invariants. Future high-energy cosmic-ray or gravitational-wave detections, if they reveal characteristic cutoffs or discrete spectra corresponding to this minimal unit, can test the prediction.

6.3 Black-Hole–Electron Hierarchical Duality Prediction (New Deepening)

Prediction 5: Information duality between black holes and electrons from multi-layer observer perspectives

Electrons and Planck-scale micro black holes (especially extreme states) are information duals of the $n = 5$ critical product in TRT: the electron is the stable endpoint after the charge phase transition (point-like, stable, non-evaporating, carrying positive α^3 power + positive $\ln(10)$ contribution); the micro black hole is the intermediate state of the horizon phase transition (possessing an event horizon, T_H can approach 0 but S is finite, carrying negative α^{-3} power + negative $\ln(10)$ contribution as a mirror image). At different observer levels the roles are interchangeable—for a more macroscopic observer, the largest black hole at cosmic-horizon scale may appear as the "basic stable charge/information excitation" of that level (electron counterpart); for a more microscopic observer, our electron may correspond to some "micro black-hole-like information-dense structure." Extreme micro black holes ($M \approx m_p$, $Q \approx q_p$ or e) are the region of strongest fusion of the two; information is preserved in purely geometric form. This provides a new perspective on the black-hole information paradox: information during micro-black-hole evaporation may be released or re-encoded in the form of "electron-like excitations." Test direction: search for characteristic peaks related to electron quantum numbers in the evaporation spectra of extreme micro black holes, or indirect constraints on Planck-scale black-hole–electron duality signals via gravitational waves or high-energy cosmic rays.

7. Discussion: Relationship with the Standard Holographic Principle and Sakharov's Induced Gravity

The holographic property derived by TRT is consistent in spirit with the standard holographic principle ('t Hooft, Susskind), but the mechanism is completely different: the standard holographic principle is often associated with AdS/CFT duality or ultraviolet cutoffs in quantum gravity, whereas TRT's holography emerges from classical geometric-topological residual networks without additional assumptions. Mechanisms such as the black-hole event horizon serving as the effective boundary of the residual network, linking-number encoding of information, and Gauss-Bonnet enforcement of surface dominance provide a geometric-topological realization of the holographic principle.

Regarding the origin of constants, $G = \mu_0 \alpha^2$ explicitly expresses the gravitational constant as the induced result of electromagnetic vacuum rigidity (bare magnetic tube) through three-layer π

topological projection, directly echoing Sakharov's 1967 "induced gravity" idea. However, within this framework a quantitative, self-consistent realization is provided through explicit topological hierarchy and residual iteration, and the geometric expression for h is naturally derived, achieving a deeper closed loop.

Electromagnetism and gravity have already achieved geometric unification at the phenomenological level (common origin, unified expression, local topological energy balance $\rho_G = \rho_E$, different topological hierarchical projections), but have not yet reached complete dynamical unification (a single Lagrangian). This points the direction for subsequent work.

8. Conclusions and Outlook

This paper has systematically demonstrated the uniqueness and necessity of the "three-layer π expansion + five residual iterations" in Topology Residual Entropy Increase Theory. From the five dimensions of geometric topology, dimensional self-consistency, critical balance, experimental agreement, and minimality, it proves that this structure is the unique minimal structure within the current framework that simultaneously satisfies all self-consistency conditions. The theory successfully achieves a self-consistent closed loop for the constants G, h, α, e, m_p, q_p, T_H, S, etc., and naturally derives the holographic property of space.

The proposal of a series of falsifiable predictions provides a clear pathway for experimental testing of the theory. Future work will include: (1) more precise numerical calculation of Factor_h and residual analysis; (2) quantitative correspondence between CMB non-Gaussianity and the evolution of $N_{\text{res}}(t)$; (3) detailed spectral predictions of primordial black-hole evaporation signals; (4) further geometric-residual explanations of the strong and weak interactions; (5) interfacing with numerical relativity and cosmological simulations.

Topology Residual Theory, with its extremely minimal foundational axiom and geometric self-consistency at its core, demonstrates powerful explanatory and predictive potential. We look forward to in-depth collaboration with experimental physicists, cosmologists, and numerical relativists to jointly test this framework.

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