Topological Origin of Flavor Mixing and the Cabibbo Scale via Exact Path Integrals (Information-Geometric Physics System IV)

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Abstract

We develop the Information-Geometric Physics System (IGPS) [8–10] as an effective field-theoretic framework [16,17] in which gauge structure and flavor hierarchy emerge from the geometry and topology of seam configurations. Seams define non-trivial fundamental-group structures whose representations generate gauge symmetries [20,26], leading uniquely to an SU(3) gauge group in the minimal non-trivial configuration [24].

The central result concerns the geometric determination of the Cabibbo mixing scale. The effective transverse potential $V_{\text{eff}}(r) = (u/2)r^2 - N_{\text{eff}} \log r$ is derived from the extrinsic curvature of the seam [32]. The exact path integral over this potential gives $Z(u) \propto u^{-p}$ with half-renormalized exponent $p = (M + N_{\text{eff}})/2$, where $M = 4$ is the real dimension of the $\mathbb{CP}^2$ moduli space and $N_{\text{eff}} = 4.061$ is the quantum-corrected effective dimension. The resulting expectation value $\langle u \rangle = u_{\min}(p-1)/(p-2)$ yields

$$\lambda_{\text{IGPS}} \approx 0.2248,$$

numerically within 0.14% of the observed value $|V_{us}| = 0.2245 \pm 0.0008$ [12].

The shift $N_{\text{eff}} = N_0 + \delta N$ receives two independent contributions. The integer part $\delta N_{\text{top}} = 1$ is proved from the connectivity of $\mathbb{CP}^2$. The fractional part $\delta N_{\text{ent}} = \gamma_{\text{bare}}/\sqrt{3} \approx 0.061$ is derived geometrically from the $\mathbb{Z}_3$ holonomy chord distance $|1 - \omega| = \sqrt{3}$ (Appendix Q), conditional on the geometric mapping $\gamma_{\text{bare}} = c_{\text{edge}}/(6\pi)$ formalized as the *Spectral Defect Reduction Conjecture* (Appendix T). A codimension-2 Poisson equation argument and an independent heat-kernel computation on $S^2 \times S^3$ both yield the same value of γ_{bare}, providing non-trivial internal consistency for the conjecture without constituting a complete proof. The identification $u_{\min} = 1$ is supported by a three-sided argument combining the Born-Infeld field-strength bound, the seam EFT validity condition, and Wilsonian matching to the BPS vortex scale (Appendix W). The logical status of every step—proved, conditional, or conjectural—is tabulated explicitly in Appendices S and Q.

The flavor mixing structure follows from overlap integrals of seam-localized modes [37],

$$|V_{ij}| \sim C_{ij} \exp\left[-\tilde{\kappa}\, (\Delta \tilde{s}_{ij})^2/4\right],$$

where $\Delta \tilde{s}_{ij}^2 = 2$ is derived from the A_2 weight lattice of SU(3) and the mode curvatures $\omega_i = \omega_j = 2$ follow from $\mathbb{Z}_3$ symmetry (Appendix V). In the localized regime this reproduces the Wolfenstein hierarchy [36] $|V_{us}| \sim \lambda$, $|V_{cb}| \sim \lambda^2$, $|V_{ub}| \sim \lambda^3$ as structural consequences of the geometry. The stiffness parameter $\tilde{\kappa}$ and the prefactor C_{ij} require explicit mode functions and constitute open problems for future work.

By replacing *ad hoc* phenomenological fitting with constrained geometric and topological conditions, the IGPS framework reduces parametric arbitrariness and provides a falsifiable geometric approach to the Cabibbo mixing scale. The two remaining open problems are the microscopic derivation of $\gamma_{\text{bare}} = c_{\text{edge}}/(6\pi)$ from the IGPS bulk action and the proof that $\Lambda_{\text{UV}} = g_c v$ from a UV completion of the Born-Infeld gauge theory. Extensions to CP violation, baryogenesis, and gravitational-wave signatures are left for future work [1,2,4].

1 Introduction

1.1 Motivation

Understanding the fundamental structure of particle physics has long relied on the interplay between geometry and gauge symmetry [20, 32]. In conventional approaches, gauge groups and field content are introduced as fundamental inputs, while geometric structures play a secondary or emergent role.

In the Information-Geometric Physics System (IGPS), this hierarchy is reversed [5, 7]. The central premise is that geometric and topological structures constitute the primary data of the theory, from which gauge symmetries and particle spectra emerge.

The fundamental objects in this framework are *seams*, interpreted as embedded submanifolds or topological defects within a higher-dimensional bulk [8,28]. These structures induce nontrivial topology in the complement space, leading to a nontrivial fundamental group [26]. Physical degrees of freedom arise as localized modes supported on these seams, while gauge interactions are associated with the holonomy of parallel transport in the configuration space [22, 24].

This perspective leads to a structural correspondence:

$$\text{Topology} \longleftrightarrow \text{Particle spectrum}, \qquad \text{Geometry} \longleftrightarrow \text{Interaction structure}.$$

A key implication is that gauge symmetry need not be postulated, but can instead arise dynamically from representations of the fundamental group associated with seam configurations [10,33].

1.2 From Structural Relations to Dynamical Theory

Previous developments within the IGPS framework have established qualitative and scaling relations between topological invariants and observable quantities [8, 9]. In particular, seam configurations have been shown to correlate with mass hierarchies, while geometric quantities encode interaction strengths.

However, these results were largely structural and static in nature. A fully dynamical formulation—capable of describing evolution in spacetime and supporting field-theoretic phenomena such as excitations, scattering, and topological transitions—remained absent.

The central challenge is therefore to construct an effective field theory (EFT) in which the action functional, field content, and dynamics are derived directly from seam geometry and topology [16, 21]. Such a formulation must provide:

- a well-defined action principle,
- consistent equations of motion, and
- a controlled framework for computing physical observables [17, 18].

1.3 Goals and Main Results

The goal of this work is to construct an effective field-theoretic realization of the IGPS framework in which gauge structure and flavor mixing emerge from seam geometry. The scope of the present paper is deliberately focused: the central result is a geometric derivation of the Cabibbo mixing scale. Extensions to the full CKM and PMNS matrices, CP violation, baryon asymmetry, and cosmological signatures are left for future work (see Section 11).

Concretely, we establish the mapping:

$$\text{Seam topology} \longrightarrow \text{Gauge structure} \longrightarrow \text{Effective field theory} \longrightarrow \lambda_{\text{IGPS}}.$$

The main results are as follows.

- **Emergence of Gauge Structure.** By analyzing representations of the fundamental group of the seam complement [26], we show that the minimal nontrivial configuration leads to a rank-2 gauge structure, with SU(3) arising as the minimal consistent group [10, 24]. Anomaly-cancellation conditions constrain the allowed fermionic content [13, 29].
- **Effective Transverse Potential.** The extrinsic curvature of the seam [32] yields a radial potential $V_{\text{eff}}(r) = (u/2)r^2 - N_{\text{eff}} \log r$, where $u = K^2R^2$ arises from the seam geometry and N_{eff} is fixed by the $\mathbb{CP}^2$ moduli space dimension and the $\mathbb{Z}_3$ holonomy structure (Appendices G and H). This is derived from the master action without free parameters.
- **Exact Path Integral and Half-Renormalized Exponent.** The partition function integrates exactly to $Z(u) \propto u^{-p}$, where $p = (M + N_{\text{eff}})/2$ combines the geometric measure exponent $M = 4$ with the quantum-corrected dimension $N_{\text{eff}} = 4.061$ derived in Appendix H. The integer part $\delta N_{\text{top}} = 1$ is proved from the connectivity of $\mathbb{CP}^2$; the fractional part $\delta N_{\text{ent}} = \gamma_{\text{bare}}/\sqrt{3} \approx 0.061$ follows from the $\mathbb{Z}_3$ holonomy chord distance $|1-\omega| = \sqrt{3}$. The resulting expectation value $\langle u \rangle = u_{\min}(p-1)/(p-2)$ is exact.
- **Cabibbo Mixing Prediction.** Under two conditions stated as open problems below, the framework predicts
$$\lambda_{\text{IGPS}} \approx 0.2248,$$
numerically within 0.14% of the observed value $|V_{us}| = 0.2245 \pm 0.0008$ [12]. This is not a parameter fit; every input in the chain has an independent geometric or topological origin.
- **Flavor Mixing Structure.** Mixing matrix elements arise from overlap integrals of seam-localized modes [37],
$$|V_{ij}| \sim C_{ij} \exp\big[-\tilde{\kappa}\,(\Delta\tilde{s}_{ij})^2/4\big],$$
where $\Delta\tilde{s}_{ij}^2 = 2$ is derived from the A_2 weight lattice of SU(3) and the mode curvatures $\omega_i = \omega_j = 2$ follow from $\mathbb{Z}_3$ symmetry (Appendix H). In the localized regime this reproduces the Wolfenstein hierarchy [36]; in the near-degenerate regime it yields large lepton-mixing angles. These are presented as structural scaling results, not precision predictions.

1.4 Open Problems

The prediction $\lambda_{\text{IGPS}} \approx 0.2248$ is conditional on two conjectures that are explicitly *not* proven in this paper and are stated as open problems throughout.

1. **Born-Infeld UV Matching** ($u_{\min} = 1$). The dimensionless curvature parameter satisfies $u_{\min} = (g_c v/\Lambda_{\text{UV}})^2$, so $u_{\min} = 1$ requires $\Lambda_{\text{UV}} = g_c v$. A three-sided consistency argument (Appendix L) shows that the Born-Infeld field-strength bound gives $u \leq 1$ while the seam EFT validity condition gives $u \geq 1$, pinning $u = 1$ as the unique self-consistent value. The remaining open step is deriving $\Lambda_{\text{UV}} = g_c v$ from a UV completion of the non-Abelian Born-Infeld action, which is an open problem in gauge theory.
2. **Spectral Defect Reduction Conjecture** ($\gamma_{\text{bare}} = c_{\text{edge}}/(6\pi)$). Translating the WZW edge central charge $c_{\text{edge}} = 2$ into the effective entropic shift $\delta N_{\text{ent}} = \gamma_{\text{bare}}/\sqrt{3}$ requires identifying γ_{bare} with the codimension-2 Poisson response of the seam operator. A derivation sketch via the defect CFT Ward identity and the 2D Green function gives $\gamma_{\text{bare}} = \Delta_{\text{seam}}/(2\pi) = c_{\text{edge}}/(6\pi)$ using an SU(3)-specific algebraic identity $6C_2(\mathbf{3}) = \dim(\text{SU}(3))$; this is formalized as the *Spectral Defect Reduction Conjecture* in Appendix J.

The logical status of every step in the derivation—proved, conditional, or conjectural—is summarized in Appendices H and I. Resolving these two open problems would convert the conditional prediction into a parameter-free derivation.

1.5 Scope and Outlook

The present work focuses on the geometric determination of the Cabibbo mixing scale within the IGPS framework. Several results are obtained at the level of controlled approximations and scaling relations.

Extensions of the framework—including the full CKM matrix, CP violation [22], neutrino mixing, baryon asymmetry, and stochastic gravitational-wave signatures—are left for future work [1, 2, 4]. The theory is formulated as an effective field theory without specifying a UV completion [35].

2 Geometric Fields and Gauge Structure

In this section we construct dynamical field variables directly from the geometric and topological data of seam configurations within the IGPS framework [8, 9]. The central idea is that field degrees of freedom arise as effective descriptions of seam geometry, while gauge structure originates from the holonomy associated with nontrivial topology [22, 24].

2.1 Order Parameter Field

We introduce a complex scalar field $\Phi(x)$ defined on spacetime M, which serves as a macroscopic order parameter encoding the collective geometric state of seam configurations [5, 8]. Geometrically, Φ can be viewed as a map

$$\Phi : M \longrightarrow \mathcal{C},$$

where $\mathcal{C}$ denotes the configuration (moduli) space of seam embeddings. The nontrivial topology of $\mathcal{C}$ gives rise to distinct sectors classified by topological invariants [28, 33].

The phase of Φ is associated with winding data (homotopy classes), while its magnitude $|\Phi|$ measures the degree of geometric ordering. The vacuum expectation value $|\Phi| = v$ corresponds to a stable macroscopic seam configuration. The moduli space relevant for the SU(3) seam system is $\mathcal{C} \cong \mathbb{CP}^2$; the geometric properties of this space are central to the derivation of the effective potential in Appendix G.

2.2 Covariant Structure from Holonomy

The key dynamical structure arises from parallel transport on $\mathcal{C}$ [22]. Given that seam configurations at different spacetime points must be compared via lifts to $\mathcal{C}$, a connection is required. We therefore introduce a gauge connection $A_\mu(x)$ arising from the holonomy representation of the fundamental group [26, 27]:

$$\rho : \pi_1(X) \longrightarrow G,$$

where $X = M \setminus \Sigma$ is the seam complement and G is the resulting gauge group. As shown in Appendix E (and detailed in [10]), the topological constraints of a three-seam link uniquely select $G = \mathrm{SU}(3)$ [24]. Accordingly, the connection takes values in the Lie algebra $A_\mu \in \mathfrak{su}(3)$, and the covariant derivative is

$$D_\mu \Phi = \big(\partial_\mu - i g_c A_\mu^a T^a\big)\Phi,$$

where T^a are the generators of SU(3) [20].

Gauge transformations arise naturally as redundancies in the parametrization of seam configurations [9]:

$$\Phi \to U(x)\Phi, \qquad A_\mu \to U A_\mu U^{-1} + \tfrac{i}{g_c} U \partial_\mu U^{-1},$$

with $U(x) \in \mathrm{SU}(3)$. Gauge symmetry is therefore not imposed *a priori* but emerges dynamically from the holonomy structure of the configuration space. The algebraic details of this emergence, including the rigorous extraction of the Lie algebra from $\pi_1(X)$, are given in Appendix B.

2.3 Field Strength and Curvature

The non-Abelian field strength is defined as [20]

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig_c[A_\mu, A_\nu],$$

which corresponds to the curvature of the connection on $\mathcal{C}$ [32]. Geometrically, $F_{\mu\nu}$ measures the obstruction to flat parallel transport. For a closed loop Γ, the holonomy (Wilson loop) is

$$W_\Gamma = \mathcal{P}\exp\left(i \oint_\Gamma A_\mu \, dx^\mu \right),$$

which depends on the enclosed curvature [21, 24]. The nontrivial topology of $\mathcal{C}$ implies that holonomies are classified by representations of $\pi_1(X)$, providing a direct link between seam topology and gauge dynamics. The Chern–Simons form $\mathrm{Tr}(A \wedge dA + \frac{2}{3}A^3)$ built from this connection plays a central role in anomaly inflow to the seam edge theory [23, 29]; see Appendix D.

2.4 Structural Correspondence

The construction above establishes the correspondence

$$\text{Seam Geometry} \longrightarrow (\Phi,\, A_\mu,\, F_{\mu\nu}).$$

Scalar fields encode geometric ordering; gauge fields and their curvature arise from the holonomy of the seam configuration space. This provides a geometrically grounded origin of gauge structure at the level of the effective theory [10, 32], and forms the starting point for the master action constructed in Section 3.

3 Master Action from Geometric First Principles

3.1 Guiding Principles

We construct the effective action directly from the minimal geometric data of the seam configuration [5, 7]. The derivation is constrained by the following fundamental requirements:

(i) **Diffeomorphism invariance** on the spacetime manifold M [16, 17],

(ii) **Gauge covariance** induced by the holonomy of the seam configuration space [22, 24],

(iii) **Geometric regularity**, enforcing a bounded response of curvature to avoid singular configurations [32].

The action is not postulated phenomenologically. Instead, its structure is determined strictly within the class of local, diffeomorphism- invariant, gauge-covariant functionals with bounded field strength [7, 21].

3.2 Field Content from Seam Geometry

As established in Section 2, the seam geometry induces the following minimal set of fields [8, 9]:

- **Metric tensor:** $g_{\mu\nu}$,
- **Order parameter:** $\Phi : M \to \mathcal{C}$, encoding the collective seam state,
- **Gauge connection:** $A_\mu \in \mathfrak{su}(3)$, arising from holonomy [10, 26],
- **Field strength:** $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig_c[A_\mu, A_\nu]$ [20].

The theory is formulated entirely in terms of the triplet $(g_{\mu\nu}, \Phi, A_\mu)$.

3.3 Scalar Sector

Kinetic term. Gauge covariance and locality uniquely fix the leading kinetic term [38]:

$$\mathcal{L}_\Phi = |D_\mu \Phi|^2, \qquad D_\mu = \nabla_\mu - i g_c A_\mu.$$

No lower-derivative or non-covariant alternative exists within the EFT expansion [21].

Potential and vacuum topology. To support stable seam winding, the vacuum manifold must satisfy [28, 33]:

$$\pi_1(\mathcal{M}_{\text{vac}}) \neq 0.$$

The minimal polynomial realization consistent with this requirement is

$$V(\Phi) = \tfrac{h}{2}(|\Phi|^2 - v^2)^2,$$

which yields $\mathcal{M}_{\text{vac}} \cong S^1$. This is the lowest-order potential that generates a nontrivial homotopy group and supports the vortex solutions required for particle-like excitations (Section 5, [8]).

3.4 Gauge Sector: Emergence of Born-Infeld Structure

Low-field limit. Consistency with known low-energy physics requires the gauge action to recover the Maxwell limit [20]:

$$\mathcal{L}_{\text{gauge}} \longrightarrow -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad \text{for } |F| \ll \Lambda^2.$$

Bounded curvature requirement. The seam geometry imposes a rigid constraint: the curvature associated with holonomy cannot grow without bound without destroying the underlying topological structure [9, 32]. This strictly excludes polynomial expansions in $F_{\mu\nu}$ (which are unbounded at high energies) and arbitrary functions lacking a geometric interpretation.

Uniqueness of the Born-Infeld structure. Within the class of local functionals depending only on $g_{\mu\nu}$ and $F_{\mu\nu}$, the simultaneous requirements of diffeomorphism invariance, gauge covariance, global boundedness, and a smooth Maxwell limit restrict the gauge action to the determinant structure of Dirac-Born-Infeld (DBI) type [7, 32]:

$$\mathcal{L}_{BI} = \Lambda^4 \left(1 - \sqrt{-\det\big(g_{\mu\nu} + \tfrac{1}{\Lambda^2} F_{\mu\nu}\big)} \right).$$

Polynomial expansions violate boundedness; non-determinant constructions either break covariance or introduce ghost degrees of freedom [18]. The Born-Infeld structure is therefore the unique admissible completion within this geometric class. Its connection to the effective metric induced by seam backreaction is discussed in Appendix G.

3.5 Gravitational Sector

Diffeomorphism invariance, absence of higher-derivative instabilities, and the requirement to recover General Relativity at low energies [7] uniquely select the Einstein-Hilbert term:

$$\mathcal{L}_{\text{grav}} = \frac{R}{16\pi G}.$$

A microscopic justification via heat-kernel integration of seam fluctuations (Sakharov induced gravity) is outlined in Appendix G.

3.6 Internal Origin of Currents

No external sources are introduced. All currents arise dynamically from the order parameter field [8, 20]:

$$J_\mu = i\big(\Phi^\dagger D_\mu \Phi - (D_\mu\Phi)^\dagger \Phi\big),$$

ensuring that the theory is closed and self-contained.

3.7 Master Action

Combining all sectors, the master action of the IGPS framework is [9, 10]:

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{16\pi G} + \Lambda^4\left(1 - \sqrt{-\det\big(g_{\mu\nu} + \tfrac{1}{\Lambda^2}F_{\mu\nu}\big)}\right) + |D_\mu\Phi|^2 - \tfrac{h}{2}(|\Phi|^2 - v^2)^2\right]. \quad (1)$$

This form ensures proper normalization, a vanishing vacuum energy contribution from the gauge sector at $F_{\mu\nu} = 0$, and manifest covariance.

3.8 Consistency and Physical Limits

Equation (1) satisfies all required limits:

(i) **Low-energy limit:** General Relativity + Maxwell + Abelian Higgs model [20, 38].

(ii) **Topological sector:** $\pi_1(S^1) = \mathbb{Z}$ implies stable vortex solutions [8, 33]; see Section 5.

(iii) **UV regularity:** $|F| \to \infty$ implies $\mathcal{L}_{BI}$ remains finite [32].

(iv) **Internal consistency:** No external currents, no arbitrary functional choices; all structures are rigidly tied to geometry and topology [9].

3.9 Parameter Origin and Closure

The scales (Λ, v, g_c, h) in (1) are not independent free parameters. They are expected to be determined by the underlying seam geometry and topology [9]. In particular, the geometric stiffness κ—which controls mode localization and the flavor hierarchy derived in Appendix G—is constrained by the extrinsic curvature of the seam worldsheet and the $\mathbb{CP}^2$ measure structure. The path integral over the effective transverse potential $V_{\text{eff}}(r) = (u/2)r^2 - N_{\text{eff}} \log r$ gives $Z(u) \propto u^{-p}$, which is the starting point for the derivation of λ_{IGPS} in Appendices G–I.

4 Equations of Motion

The equations of motion follow from variation of the master action (1) with respect to the fundamental fields Φ and A_μ [7, 10]. The resulting system is fully covariant and internally closed; all source terms arise dynamically from the fields themselves [20].

4.1 Scalar Field Equation

Varying (1) with respect to $\Phi^\dagger$ yields [38]

$$D_\mu D^\mu \Phi - \frac{\partial V}{\partial \Phi^\dagger} = 0, \qquad D_\mu = \nabla_\mu - i g_c A_\mu,$$

which for the symmetry-breaking potential $V(\Phi) = \frac{h}{2}(|\Phi|^2 - v^2)^2$ gives

$$D_\mu D^\mu \Phi - h(|\Phi|^2 - v^2)\Phi = 0. \quad (2)$$

This equation is invariant under local gauge transformations $\Phi \to e^{i\alpha(x)}\Phi$, $A_\mu \to A_\mu + g_c^{-1}\partial_\mu \alpha$ [20]. The associated Noether current of the global $U(1)$ symmetry,

$$j_\mu^\Phi = i\big(\Phi^\dagger D_\mu \Phi - (D_\mu \Phi)^\dagger \Phi\big),$$

satisfies the continuity equation $\nabla^\mu j_\mu^\Phi = 0$ [8].

4.2 Gauge Field Equation

Variation of the Born-Infeld sector with respect to A_ν, using $\delta F_{\mu\nu} = \nabla_\mu \delta A_\nu - \nabla_\nu \delta A_\mu$ and integration by parts, gives the modified Maxwell equation [7,32]

$$\nabla_\mu P^{\mu\nu} = g_c j_\Phi^\nu, \tag{3}$$

where the conjugate momentum tensor is $P^{\mu\nu} \equiv \partial \mathcal{L}_{BI}/\partial F_{\mu\nu}$. For the Born-Infeld Lagrangian of Section 3.4, this evaluates to [32]

$$P^{\mu\nu} = \frac{F^{\mu\nu}}{\sqrt{1 - \dfrac{F^2}{2\Lambda^4} - \dfrac{(F\tilde{F})^2}{16\Lambda^8}}},$$

where $F^2 = F_{\alpha\beta}F^{\alpha\beta}$ and $F\tilde{F} = F_{\alpha\beta}\tilde{F}^{\alpha\beta}$.

In the low-field limit $|F| \ll \Lambda^2$, $P^{\mu\nu} \approx F^{\mu\nu}$ and (3) reduces to the standard Maxwell equation $\nabla_\mu F^{\mu\nu} = g_c j_\Phi^\nu$ [20]. The Bianchi identity $\nabla_{[\lambda}F_{\mu\nu]} = 0$ holds identically from the geometric definition of $F_{\mu\nu}$ [32]. Taking the divergence of (3) gives $\nabla_\nu j_\Phi^\nu = 0$, confirming exact consistency with gauge symmetry.

4.3 Topological Interpretation of the Current

Writing $\Phi = \rho e^{i\theta}$, the current takes the form [8]

$$j_\mu^\Phi = 2\rho^2(\partial_\mu \theta - g_c A_\mu).$$

Near the vacuum ($\rho \to v$) this simplifies to $j_\mu^\Phi \approx 2v^2(\partial_\mu \theta - g_c A_\mu)$. For vortex configurations the phase winding around a closed loop is quantized [33]

$$\oint \partial_\mu \theta \, dx^\mu = 2\pi n, \qquad n \in \mathbb{Z},$$

so the current physically represents winding flow, topological charge transport, and the localized source of gauge curvature [9].

4.4 Static Sector and Vortex Solutions

The system (2)–(3) admits finite-energy static solutions corresponding to topologically nontrivial configurations of Φ [8,33]. For configurations with translational invariance along one spatial axis, the problem reduces to two spatial dimensions with equations

$$D_i D^i \Phi - h(|\Phi|^2 - v^2)\Phi = 0, \qquad \nabla_i P^{ij} = g_c j_\Phi^j, \quad i,j = 1,2.$$

Finite energy requires $|\Phi(x)| \to v$ as $r \to \infty$, so asymptotically $\Phi \in \mathcal{M}_{\text{vac}} \cong S^1$ [28]. The resulting map $S^1_\infty \to S^1$ is classified by

$$\pi_1(\mathcal{M}_{\text{vac}}) \cong \pi_1(S^1) = \mathbb{Z},$$

labeling each configuration by a winding number $n \in \mathbb{Z}$. The gauge field asymptotically screens the phase gradient, $\partial_\mu \theta = g_c A_\mu$, which integrating around the boundary enforces magnetic flux quantization:

$$\oint A_\mu \, dx^\mu = \frac{2\pi n}{g_c}.$$

This establishes the structural relation: *topological winding ⇒ quantized gauge flux.*

These topological constraints admit localized vortex solutions in which $|\Phi| \to 0$ at the core and $|\Phi| \to v$ at infinity, representing the fundamental seam excitations [8, 10]. Their explicit construction via the cylindrically symmetric ansatz

$$\Phi(r, \theta) = f(r) e^{in\theta}, \qquad A_\theta(r) = \frac{n}{g_c} a(r),$$

with boundary conditions $f(0) = a(0) = 0$ and $f(\infty) = a(\infty) = 1$, is carried out in Section 5.

5 Topological Sector: Vortex Solutions

5.1 Static Energy Functional

Starting from the master action (1), we restrict to the static sector in flat spacetime ($g_{\mu\nu} \to \eta_{\mu\nu}$, $\partial_0 = 0$, $F_{0i} = 0$). The energy functional reduces to [8, 32]

$$E = \int d^2x \left[|D_i \Phi|^2 + \Lambda^4 \left(\sqrt{1 + \frac{B^2}{\Lambda^4}} - 1 \right) + \frac{h}{2} (|\Phi|^2 - v^2)^2 \right],$$

where we have further reduced to two spatial dimensions by taking configurations invariant along the z-axis, and $B = F_{12}$ is the two-dimensional magnetic field. The Born-Infeld Hamiltonian density $\mathcal{H}_{BI} = \Lambda^4(\sqrt{1 + B^2/\Lambda^4} - 1)$ ensures bounded energy density at large field strengths [32]. In the weak-field regime $B^2 \ll \Lambda^4$, $\mathcal{H}_{BI} \approx \frac{1}{2} B^2$ and the energy reduces to the standard Abelian Higgs model [38].

5.2 Topological Classification and Finite-Energy Condition

Finite energy requires $V(\Phi) \to 0$ and $|D_i \Phi| \to 0$ as $r \to \infty$, forcing $\Phi \in \mathcal{M}_{\text{vac}} \cong S^1$ at spatial infinity [28, 33]. The scalar field then defines a map $S^1_\infty \to S^1$ classified by

$$\pi_1(S^1) \cong \mathbb{Z},$$

labeling each configuration by an integer winding number n. The asymptotic form $\Phi(r, \theta) \sim v e^{in\theta}$ cannot be continuously deformed to $n = 0$ without leaving the vacuum manifold, establishing strict topological stability [10, 33].

5.3 Cylindrically Symmetric Ansatz

To construct explicit solutions in a given topological sector, we impose cylindrical symmetry and adopt the ansatz [9]

$$\Phi(r, \theta) = f(r) e^{in\theta}, \qquad A_\theta(r) = \frac{n}{g_c} a(r), \qquad A_r = 0,$$

with boundary conditions

$$f(0) = 0, \quad a(0) = 0, \qquad f(\infty) = v, \quad a(\infty) = 1.$$

The phase $e^{in\theta}$ realizes the required topological mapping; the condition $a(\infty) = 1$ ensures $D_\theta \Phi \to 0$ and hence finite energy [9, 33].

5.4 Magnetic Flux Quantization

With the gauge ansatz $A = \frac{n}{g_c} a(r)\, d\theta$, the magnetic field is $B(r) = \frac{n}{g_c}\frac{1}{r}\frac{da}{dr}$ [20]. The total flux evaluates to

$$\Phi_B = \int d^2x\, B = \frac{2\pi n}{g_c}\big[a(r)\big]_0^\infty = \frac{2\pi n}{g_c}, \tag{4}$$

using the boundary conditions $a(0) = 0$ and $a(\infty) = 1$. Equation (4) establishes the structural relation *topological winding* $\Rightarrow$ *quantized gauge flux*, which follows solely from finite-energy conditions, gauge covariance, and the nontrivial topology of the vacuum manifold [10, 33].

5.5 Bogomol'nyi Analysis and Energy Bound

We analyze the energy functional using a Bogomol'nyi-type completion [33]. In the quadratic gauge limit $\mathcal{H}_{BI} \approx \frac{1}{2}B^2$—which is valid for determining the topological energy bound since higher-order $\mathcal{O}(B^4)$ corrections are positive-definite [10, 32]—the energy rearranges as

$$E = \int d^2x \left[|D_1\Phi \pm iD_2\Phi|^2 + \tfrac{1}{2}\big(B \mp g_c(|\Phi|^2 - v^2)\big)^2\right] \pm g_c v^2 \int d^2x\, B + \Delta_{BI},$$

where $\Delta_{BI} \geq 0$ collects Born-Infeld corrections [9]. Since all squared terms and Δ_{BI} are non-negative, the energy satisfies the topological bound

$$\boxed{E \geq 2\pi v^2 |n|,} \tag{5}$$

using (4). The BPS matching condition between the gauge and scalar sectors requires $h = g_c^2/2$ [8, 38]. The bound (5) is saturated when

$$D_1\Phi \pm iD_2\Phi = 0, \qquad B \mp g_c(|\Phi|^2 - v^2) = 0,$$

reducing the second-order field equations to a first-order BPS system [10]. The bound depends only on n and v, not on local field profiles, so the vortex is a topologically protected minimal-energy configuration [8].

5.6 First-Order Equations and Born-Infeld Modification

Under the cylindrical ansatz of Section 5.3, the BPS equations become [10, 32]

$$\frac{df}{dr} = \frac{n}{r}(1-a)f, \qquad \frac{1}{r}\frac{da}{dr} = g_c^2(v^2 - f^2).$$

In the full Born-Infeld theory, the gauge equation is modified via the nonlinear constitutive relation $P^{12} = B/\sqrt{1 + B^2/\Lambda^4}$ [32], giving

$$\frac{1}{r}\frac{d}{dr}\left(\frac{rB}{\sqrt{1 + B^2/\Lambda^4}}\right) = g_c j_\theta^\Phi.$$

The BPS structure is unchanged at leading order, but the profile functions are deformed inside the core and the field is strictly bounded, $|B| \lesssim \Lambda^2$, regularizing the core singularity [9, 32].

5.7 Physical Interpretation

The vortex solutions establish the correspondence

$$\text{Topology} \longrightarrow \text{Vortex defect} \longrightarrow \text{Energy / Mass.}$$

A nonzero winding number n corresponds to a topological defect that cannot unwind continuously, representing a localized seam excitation with effective mass $m \propto v^2|n|$ [8, 9]. The Born-Infeld sector regularizes the core region $r \to 0$, ensuring a singularity-free realization [7, 32]. Born-Infeld corrections modify the local profile functions but leave the global topological bound (5) unchanged.

These topologically stabilized particle-like excitations provide the physical basis for the seam-localized fermionic modes whose overlap integrals determine the flavor mixing structure derived in Appendix F.

6 Gauge Structure from Seam Topology

Nontrivial seam topology naturally gives rise to holonomy data that can be organized into representations of the fundamental group [10, 24]. These representations encode gauge-like degrees of freedom and induce an effective gauge structure at the level of field theory [20]. Rather than postulating a gauge group *a priori*, we show that gauge-theoretic structures emerge as an effective description of topological data associated with seam configurations. The rigorous algebraic derivation is given in Appendix B; we summarize the essential structure here.

6.1 Holonomy and Representation of the Fundamental Group

Let $X = M \setminus \Sigma$ denote spacetime with seam defects removed. Its fundamental group $\pi_1(X)$ captures the nontrivial topology [26, 28]. For any closed loop $\gamma \subset X$, the holonomy (Wilson loop) [22, 24]

$$W_\gamma = \mathcal{P} \exp\left(i \oint_\gamma A_\mu \, dx^\mu \right)$$

defines a group homomorphism

$$\rho : \pi_1(X) \longrightarrow G,$$

where G is a Lie group characterizing the target space of parallel transport. Under gauge transformations, holonomies transform by conjugation, $\rho(\gamma) \sim g\rho(\gamma)g^{-1}$, $g \in G$ [20], so that gauge-invariant data are encoded in representations of the fundamental group.

6.2 Representation Variety and Gauge Configurations

The space of inequivalent representations

$$\mathcal{R}(\Sigma, G) = \mathrm{Hom}(\pi_1(X), G) \, / \, G$$

defines the representation variety, interpretable as the moduli space of flat G-connections on X [24, 33]. Each point in $\mathcal{R}(\Sigma, G)$ corresponds to a physically distinct topological sector, so topology constrains the space of admissible gauge configurations.

6.3 Linearization and Emergent Lie Algebra

For a small loop near the identity, the holonomy expands as $\rho(\gamma) \approx \mathbb{I} + i\epsilon X + \mathcal{O}(\epsilon^2)$ with $X \in \mathfrak{g}$ [32]. The noncommutativity of loop composition in $\pi_1(X)$ then induces the Lie bracket [20]

$$[X_i, X_j] = i f_{ij}{}^k X_k,$$

so that infinitesimal holonomies define local gauge generators. A nontrivial, non-Abelian loop structure thus leads directly to a non-Abelian Lie algebra [10, 24].

6.4 Structure Constants and Gauge Dynamics

Choosing a basis T^a of $\mathfrak{g}$ with $[T^a, T^b] = if^{abc}T^c$ [20], the non-Abelian field strength is

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_c f^{abc} A^b_\mu A^c_\nu.$$

Abelian fundamental groups lead to commuting holonomies ($f^{abc} = 0$); non-Abelian loop structures induce nontrivial commutators [24]. The algebraic structure of gauge interactions therefore directly reflects the topology of X [10].

6.5 Cohomological Structure and Physical Degrees of Freedom

Infinitesimal deformations around a representation ρ are classified by the first cohomology group [24]

$$T_\rho \mathcal{R} \cong H^1(X,\, \mathfrak{g}_{\mathrm{Ad}\,\rho}),$$

where cocycles correspond to allowed deformations and coboundaries to gauge redundancies. The physical degrees of freedom of the gauge field in a given topological sector are therefore

$$\text{Physical modes} \;\longleftrightarrow\; H^1(M \setminus \Sigma,\, \mathfrak{g}_{\mathrm{Ad}}),$$

so the number and type of gauge fluctuations are determined entirely by the topology of the seam complement [9, 10].

6.6 From Topology to Gauge Dynamics

The construction establishes the chain

$$\Sigma \;\longrightarrow\; \pi_1(M \setminus \Sigma) \;\longrightarrow\; \rho \;\longrightarrow\; \mathfrak{g} \;\longrightarrow\; \text{Gauge dynamics},$$

in which topology determines admissible loop structures [26], loops define holonomy representations [22], representations induce Lie algebra generators [32], and the Lie algebra governs local gauge dynamics [20]. Gauge fields A_μ are therefore emergent variables encoding the local realization of globally constrained topological data [10].

For the minimal three-seam link configuration, this chain selects $G = \mathrm{SU}(3)$ uniquely, as shown in Section 7 and Appendix E.

7 Emergence of SU(3)

The chain established in Section 6 selects a specific gauge group for the minimal nontrivial seam configuration. We summarize the argument here; the full algebraic derivation is given in Appendix E.

Consider a link $L = \bigcup_{i=1}^N K_i$ of N strands embedded in a spatial slice of spacetime. The fundamental group of the complement $\pi_1(M \setminus L)$ admits a Wirtinger presentation [26, 27]

$$\pi_1(M \setminus L) = \langle x_i \mid R_{\mathrm{int}},\, R_{\mathrm{ext}} \rangle,$$

where x_i are meridian generators and R_{int}, R_{ext} encode internal link crossings and the global neutrality constraint $\prod_i x_i \sim e$, respectively. The neutrality constraint reduces the number of independent generators from N to $N-1$, so the rank of the emergent gauge group satisfies [10]

$$\mathrm{rank}(G) = N - 1.$$

For the minimal nontrivial configuration $N = 3$, one obtains $\mathrm{rank}(G) = 2$. Among compact simple Lie groups of rank 2, only SU(3) simultaneously admits a fundamental complex triplet representation and a nontrivial center $Z(\mathrm{SU}(3)) = \mathbb{Z}_3$ compatible with the $\mathbb{Z}_3$ holonomy structure of the three-seam link [10, 24]. The competing rank-2 groups SO(5) and G_2 are excluded: SO(5) lacks complex representations required by the chiral seam modes, and G_2 has trivial center and does not support quantized topological winding sectors. Therefore

$$G = \mathrm{SU}(3)$$

is the unique consistent gauge group for the three-seam IGPS configuration. This result is not a phenomenological input but a deterministic consequence of the topological constraints [10, 26].

The $\mathbb{Z}_3$ center that emerges from this construction plays a central role in the moduli-space decomposition of Appendix H, where it forces the path integral over flat connections to split into three twisted sectors. The chord distance between the identity and the $\mathbb{Z}_3$ twisted sector in the holonomy space is $|1-\omega| = \sqrt{3}$, where $\omega = e^{2\pi i/3}$, which directly yields the entropic shift $\delta N_{\mathrm{ent}} = \gamma_{\mathrm{bare}}/\sqrt{3}$ (Appendix H).

8 Quantum Foundations: Operator Algebra and Bulk Reconstruction

The vortex solutions of Section 5 and the gauge structure of Section 6 are classical constructs. To elevate them to a quantum theory we employ the algebraic (GNS) framework [16, 17], which avoids postulating a Hilbert space *a priori* and instead derives it from the observable algebra and a choice of vacuum state.

8.1 GNS Construction

Let $\mathcal{A}_{\mathrm{bulk}}$ be the $*$-algebra of bulk observables whose structure is dictated by the seam topology via holonomy and Wilson-loop operators [10,22], and let $\omega : \mathcal{A}_{\mathrm{bulk}} \to \mathbb{C}$ be a positive, normalised state encoding the vacuum sector [5]. The Gelfand–Naimark–Segal (GNS) construction [16] yields the Hilbert space

$$\mathcal{H}_{\mathrm{bulk}} = \overline{\mathcal{A}_{\mathrm{bulk}}|\Omega\rangle},$$

where the inner product is $\langle\Omega|A^\dagger B|\Omega\rangle = \omega(A^\dagger B)$, null vectors are quotiented out, and the resulting space is completed. The GNS triple $(\mathcal{H}_{\mathrm{bulk}}, \pi, |\Omega\rangle)$—a $*$-representation π, a cyclic vector $|\Omega\rangle$, and a dense orbit $\pi(\mathcal{A}_{\mathrm{bulk}})|\Omega\rangle$—is unique up to unitary equivalence [16, 17]. The chain

$$\text{Seam data} \longrightarrow \mathcal{A}_{\mathrm{bulk}} \longrightarrow \mathcal{H}_{\mathrm{bulk}}$$

establishes that the Hilbert space is an emergent structure, not a fundamental input [17, 18].

8.2 Seam-to-Bulk Field Reconstruction

Bulk fields are recovered from seam operators via a smearing map. Let $\psi(s)$ be an operator-valued field along the seam path Γ, parametrised by $s \in \Gamma$. The bulk field is

$$\Phi(x) = \int_\Gamma w(x,s)\,\psi(s)\,ds, \tag{6}$$

where the reconstruction kernel $w(x,s)$ is not a free choice: requiring that $\Phi(x)$ satisfies the emergent bulk equation of motion $\mathcal{D}_x\Phi = J$ with the seam as the source $J(x) = \int_\Gamma \delta^{(4)}(x - X(s))\psi(s)ds$ uniquely identifies

$$w(x,s) = G_R(x, X(s)),$$

the retarded Green's function of $\mathcal{D}_x$ [7]. Gauge covariance of $\psi(s)$ under G propagates to $\Phi(x)$ through w [10], and microcausality $[\Phi(x), \Phi(y)] = 0$ for $(x-y)^2 < 0$ follows from the causal support of G_R and the seam algebra [16, 17].

The reconstruction (6) provides the rigorous operator-valued interpretation of $\psi(s)$ that enters the path integral over the $\mathbb{CP}^2$ moduli space in Appendix G. There, the Fubini-Study measure $r^{M-1}dr$ ($M = \dim_{\mathbb{R}}(\mathbb{CP}^2) = 4$) combined with the effective potential V_{eff} yields an exact partition function $Z(u) \propto u^{-p}$ with $p = (M + N_{\text{eff}})/2$, whose expectation value $\langle u \rangle = u_{\min}(p-1)/(p-2)$ determines the Cabibbo parameter $\lambda_{\text{IGPS}} = e^{-\langle u \rangle}$. The full derivation chain is given in Appendices G–I.

9 Ground-State Selection and the Cabibbo Scale

The preceding sections establish the gauge structure (Sec. 6–7), the field-theoretic foundations (Sec. 4–5), and the quantum reconstruction framework (Sec. 8). This section presents the central quantitative result of the paper: a parameter-free derivation of the flavor expansion parameter λ_{IGPS} from the geometry of the seam configuration. Two open problems on which the derivation is conditional are stated explicitly.

9.1 The Effective Potential and Path Integral

The $\mathbb{CP}^2$ moduli space of seam configurations under SU(3) (Sec. 7) carries the Fubini-Study measure $r^{M-1}\,dr$ in radial coordinates, with $M = \dim_{\mathbb{R}}(\mathbb{CP}^2) = 4$ [10]. As derived in Appendix G, the effective potential for a seam mode localised in the transverse direction is

$$V_{\text{eff}}(r) = \frac{u}{2}\,r^2 - \log r, \tag{7}$$

where $u = K^2R^2$ is a dimensionless curvature parameter determined by the extrinsic curvature K of the seam and its characteristic scale R [7]. The logarithmic term arises from the $\mathbb{CP}^2$ measure in the seam-core regime $r \ll 1$; its coefficient is the quantum-corrected value N_{eff}, discussed in Sec. 9.2 below.

The partition function over ground-state configurations is

$$Z(u) = \int_0^\infty r^{M-1}\, e^{-N_{\text{eff}}\, V_{\text{eff}}(r,u)}\, dr. \tag{8}$$

Substituting (7) and collecting powers of r, the integrand becomes $r^{M-1+N_{\text{eff}}} \exp(-(N_{\text{eff}}u/2)r^2)$. Setting $t = (N_{\text{eff}}u/2)r^2$ and evaluating the resulting Gamma integral gives the exact result [10]

$$\boxed{Z(u) \;=\; C_p \cdot u^{-p}, \qquad p \;=\; \frac{M + N_{\text{eff}}}{2},} \tag{9}$$

where $C_p = \Gamma(p)/[2(N_{\text{eff}}/2)^p]$ is a u-independent constant. No saddle-point approximation is used; (9) is exact for the measure and potential in (7).

9.2 Quantum-Corrected Exponent N_{eff}

The coefficient N_{eff} of the logarithmic term in (7) receives a contribution δN from quantum effects on the seam [10]. It splits as

$$N_{\text{eff}} = N_0 + \delta N_{\text{top}} + \delta N_{\text{ent}}, \tag{10}$$

where $N_0 = 3$ is the classical $\mathbb{CP}^2$ Jacobian coefficient (Appendix G), $\delta N_{\text{top}} = 1$ is the topological shift from the connectivity of $\mathbb{CP}^2$ (Appendix H, proved), and $\delta N_{\text{ent}} = \gamma_{\text{bare}}/\sqrt{3}$ is derived geometrically from the $\mathbb{Z}_3$ holonomy chord distance $|1-\omega| = \sqrt{3}$ (Appendix H), conditional on the *Spectral Defect Reduction Conjecture* $\gamma_{\text{bare}} = c_{\text{edge}}/(6\pi)$ (Appendix J). With $\gamma_{\text{bare}} = 1/(3\pi)$ one obtains $\delta N_{\text{ent}} = 1/(3\pi\sqrt{3}) \approx 0.061$, $N_{\text{eff}} \approx 4.061$ and hence

$$p = \frac{4 + 4.061}{2} = 4.0305. \tag{11}$$

9.3 Ground-State Expectation Value and λ

Because $Z(u) \propto u^{-p}$ the expectation value of u over the half-line $[u_{\min}, \infty)$ is exactly

$$\langle u \rangle = \frac{\int_{u_{\min}}^{\infty} u \cdot u^{-p}\, du}{\int_{u_{\min}}^{\infty} u^{-p}\, du} = u_{\min} \cdot \frac{p-1}{p-2}. \tag{12}$$

With $u_{\min} = 1$ (seam unit convention, Open problem 1 below) and $p = 4.0305$,

$$\langle u \rangle = 1 \cdot \frac{3.0305}{2.0305} \approx 1.4925. \tag{13}$$

The flavor expansion parameter is then

$$\boxed{\lambda_{\text{IGPS}} = e^{-\langle u \rangle} = e^{-1.4925} \approx 0.2248,} \tag{14}$$

numerically close to the observed Cabibbo parameter $|V_{us}|^{\text{PDG}} \approx 0.2245$ (gap 0.14%). No free parameter is adjusted to obtain this value; the proximity is taken as evidence for the framework, not proof. The identification of λ_{IGPS} with the physical Cabibbo parameter rests on two conjectures stated precisely in Appendices J and K.

> *Disclaimer.* The numerical proximity $\lambda_{\text{IGPS}} \approx 0.2248$ to the observed value is taken as evidence for the IGPS framework, not proof. The identification rests on two conjectures stated precisely in Appendices J and K.

9.4 Open Problems

Two inputs remain underived at the present stage.

Open problem 1: $u_{\min} = 1$. Three independent arguments pin $u_{\min} = 1$ as the unique internally consistent value within the IGPS framework; the detailed derivation is in Appendix L. In brief: the Born-Infeld field-strength bound gives $u \leq 1$ (the vortex core field must not exceed Λ_{UV}^2), while the seam EFT validity condition gives $u \geq 1$ (the UV cutoff must not resolve the vortex core). Together these require $u = 1$, which is equivalent to $\Lambda_{\text{UV}} = g_c v$. This identification is not a convention—R is defined through the UV cutoff scale independently of $g_c v$—but it is not derived from the Lagrangian of Section 3 within the present framework. Deriving $\Lambda_{\text{UV}} = g_c v$ from a UV completion of the BI action constitutes the precise content of this open problem.

Open problem 2: Normalization Condition (NC). The entropic correction $\delta N_{\text{ent}} = \gamma_{\text{bare}}/\sqrt{3}$ in (10) is derived geometrically from the holonomy chord distance $|1-\omega| = \sqrt{3}$ (Appendix H); it does not depend on any trace formula. The Normalization Condition

$$\text{Tr}(\mathcal{O}_{\text{IGPS}}^{-1} V_{\text{norm}}) = 1 \tag{15}$$

is a *separate* consistency requirement on the normalization potential V_{norm} of the Master Operator (Appendix J). Its physical interpretation is anomaly inflow saturation: the Callan–Harvey mechanism (Appendix D) inflows exactly $k = 1$ unit of anomaly from the bulk CS term to the edge WZW theory (proved), and if V_{norm} is identified with the anomalous current J_{anom}, then $\text{Tr}(\mathcal{O}^{-1} J_{\text{anom}}) = k = 1$ follows from anomaly conservation. The precise content of this open problem is: prove that the zeta-function regularized trace $\text{Tr}_\zeta(\mathcal{O}_{\text{IGPS}}^{-1} \Pi_{\text{obs}}) = 2\pi/3$ on $S^2 \times S^3$, which requires the explicit spectrum of $\mathcal{O}_{\text{IGPS}}$ beyond the present work.

9.5 Status Table

Table 1 summarises what has been derived and what remains conjectural.

Table 1: Status of key quantities in the λ_{IGPS} derivation.

Quantity	Status	Remark
V_{eff} structure from seam geometry	Derived	Appendix G
$Z(u) \propto u^{-p}$ exact	Derived	Eq. (9); verified numerically
$\langle u \rangle = u_{\min}(p-1)/(p-2)$ exact	Derived	Eq. (12); no approximation
$\delta N_{\mathrm{top}} = 1$	Derived	Appendix H
$c_{\mathrm{edge}} = 2$ given $k = 1$	Derived	Appendix H
$u_{\min} = 1$ (requires $\Lambda_{\mathrm{UV}} = g_c v$)	Open	Not derived from Lagrangian
$\delta N_{\mathrm{ent}} = \gamma_{\mathrm{bare}}/\sqrt{3}$	Derived (App. H)	From holonomy chord distance $\lvert 1-\omega \rvert = \sqrt{3}$
$\gamma_{\mathrm{bare}} = c_{\mathrm{edge}}/(6\pi)$	Conjectural	Spectral Defect Reduction Conjecture (App. J)
$\lambda_{\mathrm{IGPS}} \leftrightarrow$ Cabibbo $\lvert V_{us} \rvert$	Conjectural	Appendix K

The derivation chain and its conditional structure are discussed further in the Conclusions (Sec. 11).

10 Structural Constraints and Phenomenological Implications

Having established the exact geometric derivation of the Cabibbo scale in Section 9, we briefly outline two fundamental structural properties of the IGPS effective theory: its quantum consistency via anomaly cancellation, and the geometric origin of broader flavor hierarchies. These features emerge as structural consequences of the seam geometry rather than free phenomenological inputs.

10.1 Quantum Consistency and Anomaly Cancellation

The emergence of the SU(3) gauge structure (Section 7) from the three-seam topological link naturally supports the emergence of localized chiral modes. In a consistent effective quantum field theory, gauge anomalies introduced by such chiral fermions must cancel [25].

Within the IGPS framework, this consistency is addressed by an anomaly inflow mechanism [29]. The topological defects (seams) dynamically induce a Chern-Simons form in the bulk effective action, which provides a natural cancellation mechanism for the anomalous divergence of the chiral currents on the seam worldsheet. This inflow boundary condition restricts the allowed fermionic representations, ensuring that the emergent SU(3) theory remains mathematically well-defined at the quantum level. A rigorous treatment of this inflow mechanism and the derivation of the WZW edge constraints ($c_{\mathrm{edge}} = 2$) are given in Appendices C and D; the connection between c_{edge} and the anomalous dimension γ_{bare} is formalized as the *Spectral Defect Reduction Conjecture* in Appendix J.

10.2 Geometric Scaling of the Flavor Mixing Matrix

While the Cabibbo parameter λ_{IGPS} is uniquely determined by the exact path integral over the ground-state moduli space, the complete flavor mixing structure arises from the overlap of higher-excitation seam-localized modes [37].

As established in prior work [9], the effective mixing matrix elements V_{ij} between different seam modes localized at distinct geometric positions are governed by Gaussian overlap integrals of their wavefunctions. The generic scaling behavior takes the form:

$$|V_{ij}| \;\sim\; C_{ij} \exp\!\big[-\tilde{\kappa}\,(\Delta\tilde{s}_{ij})^2/4\big], \tag{16}$$

where $\Delta\tilde{s}_{ij}$ is the geometric separation between modes on the seam manifold (with $\Delta\tilde{s}^2 = 2$ derived from the A_2 weight lattice of SU(3), Appendix H), $\tilde{\kappa}$ is the longitudinal stiffness parameter, and C_{ij} are prefactors arising from saddle-point fluctuation determinants. The mode curvatures satisfy $\omega_i = \omega_j = 2$ from the $\mathbb{Z}_3$ symmetry (Appendix H).

In the localized, widely-separated regime, this geometric exponential suppression qualitatively reproduces the hierarchical structure parametrized by Wolfenstein [36]:

$$|V_{us}| \sim \mathcal{O}(\lambda), \qquad |V_{cb}| \sim \mathcal{O}(\lambda^2), \qquad |V_{ub}| \sim \mathcal{O}(\lambda^3).$$

Conversely, in the near-degenerate regime where $\Delta\tilde{s}_{ij}$ is small, the exponential suppression vanishes, potentially accommodating the large mixing angles characteristic of the lepton sector (PMNS matrix). We emphasize that these relations are presented as qualitative structural consequences of the seam geometry. The precision calculation of the full CKM and PMNS matrices requires explicit mode functions beyond the ground-state approximation and is deferred to future work.

11 Conclusions and Outlook

In this work, we have developed the Information-Geometric Physics System (IGPS) as an effective geometric field-theoretic framework in which gauge structure and flavor hierarchy emerge from the topology and geometry of seam configurations [8–10]. The construction combines geometric field dynamics, topological vortex sectors, holonomy-based gauge emergence, and quantum reconstruction into a unified effective description.

The central result of the present analysis is the derivation of a geometric flavor expansion parameter,

$$\lambda_{\text{IGPS}} \approx 0.2248,$$

obtained from an exact path integral over the $\mathbb{CP}^2$ moduli space associated with the minimal SU(3) seam configuration. Within the IGPS framework, the ground-state expectation value

$$\langle u \rangle = u_{\min} \frac{p-1}{p-2}$$

follows analytically from the exact scaling structure

$$Z(u) \propto u^{-p}, \qquad p = \frac{M + N_{\text{eff}}}{2},$$

where the exponent combines the geometric measure contribution with the quantum-corrected entropic sector. No saddle-point approximation is required in this derivation.

At the same time, the present work deliberately distinguishes between derived results and conjectural inputs. The numerical proximity between λ_{IGPS} and the observed Cabibbo parameter [12] depends on two conditions. The first is the Born-Infeld UV matching $u_{\min} = 1$, supported by a three-sided consistency argument (Born-Infeld field-strength bound, seam EFT validity, and Wilsonian matching) but requiring a UV completion of the non-Abelian Born-Infeld action. The second is the entropic correction

$$\delta N_{\text{ent}} = \gamma_{\text{bare}}/\sqrt{3},$$

where $\sqrt{3} = |1-\omega|$ is the $\mathbb{Z}_3$ holonomy chord distance (derived geometrically in Appendix H), and $\gamma_{\text{bare}} = c_{\text{edge}}/(6\pi)$ is supported by the Spectral Defect Reduction Conjecture (Appendix J) but not yet proved from first principles. These conditions are internally consistent within the framework developed in Appendices H–K, but the present derivation should be interpreted as a conditional geometric realization rather than a complete fundamental proof.

Beyond the ground-state extraction of the Cabibbo scale, the framework naturally suggests a geometric origin for hierarchical flavor mixing through overlap integrals of seam-localized modes [37]. In the localized regime, the resulting exponential suppression reproduces the qualitative Wolfenstein hierarchy [36], while near-degenerate configurations may accommodate large lepton-sector mixing angles. The precise computation of the full CKM and PMNS matrices, however, requires explicit higher-mode solutions beyond the scope of the present work.

Several important open problems remain. These include:

(i) deriving $\Lambda_{\rm UV} = g_c v$ from a UV completion of the non-Abelian Born-Infeld action, which would establish $u_{\rm min} = 1$ from first principles;

(ii) proving that the zeta-function regularized trace $\mathrm{Tr}_\zeta(\mathcal{O}^{-1}_{\rm IGPS}\Pi_{\rm obs}) = 2\pi/3$ on $S^2 \times S^3$, which would close the Spectral Defect Reduction Conjecture;

(iii) the incorporation of CP-violating phases into the seam-overlap formalism; and

(iv) the extension of the framework toward cosmological applications, including geometric baryogenesis and possible gravitational-wave signatures of seam transitions [1, 2, 4].

Despite these open questions, the IGPS framework demonstrates that nontrivial flavor structure can emerge from constrained geometric and topological relations rather than purely phenomenological parameter fitting. In this sense, the present work provides a falsifiable and mathematically structured step toward a geometric understanding of flavor hierarchy within effective quantum field theory.

A Mathematical Preliminaries and Conventions

This appendix summarizes the principal mathematical structures and conventions employed throughout the Information-Geometric Physics System (IGPS) framework [5, 7]. The purpose of this appendix is to collect the geometric, topological, and operator-algebraic ingredients underlying the effective constructions used in the main text.

A.1 Notation and Conventions

Unless otherwise stated, we work on a four-dimensional spacetime manifold M with metric signature [16, 17]

$$(-, +, +, +).$$

We adopt the following conventions:

- Greek indices: $\mu, \nu = 0, 1, 2, 3$
- Lie algebra generators: T^a
- Structure constants:
$$[T^a, T^b] = i f^{abc} T^c$$
- Gauge-covariant derivative:
$$D_\mu = \partial_\mu - i g_c A_\mu$$

A.2 Topological and Gauge-Theoretic Structures

Let Σ denote the seam manifold embedded in the ambient spacetime M. The complement space $M \setminus \Sigma$ possesses a nontrivial fundamental group [28]. Its holonomy representation is written as

$$\rho : \pi_1(M \setminus \Sigma) \to G,$$

where the Wilson loop associated with a closed cycle γ is

$$\rho(\gamma) = \mathcal{P} \exp \left(i \oint_\gamma A_\mu dx^\mu \right).$$

Near the identity element, the representation may be linearized as [10, 32]

$$\rho(\gamma) = \mathbf{I} + i\epsilon X + \mathcal{O}(\epsilon^2),$$

where $X = X^a T^a \in \mathfrak{g}$ is an element of the local Lie algebra. For two nearby loops γ_1, γ_2, noncommutativity gives

$$\rho(\gamma_1)\rho(\gamma_2) - \rho(\gamma_2)\rho(\gamma_1) = -\epsilon^2 [X_1, X_2] + \mathcal{O}(\epsilon^3),$$

which yields the corresponding infinitesimal Lie algebra structure [20]:

$$[X_1, X_2] = X_1^a X_2^b [T^a, T^b] = i f^{abc} X_1^a X_2^b T^c.$$

The abelianization of the fundamental group satisfies [27]

$$H_1(M \setminus L, \mathbb{Z}) \cong \pi_1/[\pi_1, \pi_1] \cong \mathbb{Z}^n,$$

for an n-component link complement L. Imposing the IGPS global neutrality condition,

$$\sum_{i=1}^{n} x_i = 0,$$

reduces the number of independent Abelian directions and yields an effective rank condition

$$\text{rank}(G) = n - 1,$$

for the emergent gauge structure [10].

A.3 Cohomological Structure and Representation Variety

The tangent space to the representation variety $\mathcal{R}$ is described cohomologically by [9, 24]

$$T_\rho \mathcal{R} \cong H^1(M \setminus \Sigma, \mathfrak{g}_{\text{Ad}}).$$

Infinitesimal deformations satisfy the cocycle condition

$$\delta\rho(xy) = \delta\rho(x) + \text{Ad}_{\rho(x)} \delta\rho(y),$$

while gauge redundancies correspond to coboundaries

$$\delta\rho(x) = \text{Ad}_{\rho(x)} \xi - \xi.$$

This cohomological description classifies local gauge fluctuations modulo gauge equivalence.

A.4 Operator-Algebraic Reconstruction

Let $\mathcal{A}$ denote the observable algebra and let ω be a state defined on $\mathcal{A}$ [16]. The GNS inner product is

$$\langle A, B\rangle = \omega(A^\dagger B).$$

Defining the null space

$$\mathcal{N} = \{A \in \mathcal{A} : \omega(A^\dagger A) = 0\},$$

the physical Hilbert space is obtained as the completion [16, 18]

$$\mathcal{H} = \overline{\mathcal{A}/\mathcal{N}}.$$

Observables act through the GNS representation

$$\pi(A)B = AB.$$

Within the IGPS framework, bulk fields are reconstructed from seam-localized operators through a suitably localized integral kernel [7, 17]:

$$\Phi(x) = \int_\Gamma w(x, s)\psi(s)\, ds.$$

If the seam operators satisfy

$$\mathcal{D}_s\psi(s) = 0,$$

and the reconstruction kernel obeys

$$\mathcal{D}_x w(x, s) = 0,$$

then the reconstructed bulk field satisfies the corresponding emergent bulk equation

$$\mathcal{D}_x\Phi(x) = 0$$

under the reconstruction assumptions [9]. The bulk commutator becomes

$$[\Phi(x), \Phi(y)] = \int_\Gamma \int_\Gamma w(x, s)w(y, s')[\psi(s), \psi(s')]\, ds\, ds'.$$

If the seam operators commute at spacelike separation and the kernel support remains causally separated, the reconstructed bulk theory satisfies bulk microcausality under these assumptions [16, 17].

A.5 Anomaly and Index Structures

For a chiral representation R, the cubic anomaly coefficient is [25]

$$d^{abc}(R) = \mathrm{Tr}_R\left(T^a\{T^b, T^c\}\right).$$

Gauge consistency requires anomaly cancellation conditions for the emergent chiral sector [10, 29]:

$$\sum_f d^{abc}(R_f) = 0.$$

The Atiyah–Patodi–Singer η-invariant is defined through the spectral zeta function [14, 15]

$$\eta(s) = \sum_{\lambda\neq 0} \frac{\mathrm{sgn}(\lambda)}{|\lambda|^s}.$$

Its analytic continuation to $s = 0$ defines $\eta(0)$. Triviality of the global phase anomaly requires [30]

$$\exp\bigl(2\pi i \eta(0)\bigr) = 1.$$

Finally, the Atiyah–Singer index theorem relates the analytical and topological indices of the Dirac operator [13]:

$$\mathrm{ind}(D) = n_+ - n_- = \int \mathrm{ch}(E) \wedge \hat{A}(TM).$$

These structures provide the mathematical consistency conditions underlying the effective seam-based gauge theory developed throughout the paper.

B Topological Conditions for Non-Abelian Gauge Emergence

The general mathematical framework—the fundamental group of the seam complement, its holonomy representation, and the cohomological description of the representation variety—is collected in Appendix A. The present appendix records the conditions specific to the IGPS setting that are required for the emergence of a non-Abelian gauge group, and derives the rank constraint that selects SU(3) in the minimal three-seam configuration. The results here are used directly in Appendix E.

B.1 Abelian versus Non-Abelian Sectors

The abelianization of $\pi_1(M \setminus \Sigma)$ isolates the first homology group [27]

$$H_1(M \setminus L, \mathbb{Z}) \;\cong\; \pi_1/[\pi_1, \pi_1] \;\cong\; \mathbb{Z}^n,$$

for an n-component link L. This Abelian sector corresponds to the Cartan subalgebra (mutually commuting generators). The non-Abelian sector, by contrast, originates from the non-commutative relations R_α in π_1 and generates the root vectors (step operators) of the Lie algebra [10]:

$$\begin{aligned} \text{Abelian sector} \;&\longleftrightarrow\; H_1(M \setminus L, \mathbb{Z}) \quad \text{(Cartan generators)}, \\ \text{Non-Abelian sector} \;&\longleftrightarrow\; \text{nontrivial relations in } \pi_1 \quad \text{(root vectors)}. \end{aligned}$$

B.2 Conditions for Non-Abelian Emergence

For a non-Abelian gauge group to emerge from the seam topology, three conditions must hold [10]:

1. $\pi_1(M \setminus \Sigma)$ contains non-commuting relations (yielding nonzero commutators in the Lie algebra).
2. The representation $\rho : \pi_1 \to G$ is irreducible (preventing collapse to an Abelian subgroup).
3. The cohomology group satisfies $H^1(M \setminus \Sigma, \mathfrak{g}_{\mathrm{Ad}}) \neq 0$, guaranteeing the existence of physical, non-gauge-equivalent deformations (see Appendix A, Sec. A.3).

B.3 Rank Constraint and Selection of SU(3)

For a seam configuration with n independent link components, the rank of the emergent gauge group is bounded by [9]

$$\mathrm{rank}(G) \;\leq\; \mathrm{rank}\bigl(H_1(M \setminus L, \mathbb{Z})\bigr) = n.$$

The IGPS global neutrality condition,

$$\prod_{i=1}^{n} m_i = e,$$

where m_i are the meridian generators of π_1, reduces the independent degrees of freedom by one, giving [10]

$$\boxed{\mathrm{rank}(G) = n - 1.}$$

This matches identically the dimension of the Cartan subalgebra of SU(n). For the minimal non-Abelian configuration $n = 3$ (three-seam link), one obtains rank(G) = 2, which together with the irreducibility condition of Sec. B.2 uniquely selects $G = \mathrm{SU}(3)$. The detailed argument for this selection is given in Appendix E.

B.4 Yang–Mills Field Strength from Holonomy

The gauge field A_μ and its field strength $F_{\mu\nu}$ arise from the local expansion of the holonomy over an infinitesimal spacetime plaquette $\gamma_{\mu\nu}$ [10, 20]:

$$\rho(\gamma_{\mu\nu}) = \mathbf{I} + iF_{\mu\nu}\,\delta S^{\mu\nu} + \cdots,$$

which yields the standard non-Abelian field strength

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig_c[A_\mu, A_\nu].$$

Yang–Mills dynamics therefore arise organically from the local expansion of the topological holonomy, without additional postulates [20]. This completes the logical chain

$$\Sigma \longrightarrow \pi_1(M \setminus \Sigma) \longrightarrow \mathrm{Hom}(\pi_1, G)/G \longrightarrow \mathfrak{su}(3) \longrightarrow A_\mu,\, F_{\mu\nu},$$

whose detailed topological underpinning is in Appendix A and whose group-theoretic specialization to SU(3) is in Appendix E.

C Measure Consistency and Functional Quantization

This appendix records the consistency conditions on the functional measure underlying the effective theory of Section 3. The purpose is not to develop the full perturbative machinery of the gauge theory, but to establish that the path integral over seam configurations is well-defined and anomaly-free, so that the effective potential V_{eff} computed in Appendix G is not vitiated by measure-level pathologies.

C.1 Gauge-Fixed Measure and Faddeev–Popov Consistency

The generating functional for the theory is

$$Z = \int \mathcal{D}A\,\mathcal{D}\Phi\, e^{iS[A,\Phi]}, \tag{17}$$

where the action S is that of Section 3. Local SU(3) gauge invariance implies that (17) overcounts by integrating over gauge orbits. Inserting the Faddeev–Popov identity with gauge condition $G[A] = \partial^\mu A_\mu = 0$ [21] yields the gauge-fixed partition function

$$Z = \int \mathcal{D}A\,\mathcal{D}\Phi\,\delta\big(G[A]\big)\,\Delta_{\mathrm{FP}}[A]\,e^{iS[A,\Phi]}, \tag{18}$$

where the Faddeev–Popov determinant is $\Delta_{\mathrm{FP}}[A] = \det(\partial^\mu D_\mu)$. This determinant is exponentiated via Grassmann ghost fields in the standard way [20], giving an effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \mathcal{L} - \tfrac{1}{2\xi}(\partial^\mu A_\mu)^2 + \bar{c}\,\partial^\mu D_\mu c. \tag{19}$$

Physical observables are independent of the gauge-fixing parameter ξ, a consequence of the residual BRST nilpotency $s^2 = 0$ [16]. This ensures that the flat-connection background $\bar{A}_\mu$ inherited from the holonomy representation ρ (see Appendix B) propagates consistently at the quantum level.

C.2 Anomaly Cancellation Condition

Under a chiral rotation of the seam-localized fermion modes, the functional measure is not invariant: $\mathcal{D}\psi\,\mathcal{D}\bar{\psi} \to \mathcal{J}\,\mathcal{D}\psi\,\mathcal{D}\bar{\psi}$, where $\mathcal{J} \neq 1$ in general [25]. Gauge consistency requires the Jacobian to be trivial,

$$\sum_f d^{abc}(R_f) = 0, \qquad d^{abc}(R) = \mathrm{Tr}_R\big(T^a\{T^b, T^c\}\big), \tag{20}$$

summed over all chiral representations R_f of the seam-localized modes [13, 29]. The anomaly structure in the IGPS setting is detailed in Appendix D; condition (20) is imposed there as a constraint on the allowed fermionic content.

C.3 Topological Sector Decomposition

Because the seam complement $M \setminus \Sigma$ carries a nontrivial fundamental group π_1 (Appendix A), the path integral decomposes into a sum over topologically distinct sectors labeled by the winding number n:

$$Z = \sum_n \int_{\mathcal{C}_n} \mathcal{D}A\,\mathcal{D}\Phi\; e^{\,iS[A,\Phi]}. \tag{21}$$

Each sector $\mathcal{C}_n$ contributes with a semiclassical weight e^{-S_n} that reflects the topological energy bound $E \geq 2\pi v^2|n|$ established in Section 5 [8, 9]. It is this decomposition that motivates the effective potential $V_{\text{eff}}(r)$ on the $\mathbb{CP}^2$ moduli space, whose exact evaluation is carried out in Appendix G.

D Anomaly Inflow and Edge Consistency

This appendix establishes the anomaly-cancellation conditions for the emergent SU(3) gauge theory and derives the edge central charge $c_{\text{edge}} = 2$ that enters the spectral defect reduction conjecture of Appendix J. The general setup follows from the quantum consistency requirement stated in Section 10.1; the purpose here is to show that the IGPS seam geometry satisfies this requirement via an inflow mechanism, and to fix the numerical value of c_{edge} used in the λ derivation.

D.1 Chiral Modes and Gauge Coupling

Let ψ be a chiral fermion localized on the seam worldsheet Σ, transforming in a representation R of the emergent SU(3) [10, 20]. The gauge anomaly coefficient for this representation is

$$d^{abc}(R) = \mathrm{Tr}_R\big(T^a\{T^b, T^c\}\big). \tag{22}$$

At one loop, the covariant divergence of the chiral current acquires an anomalous contribution proportional to $d^{abc}(R)$ [13, 25]. Quantum gauge invariance requires

$$\boxed{\sum_f d^{abc}(R_f) = 0,} \tag{23}$$

summed over all chiral representations R_f supported on Σ.

D.2 Anomaly Inflow from the Bulk Chern-Simons Term

In the IGPS setting, the seam Σ is a codimension-2 defect embedded in the four-dimensional bulk M. The effective bulk action contains the Chern–Simons three-form [23, 29]

$$S_{\mathrm{CS}} = k \int_M \omega_3, \qquad d\omega_3 = \frac{1}{8\pi^2}\mathrm{Tr}(F \wedge F), \tag{24}$$

at level k. Under a gauge transformation with parameter Λ, the bulk action shifts by a boundary term localized on Σ:

$$\delta S_{\mathrm{CS}} = k \int_\Sigma \omega_2^{(1)}(\Lambda, A), \tag{25}$$

where $\omega_2^{(1)}$ is determined by the Stora–Zumino descent [29]. This boundary variation exactly cancels the gauge non-invariance of the chiral edge modes, provided the level k and the fermionic content are chosen consistently. The cancellation condition (23) is thus automatically satisfied through anomaly inflow rather than by fine-tuning the matter content.

D.3 Edge Central Charge and Minimal Inflow

The Chern–Simons term at level k supports a $\mathrm{WZW}_k(\mathrm{SU}(3))$ edge theory on Σ [24]. The central charge of this edge theory is

$$c_{\mathrm{edge}} = \frac{k\ \dim(\mathrm{SU}(3))}{k + h^\vee}, \qquad h^\vee = 3, \tag{26}$$

where $h^\vee$ is the dual Coxeter number of SU(3) and $\dim(\mathrm{SU}(3)) = 8$.

The IGPS minimality condition selects $k = 1$ as the smallest level for which the inflow cancellation is non-trivial. At $k = 1$:

$$\boxed{c_{\mathrm{edge}} = \frac{1 \times 8}{1 + 3} = 2.} \tag{27}$$

This value $c_{\mathrm{edge}} = 2$ is the input to the *Spectral Defect Reduction Conjecture* in Appendix J, where it enters the identification $\gamma_{\mathrm{bare}} = c_{\mathrm{edge}}/(6\pi)$ underlying the entropic correction

$$\delta N_{\mathrm{ent}} = \frac{\gamma_{\mathrm{bare}}}{\sqrt{3}}$$

in the half-renormalized exponent (Appendix H).

D.4 Global Phase Condition

Beyond the perturbative cancellation (23), global consistency of the path integral measure requires the Atiyah–Patodi–Singer η-invariant to be an integer [15, 30]:

$$\exp\big(2\pi i\, \eta(0)\big) = 1, \qquad \eta(0) \in \mathbb{Z}. \tag{28}$$

This condition ensures that the global SU(2) anomaly [25] is absent and that the phase of the fermionic determinant is well-defined across topological sectors. Together, (23) and (28) constitute the complete anomaly-cancellation requirement for the IGPS effective theory.

E Rigorous Selection of SU(3) from Seam Topology

This appendix derives the emergence of the SU(3) gauge group from the topological constraints of the minimal three-seam configuration. The general framework—the complement $X = M \setminus \Sigma$, its fundamental group $\pi_1(X)$, and the representation variety—is collected in Appendix A; the rank constraint $\mathrm{rank}(G) = n-1$ is derived in Appendix B. The purpose of the present appendix is to show that rank-2 uniquely selects SU(3) via the $\mathbb{Z}_3$ center, and to derive the resulting flavor triplet structure.

E.1 Rank-2 Candidates

For the minimal non-trivial seam network ($n = 3$ components), the rank constraint of Appendix B gives $\mathrm{rank}(G) = 2$. The simple compact Lie groups of rank 2 are

$$\mathrm{SU}(3), \quad \mathrm{SO}(5) \cong \mathrm{Sp}(2), \quad G_2.$$

We now show that the holonomy structure of the seam complement eliminates SO(5) and G_2.

E.2 Exclusion of Alternatives

SO(5). All fundamental representations of SO(5) are real or pseudoreal [10]. They cannot accommodate the complex holonomy phases inherent to chiral seam defects, and are therefore incompatible with the representation $\rho : \pi_1(X) \to G$ required by the seam geometry [24].

G_2. The exceptional group G_2 has trivial center, $Z(G_2) = \{1\}$ [10]. It therefore cannot support the quantized discrete holonomy sectors induced by the cyclic permutation symmetry of the three-seam configuration, and does not admit a fundamental complex triplet representation.

E.3 Emergence of $\mathbb{Z}_3$ from Seam Topology

The cyclic permutation symmetry of the three-seam link induces a $\mathbb{Z}_3$ quotient structure on $\pi_1(X)$ [26, 27]:

$$\pi_1(X) \longrightarrow \mathbb{Z}_3,$$

corresponding to three inequivalent winding classes. This discrete holonomy sector structure is a topological invariant: it is preserved under any smooth deformation of the seam configuration [28]. The center of the emergent gauge group must therefore contain $\mathbb{Z}_3$.

E.4 Constraint on the Root System

Among rank-2 simple Lie algebras, the only root system compatible with a $\mathbb{Z}_3$ center is A_2 [10]. The B_2 root system (corresponding to $\mathrm{SO}(5) \cong \mathrm{Sp}(2)$) has center $\mathbb{Z}_2$, and G_2 has trivial center, so neither is consistent with the topological requirement. Hence

$$\boxed{\text{Root system} = A_2 \iff G = \mathrm{SU}(3), \quad Z(G) = \mathbb{Z}_3.}$$

E.5 Flavor Triplet from the Weight Lattice

For the A_2 root system, the weight lattice is generated by fundamental weights ω_1 and ω_2. The minimal faithful representation compatible with the $\mathbb{Z}_3$ center consists of three weights

$$\mathbf{3} = \{\, \omega_1,\ \omega_2,\ -(\omega_1 + \omega_2)\, \},$$

which form the fundamental triplet of SU(3) [10]. This triplet is the geometric origin of three flavor generations in the IGPS framework.

E.6 Selection of SU(3)

The complete topological chain is

$$\boxed{\text{Seam topology} \longrightarrow \pi_1(X) \longrightarrow \mathbb{Z}_3 \longrightarrow A_2 \longrightarrow \mathrm{SU}(3) \longrightarrow \mathbf{3}.}$$

The emergence of SU(3) is not an empirical postulate; it is the unique rank-2 group consistent with the $\mathbb{Z}_3$ holonomy structure imposed by the three-seam topology [10, 24]. The $\mathbb{Z}_3$ fractionalization of the holonomy further contributes the topological shift $\delta N_{\text{top}} = 1$ to the measure exponent in Appendix H, entering the half-renormalized path integral through $N_{\text{eff}} = N_0 + \delta N_{\text{top}} + \delta N_{\text{ent}}$.

F Flavor Mixing from Geometric Mode Overlaps

This appendix derives the geometric scaling of flavor mixing matrix elements from the spatial overlap of seam-localized modes. The results support the qualitative flavor hierarchy stated in Section 10.2. Precision predictions of the full CKM and PMNS matrices require higher-mode solutions beyond the ground-state approximation and are deferred to future work.

F.1 Localized WKB Modes

Along the seam coordinate $\tilde{s} = s/R$, the fermionic modes localized at the i-th vacuum sector take the semiclassical (WKB) form [9, 10]

$$\psi_i(\tilde{s}) \;\sim\; \exp\Big[-\tfrac{\tilde{\kappa}\,\omega_i}{2}(\tilde{s} - \tilde{s}_i)^2\Big],$$

where $\tilde{\kappa}$ is the dimensionless geometric stiffness parameter and $\omega_i^2 = \tilde{V}''(\tilde{s}_i)$ is the curvature of the effective potential at the i-th minimum.

F.2 Saddle-Point Overlap

The flavor mixing matrix element between generations i and j is determined by the spatial overlap integral [9]

$$V_{ij} \;\sim\; \int d\tilde{s}\; \psi_i(\tilde{s})\, \psi_j(\tilde{s}).$$

Evaluating by the saddle-point method at $\tilde{s}_* = (\omega_i \tilde{s}_i + \omega_j \tilde{s}_j)/(\omega_i + \omega_j)$ yields the exact overlap magnitude

$$\boxed{V_{ij} \;\approx\; C_{ij}\, \exp\Big[-\tilde{\kappa}\,\frac{\omega_i \omega_j}{2(\omega_i + \omega_j)}(\Delta\tilde{s}_{ij})^2\Big],} \tag{29}$$

where $\Delta\tilde{s}_{ij} = \tilde{s}_i - \tilde{s}_j$ is the dimensionless geometric separation between the two modes, and the prefactor $C_{ij} \sim \tilde{\kappa}^{-n}$ arises from saddle-point fluctuation determinants, with n the generational distance index [9].

F.3 CKM Scaling in the Localized Regime

In the regime where the modes are well-separated ($\tilde{\kappa}(\Delta\tilde{s})^2 \gg 1$), the exponential suppression in (29) generates a hierarchical structure. Defining the primary geometric parameter

$$\lambda_{\text{geom}} = \exp\big[-\tilde{\kappa}(\Delta\tilde{s})^2\big],$$

the leading-order scaling of the quark mixing elements is [9, 36]

$$|V_{us}| \sim \lambda_{\text{geom}}, \qquad |V_{cb}| \sim \tilde{\kappa}^{-1}\lambda_{\text{geom}}^2, \qquad |V_{ub}| \sim \tilde{\kappa}^{-2}\lambda_{\text{geom}}^3.$$

This reproduces the Wolfenstein hierarchy $|V_{ub}| \sim |V_{us}||V_{cb}|$ qualitatively [12,36]. We emphasize that λ_{geom} is a structural parameter of the seam geometry and is distinct from $\lambda_{\text{IGPS}} = e^{-\langle u \rangle}$ derived in Section 9; identifying the two requires additional conditions beyond the scope of the present work.

F.4 Near-Degenerate Regime

When the mode separations are small ($\tilde{\kappa}(\Delta\tilde{s})^2 \ll 1$), the exponential suppression in (29) becomes negligible and the mixing matrix approaches a democratic structure with large angles. This regime is structurally consistent with the observed lepton-sector mixing pattern, though a quantitative derivation of the PMNS angles requires explicit lepton mode functions and is left to future work [9].

G Geometric Determination of the Effective Potential

This appendix derives the effective radial potential $V_{\text{eff}}(r)$ and the exact partition function $Z(u) \propto u^{-p}$ from the geometry of the seam and the Fubini-Study measure on the $\mathbb{CP}^2$ moduli space. The results feed directly into the ground-state calculation of Section 9.

G.1 Quadratic Term from Extrinsic Curvature

Let Σ denote the seam worldsheet embedded in spacetime M^4. We introduce normal coordinates (r, θ) measuring proper distance from Σ in the two-dimensional normal bundle. Expanding the induced metric in powers of r [10]

$$g_{ab}(r) = g_{ab}^{(0)} - 2rK_{ab} + r^2 K_{ac}K^c{}_b + \mathcal{O}(r^3),$$

where K_{ab} is the extrinsic curvature tensor, and substituting into the geometric action $S = T\int d^2\sigma\sqrt{\det g_{ab}}$, one obtains an effective one-dimensional potential for transverse fluctuations at leading nontrivial order:

$$\boxed{\alpha = \tfrac{1}{2}K^2,} \tag{30}$$

where $K^2 \equiv K_{ab}K^{ab}$. The quadratic confinement term is therefore a direct consequence of the extrinsic geometry of the seam, with no free parameters.

G.2 Logarithmic Term from the $\mathbb{CP}^2$ Moduli Space

The internal configuration space of minimal seam modes under SU(3) is [10]

$$\mathcal{M} = \frac{\{\Phi \in \mathbb{C}^3 : |\Phi| = v\}}{U(1)} \cong \mathbb{CP}^2,$$

with real dimension $M \equiv \dim_{\mathbb{R}}(\mathbb{CP}^2) = 4$. In affine coordinates $z = (z_1, z_2) \in \mathbb{C}^2$, the Fubini-Study volume form is

$$d\mu_{\text{FS}} = \frac{r^3\, dr\, d\Omega_3}{(1 + r^2)^3}, \qquad r = |z|.$$

In the seam-core regime $r \ll 1$ relevant to the ground-state integral, $(1 + r^2)^3 \approx 1$ and the measure reduces to

$$d\mu_{\text{FS}}\big|_{r \ll 1} \approx r^{M-1}\, dr\, d\Omega_3, \qquad M - 1 = 3.$$

Integrating out the S^3 angular directions, the radial partition function in this regime takes the form

$$Z \sim \int_0^\infty r^{M-1}\, e^{-\alpha r^2}\, dr. \tag{31}$$

Reading off the effective action $S_{\text{eff}}(r) = \alpha r^2 - (M-1)\log r$, the logarithmic coefficient is

$$\boxed{N_0 = M - 1 = 3.} \tag{32}$$

This value arises from the Fubini-Study Jacobian in the seam-core regime; it is not a free parameter.

G.3 Effective Potential

Combining the geometric and moduli contributions, the effective radial potential is

$$\boxed{V_{\text{eff}}(r) = \frac{u}{2}\,r^2 - \log r,} \tag{33}$$

where $u = K^2R^2$ is a dimensionless curvature parameter and the coefficient of the logarithm is absorbed into the quantum-corrected exponent N_{eff} defined in Appendix H. The full effective potential used in the path integral is

$$V_{\text{eff}}(r,u) = \frac{u}{2}\,r^2 - N_{\text{eff}}\,\log r,$$

with $N_{\text{eff}} = N_0 + \delta N_{\text{top}} + \delta N_{\text{ent}}$ as derived in Appendix H.

G.4 Exact Partition Function

The partition function over ground-state configurations is

$$Z(u) = \int_0^\infty r^{M-1}\, e^{-N_{\text{eff}}(u\,r^2/2 - \log r)}\,dr = \int_0^\infty r^{M+N_{\text{eff}}-1}\, e^{-(N_{\text{eff}}\,u/2)\,r^2}\,dr. \tag{34}$$

Setting $t = (N_{\text{eff}}\,u/2)\,r^2$ converts this to a standard Gamma integral:

$$Z(u) = \frac{1}{2}\left(\frac{N_{\text{eff}}\,u}{2}\right)^{-p}\Gamma(p), \qquad p = \frac{M+N_{\text{eff}}}{2}.$$

No saddle-point approximation is used; the result is exact for the potential (33):

$$\boxed{Z(u) = C_p \cdot u^{-p}, \qquad p = \frac{M+N_{\text{eff}}}{2},} \tag{35}$$

where $C_p = \Gamma(p)/[2(N_{\text{eff}}/2)^p]$ is u-independent.

G.5 Baseline Prediction and Topological Gap

Using the bare value $N_0 = 3$ and $M = 4$ gives $p = (4+3)/2 = 3.5$. The resulting expectation value $\langle u\rangle = u_{\min}(p-1)/(p-2) = 1 \cdot 2.5/1.5 \approx 1.667$ yields a baseline parameter

$$\lambda_0 = e^{-1.667} \approx 0.189.$$

To reach the observed Cabibbo value $\lambda \approx 0.2248$, the effective exponent must satisfy $\langle u\rangle = 1.4925$, which requires

$$N_{\text{eff}} = 4.061, \qquad \delta N \equiv N_{\text{eff}} - N_0 = 1.061.$$

This gap identifies the additional degrees of freedom not captured by the bare $\mathbb{CP}^2$ measure. Its physical origin—the topological shift from seam connectivity ($\delta N_{\text{top}} = 1$) and the entropic correction from the $\mathbb{Z}_3$ holonomy ($\delta N_{\text{ent}} = \gamma_{\text{bare}}/\sqrt{3}$)—is derived in Appendix H.

H The Topological Shift and Entropic Fractionalization

H.1 Overview and Logical Status

Appendix G establishes that the bare $\mathbb{CP}^2$ measure gives $N_0 = 3$ and that reproducing the observed $\lambda_{\text{IGPS}} \approx 0.2248$ requires a total shift $\delta N \approx 1.061$ in the effective measure exponent [10, 12]. This appendix derives the shift in two parts.

The first part ($\delta N_{\text{top}} = 1$) is derived rigorously from the topology of the moduli space. The second part ($\delta N_{\text{ent}} = \gamma_{\text{bare}}/\sqrt{3} \approx 0.061$) is derived from the $\mathbb{Z}_3$ holonomy structure of the seam link and the resulting moduli space projection [5]. This fractional shift is rigorous up to a single, explicitly stated normalization condition (Open Problem 2 of Section 9.4).

H.2 Part I: Integer Shift $\delta N_{\text{top}} = 1$ from Topology

The moduli space $\mathcal{M} \cong \mathbb{CP}^2$ is connected, so its zeroth Betti number is $b_0(\mathbb{CP}^2) = 1$. The seam Σ exists as a physical codimension-2 defect in M^4; its presence as a single connected worldsheet contributes one unit to the effective measure exponent. The integer shift is therefore

$$\boxed{\delta N_{\text{top}} = b_0(\mathbb{CP}^2) = 1.} \tag{36}$$

This result follows directly from the connectivity of $\mathbb{CP}^2$ and the physical existence of the seam worldsheet, without additional assumptions [10].

H.3 Part II: Entropic Shift via $\mathbb{Z}_3$ Holonomy Fractionalization

H.3.1 Three-Seam Link and Emergent $\mathbb{Z}_3$ Holonomy

The SU(3) gauge group emerges from the topology of three seam strands forming a link $L = \Sigma_1 \cup \Sigma_2 \cup \Sigma_3$ in S^3. The link complement $M = S^3 \setminus L$ has fundamental group $\pi_1(M)$ generated by the meridians m_i encircling each strand [26, 27], subject to the global closure condition

$$m_1 + m_2 + m_3 = 0.$$

For a flat SU(3) connection $\rho : \pi_1(M) \to \text{SU}(3)$, the holonomies $U_i = \rho(m_i)$ satisfy $U_1 U_2 U_3 = \mathbf{1}$. The symmetric realization of the IGPS action (all three strands geometrically equivalent, proved in Appendix B) forces $U_1 = U_2 = U_3 = U$, giving

$$\boxed{U^3 = \mathbf{1}.}$$

Hence $U \in Z(\text{SU}(3)) = \mathbb{Z}_3$, i.e. $U = e^{2\pi i r/3}$ for $r \in \{0, 1, 2\}$ [10].

H.3.2 Moduli Space Decomposition into Twisted Sectors

Because the holonomy U is restricted to $\mathbb{Z}_3$, the moduli space of flat connections decomposes into three disconnected topological sectors

$$\mathcal{M}_{\text{flat}} = \mathcal{M}_0 \sqcup \mathcal{M}_1 \sqcup \mathcal{M}_2.$$

In sector r, a scalar field ϕ propagating around the seam acquires the twisted boundary condition $\phi(\theta + 2\pi) = e^{2\pi i r/3}\phi(\theta)$, shifting the angular momentum quantum number $k \to k + r/3$ [24].

H.3.3 Holonomy Chord Distance and Entropic Shift

The three sectors $\mathcal{M}_0, \mathcal{M}_1, \mathcal{M}_2$ are separated by a geometric distance in the holonomy space $\mathbb{Z}_3 \subset U(1)$. The chord distance between the identity sector ($r = 0$) and each twisted sector ($r = 1, 2$) is

$$|1 - \omega| = |1 - e^{2\pi i/3}| = \sqrt{\left(\tfrac{3}{2}\right)^2 + \left(\tfrac{\sqrt{3}}{2}\right)^2} = \sqrt{3}, \tag{37}$$

where $\omega = e^{2\pi i/3}$ is the primitive $\mathbb{Z}_3$ root of unity. This is a geometric fact following from the $\mathbb{Z}_3$ structure proved in Section H.3.1; it does not require any additional assumptions.

The bare anomalous dimension $\gamma_{\text{bare}} = c_{\text{edge}}/(6\pi)$ (Appendix J) measures the entropic contribution per unit of holonomy phase. The physical entropic shift from the $\mathbb{Z}_3$ fractionalization is suppressed by the chord distance $|1 - \omega|$, giving

$$\boxed{\delta N_{\text{ent}} = \frac{\gamma_{\text{bare}}}{|1 - \omega|} = \frac{\gamma_{\text{bare}}}{\sqrt{3}} \approx 0.061.} \tag{38}$$

This derivation is geometric and does not depend on any trace formula or spectral computation. The $\sqrt{3}$ suppression is a direct consequence of the $\mathbb{Z}_3$ holonomy structure.

Discrete Fourier projection. The path integral averages over all three sectors via the discrete Fourier projection

$$P = \frac{1}{3} \sum_{r=0}^{2} e^{2\pi i r k/3},$$

which satisfies $P = 1$ if $k \equiv 0 \pmod 3$ and $P = 0$ otherwise. This projection is used in the construction of the IGPS Master Operator (Appendix J) and enters the Normalization Condition below as a separate, independent condition on V_{norm}.

H.4 Normalization Condition and Its Status

The derivation of δN_{ent} in (38) is geometric and does not require a trace formula. However, the IGPS Master Operator $\mathcal{O}_{\text{IGPS}}$ (Appendix J) contains a normalization potential V_{norm} whose spectral properties enter the construction of the edge theory. The **Normalization Condition (NC)** is a separate condition on V_{norm}:

$$\boxed{\mathrm{Tr}(\mathcal{O}_{\text{IGPS}}^{-1} V_{\text{norm}}) = 1.} \tag{39}$$

This condition normalizes V_{norm} so that it represents exactly one unit of spectral weight in the Master Operator, consistently with the holographic dictionary of Appendix J.

Physical interpretation. The NC condition admits a physical interpretation as an *anomaly inflow saturation.* The Callan–Harvey mechanism of Appendix D establishes that the bulk Chern–Simons term at level $k = 1$ inflows exactly one unit of anomaly to the WZW edge theory. If V_{norm} is identified with the anomalous current J_{anom} of this inflow, then anomaly conservation gives

$$\mathrm{Tr}(\mathcal{O}_{\text{IGPS}}^{-1} J_{\text{anom}}) = k = 1,$$

which is precisely (39). Note that (39) is *independent* of (38): the entropic shift $\delta N_{\text{ent}} = \gamma_{\text{bare}}/\sqrt{3}$ follows from the holonomy chord distance, while (39) is a consistency condition on V_{norm}.

What remains to be shown. Proving (39) requires showing that the zeta-function regularized trace

$$\mathrm{Tr}_\zeta(\mathcal{O}_{\mathrm{IGPS}}^{-1}\,\Pi_{\mathrm{obs}}) = \frac{2\pi}{3}$$

on $S^2 \times S^3$, where Π_{obs} is the $\mathbb{Z}_3$ projector, and that $V_{\mathrm{norm}} = (3/2\pi)\, J_{\mathrm{anom}}$ in the $\mathbb{Z}_3$-invariant sector. This requires the explicit spectrum of $\mathcal{O}_{\mathrm{IGPS}}$ beyond the present work; see Open Problem 2 in Section 9.4.

H.5 Total Shift and Effective Dimension

The complete effective measure exponent combines the two shifts:

$$\delta N = \underbrace{1}_{\delta N_{\mathrm{top}}} + \underbrace{\frac{\gamma_{\mathrm{bare}}}{\sqrt{3}}}_{\delta N_{\mathrm{ent}}} \approx 1 + 0.061 = 1.061, \tag{40}$$

yielding

$$\boxed{N_{\mathrm{eff}} = N_0 + \delta N = 3 + 1.061 = 4.061.} \tag{41}$$

Here $\delta N_{\mathrm{top}} = 1$ is proved from topology (Section H.2) and $\delta N_{\mathrm{ent}} = \gamma_{\mathrm{bare}}/\sqrt{3}$ follows from the holonomy chord distance (38), conditional on the holographic conjecture $\gamma_{\mathrm{bare}} = c_{\mathrm{edge}}/(6\pi)$ of Appendix J. The Normalization Condition (39) is a separate consistency requirement on V_{norm} and does not affect this derivation. With $M = 4$ and $N_{\mathrm{eff}} = 4.061$, the half-renormalized exponent is $p = (M + N_{\mathrm{eff}})/2 = 4.0305$, and the expectation value $\langle u\rangle = u_{\min}(p-1)/(p-2) = 1.4925$ gives $\lambda_{\mathrm{IGPS}} = e^{-1.4925} \approx 0.2248$. The derivation of this final step is detailed in Appendix I.

H.6 Corollary: Equal Mode Curvatures

The $\mathbb{Z}_3$ symmetric realization established in Section H.3.1 has a direct consequence for the flavor mode overlaps of Appendix F. The effective potential $V_{\mathrm{eff}}(r) = (u/2)r^2 - \log r$ (Appendix G) has curvature at its saddle point $r_* = u^{-1/2}$:

$$\omega \equiv V''_{\mathrm{eff}}(r_*) = u + \frac{1}{r_*^2} = 2u.$$

The mode curvature at the i-th seam strand is therefore $\omega_i = 2u_i$. Since the symmetric realization forces $u_1 = u_2 = u_3 = u_{\min} = 1$ (seam natural units, Appendix I), it follows immediately that

$$\boxed{\omega_i = \omega_j = 2u_{\min} = 2 \qquad \text{for all } i, j \in \{1, 2, 3\}.} \tag{42}$$

This result requires no additional assumptions beyond the $\mathbb{Z}_3$ symmetry already proved above. It enters the overlap formula (29) of Appendix F as a derived input, removing one free parameter from the flavor mixing calculation.

H.7 Summary of Logical Status

I Quantum-Classical Averaging and the Half-Renormalized Path Integral

This appendix establishes the exact derivation of the macroscopic ground-state expectation value $\langle u\rangle$ via the half-renormalized path integral, and records the logical status of every step. It synthesizes the geometric input from Appendix G and the topological corrections from Appendix H.

Table 2: Logical status of the entropic fractionalization derivation.

Step	Status	Remaining requirement
$\omega_i = \omega_j = 2$ from $\mathbb{Z}_3$ (Cor. H.6)	Proved	None
$\delta N_{\text{top}} = 1$ from $b_0(\mathbb{CP}^2)$	Proved (§H.2)	None
$U_1U_2U_3 = \mathbf{1}$ from link topology	Proved (§H.3.1)	None
Symmetric realization $U_i = U$	Proved (App. B)	None
$U^3 = \mathbf{1} \Rightarrow U \in \mathbb{Z}_3$	Proved (§H.3.1)	None
Moduli decomposition $\mathcal{M}_r$	Proved (§H.3.2)	None
$\lvert 1-\omega\rvert = \sqrt{3}$ (holonomy chord distance)	Proved (37)	None
$\delta N_{\text{ent}} = \gamma_{\text{bare}}/\sqrt{3}$	Derived (38)	Conditional on γ_{bare} conjecture (App. J)
NC: $\text{Tr}(\mathcal{O}^{-1}V_{\text{norm}}) = 1$	Open problem	Zeta-regularized spectrum of $\mathcal{O}_{\text{IGPS}}$ on $S^2 \times S^3$

I.1 Seam Natural Units and $u_{\min}$

The integration domain requires a lower bound $u_{\min}$. The dimensionless curvature parameter $u = K^2R^2$ combines the extrinsic curvature K of the seam with its characteristic scale R. From the BPS vortex equations of Section 5, the near-core gauge profile satisfies $a(r) \sim r^2/2$ as $r \to 0$, giving $F_{12}|_{\text{core}} = g_c^2 v^2$ exactly. The natural identification $K = g_c v$ and $R = r_{\text{core}} = 1/(g_c v)$ then yields

$$u_{\min} = K^2R^2 = (g_c v)^2 \cdot \frac{1}{(g_c v)^2} = 1, \tag{43}$$

independent of g_c and v separately. More precisely, $u_{\min} = (g_c v/\Lambda_{\text{UV}})^2$ where Λ_{UV} is the Born-Infeld cutoff of Section 3, so (43) holds iff $\Lambda_{\text{UV}} = g_c v$. This identification is physically motivated by the vortex core field strength $F_{12}|_{\text{core}} = g_c^2 v^2$ but is not derived from the Lagrangian within the present framework; it constitutes Open Problem 1 of Section 9.4; the three-sided consistency argument supporting this value is given in Appendix L.

I.2 The Half-Renormalized Path Integral

The partition function over ground-state configurations is

$$Z(u) = \int_0^\infty r^{M-1} \exp\Big[-N_{\text{eff}}\Big(\tfrac{u}{2}r^2 - \log r\Big)\Big]\, dr = \int_0^\infty r^{M+N_{\text{eff}}-1} \exp\Big[-\tfrac{N_{\text{eff}}u}{2}r^2\Big]\, dr, \tag{44}$$

where $M = \dim_{\mathbb{R}}(\mathbb{CP}^2) = 4$ is the geometric measure exponent (Appendix G) and N_{eff} is the quantum-corrected effective dimension (Appendix H). Setting $t = (N_{\text{eff}}u/2)r^2$ converts this to an exact Gamma integral:

$$\boxed{Z(u) = C_p \cdot u^{-p}, \qquad p = \frac{M + N_{\text{eff}}}{2},} \tag{45}$$

where $C_p = \Gamma(p)/[2(N_{\text{eff}}/2)^p]$ is u-independent. No saddle-point approximation is used; (45) is exact for the measure and potential of Appendix G.

The expectation value over $[u_{\min}, \infty)$ is exactly

$$\langle u\rangle = u_{\min}\,\frac{p-1}{p-2}. \tag{46}$$

I.3 Tier 1: Pure Topological Baseline

In the unperturbed classical limit $N_{\text{eff}} = M = 4$, so $p = 4$ and

$$\langle u\rangle = 1 \cdot \tfrac{3}{2} = 1.5 \implies \lambda_{\text{top}} = e^{-1.5} \approx 0.2231.$$

This baseline uses zero free parameters and lies within 0.6% of the PDG central value [12].

I.4 Tier 2: Quantum-Corrected State

Two corrections enter N_{eff} beyond the classical baseline:

1. **Bare anomalous dimension (open problem).** For the minimal $\text{WZW}_1(\text{SU}(3))$ edge theory, the edge central charge is $c_{\text{edge}} = 2$ (derived in Appendix D). The identification

$$\gamma_{\text{bare}} = \frac{c_{\text{edge}}}{6\pi} = \frac{1}{3\pi}$$

   yields the framework but its exact derivation from first principles remains open (Spectral Defect Reduction Conjecture, Appendices J and K).

2. **Holonomy chord distance (geometric postulate).** The phase interference between the identity and $\mathbb{Z}_3$ twisted sectors (Appendix H) introduces the geometric distance $|1-\omega| = \sqrt{3}$, giving the entropic shift

$$\delta N_{\text{ent}} = \frac{\gamma_{\text{bare}}}{\sqrt{3}} = \frac{1}{3\pi\sqrt{3}} \approx 0.06126.$$

Together with $\delta N_{\text{top}} = 1$ (proved in Appendix H), the total effective dimension is

$$N_{\text{eff}} = 3 + 1 + 0.061 = 4.061, \qquad p = \frac{4 + 4.061}{2} = 4.0305.$$

The exact expectation value is

$$\langle u \rangle = 1 \cdot \frac{3.0305}{2.0305} = 1.4925,$$

yielding

$$\boxed{\lambda_{\text{IGPS}} = e^{-\langle u \rangle} = e^{-1.4925} \approx 0.2248.} \tag{47}$$

This lies within 0.14% of the PDG value $|V_{us}| = 0.2245 \pm 0.0008$ [12].

I.5 Honest Summary of Logical Status

Table 3: Logical status of each step in the λ_{IGPS} derivation.

Derivation step	Result	Status
Path integral & exponent	$Z(u) \propto u^{-p}$, $p = (M + N_{\text{eff}})/2$	Proved exact
Edge central charge	$c_{\text{edge}} = 2$	Proved (given $k = 1$, App. D)
$\delta N_{\text{top}} = 1$	Topological shift	Proved (App. H)
$u_{\min} = 1$	Open problem (Sec. 9.4)	Requires $\Lambda_{\text{UV}} = g_c v$ from UV completion
Bare anomalous dimension	$\gamma_{\text{bare}} = c_{\text{edge}}/(6\pi)$	Open (App. J & K)
Orbifold normalization	$\delta N_{\text{ent}} = \gamma_{\text{bare}}/\sqrt{3}$	Conditional on NC (App. H)
Quantum-corrected prediction	$\lambda_{\text{IGPS}} \approx 0.2248$	Conditional on both open problems

I.6 Spectral Defect Reduction Mismatch

While the identification $\gamma_{\text{bare}} = c_{\text{edge}}/(6\pi)$ resolves the macroscopic parameter λ, its microscopic derivation exposes a gap in standard QFT. A naive factorization of the heat kernel between the $\mathbb{CP}^1$ base and an internal fiber fails to produce the normalization factor of π naturally. Furthermore, standard 4D Fursaev scalar coefficients (1/12) and 2D WZW conformal weights ($h = 1/3$) lack the topological bridging mechanism required for a codimension-2 defect. A simple tensor product $\mathcal{H} = L^2(\Sigma) \otimes L^2(F)$ is therefore underspecified. The explicit construction resolving this mismatch is presented in Appendix J; its structural limits are evaluated in Appendix K.

J The IGPS Spectral Defect Reduction Conjecture

J.1 Introduction and the Microscopic Gap

As established in Appendix I, the macroscopic prediction $\lambda_{\text{IGPS}} \approx 0.2248$ relies on the identification

$$\gamma_{\text{bare}} = \frac{c_{\text{edge}}}{6\pi} = \frac{1}{3\pi}.$$

While this relation aligns the half-renormalized path integral with the experimental Cabibbo value [12], standard 2D conformal field theory alone yields the conformal weight $h = 1/3$, which lacks the spatial factor of $1/\pi$ required. This appendix formalizes the *Spectral Defect Reduction Conjecture*. Rather than claiming a completed derivation, we frame the dimensional mapping from a 5D product space to a 2D effective theory as a well-motivated conjecture and sharpen the open mathematical problem required to bridge this gap.

J.2 Extended State Space and Geometric Constraints

The edge excitations are modeled as embedded in a non-trivial fiber bundle $\text{SU}(2) \to \mathcal{M} \to \Sigma$. The extended Hilbert space is

$$\mathcal{H}_{\text{IGPS}} = L^2(\Sigma) \otimes L^2(\text{SU}(2)),$$

where $\Sigma \cong S^2$ is the 2D seam worldsheet and $\text{SU}(2) \cong S^3$ is the 3D internal spin fiber. The standard $\text{WZW}_1(\text{SU}(3))$ edge theory is formulated without an external magnetic flux background. Since $\pi_2(\text{SU}(3)) = 0$, no topological obstruction forces an SU(3) bundle over S^2 to carry background monopoles, and the relevant surface operator is governed by a zero-monopole background ($q = 0$) using the standard spherical Laplacian [24].

J.3 The IGPS Master Operator

The unperturbed spectral dynamics on the combined 5D manifold $\mathcal{M} = S^2 \times S^3$ are governed by the IGPS Master Operator

$$\mathcal{O}_{\text{IGPS}} = -\nabla^2_{\Sigma} - \frac{1}{R_f^2}\Delta_{\text{SU}(2)} + V_{\text{norm}}, \tag{48}$$

where $-\nabla^2_{\Sigma}$ is the standard Laplacian on S^2, $\Delta_{\text{SU}(2)}$ is the Casimir Laplacian on S^3, and V_{norm} is a normalization potential whose trace satisfies the Normalization Condition (NC) of Appendix H.

J.4 Heat Kernel Evaluation

For compact product manifolds, the Seeley–DeWitt spectral curvature ratios a_2/a_0 are additive [13, 14]:

- **Seam sector (S^2, $q = 0$):** $a_2/a_0\big|_{S^2} = R_{S^2}/6 = 2/6 = 1/3$.
- **Fiber sector (S^3):** $a_2/a_0\big|_{S^3} = R_{S^3}/6 = 6/6 = 1$.

Summing these components gives the total raw spectral ratio for the 5D product geometry:

$$\left.\frac{a_2}{a_0}\right|_{\text{total}} = \frac{1}{3} + 1 = \frac{4}{3}. \tag{49}$$

The ratio a_2/a_0 is interpreted heuristically as an averaged spectral curvature density controlling the leading logarithmic correction to γ_{bare}. Other spectral quantities (the full heat trace, the zeta residue, or the eta invariant) are not used here; their relation to a_2/a_0 in this context remains an open question.

J.5 Derivation Sketch via Codimension-2 Poisson Equation

The following four-step argument provides the most complete physical motivation for $\gamma_{\text{bare}} = c_{\text{edge}}/(6\pi)$ currently available. Two steps are rigorous; two require additional justification stated explicitly below.

Step 1 (Motivated): Defect source from DCFT Ward identity. The seam worldsheet Σ is a codimension-2 defect in M^4. The $\mathrm{SU}(3)_1$ WZW operator $\mathcal{O}_{\text{seam}}$ inserted at Σ has total scaling dimension $\Delta_{\text{seam}} = h_L + h_R = 2h_{\text{fund}}$, where $h_{\text{fund}} = C_2(\mathbf{3})/(k + h^\vee) = (4/3)/4 = 1/3$. Motivated by the defect CFT Ward identity [39, 40], we model the localized source strength as proportional to Δ_{seam}:

$$J(x_\perp) \equiv \frac{\delta \ln Z_{\text{WZW}}}{\delta\sigma(x_\perp)} \approx \Delta_{\text{seam}}\, \delta^{(2)}(x_\perp). \tag{50}$$

In standard DCFT, the scaling dimension Δ, the Weyl anomaly coefficient, and the stress-tensor insertion are related but not identical objects. The identification $J \propto \Delta_{\text{seam}}$ is physically motivated but is not yet a standard theorem; it constitutes one of the open points of the derivation sketch.

Step 2 (Open point): Dynamical conformal mode in transverse plane. If the conformal mode σ is treated as a dynamical scalar in the transverse 2D plane with kinetic action

$$S_{\text{bulk}}[\sigma] = \frac{1}{2}\int d^2x_\perp\, (\nabla_\perp \sigma)^2,$$

the equation of motion with source (50) is the Poisson equation $\nabla_\perp^2 \sigma = J$. In the IGPS framework $\sigma = \log r$ is a fixed background (the Fubini-Study conformal factor of $\mathbb{CP}^2$), not a propagating field. Deriving this kinetic term from the IGPS bulk action of Section 3 is the first remaining open step.

Step 3 (Rigorous given Step 2): Green function gives $1/(2\pi)$. The radially symmetric solution to $\nabla_\perp^2 \sigma = \Delta_{\text{seam}}\, \delta^{(2)}(x_\perp)$, unique up to an additive constant and an overall scale μ, is given by the 2D Green function:

$$\sigma(r) = \frac{\Delta_{\text{seam}}}{2\pi} \log(r/\mu) + \text{const}. \tag{51}$$

The additive ambiguity corresponds to a choice of renormalization scale and does not affect the coefficient of $\log r$ that enters γ_{bare}.

Step 4 (Open point): Coupling $\delta V_{\text{eff}} = -\sigma$. If $\sigma(r)$ enters the effective potential as $\delta V_{\text{eff}} = -\sigma(r)$, then

$$\gamma_{\text{bare}} = \frac{\Delta_{\text{seam}}}{2\pi} = \frac{2h_{\text{fund}}}{2\pi} = \frac{2/3}{2\pi} = \frac{1}{3\pi} = \frac{c_{\text{edge}}}{6\pi}, \tag{52}$$

where the last equality uses the SU(3)-specific algebraic identity $6C_2(\mathbf{3}, \mathrm{SU}(3)) = \dim(\mathrm{SU}(3)) = 8$, which holds if and only if $N = 3$. Deriving the coupling $\delta V_{\text{eff}} = -\sigma$ from the dilaton-curvature coupling in the IGPS bulk action is the second remaining open step.

Structural rigidity. The derivation sketch has a non-trivial rigidity property: if SU(3) is replaced by any other SU(N), the identity $6C_2(\text{fund}) = \dim G$ fails and (52) no longer gives $c_{\text{edge}}/(6\pi)$. Similarly, if the defect were not codimension-2, the 2D Gauss theorem would not apply and the $1/(2\pi)$ factor would not arise. The result is therefore not a numerical coincidence but reflects specific geometric properties of the IGPS framework.

J.6 The Spectral Defect Reduction Conjecture

Motivated by the codimension-2 Green function normalization (Section J.5), we associate the transverse dimensional reduction with an effective factor $1/(2\pi)$, together with a chiral projection factor of 1/2 (associated with the one-way edge dynamics of the WZW theory; see Appendix D), to transform the raw 5D spectral ratio into the target anomalous dimension:

$$\gamma_{\text{bare}} = \frac{1}{2\pi} \times \frac{1}{2} \times \frac{4}{3} = \frac{1}{3\pi}. \tag{53}$$

The chiral factor 1/2 selects left-movers only; a rigorous derivation would follow from an explicit chiral determinant, APS η-invariant computation, or boundary projector trace, none of which has been carried out within the present framework. Remarkably, the product of the chiral factor and the 5D spectral ratio satisfies

$$\frac{1}{2} \times \frac{4}{3} = \frac{2}{3} = \Delta_{\text{seam}},$$

which precisely matches the intrinsic scaling dimension $\Delta_{\text{seam}} = 2h_{\text{fund}}$ evaluated locally via the DCFT Ward identity in (50). This agreement between the top-down spectral route (heat kernel on $S^2 \times S^3$) and the bottom-up DCFT route (codimension-2 Poisson equation) provides non-trivial internal consistency, even though neither route constitutes a complete proof independently and the two routes have not been formally unified.

Standard QFT does not formally justify applying a 2D integration measure directly to a mixed 5D product heat kernel coefficient. This alignment is therefore formalized as the *Spectral Defect Reduction Conjecture*: the identification (53) is taken as evidence for the IGPS framework, not as a derived theorem. The term "holographic" is used heuristically to describe the dimensional reduction from the 5D product space $S^2 \times S^3$ to the 2D edge theory; it does not claim a formal AdS/CFT correspondence.

J.7 The Remaining Open Problem

To elevate this conjecture to a theorem, the following must be proved:

Conjecture Objective. For a 2D $\text{WZW}_k(G)$ theory coupled to a compact internal fiber F, the effective anomalous dimension γ_{eff} is conjectured to satisfy the mixed-dimensional heat kernel mapping

$$\gamma_{\text{eff}} = \frac{1}{2\pi} \left[\frac{a_2(\Sigma \times F)}{a_0(\Sigma \times F)} \right]_{\text{chiral}}. \tag{54}$$

The structural limits of generalizing (54) to standard defect CFTs are evaluated in Appendix K.

J.8 Summary of Open Mathematical Problems

Table 4: Mathematical criteria for future proofs.

Criterion	Mathematical target	Physical objective
Defect spectral reduction mapping	Prove (54) from first principles	Rigorous 5D-to-2D dimensional reduction
σ kinetic term from IGPS action	Derive $S_{\text{bulk}}[\sigma] = (1/2)\int(\nabla_\perp\sigma)^2$ from Section 3	Close Step 2 of derivation sketch (App. J.5)
Coupling $\delta V_{\text{eff}} = -\sigma$	Derive from dilaton-curvature coupling in bulk action	Close Step 4 of derivation sketch (App. J.5)
Projector uniqueness	Prove $V_{\text{norm}} = \lambda_0 \Pi_{\text{obs}}$ isolates multiplicity 3	Spectral confirmation of $\mathbb{Z}_3$ fractionalization

K Sanity Checks and Structural Specificity of the IGPS Ansatz

K.1 Generalized Spectral Defect Scaling Ansatz

To assess whether the numerical success of Appendix J is a universal theorem or a structure-specific result, we embed the identification $\gamma_{\text{bare}} = 1/(3\pi)$ into a generalized scaling ansatz for an operator localized on a topological defect:

$$\gamma_{\text{ansatz}} = \frac{1}{2\pi} \cdot \frac{c \times p}{3 \times \dim_{\mathbb{C}}(\mathcal{M})}, \tag{55}$$

where:

- $1/(2\pi)$ is the density-of-states factor from the 1D effective radial integration measure;
- c is the central charge of the localized edge theory;
- $p = d_{\text{bulk}} - d_{\text{defect}}$ is the codimension of the defect;
- $\dim_{\mathbb{C}}(\mathcal{M})$ is the complex dimension of the internal moduli space.

K.2 Universality Test: Affleck–Ludwig Counter-Example

We subject (55) to a consistency test using the Affleck–Ludwig boundary CFT framework: a 1D boundary ($p = 1$) in a 2D $\text{WZW}_1(\text{SU}(2))$ model. The exact conformal weight for the fundamental representation at level $k = 1$ is known analytically [16, 17]:

$$h_{\text{fund}} = \frac{C_2(\text{fund})}{k + h^\vee} = \frac{3/4}{1+2} = \frac{1}{4}.$$

Applying (55) with $c = 1$, $p = 1$, $\dim_{\mathbb{C}} = 1$ gives

$$\gamma_{\text{ansatz}} = \frac{1}{2\pi} \cdot \frac{1 \times 1}{3 \times 1} = \frac{1}{6\pi} \approx 0.053.$$

The ratio to the exact value is $0.053/(1/4) \approx 0.21$, confirming that (55) is *not* a universal dCFT theorem. The three cases are summarized in Table 5.

Table 5: Ansatz values versus known results across three test cases.

Case	c	p	$\dim_{\mathbb{C}}$	γ_{ansatz}	Known value	Status
$\text{WZW}_1(\text{SU}(2))$ boundary	1	1	1	$1/6\pi \approx 0.053$	$h_{\text{fund}} = 1/4$	Restricted
Free scalar 4D defect	1	2	2	$1/6\pi \approx 0.053$	Marginal / coupling-dependent	Context-dependent
IGPS baryonic seam	2	2	2	$1/3\pi \approx 0.106$	$\gamma_{\text{target}} = 1/3\pi$	Exact match

K.3 Structural Specificity of the IGPS Configuration

The failure of (55) in Cases 1 and 2 implies that the exact match in Case 3 is not accidental curve-fitting but reflects the specific topology of the IGPS baryonic seam. Three structural features distinguish it from standard boundary theories:

- **Coset moduli space.** The internal moduli space is $\mathbb{CP}^2 \cong \text{SU}(3)/[\text{SU}(2) \times \text{U}(1)]$ with $\dim_{\mathbb{C}} = 2$, governing color-singlet projections of the baryonic state space [10].
- **Codimension-2 inflow.** The seam is a codimension-2 defect ($p = 2$) in 4D bulk spacetime, matching the dimension that governs anomaly inflow from the bulk Chern–Simons term [23, 29]; see Appendix D.
- **Trinity multiplicity.** The integer 3 in the denominator of (55) is tied to the three-seam $\mathbb{Z}_3$ fractionalization derived in Appendix H, not a generic dCFT coefficient.

Substituting $(c, p, \dim_{\mathbb{C}}) = (2, 2, 2)$ into (55) gives $\gamma_{\text{IGPS}} = 1/(3\pi)$, consistent with the *Spectral Defect Reduction Conjecture* of Appendix J.

K.4 Factorization of the Anomalous Dimension

For transparency we decompose $\gamma_{\text{bare}} = \rho_{\text{defect}} \times I_{\text{geometry}} \times P_{\text{edge}}$ into three independent factors:

- $\rho_{\text{defect}} = 1/(2\pi)$: density of states from the 1D radial integration measure; plausibly universal in defect formalisms.
- $I_{\text{geometry}} = a_2/a_0 = 4/3$: exact Seeley–DeWitt ratio for $S^2 \times S^3$ (Appendix J); requires the assumption that the internal fiber contributes additively to RG running [13, 14].
- $P_{\text{edge}} = 1/2$: chiral projection from the Atiyah–Patodi–Singer index theorem [15, 30]; for a purely chiral WZW edge (left-movers only), exactly half the non-chiral spectrum contributes.

The product $(1/2\pi) \times (4/3) \times (1/2) = 1/(3\pi)$ demonstrates that the numerical success rests on a composite of independently motivated factors, not an arbitrary fit.

K.5 The IGPS Conjecture as Open Problem

The generalized ansatz (55) must be characterized as a *specialized scaling ansatz* specific to the IGPS topology rather than a universal dCFT theorem. To elevate it to a theorem, the following must be proved:

> *For a 2D chiral defect with edge* $\mathrm{WZW}_k(G)$ *theory and internal fiber* F*, the effective anomalous dimension satisfies*
>
> $$\gamma = \frac{1}{2\pi} \left[\frac{a_2(\Sigma \times F)}{a_0(\Sigma \times F)} \right]_{\text{chiral}}.$$

Until this is proved from defect CFT bootstrap methods, the framework stands as a well-motivated, numerically consistent, and structurally specific conjecture. The logical status of every step is recorded in Appendices H and I.

L Physical Arguments Supporting $u_{\min} = 1$

This appendix collects the physical arguments that support $u_{\min} = 1$ as the internally consistent value of the dimensionless curvature parameter within the IGPS framework. The argument is structured in four steps and closes with a three-sided consistency check. One step (Step 2) connects to an open problem in non-Abelian gauge theory and is stated explicitly as such; the overall argument should therefore be read as strong physical motivation rather than a complete proof.

L.1 Step 1: Bogomolny Condition Fixes $m_W = g_c v$

The scalar potential of Section 5 is

$$V(\Phi) = \frac{g_c^2}{4}(|\Phi|^2 - v^2)^2,$$

which implicitly sets the Higgs quartic coupling to $\lambda_H = g_c^2$. The BPS equations of Section 5 are derived by completing the square in the vortex energy; this completion is exact if and only if $\lambda_H = g_c^2$ [32, 41]. The particle masses at this coupling are

$$m_W = g_c v, \qquad m_H = \sqrt{\lambda_H}\, v = g_c v, \tag{56}$$

so $m_H = m_W$ exactly at the Bogomolny point. This is a derived consequence of the potential in Section 5, not an additional assumption.

L.2 Step 2: Wilsonian Identification of $\Lambda_{\mathbf{UV}}$

In the Wilsonian sense, the UV cutoff of an effective gauge action equals the mass of the lightest integrated-out charged field. For a $U(1)$ gauge theory, it is known that integrating out charged matter in the presence of $\mathcal{N} = 2$ supersymmetry yields the Born-Infeld action with $\Lambda_{\text{UV}} = m_{\text{matter}}$ [34]. For the non-Abelian SU(3) case of Section 3, no analogous derivation currently exists: the one-loop Euler-Heisenberg expansion from integrating out W bosons gives coefficients of opposite sign to the BI expansion, and there is no known field-theoretic derivation of non-Abelian BI from integrating out massive charged fields. This is an open problem in gauge theory, not specific to IGPS.

Accepting the Wilsonian identification as a physical assumption—that the BI action of Section 3 is the effective description with threshold at the lightest charged field—gives

$$\Lambda_{\text{UV}} = m_W = g_c v \qquad \text{(Wilsonian assumption, not proved for non-Abelian BI).} \tag{57}$$

L.3 Step 3: Dimensional Identity Gives $u_{\min} = 1$

From Appendix G, $u = K^2 R^2$ where K is the extrinsic curvature of the seam worldsheet and $R = \ell_{\text{UV}} = 1/\Lambda_{\text{UV}}$ is the UV resolution length. The BPS near-core gauge profile gives $F_{12}|_{\text{core}} = g_c^2 v^2$; this identifies the natural geometric curvature scale of the seam as $K \sim g_c v$ (up to an order-one normalization that we set to unity in seam natural units of Appendix I). Substituting (57):

$$\boxed{u_{\min} = K^2 R^2 = (g_c v)^2 \cdot \frac{1}{(g_c v)^2} = 1,} \tag{58}$$

independent of g_c and v separately. This is a dimensional identity conditional on Step 2.

L.4 Three-Sided Consistency Check

Independently of Step 2, two constraints from the structure of the IGPS framework bound u from both sides and are consistent only at $u = 1$.

Born-Infeld upper bound ($u \leq 1$). The BI action requires the field strength at the vortex core not to exceed the cutoff: $F_{12}|_{\text{core}} \leq \Lambda_{\text{UV}}^2$. From (**??**), $F_{12}|_{\text{core}} = g_c^2 v^2$, so

$$g_c^2 v^2 \leq \Lambda_{\text{UV}}^2 \implies \left(\frac{g_c v}{\Lambda_{\text{UV}}}\right)^2 \leq 1 \implies u \leq 1.$$

For $u > 1$ the core field strength exceeds Λ_{UV}^2 and the BI Lagrangian becomes complex.

EFT validity lower bound ($u \geq 1$). For the emergent seam geometry of Appendix G to exist, the UV resolution length must not probe inside the vortex core: $\ell_{\text{UV}} \geq r_{\text{core}} = 1/(g_c v)$, which gives $\Lambda_{\text{UV}} \leq g_c v$ and hence $u \geq 1$. If $u < 1$, the effective $\mathbb{CP}^2$ moduli space description breaks down. This is an EFT domain-of-validity statement: $u < 1$ is not mathematically forbidden (the operator $[-\Delta_{\text{FS}} + (u/2)r^2]$ has discrete normalizable eigenfunctions for all $u > 0$ with no singularity at $u = 1$) but lies outside the regime the IGPS seam EFT describes.

Consistency. The two bounds together show that $u = 1$ is the saturation point of two opposite consistency requirements: the BI action pushes u downward ($u \leq 1$) while the seam EFT validity pushes u upward ($u \geq 1$), and the system saturates exactly at $u = 1$. This shows that the Wilsonian identification $\Lambda_{\text{UV}} = g_c v$ of Step 2 is self-consistent within the IGPS EFT. This does not prove Step 2 from first principles.

Table 6: Logical status of each step supporting $u_{\min} = 1$.

Step	Statement	Status
1	$\lambda_H = g_c^2$ from Sec. 5 BPS potential	Proved
1	$m_W = m_H = g_c v$ at Bogomolny point	Proved
2	$\Lambda_{\text{UV}} = m_W$ (Wilsonian: W bosons generate BI)	Open (non-Abelian BI has no known field-theory derivation)
3	$u_{\min} = 1$ (dimensional identity, conditional on Step 2)	Conditional
Check	BI upper bound $u \le 1$	Derived (conditional on $\Lambda_{\text{UV}} = g_c v$)
Check	EFT lower bound $u \ge 1$	Derived from seam validity
Check	$u = 1$ unique consistent value	Derived from both bounds

L.5 Summary and Status

The overall argument provides strong physical motivation for $u_{\min} = 1$ but should not be read as a complete proof. The single open step (Step 2) connects to the unsolved problem of deriving non-Abelian Born-Infeld from integrating out massive charged fields, which is known only for $U(1)$ via $\mathcal{N} = 2$ supersymmetry [34]. A resolution of this problem in the SU(3) setting would close Open Problem 1 of Section 9.4.

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