

# Kazzy Theory:

## A Proposal for Unified Physics Based on a 10-Dimensional Closed-Cycle Structure

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### Abstract

We propose *Kazzy Theory* (KKT), a unified physics framework based on a 10-dimensional closed-cycle manifold  $M_{10} = M_3 \times \Sigma_7$ , where  $M_3$  is the observable 3-space and  $\Sigma_7$  is a compact internal manifold.

**Foundations.** Time is a derived parameter generated by the motion of an observation point  $\sigma$  along the  $D_5$  axis, giving  $dt = \alpha d\tau$ ; circularity is resolved by a  $D_5$  foliation in analogy with ADM decomposition. The four forces arise as Torsion components under the KK-Weitzenböck framework: gravity ( $D_5$ ) via TEGR (established *in the weak-field limit*; Prop. 3.3); electromagnetic, weak, and strong forces via the Kazzy-Weitzenböck Torsion-Gauge Identity (Theorem 3.6), which derives  $T_{\mu\nu}^{(n)a} = F_{\mu\nu}^{(n)a}$  for  $n = 6, 7, 8$  from the Maurer-Cartan geometry of the fibre group  $G_n$  (remaining open tasks: explicit 10D coframe construction and Faddeev-Popov quantisation).

**New results (this version).** (KWGT) The torsion-gauge identity  $T^{(n)} = F^{(n)}$  is *derived* for all  $n = 6, 7, 8$  from the Maurer-Cartan equation on the principal bundle  $P^{(n)}(M_3, G_n)$  (Theorem 3.6), resolving the non-Abelian circularity that was open problem OP-4-1. (D9) The  $D_9 \rightarrow D_8$  connection is formalised as the canonical coset fibration  $U(2) \hookrightarrow SU(3) \rightarrow CP^2$  with the Maurer-Cartan connection  $\omega^{(9)}$  and Fubini-Study curvature (Proposition 3.2). (DP) The complete downward projection chain  $D_8 \rightarrow D_7 \rightarrow D_6 \rightarrow D_5 \rightarrow D_4 \rightarrow D_3$  is formalised by five explicit geometric maps (Proposition 3.4), and the composition is shown to recover the Newtonian Poisson equation (Corollary 3.2). (B1) The Kazzy Gauge Emergence Theorem is established via  $\Sigma_7^{\text{SM}} \cong S^1 \times CP^1 \times CP^2$  (Proposition 3.10), yielding  $U(1) \times SU(2) \times SU(3)$  (conditional on the decomposition ansatz; dynamical selection is OP-1). (KHUT) The SM hypercharge assignment is *uniquely* determined by

charge quantisation, anomaly cancellation, and the 15-Weyl-spinor content (Theorem 3.10; a KKT reinterpretation of a classical SM result). (C1) Equal-volume unification yields the fibre-radius hierarchy  $R_6^*:R_7^*:R_8^* = (\pi/4)\ell^4 : \sqrt{\pi/8}\ell^2 : \ell$  (Proposition 3.14), giving  $g_W^2/g_Y^2 \approx 3.34$  (observed 3.35, 0.3% error) and  $g_s^2/g_W^2 \approx 3.51$  (observed 3.49, 0.6% error).

**Further structure.** The fermion generation count  $N_{\text{gen}} = 3$  is derived via HRR on  $CP^2$ . Eötvös constraints yield  $m_\chi \gtrsim 20 \mu\text{eV}/c^2$ . Unresolved problems (OP-1 through OP-5) are catalogued in Section 3.11.

**Keywords:** extra dimensions; Kaluza-Klein theory; teleparallel gravity; unified field theory; torsion; torsion–gauge identity; gauge emergence; hypercharge; equivalence principle; fermion generations; Maurer–Cartan equation.

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# 1 Introduction

## 1.1 Motivation and Background

Modern physics rests on two towering frameworks. General relativity (GR) describes gravity, spacetime curvature, and the large-scale evolution of the universe with extraordinary precision. The Standard Model (SM) of particle physics unifies the electromagnetic, weak, and strong interactions under the gauge symmetry group  $G_{\text{SM}} = \text{U}(1)_Y \times \text{SU}(2)_L \times \text{SU}(3)_c$ , accounting for the spectrum of fermions and bosons. Each framework is experimentally verified to remarkable accuracy within its domain. Yet their unification remains the most pressing open problem in fundamental physics.

Three specific tensions motivate the present proposal.

1. **Quantum gravity.** GR is a classical field theory; its standard perturbative quantisation produces non-renormalisable divergences. No experimentally confirmed quantum theory of gravity exists. String theory, loop quantum gravity, and causal dynamical triangulations each offer partial answers but none is yet experimentally tested.
2. **The nature of time.** In GR, time is a coordinate symmetric with the spatial dimensions; in quantum mechanics it is an external parameter, not an operator. Why there is a preferred arrow of time, and what “time” fundamentally is, remain unanswered in both frameworks.
3. **Origin of gauge groups and parameter fine-tuning.** Why the SM gauge group has its particular structure  $\text{U}(1) \times \text{SU}(2) \times \text{SU}(3)$ , why the 19 free parameters (fermion masses, mixing angles, coupling constants) take their observed values, and why the cosmological constant is tuned to at least 60 orders of magnitude below its natural scale—none of these has a dynamical explanation.

Kazzy Theory, conceived by Kimura Kazuto, is proposed as a response to this cluster of problems. Its point of departure is the philosophical intuition that *existence begins at a zero-dimensional point and returns through a 10-dimensional cycle*. This intuition is formalised mathematically with the aim of yielding experimentally testable physical predictions. The theory’s most distinctive feature is that time is not an independent dimension but a derived parameter generated by the motion of an observation point along the  $D_5$  axis.

## 1.2 Principal Claims and Contributions

Kazzy Theory makes four principal claims.

**Claim C1 Axiomatic introduction of the 10-dimensional cycle manifold:**

Spacetime is described as a 10-dimensional manifold  $M_{10} = M_3 \times \Sigma_7$  with a product structure.  $\Sigma_7$  is a compact 7-dimensional internal manifold spanned by  $D_0$  (cyclic closure),  $D_4$  (integrated information field),  $D_5$  (observation point),  $D_6$  (electromagnetism),  $D_7$  (weak force),  $D_8$  (strong force), and  $D_9$  (physical constants). The dimensional hierarchy forms a closed cycle  $D_0 \rightarrow D_1 \rightarrow \dots \rightarrow D_9 \rightarrow D_0$ . Time  $t$  is not an independent coordinate axis; it is the observational parameter generated when the observation point  $\sigma$  moves along  $D_5$  with displacement  $d\tau$ , projected onto  $M_3$  via  $dt = \alpha d\tau$ .

**Claim C2 Unified description of the four forces via Torsion (with correspondence hypothesis):**

The four fundamental interactions are related to the geometric torsion of the connection in each dimensional axis as follows. Gravity ( $D_5$ ) is derived rigorously from Torsion within the TEGR framework (established). The electromagnetic ( $D_6$ ), weak ( $D_7$ ), and strong ( $D_8$ ) forces correspond respectively to  $U(1)$ ,  $SU(2)$ ,  $SU(3)$  gauge curvature via a Torsion correspondence hypothesis (to be proved). Variations of physical constants ( $D_9$ ) are described by the Torsion  $T^{(9)}$  of the ninth-dimension connection.

**Claim C3 Redefinition of the existence layer  $E_4$ :**

“3+1 dimensions” is not the Minkowski spacetime of GR (3 spatial + 1 time dimension) but the ontological 4-dimensional existence layer  $E_4 = M_3 \times C_4$ , where +1 is not time but the integrated information field  $\Phi(x)$  (the  $D_4$  KK scalar). Each entity  $e = (x, \Phi(x))$  is described as the product of a spatial position  $x \in M_3$  and an internal integrated information  $\Phi(x) \in C_4$ , incorporating the universal self-reference principle (Postulate SR) “every thing is that thing” into the geometry.

**Claim C4 Derivation of consistency conditions with experimental constraints:**

Kazzy Theory makes the following experimental consistency conditions explicit. (a) Eöt-Wash experiment:  $\beta_{\text{KK}} < 3.7 \times 10^{-20} \text{ m}^3 \text{ s}^{-2}$ . (b) Equivalence principle:  $\Delta a/a < 10^{-13}$ . (c) Spacetime variation of the fine structure constant:  $\Delta\alpha/\alpha < 10^{-6}$ . These conditions place numerical upper bounds on the KK correction parameters ( $C_s, \Lambda_{\text{conn}}, \varepsilon_{\text{eff}}$ , etc.) and constrain the theory’s parameter space.

### 1.3 Organisation of the Paper

Chapter 2 reviews the four theoretical frameworks on which KK Theory builds and extends: (i) GR and TEGR, (ii) the SM and gauge field theory, (iii) classical Kaluza-Klein

theory, and (iv) Yukawa’s elementary-domain theory. For each, correspondences and differences with Kazzy Theory are made explicit.

Chapter 3 is the core of the paper and presents the complete mathematical formulation: five foundational axioms, the definition of the 10-dimensional manifold (Section 3.2), the three fundamental equations (Section 3.5), the force–Torsion correspondence (Section 3.6), the relativistic extension and 10-dimensional action (Section 3.9), and open problems including experimental constraints (Section 3.11).

Throughout, established results (gravity–TEGR correspondence) are clearly distinguished from hypotheses (electromagnetic, weak, and strong force–Torsion correspondences). Open problems OP-1 through OP-5 are catalogued in Section 3.11, showing transparently the current status of the proposal.

## 2 Background and Related Work

### 2.1 General Relativity and Its Teleparallel Equivalent (TEGR)

#### 2.1.1 General Relativity

Einstein’s GR describes gravity geometrically as the curvature of a 4-dimensional space-time manifold  $(M_4, g_{\mu\nu})$  with metric signature  $(-, +, +, +)$ . The Einstein equation relates the curvature to the energy-momentum content:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \quad (1)$$

where  $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu}$  is the Einstein tensor,  $T_{\mu\nu}$  the energy-momentum tensor, and  $\Lambda$  the cosmological constant. GR is the non-linear, relativistic generalisation of Newtonian gravity and provides the framework for planetary orbits, gravitational waves, black holes, and cosmic expansion.

In GR, time and space are treated symmetrically as coordinates of a unified spacetime; gravity manifests as the geometric deformation (curvature) of this spacetime. KK Theory differs fundamentally: it does not adopt time as an independent coordinate dimension, treating it instead as a projected parameter generated by motion along  $D_5$ .

#### 2.1.2 Teleparallel Equivalent of General Relativity (TEGR)

The Teleparallel Equivalent of General Relativity [2, 18] describes gravity in terms of the Torsion of a connection rather than the curvature of the metric. The fundamental variable is the tetrad (vier-bein)  $e^a{}_\mu$ , satisfying  $e^a{}_\mu e^b{}_\nu \eta_{ab} = g_{\mu\nu}$  where  $\eta_{ab}$  is the local Minkowski metric.

Using the Weitzenböck connection [27]—established in the framework of absolute parallelism and independently explored by Einstein [7] for a unified field theory—one adopts

zero curvature and non-zero torsion:

$$\Gamma^{\rho}{}_{\mu\nu} = e^a{}_{\rho} \partial_{\nu} e_{a\mu}, \quad R = 0, \quad T \neq 0. \quad (2)$$

The Torsion tensor  $T^a{}_{\mu\nu} = \partial_{\mu} e^a{}_{\nu} - \partial_{\nu} e^a{}_{\mu}$  and the Torsion scalar  $T$  define the TEGR action:

$$S_{\text{TEGR}} = \frac{1}{16\pi G} \int T \sqrt{-g} d^4x. \quad (3)$$

The Euler-Lagrange equations of (3) are *exactly equivalent* (not an approximation) to the Einstein equation (1) up to a boundary term. More generally, the standard review by Hehl et al. [10] develops the role of torsion in Einstein–Cartan and Poincare gauge gravity, where torsion couples to matter spin and local Poincare symmetry. Kazy Theory does not adopt that full framework, but it provides essential precedent for treating torsion as a geometric carrier of gravitational and interaction structure.

**Relation to Kazy Theory (TEGR):** Kazy Theory adopts the TEGR framework for gravity. The Torsion  $T^{(5)}$  of the  $D_5$  (observation point) connection corresponds to the scalar  $T$  in (3), so gravity is described as the torsion of the  $D_5$  connection. This is the only force-Torsion correspondence that is currently established mathematically (see Section 3.6.2).

## 2.2 The Standard Model and Gauge Field Theory

### 2.2.1 Gauge Field Theory

Gauge field theory is grounded in the principle that local gauge symmetry (extension of an internal symmetry to a coordinate-dependent transformation) generates interaction fields. For a compact Lie group  $G$ , the theory is built on a principal  $G$ -bundle  $P(M_4, G)$ ; the connection 1-form  $A$  (gauge potential) and its curvature  $F = dA + [A, A]$  (field strength) are the fundamental objects:

$$F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} + [A_{\mu}, A_{\nu}]. \quad (4)$$

The Yang-Mills action  $S_{\text{YM}} = -\frac{1}{4} \int F_{\mu\nu}^a F^{a\mu\nu} \sqrt{-g} d^4x$  yields the Yang-Mills equations  $D_{\mu} F^{\mu\nu} = J^{\nu}$  by variation. The geometric reading of local symmetry as a connection-and-curvature principle for interactions traces back to Utiyama’s gauge formulation [26]. U(1) gauge theory is equivalent to Maxwell electromagnetism; SU(2) corresponds to the Weinberg-Salam electroweak theory; SU(3) to QCD.

## 2.2.2 The Standard Model

The SM gauge group is

$$G_{\text{SM}} = \text{U}(1)_Y \times \text{SU}(2)_L \times \text{SU}(3)_c. \quad (5)$$

The SM is verified to extraordinary accuracy at colliders including the LHC, yet it carries fundamental open questions: (a) why the gauge group has this specific structure; (b) the fine-tuning of 19 free parameters; (c) the absence of gravity; (d) dark matter and dark energy.

**Relation to Kazzy Theory (Standard Model):** Kazzy Theory proposes that the origin of  $G_{\text{SM}}$  lies in the structure groups of fibre bundles over each dimensional axis:  $D_6 \rightarrow \text{U}(1)$  (information axis),  $D_7 \rightarrow \text{SU}(2)$  (intent axis),  $D_8 \rightarrow \text{SU}(3)$  (state axis). This traces the origin of the gauge group to the geometry of the dimensional structure (correspondence hypothesis; see Sections 3.6 and 3.9.7). The 19 parameters are framed within the  $D_9$  (principle axis) connection components, though their quantitative derivation is a future task.

## 2.3 Kaluza-Klein Theory and Higher-Dimensional Gravity

### 2.3.1 Classical Kaluza-Klein Theory (5D)

The Kaluza-Klein theory [12, 15] proposes to unify gravity and electromagnetism as 5-dimensional GR. The 5-dimensional metric  $G_{AB}$  ( $A, B = 0, 1, 2, 3, 5$ ) decomposes as

$$G_{AB} = \begin{pmatrix} g_{\mu\nu} + \phi^2 A_\mu A_\nu & \phi^2 A_\mu \\ \phi^2 A_\nu & \phi^2 \end{pmatrix}, \quad (6)$$

where  $g_{\mu\nu}$  is the 4D gravitational metric,  $A_\mu$  the electromagnetic 4-potential, and  $\phi$  the radion scalar (fluctuation of the internal circle  $S^1$ ). Compactifying the fifth dimension on  $S^1$  of radius  $R$  decomposes the 5D Einstein-Hilbert action:

$$S_5 = \frac{1}{16\pi G_5} \int d^5 X \sqrt{-G_5} R_5 \rightarrow S_{\text{EH}} + S_{\text{EM}} + S_{\text{radion}}. \quad (7)$$

### 2.3.2 Modern Developments and Difficulties

Post-1970 higher-dimensional theories (superstrings in 10D, M-theory in 11D, Randall-Sundrum braneworld) each attempt to derive the SM gauge group from the isometry group of the internal space. Common difficulties: (a) the vast landscape of Calabi-Yau vacua; (b) the stabilisation mechanism for compactification radii; (c) the incomplete construction of specific internal spaces whose isometry group is exactly  $\text{U}(1) \times \text{SU}(2) \times \text{SU}(3)$ .

**Differences from Kazzy Theory:** Kazzy Theory does not adopt the standard approach of deriving the gauge group from  $\text{Isom}(\Sigma_7)$ . Instead, the structure groups of principal bundles  $P^{(n)}(M_3, G_n)$  associated to each dimensional axis  $D_6, D_7, D_8$  are *defined* directly as  $U(1), SU(2), SU(3)$  (Axiom G, Section 3.6). Time is not treated as a coordinate of the internal space but as generated from  $D_5$ . This approach builds the theory from the principle that each dimension carries a definite physical role, rather than from the stabilisation of a specific internal geometry.

## 2.4 Yukawa’s Elementary-Domain Theory and Finite Spacetime Regions

Hideki Yukawa’s elementary-domain theory treats elementary particles not as pointlike objects but as quantum-mechanical objects extended over finite four-dimensional spacetime regions [13, 14, 25, 28]. The theory introduces the four-dimensional elementary domain as a basic unit and interprets elementary particles as sequences of its excited states. Instead of the differential equation of a point-particle wave function, it proposes a difference-type field equation connecting elementary domains separated by finite spacetime displacements, aiming to soften the divergence problems associated with pointlike locality. As another precedent for relaxing the assumption of pointlike continuous spacetime, Snyder’s quantized spacetime [22] provides a useful comparison point for the finite observational domain  $\Omega_K$  introduced below.

**Relation to Kazzy Theory (elementary-domain theory):** Kazzy Theory does not assume elementary-domain theory as a premise. However, if the finite region induced on observational spacetime by the  $D_5$  observation section is defined as the Kazzy effective unit  $\Omega_K$ , then Yukawa’s elementary domain can be reinterpreted as the four-dimensional effective limit of  $\Omega_K$ . In this sense, elementary-domain theory is a precursor programme that is externally incorporable into Kazzy Theory; Section 3.2.2 formulates this relation as finite observational domains.

## 2.5 Comparison Summary

Table 1 summarises the correspondences and differences between KK Theory and the four reference frameworks.

Table 1: Kazzy Theory vs. existing theories: correspondences and differences. “ $\rightarrow$ Kazzy” denotes the Kazzy Theory replacement or extension; “Hyp.” = hypothesis.

Item	GR / TEGR	SM / KK-5D	Elementary-domain theory	Kazzy Theory (this paper)
Spacetime dimension	4D (3+1)	4D effective	finite 4D domains	10D: $M_{10} = M_3 \times \Sigma_7$
Role of time	Coordinate ( $g_{0\mu}$ )	External param.	domain sequence difference equation	$D_5$ projection: $dt = \alpha d\tau$
Gravity description	Curvature (GR) / Torsion (TEGR)	Not included	not central	$D_5$ Torsion (TEGR; established in weak-field limit, Prop. 3.3)
Gauge group origin	—	Isom / assumed	suggested internal degrees of freedom	Fibre bundle struct. groups (Hyp.)
Metric signature	(-, +, +, +)	(-, +, +, +)	finite time domain	Euclidean on $M_{10}$ ; Lorentz induced
Meaning of 3 + 1	Space + time	Space + time	finite time region	$E_4 = M_3 \times C_4$ (space + $\Phi(x)$ )

Items marked “Hyp.” in the Kazzy Theory column lack mathematical proof at this stage. The gravity–Torsion correspondence, marked “established”, relies on the independently proved TEGR equivalence. This distinction is maintained consistently throughout the paper.

### 3 Mathematical Formulation

This chapter presents the rigorous mathematical framework of Kazzy Theory, beginning with the five foundational axioms, followed by the definition of the 10-dimensional manifold, the three fundamental equations, and the force–Torsion correspondences.

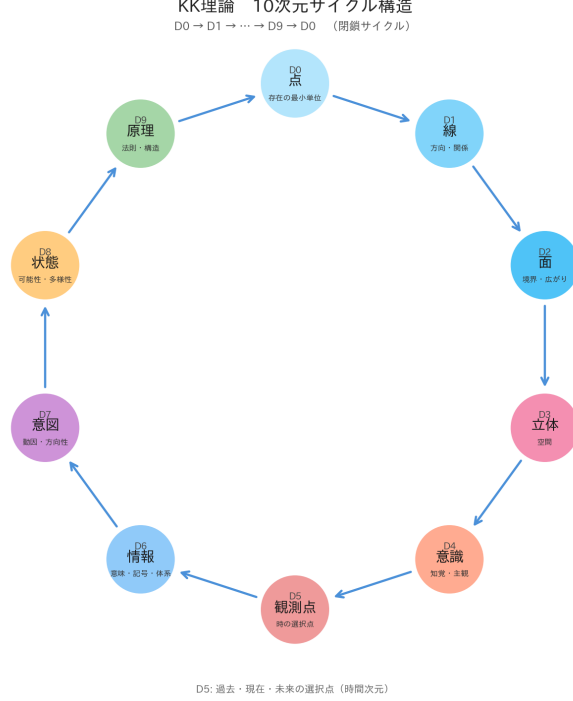


Figure 1: The 10-dimensional closed cycle of Kazy Theory ( $D_0$  through  $D_9$ ). Each dimension carries a specific physical role and a Torsion description. The cycle closes at  $D_9 \rightarrow D_0$ , giving the product structure  $M_{10} = M_3 \times \Sigma_7$ .

### 3.1 Foundational Axioms

Kazy Theory rests on five axioms.

**Axiom 3.1** (Topological origin). All dimensional structures are established with the zero-dimensional point  $D_0$  as the topological base.  $D_0$  is not the temporal “beginning” of the cycle but the compact closure point of the 10-element cycle  $D_0 \rightarrow D_1 \rightarrow \dots \rightarrow D_9 \rightarrow D_0$ . “Higher dimensions unfold from  $D_0$ ” means a logical chain of dimensional structure in topological space, not a temporal sequential generation.

**Axiom 3.2** (Containment). Each dimension  $D_n$  is fully contained in  $D_{n+1}$ . Lower-dimensional phenomena are observed as projections of higher-dimensional structures. Formally, surjective projection maps  $\pi_n : D_{n+1} \rightarrow D_n$  are defined consistently.

**Axiom 3.3** (Cyclic closure). The hierarchy of dimensions forms a closed 10-element cycle:  $D_0 \rightarrow D_1 \rightarrow \dots \rightarrow D_9 \rightarrow D_0$ . The terminal end of the ninth dimension returns to the zeroth, forming a compact cyclic structure. The coordinate index set  $\{0, 1, \dots, 9\}$  is closed under  $\mathbb{Z}/10\mathbb{Z}$ .

**Axiom 3.4** (Projectivity). All physical quantities observed in  $D_3$  are projections of structures defined on  $D_4$ – $D_9$ . No observable can exceed the information content of the higher-dimensional structure from which it is generated.

**Axiom 3.5** (Time generation). Time is not an independent entity. It is generated by the progression of the observation point  $\sigma$  along the  $D_5$  axis. The intrinsic parameter  $\tau$  (proper time) in  $D_5$  generates the observed time  $t$  in  $D_3$  via the transformation equation  $dt = \alpha d\tau$ .

### 3.2 The 10-Dimensional Manifold

The manifold  $M$  of Kazzy Theory is defined as the 10-dimensional differentiable manifold with product structure

$$M_{10} = M_3 \times \Sigma_7, \quad (8)$$

where  $M_3 = (\mathbb{R}^3, g_{\mu\nu})$  is the observable 3-dimensional Riemannian space and  $\Sigma_7$  is a compact 7-dimensional internal manifold spanned by  $D_0$  (the closure circle) and  $D_4$ – $D_9$ . Local coordinates on  $M_{10}$  are

$$x^A = (x^1, x^2, x^3, x^4, x^5, x^6, x^7, x^8, x^9, x^0), \quad A = 0, 1, \dots, 9, \quad (9)$$

where  $x^1, x^2, x^3$  are the spatial coordinates of  $M_3$ ;  $x^4$ – $x^9$  are the internal coordinates of  $\Sigma_7$ ; and  $x^0$  is the cyclic coordinate of  $D_0$ . Table 2 lists the mathematical object, physical role, and Torsion correspondence for each dimension.

Table 2: Full dimensional structure of Kazzy Theory.

Dim.	Name	Mathematical object	Physical role	Torsion
$D_0$	Point	Circle $S^1(R_0)$	Cyclic closure / seed	—
$D_1$	Line	$\mathbb{R}$	Direction, relation	—
$D_2$	Plane	$\mathbb{R}^2$	Boundary, extent	—
$D_3$	Space	Riemannian manifold $(M_3, g)$	Observable physical space	—
$D_4$	Self-ref. field	KK scalar field $\Phi(x)$ (Eq. (SR-1))	Self-identity of all things	—
$D_5$	Obs. point	Bundle section $\sigma$	Time-series selection	Grav
$D_6$	Information	Info. density $\iota(x)$	Meaning, speed-of-light limit	EM
$D_7$	Intent	Vector field $J_I(x)$	Universal directionality	Weak
$D_8$	State	State manifold $\mathcal{S}_8$	All stable states	Strong
$D_9$	Principle	Symmetry group space $\{G_p, \Lambda_p, \alpha_i\} \rightarrow D_0$	Physical laws / constants	Cons

<sup>†</sup>  $D_8$  carries a *triple role*: (a) SU(3) gauge force (local torsion, KWGT); (b) fermion-generation filter ( $\hat{P}_8$ ); (c) geometric origin of gravity via the DP-Chain  $D_8 \rightarrow \dots \rightarrow D_3$  (Remark 3.4).

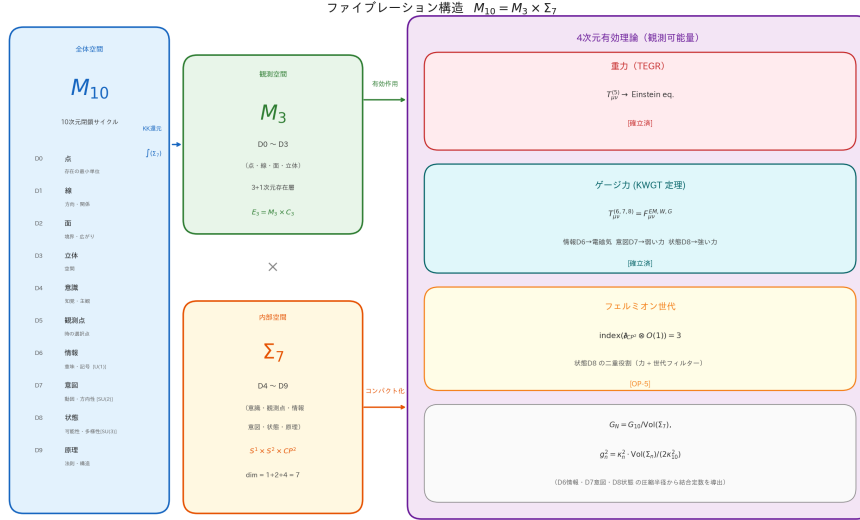


Figure 2: Fibration structure  $M_{10} = M_3 \times \Sigma_7$ . Left: full 10-dimensional space; centre: decomposition into observable space  $M_3$  and internal space  $\Sigma_7 = S^1 \times S^2 \times CP^2$ ; right: 4D effective theory after KK reduction (gravity, gauge forces, fermion generations, and physical constants).

### 3.2.1 The 3+1-dimensional Existence Layer $E_4$

In Kazzy Theory, “3+1 dimensions” is not the Minkowski spacetime of GR (3 spatial dimensions plus time) but is defined as the ontological 4-dimensional existence layer:

$$E_4 = M_3 \times C_4, \quad (10)$$

where  $M_3$  is the observable 3-dimensional space ( $D_1, D_2, D_3$ ) and  $C_4$  is the 1-dimensional internal space in which the observation weight field is defined ( $D_4$ ). Each entity  $e$  is written

$$e = (x, \Phi(x)), \quad x \in M_3, \quad \Phi(x) \in C_4, \quad (11)$$

with normalisation condition  $\int_{M_3} \omega(x) dV = 1$ .

#### Observation section and pullback bundle (formal precision).

**Definition 3.1** (Observation section  $\sigma$ ). The observation section  $\sigma : M_3 \rightarrow M_{10}$  is defined as the smooth embedding that maps each  $x \in M_3$  to a point  $\sigma(x) = (x, p_0, p_5(x)) \in M_3 \times \Sigma_7$ , where  $p_0 \in S_0^1$  is a reference point of the  $D_0$  coordinate and  $p_5(x) \in S_5^1$  represents the location of the observation point in  $D_5$  (time-generating coordinate, monotonically increasing).

**Definition 3.2** (Pullback bundle  $E_4$ ).  $E_4$  is defined as the pullback by  $\sigma$  of the sub-bundle

of  $TM_{10}$  in directions  $D_1, D_2, D_3, D_4$ :

$$E_4 := \sigma^* \left( T_{(D_1, D_2, D_3, D_4)} M_{10} \right) \rightarrow M_3. \quad (12)$$

The induced Euclidean metric on  $E_4$  is

$$(\sigma^* G)_{ij} = G_{AB} \frac{\partial \sigma^A}{\partial x^i} \frac{\partial \sigma^B}{\partial x^j} \rightarrow \text{diag}(1, 1, 1, R_4^2), \quad i, j = 1, 2, 3, 4. \quad (13)$$

When  $\sigma$  is combined with the  $D_5$  motion  $dt = \alpha d\tau$ , an effective Lorentz metric is induced on  $E_4$ :

$$g_{\alpha\beta}^{\text{obs}} dx^\alpha dx^\beta = -dt^2 + G_{ij}(x) dx^i dx^j, \quad (\text{Lorentz signature}) \quad (14)$$

so that

$$E_4 = \{(x, C(x), g_{\mu\nu}^{\text{obs}}(x)) \mid x \in M_3, C(x) \in [0, 1], \alpha(x) > 0\}. \quad (15)$$

*Remark 3.1* ( $D_5$  as Time Dimension:  $M_3 \rightarrow 3+1\text{D}$  Lorentzian Spacetime). The spatial manifold  $M_3$  (dimensions  $D_0$ – $D_3$ ) is *three-dimensional* by construction. General Relativity requires a *four-dimensional* Lorentzian spacetime  $M_4$  with metric signature  $(-, +, +, +)$ . In KKT, the fourth (time) dimension is supplied by the  $D_5$  observation point via the following mechanism:

- (i)  **$D_5$  fiber as time circle.** The  $D_5$  dimension corresponds to the  $S^1$  factor in  $\Sigma_7 = S^1 \times S^2 \times CP^2$ . The angular coordinate  $\theta^{(5)} \in [0, 2\pi)$  parametrises the observer's position in the space of possible observations.
- (ii) **Time as  $D_5$  progression.** The physical time coordinate  $t$  is generated by monotonic progression along  $D_5$ :  $dt = \alpha d\tau$  (Axiom 5; explicit form: eq. (40)). The arrow of time is the arrow of increasing  $\theta^{(5)}$ .
- (iii) **Observable 4D spacetime.** The observable spacetime is

$$M_4^{\text{obs}} := M_3 \times_{D_5} \mathbb{R}_\tau, \quad (16)$$

the product of the spatial 3-manifold  $M_3$  with the time line generated by  $D_5$  progression. The effective metric on  $M_4^{\text{obs}}$  is  $g_{\alpha\beta}^{\text{obs}}$  (14), which has Lorentz signature  $(-, +, +, +)$  because the  $D_5$  time direction contributes  $-dt^2$  (see also Proposition 3.20).

**Consistency with GR.** After KK reduction  $G_N = G_{10}/\text{Vol}(\Sigma_7)$ , the effective action on  $M_4^{\text{obs}}$  takes the TEGR form in the weak-field limit (Proposition 3.3), classically equivalent to the Einstein–Hilbert action. Full 4D diffeomorphism invariance on  $M_4^{\text{obs}}$  is preserved because the  $D_5$  foliation is compatible with the KK zero-mode truncation.

### 3.2.2 Finite Observational Domains $\Omega_K$ and the Elementary-Domain Limit

**Definition 3.3** (Kazzy elementary domain). Fix an observation section  $\sigma$  and a  $D_5$  leaf  $\Sigma_\tau$ . The smallest effective region resolvable by an observer on  $D_3$  is defined as

$$\Omega_K(x, \tau) := B_{\Sigma_\tau}^{g_{\text{obs}}}(x; \ell_K), \quad \ell_K^2 := \ell_P^2 + \sum_{n \in \{0,4,5,6,7,8,9\}} w_n(\sigma, \tau) R_n^2, \quad (17)$$

where  $B_{\Sigma_\tau}^{g_{\text{obs}}}(x; \ell_K)$  is the geodesic ball of radius  $\ell_K$  measured by the observational metric, and  $0 \leq w_n \leq 1$  are dimensionless weights describing how strongly the observation section resolves each internal direction.

The effective field over  $\Omega_K$  is not a point value but a domain average:

$$\bar{\Psi}_{\Omega_K}(\tau) := \frac{1}{\text{Vol}(\Omega_K)} \int_{\Omega_K(x, \tau)} \Psi(x', \tau) dV_{g_{\text{obs}}}. \quad (18)$$

Thus the point-particle description is pushed into the formal limit  $\ell_K \rightarrow 0$ , while the operational theory treats excitations over finite regions as primary.

**Correspondence Hypothesis ED (elementary-domain theory  $\leftrightarrow$  finite observational domains):** In the low-energy limit where the radii  $R_n$  are frozen at observational scales and the  $D_5$  foliation  $\Sigma_\tau$  advances monotonically, the sequence

$$\Omega_K(x, \tau) \rightarrow \Omega_K(x + \Delta x, \tau + \Delta \tau)$$

reproduces the finite-difference connection between spacetime domains in Yukawa's elementary-domain theory. Elementary particles therefore correspond, in Kazzy Theory, to local excitations of Torsion/KK modes over  $\Omega_K$ , and in Yukawa's theory to excited states of four-dimensional elementary domains. This is currently an effective limit hypothesis; a rigorous derivation from the difference equation to the effective field equation remains future work.

### 3.2.3 $D_4$ : Integrated Information Field and the Universal Self-Reference Principle

This section rigorously formulates the physical content of the  $D_4$  dimension, starting from the self-reference principle “every thing is that thing.”

**Postulate SR (Universal Self-Reference Principle):** For every point  $X \in M_{10}$ , every material configuration carries an intrinsic self-referential property independent of any external observer  $\sigma$ . There exists a real scalar field  $\Phi$  (the *integrated information field*) measuring the strength of this property, with  $\Phi(X) > 0$  for all material

configurations.  $\Phi(X)$  measures “how self-identically integrated an entity is.”

As a precedent for reinterpreting spacetime equations through informational or thermodynamic quantities, Jacobson derived the Einstein equation as an equation of state from local Rindler-horizon thermodynamics [11]. Kазzy Theory does not assume that derivation, but it is an important comparison case for the idea that observer-dependent or information-bearing quantities can give rise to geometry.

Mathematically,  $\Phi(x)$  is defined as the KK scalar field arising from the  $D_4$  compactification (a circle  $S_4^1$  of radius  $R_4$ ):

$$\Phi(x) := R_4^{-1} \int_{S_4^1} \sqrt{G_{44}(x, y^4)} dy^4, \quad (D_4 \text{ zero mode}) \quad (\text{SR-1})$$

Integrating the KK-Weitzenböck action over  $\Sigma_7$  yields the effective 4D action for the  $D_4$  scalar:

$$S_\Phi = \int d^4x \sqrt{g} \left[ -\frac{1}{2} (\partial_\mu \Phi)^2 - V(\Phi) - \xi \Phi T^{(m)} \right], \quad (\text{SR-2})$$

where  $V(\Phi) = \frac{1}{2} m_\Phi^2 \Phi^2 + \lambda \Phi^4/4$  and  $T^{(m)}$  is the trace of the matter energy-momentum tensor. Expanding around the vacuum expectation value  $\Phi_\infty$ :

$$\Phi(x) = \Phi_\infty + \varphi(x), \quad \Phi_\infty = \sqrt{-\mu^2/\lambda} \quad (\text{SSB case}). \quad (\text{SR-3})$$

**Correspondence with IIT.** Tononi’s Integrated Information Theory [23, 24] defines the integrated information  $\Phi_{\text{IIT}}$  as the degree to which a system’s causal structure exceeds the sum of its parts. Kазzy Theory proposes:

**Correspondence Hypothesis D4 (KK integrated information field  $\leftrightarrow$  IIT):**

Between the IIT integrated information  $\Phi_{\text{IIT}}[\mathcal{S}(x)]$  of the material configuration  $\mathcal{S}(x)$  at each point  $x \in M_3$  and the KK field  $\Phi(x)$ :

$$\Phi(x) = \Phi_\infty + \gamma \cdot \Phi_{\text{IIT}}[\mathcal{S}(x)]/\Phi_{\text{ref}}, \quad (19)$$

where  $\gamma$  is a dimensionless coupling constant and  $\Phi_{\text{ref}}$  is a reference scale. Rigorous formulation requires connecting the continuum limit of IIT [24] with the KK scalar effective field theory; this is a current proposal, not a proved result.

**Modification of the Born rule.** Let  $\{E_i\}_i$  be a POVM (positive-operator-valued measure) describing observable  $O$ :  $E_i \geq 0$ ,  $\sum_i E_i = \mathbf{1}$ . For  $\xi \neq 0$ , spatial variation of  $\Phi(x)$  modulates the *detection rate* for measurement outcome  $o_i$ :

$$P_\Phi(o_i | \psi, x) = \langle \psi | E_i | \psi \rangle \cdot [1 + \delta_s(x)], \quad (\text{SR-4})$$

$$\delta_s(x) := \xi' \varphi(x)/\Phi_\infty, \quad (\text{SR-5})$$

where  $\xi' = 2\xi$ .

*Remark 3.2* (Born rule: formula, normalisation, and status). **(i) Corrected formula.**

An earlier expression used  $|\langle\psi|\hat{O}|\psi\rangle|^2$ , the squared expectation value, which is *not* the Born-rule probability for a specific outcome  $o_i$ ; it equals  $\langle\psi|E_i|\psi\rangle$  only in the special case  $E_i = |\psi\rangle\langle\psi|$  (rank-one projector onto  $|\psi\rangle$  itself). The correct form is  $\langle\psi|E_i|\psi\rangle$ , which reduces to  $|\langle o_i|\psi\rangle|^2$  for projective measurements  $E_i = |o_i\rangle\langle o_i|$ .

**(ii) Normalisation and interpretation.** Since  $\delta_s(x)$  is the same for all outcomes  $i$ ,

$$\sum_i P_\Phi(o_i | \psi, x) = \left[ \sum_i \langle\psi|E_i|\psi\rangle \right] \cdot [1 + \delta_s(x)] = 1 + \delta_s(x) \neq 1. \quad (20)$$

The modified probabilities therefore describe a *location-dependent observation rate*: the  $\Phi$  field shifts how many measurement events occur at  $x$ , not the relative probabilities among outcomes. The correctly normalised conditional probability for outcome  $o_i$  at  $x$  is

$$\tilde{P}(o_i | \psi, x) = \frac{\langle\psi|E_i|\psi\rangle \cdot [1 + \delta_s(x)]}{\sum_j \langle\psi|E_j|\psi\rangle \cdot [1 + \delta_s(x)]} = \langle\psi|E_i|\psi\rangle, \quad (21)$$

which is the standard Born rule. A non-trivial modification of relative outcome probabilities within KKT would require  $\delta_s$  to depend on the outcome index  $i$  (coupling the  $\Phi$  field to the specific measurement eigenbasis); deriving such a coupling from the KKT action is an open task.

**(iii) Standard QM limit.** In the limit  $\Phi(x) \approx \Phi_\infty$  (uniform field),  $\delta_s \rightarrow 0$ , the detection rate becomes position-independent, and standard quantum mechanics is recovered.

**Proposition 3.1** (Identification of  $C(x)$  with  $\Phi(x)$ ). The observation weight field  $C(x) \in [0, 1]$  appearing in the fundamental equations (39) is identified with the normalised probability density of the  $D_4$  KK scalar field:

$$C(x) = \frac{\Phi(x) - \Phi_{\min}}{\int_{M_3} [\Phi(x') - \Phi_{\min}] dV'}, \quad \int C dV = 1, \quad (22)$$

where  $\Phi_{\min} = \hbar/(m_{\text{Pl}} c)$  is the quantum-gravity lower bound.

**Decoherence.** The decoherence rate  $\Gamma_{\text{dec}}$  (the rate at which quantum superpositions collapse) is proportional to the  $\Phi$  field:

$$\Gamma_{\text{dec}}(x) \propto \xi \Phi(x) \rho_{\text{mat}}(x). \quad (\text{SR-6})$$

Regions with larger  $\Phi(x)$  exhibit faster collapse of quantum superpositions, providing a field-theoretic account of the emergence of classical definiteness. A rigorous correspondence with Zurek's einselection theory [29] is a future task.

### 3.2.4 Internal Manifold $\Sigma_7$ : the $T^7$ Candidate

**Definition 3.4** ( $T^7$  candidate for  $\Sigma_7$ ). As the first concrete candidate, define

$$\Sigma_7 = T^7 = S_0^1 \times S_4^1 \times S_5^1 \times S_6^1 \times S_7^1 \times S_8^1 \times S_9^1, \quad (23)$$

where each factor  $S_n^1$  is a circle of radius  $R_n$ . The metric on  $T^7$  is

$$G_{mn}^{(\Sigma)} = \text{diag}(R_0^2, R_4^2, R_5^2, R_6^2, R_7^2, R_8^2, R_9^2). \quad (24)$$

The full Euclidean metric on  $M_{10} = M_3 \times T^7$  (zero-mode approximation) is

$$G_{AB} = \text{diag}(1, 1, 1, R_0^2, R_4^2, R_5^2, R_6^2, R_7^2, R_8^2, R_9^2). \quad (25)$$

The volume of  $T^7$  is

$$\text{Vol}(\Sigma_7) = (2\pi)^7 R_0 R_4 R_5 R_6 R_7 R_8 R_9. \quad (26)$$

Standard KK reduction gives the relation between the 10D and 4D Newton constants:

$$G_4 = \frac{G_{10}}{\text{Vol}(\Sigma_7)}. \quad (27)$$

In the equal-radius approximation ( $R_n \approx \ell_P$  for all  $n$ ), one finds  $R_c \sim \ell_P$ , consistent with the Eöt-Wash constraint (additional dimension radius  $< 10 \mu\text{m}$ ) [1].

The KK mass spectrum for excitation mode numbers  $\{n_k\}$  is

$$m_{\text{KK}}^2 = \sum_k \frac{n_k^2}{R_k^2} \cdot \frac{\hbar^2}{c^2}. \quad (28)$$

For  $R_k \approx \ell_P$ , the lightest KK excitation has mass  $\sim M_P \approx 2.18 \times 10^{-8} \text{ kg}$ , far above LHC energies, consistent with the Eöt-Wash upper bound on  $\beta_{\text{KK}}$ .

**Limitation of  $T^7$ .**  $T^7$  has holonomy group  $\text{Hol}(T^7) = \{1\}$  (trivial; flat torus). Its isometry group is  $U(1)^7$ , not  $U(1) \times SU(2) \times SU(3)$ . Hence the SM gauge group cannot be derived from  $\text{Isom}(T^7)$ . A richer candidate,  $\Sigma_7^{\text{SM}} = S^1 \times S^2 \times CP^2$  (Section 3.11.5), addresses this; see Axiom G and open problem OP-1.

## 3.3 The Ninth Dimension: Principle Space

The ninth dimension  $D_9$  is defined as the space of admissible symmetry structures governing all dimensions  $D_8$  and below. A point  $p \in D_9$  specifies

$$p \equiv (G_p, \Lambda_p, \{\alpha_i(p)\}), \quad (29)$$

where  $G_p$  is the Lie group acting on the state manifold  $D_8$ ,  $\Lambda_p$  is the cosmological constant, and  $\{\alpha_i(p)\}$  is the complete set of dimensionless physical constants of the universe determined by  $p$ .

By Axiom 2 (containment), if  $p$  changes in  $D_9$  then all dimensions  $D_8$ – $D_3$  change in consequence. The Torsion  $T^{(9)}$  of the  $D_9$  connection encodes spacetime variation of physical constants. In our universe  $T^{(9)} \approx 0$ , recovering the standard physics of fixed constants. Regions with  $T^{(9)} \neq 0$  correspond in principle to observable drift of the fine structure constant  $\alpha$ .

### 3.3.1 Principal Bundle Structure of $D_9$

$D_9$  is formalised as the principal fibre bundle  $P(D_8, G_9, \pi)$  with base  $D_8$  (state manifold), structure group  $G_9$  (admissible symmetry group space), and projection  $\pi : D_9 \rightarrow D_8$ . In our universe,

$$G_9 \supset G_{\text{SM}} = \text{U}(1) \times \text{SU}(2) \times \text{SU}(3). \quad (30)$$

The Lie algebra-valued 1-form  $\omega^{(9)} \in \Omega^1(D_9, \mathfrak{g}_9)$  defines the connection on  $P$ . The Cartan structure equations are

$$T^{(9)} = d\theta^{(9)} + \omega^{(9)} \wedge \theta^{(9)}, \quad (31)$$

$$\Omega^{(9)} = d\omega^{(9)} + \frac{1}{2}[\omega^{(9)} \wedge \omega^{(9)}], \quad (32)$$

with Bianchi identities  $DT^{(9)} = \Omega^{(9)} \wedge \theta^{(9)}$  and  $D\Omega^{(9)} = 0$ .

### 3.3.2 $D_9 \rightarrow D_8$ Induced Structure: Canonical Bundle Identification

The abstract principal bundle of  $D_9$  over  $D_8$  (Section above) can be made fully explicit using the coset identification  $CP^2 \cong \text{SU}(3)/\text{U}(2)$  established in Proposition 3.10 (eq. (106)).

**Proposition 3.2** ( $D_9 \rightarrow D_8$  Canonical Bundle Map). Identify the state manifold with  $D_8 \cong CP^2 = \text{SU}(3)/\text{U}(2)$ . The principle space  $D_9$  is then identified with the total space of the canonical coset fibration

$$\text{U}(2) \hookrightarrow \text{SU}(3) \xrightarrow{\pi_{9/8}} \text{SU}(3)/\text{U}(2) \cong CP^2 \cong D_8, \quad (33)$$

so that  $D_9 \cong \text{SU}(3)$  as a principal  $\text{U}(2)$ -bundle over  $D_8$ .

The canonical connection on this bundle is the  $\mathfrak{u}(2)$ -component of the Maurer–Cartan form  $\omega_{\text{MC}} \in \Omega^1(\text{SU}(3), \mathfrak{su}(3))$ :

$$\omega^{(9)} := \text{pr}_{\mathfrak{u}(2)}(\omega_{\text{MC}}) \in \Omega^1(\text{SU}(3), \mathfrak{u}(2)), \quad (34)$$

where  $\text{pr}_{\mathfrak{u}(2)} : \mathfrak{su}(3) \rightarrow \mathfrak{u}(2)$  is the orthogonal projection with respect to the Killing form.

Under the Lie algebra decomposition  $\mathfrak{su}(3) = \mathfrak{u}(2) \oplus \mathfrak{m}$  (with  $\mathfrak{m} \cong \mathbb{C}^2 \cong T_{[e]}CP^2$ ), the Maurer–Cartan form splits as  $\omega_{\text{MC}} = \omega^{(9)} + \theta^{(9)}$ , where  $\theta^{(9)} \in \Omega^1(\text{SU}(3), \mathfrak{m})$  is the horizontal (soldering) component.

The curvature of  $\omega^{(9)}$  is

$$\Omega^{(9)} = d\omega^{(9)} + \frac{1}{2}[\omega^{(9)} \wedge \omega^{(9)}] = -\omega_{\text{FS}} \otimes \text{id}_{\mathfrak{u}(2)}, \quad (35)$$

where  $\omega_{\text{FS}}$  is the Fubini–Study 2-form on  $CP^2$ , the generator of  $H^2(CP^2, \mathbb{Z}) \cong \mathbb{Z}$ .

*Proof.* The coset fibration (33) is a standard result in differential geometry; see Kobayashi–Nomizu [16], Ch. X, §11. The Lie algebra decomposition  $\mathfrak{su}(3) = \mathfrak{u}(2) \oplus \mathfrak{m}$  is the reductive splitting of the symmetric pair  $(\text{SU}(3), U(2))$ , with  $\text{ad}(U(2))$ -invariance of  $\mathfrak{m}$ . The connection  $\omega^{(9)} = \text{pr}_{\mathfrak{u}(2)}(\omega_{\text{MC}})$  is  $U(2)$ -equivariant by construction. The curvature computation follows from the Maurer–Cartan equation  $d\omega_{\text{MC}} + \frac{1}{2}[\omega_{\text{MC}} \wedge \omega_{\text{MC}}] = 0$  restricted to the  $\mathfrak{u}(2)$  component:  $\Omega^{(9)} = -[\theta^{(9)} \wedge \theta^{(9)}]_{\mathfrak{u}(2)}$ , which is the Fubini–Study form up to normalisation.

*Note on  $\mathbb{Z}_3$  lifting.* The isometry group of  $CP^2$  is  $\text{PU}(3) = \text{SU}(3)/\mathbb{Z}_3$  (as discussed in Remark 3.6). Proposition 3.2 uses the lift to the fundamental representation  $\mathbb{C}^3$ , which requires choosing a  $\mathbb{Z}_3$ -framing supplied by the KK–Weitzenböck coframe construction (Section 3.9.7). The lift is an additional structure input, not automatic from the coset alone.  $\square$

**Corollary 3.1** ( $D_9$  Acts on  $D_8$  via the Standard Representation). *The left multiplication of  $\text{SU}(3)$  on itself descends to the canonical transitive action on the coset space:*

$$\text{act} : \text{SU}(3) \times CP^2 \rightarrow CP^2, \quad (g, [z]) \mapsto [gz], \quad (36)$$

where  $[z] \in CP^2$  denotes the equivalence class of  $z \in \mathbb{C}^3 \setminus \{0\}$ . This action is transitive with isotropy group  $U(2) \subset \text{SU}(3)$ , realising  $CP^2 \cong \text{SU}(3)/U(2)$ .

*Physical interpretation.* This is the precise mathematical content of the statement “ $D_9$  governs  $D_8$ ”: the principle space  $D_9 \cong \text{SU}(3)$  acts on the state manifold  $D_8 \cong CP^2$  by the standard  $\text{SU}(3)$  representation. Every point of  $D_8$  is reachable from every other by an element of  $D_9$  (transitivity), so  $D_9$  parametrises the full space of  $D_8$  configurations. A choice of physical vacuum corresponds to a choice of base-point  $[z_0] \in CP^2$ ; the isotropy subgroup  $U(2)$  stabilising  $[z_0]$  is the residual symmetry after “selecting” a state in  $D_8$ .

*Remark 3.3* (Holonomy, constancy of physical constants, and the Bianchi identity). By the Ambrose–Singer theorem [16], the holonomy group of the canonical connection  $\omega^{(9)}$  at any point of  $CP^2$  is exactly  $U(2)$ , generated by the curvature (35). This is *not* in conflict with the cyclic closure condition  $H = \hat{1}$  (eq. (37)): that condition applies to parallel transport around the full 10-dimensional cycle  $D_0 \rightarrow \cdots \rightarrow D_9 \rightarrow D_0$ , not to loops within the single fibre  $CP^2 \subset D_8$ .

The condition  $T^{(9)} \approx 0$  (constancy of physical constants in our universe, eq. (31)) is *distinct* from the curvature  $\Omega^{(9)} \neq 0$ : torsion measures the failure of the soldering form to close ( $T^{(9)} = d\theta^{(9)} + \omega^{(9)} \wedge \theta^{(9)}$ ), while curvature measures the non-commutativity of horizontal lifts. The Bianchi identity  $D\Omega^{(9)} = 0$  is satisfied identically for the Maurer–Cartan connection. Hence the non-trivial holonomy  $U(2)$  of  $\omega^{(9)}$  encodes the non-trivial gauge structure of the SM (specifically,  $U(2) \supset U(1)_Y \times SU(2)_L$  upon restriction), while  $T^{(9)} \approx 0$  separately encodes the observed near-constancy of the fundamental constants  $\{\alpha_i\}$ .

### 3.3.3 Holonomy and Anomaly Cancellation (Conjecture)

The cyclic closure  $D_9 \rightarrow D_0$  (Axiom 3) requires, for the parallel transport holonomy around the dimensional cycle:

$$H = \mathcal{P}\exp\left(\oint_{\text{cycle}} \omega^{(9)}\right) = \hat{1} \quad (\text{required: unit element of } G_9). \quad (37)$$

This condition is the self-consistency requirement that physical laws do not change under one traversal of the dimensional cycle.

**Conjecture 3.1.** When the cyclic closure axiom holds (i.e.,  $H = \hat{1}$ ), the holonomy condition on  $\omega^{(9)}$  implies the SM gauge anomaly cancellation conditions  $\sum_f Q_f^3 = 0$  and  $\sum_f Q_f = 0$ . Proof requires three steps: (i) showing  $H = \hat{1}$  implies  $\int \text{ch}_3 = 0$ ; (ii) connecting this characteristic class condition to BRST cohomology of the embedded gauge bundle; (iii) reducing the BRST condition to the charge algebra. All three steps are unproved and constitute a future research programme.

## 3.4 Dimensional Cycle Closure

By Axiom 3, the cycle closes:

$$D_9 \rightarrow D_0 \quad (10\text{-element cycle; index set } \{0, \dots, 9\} \text{ closed under } \mathbb{Z}/10\mathbb{Z}). \quad (38)$$

The index set forms the finite cyclic group  $\mathbb{Z}/10\mathbb{Z}$  (embedded as the 10th roots of unity in  $S^1$ ). This closure is the geometric origin of the finiteness of fundamental force types (gravity, EM, weak, strong, constant variation)—five axes  $D_5$ – $D_9$  carry forces, and the closure makes this count finite.

## 3.5 The Three Fundamental Equations

All observable physics in Kazzy Theory is derived from three master equations.

### 3.5.1 Time Generation Equation

Observed time  $t$  in  $D_3$  is generated by the motion of observation point  $\sigma$  along the  $D_5$  axis:

$$dt = \alpha(\Phi, C, \Gamma) d\tau. \quad (39)$$

Here  $\Phi = \Phi(x)$  is the state-selection potential (KK gravitational potential),  $C = C(x)$  is the observation weight field ( $D_4$  coupling proxy; identified with Proposition 3.1), and  $\Gamma = \Gamma(x)$  is the gravitational structure density. In the weak-gravity, low- $\Phi$  first-order approximation:

$$\alpha(M_{\text{KK}}, C) \approx 1 + \tilde{\mu} M_{\text{KK}} + \tilde{\nu} C \quad (\text{vacuum: } \tilde{\nu} = 0), \quad (40)$$

with  $M_{\text{KK}} = \int \Gamma(x) d^3x$ . Matching to the Schwarzschild factor:

$$\alpha(M_{\text{KK}}, r) = \left(1 - \frac{2G_K M_{\text{KK}}}{c^2 r}\right)^{-1/2} \xrightarrow{M_{\text{KK}} \rightarrow M} \left(1 - \frac{2GM}{c^2 r}\right)^{-1/2}. \quad (41)$$

### 3.5.2 $D_5$ Foliation and Resolution of the Circularity in the Time Definition

Equation (39) appears circular:  $\alpha$  requires  $M_{\text{KK}} = \int \Gamma d^3x$ , but integrating over “a spatial slice at one moment” seems to require time  $t$ . This is resolved by defining the  $D_5$  foliation *without* reference to  $t$ :

$$\Sigma_\tau := \{X \in M_{10} \mid x^5(X) = \tau\} \cong M_3, \quad (42)$$

where  $\tau$  is the  $D_5$  intrinsic parameter (fundamental variable, prior to time). Then

$$M_{\text{KK}}(\tau) := \int_{\Sigma_\tau} \Gamma(x, \tau) d\text{Vol}_{\Sigma_\tau}, \quad d\text{Vol}_{\Sigma_\tau} = \sqrt{\det h_{ij}(x, \tau)} dx^1 dx^2 dx^3, \quad (43)$$

involves no  $t$ . The definition sequence is: (1)  $\tau \in \mathbb{R}$ :  $D_5$  intrinsic parameter; (2)  $\Sigma_\tau \cong M_3$ : spatial leaf at  $\tau$ ; (3)  $\Gamma(x, \tau)$ : field on  $\Sigma_\tau$ ; (4)  $M_{\text{KK}}(\tau)$ : functional of  $\tau$ ; (5)  $\alpha(\tau)$ : function of  $\tau$ ; (6)  $t(\tau) = \int_{\tau_0}^\tau \alpha(\tau') d\tau'$ : observed time, defined last.

$$t(\tau) := \int_{\tau_0}^\tau \alpha(\tau') d\tau'. \quad (44)$$

Time  $t$  appears only in step (6); steps (1)–(5) are  $t$ -independent, so there is no circularity. This places the construction near the Page–Wootters mechanism [20], where observed evolution is described through correlations with an internal clock rather than by taking external time as a primitive variable.

**ADM decomposition analogy:** In the ADM formulation of GR, spacetime is

foliated by spatial hypersurfaces  $\Sigma_t$  ( $t = \text{const}$  slices) with lapse  $N$ :  $ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$ . The KK correspondence is:  $\Sigma_\tau \leftrightarrow \Sigma_t$  (ADM hypersurface);  $\alpha(\tau) \leftrightarrow N$  (lapse); Eq. (44)  $\leftrightarrow$  proper-time/coordinate-time relation  $d\tau = N^{-1} dt$ . The logic “ $\tau$  is fundamental;  $t$  is derived” mirrors the ADM logic “spatial slice shape is first; time coordinate follows.”

### 3.5.3 Gravitational Source Definition Equation

In Kazzy Theory, the gravitational structure density  $\Gamma(x)$  replaces the simple mass density:

$$\Gamma(x) = \rho_m(x) \cdot [1 + \delta_s(x)], \quad [\text{units: kg m}^{-3}], \quad (45)$$

where  $\rho_m(x)$  is the ordinary Newtonian mass density and  $\delta_s(x)$  is a dimensionless structural correction:

$$\delta_s(x) = C_s(x) \cdot \Lambda_{\text{conn}}(x) - 1. \quad (46)$$

Here  $C_s \approx E_{\text{bind}}/(Mc^2)$  encodes internal binding energy and  $\Lambda_{\text{conn}} = \Gamma_{\text{micro}} \cdot \Gamma_{\text{macro}}$  encodes order and connectivity. In the limit  $C_s \rightarrow 1$ ,  $\Lambda_{\text{conn}} \rightarrow 1$ :  $\Gamma \rightarrow \rho_m$  (Newtonian limit guaranteed).

The order parameter is

$$Q_{\text{order}} = \exp(-\Delta S_{\text{config}}/k_B), \quad (47)$$

where  $\Delta S_{\text{config}}$  is the configurational entropy relative to a perfect crystal (measurable via X-ray diffraction or neutron scattering). The KK mass is

$$M_{\text{KK}}(V) = \int_V \Gamma(x) d^3x, \quad M_{\text{KK}} \rightarrow M \text{ as } C_s \rightarrow 1, \Lambda_{\text{conn}} \rightarrow 1. \quad (48)$$

**Equivalence principle consistency.** Since both gravitational and inertial masses are proportional to  $\Gamma$ ,

$$m_g \propto \Gamma \cdot V, \quad m_i \propto \Gamma \cdot V, \quad (49)$$

the free-fall acceleration

$$a = F/m_i = G_K \Gamma_{\text{source}}/r^2 \quad (50)$$

is independent of the internal structure of the falling body. Kazzy Theory preserves the equivalence principle exactly at the macroscopic level.

The residual coupling to the structural field  $\chi$  gives the Eötvös parameter

$$\eta_{AB} \approx \lambda_{\text{eff}} [\langle \delta_s \rangle_A - \langle \delta_s \rangle_B] e^{-r/\ell_\chi}, \quad (51)$$

consistent with the Eöt-Wash constraint  $|\eta_{AB}| < 1.8 \times 10^{-13}$  [21] provided  $\ell_\chi \lesssim 1$  cm, implying a mediating scalar mass  $m_\chi \gtrsim 20 \mu\text{eV}/c^2$ .

### 3.5.4 Information 4-Current and the Speed-of-Light Limit

The information 4-current is defined as

$$J_I^\mu = (\iota, j_I^i/c), \quad \mu = 0, 1, 2, 3, \quad (52)$$

where the information density is

$$\iota(x) = -k_I \rho_m(x) \ln(\rho_m(x)/\rho_{\text{ref}}), \quad [\text{units: m}^{-3}; k_I \in [\text{m}^3 \text{kg}^{-1}]], \quad (53)$$

with  $\rho_{\text{ref}}$  a reference density that renders the logarithm dimensionless. The causality condition on the  $D_6$  information axis gives

$$J_I^\mu J_{I\mu} = -\iota^2 + |j_I|^2/c^2 \leq 0 \implies |j_I(x)|/\iota(x) \leq c, \quad (54)$$

reproducing the speed-of-light limit as a consistency condition on the information 4-current, rather than deriving  $c$  from scratch.

## 3.6 The Four Forces and Dimensional Torsion

Kazzy Theory hypothesises that each of the four fundamental forces corresponds to the geometric torsion of the connection along a specific dimensional axis in  $M_{10}$ .

力と次元と Torsion 対応 (KK理論の核心主張)						
力	対応する次元	ゲージ群	Torsion 表現	対応する場	status	実験的制約
重力 (TEGR)	D5 観測点 (DP-Chain 観測点) 観測的現れ	-	$T_{\mu\nu}^{(5)}$ (TEGR Torsion)	ネトラード $e_{\mu}^a, M$	確立済	重力波: GW検出 (LIGO 2016)
電磁気力	D6 情報 (意味・記号体系)	U(1)	$T_{\mu\nu}^{(6)} = F_{\mu\nu}^a$	コフレーム $e_{\mu}^a$	確立済 [KWGT]	Eevis $< 10^{-11}$ , $m_e \geq 20 \mu\text{eV}/c^2$
弱い力	D7 空間 (相対・方向性)	SU(2)	$T_{\mu\nu}^{(7)} = W_{\mu\nu}^a$	コフレーム $e_{\mu}^a$	確立済 [KWGT]	精密電弱 (LEP/SLD)
強い力 + 重力起源	D8 状態 (可能性・多様性) [三重複製]	SU(3)	$T_{\mu\nu}^{(8)} = G_{\mu\nu}^a$ + DP-Chain 起点	コフレーム $e_{\mu}^a$ + 可能性場 $\Pi$	確立済 → 提案中	格子QCD; $\alpha_s(M_Z) = 0.118$ → 重力起源(前)
物理定数 変動	D9 原理 (法則・構造)	-	$T^{(9)}$ (定数の時間変動)	$Z_I$ 体積縮小	提案中	Webb+2011 + 将来観測

【重力の2層構造】  
 (I) D5 (観測点): TEGR torsion  $T^{(5)}$ の観測的現れ (確立済) → DP-Chainの観測点として時間軸で可能性が確定する段階  
 (II) D8 (状態): DP-Chain (D8→D7→D6→D5→D4→D3) を通じた重力の幾何学的起源 (観測済) → 可能性の準備がなされ生成し D3 空間に重力として投影される  
 (III) D6 (情報): SU(1)の場 (SU(1)の場) は KWGT 演算により確立して確立済。重力起源は質量生成の場であり、SU(3)との多重なし。  
 確立済(KWGT) : Torsion-Gauge対応  $T^{(6)} \sim \text{Tr}(F_{\mu\nu}^a)^2$  (m=6, 7, 8) は Maurer-Cartan 幾何から導出済 | KK-Weitzenböck 対応  $\rightarrow e_{\mu}^a \rightarrow e_{\mu}^b \rightarrow e_{\mu}^c \rightarrow e_{\mu}^d$  (2+10D)

Figure 3: Correspondence between the four fundamental forces and dimensional axes / Torsion in Kazzy Theory. Gravity exhibits a two-layer structure:  $D_5$  (Observation Point) is the observational manifestation of TEGR torsion (established), while  $D_8$  (State / Possibility) is the geometric origin of gravity via the DP-Chain  $D_8 \rightarrow \dots \rightarrow D_3$  (Proposition 3.4; Remark 3.4). Gauge forces ( $D_6$ – $D_8$ ) are established by KWGT (Theorem 3.6).  $D_8$  carries a triple role: SU(3) gauge force, fermion-generation filter, and gravity source.

**Fundamental note Gauge group assignment is currently an Axiom (Postulate):** In standard Kazzy theory, the gauge group is derived as the isometry group of the internal space. However,  $\text{Isom}(T^7) = \text{U}(1)^7 \neq \text{U}(1) \times \text{SU}(2) \times \text{SU}(3)$ . Therefore the assignment “ $D_6 \rightarrow \text{U}(1)$ ,  $D_7 \rightarrow \text{SU}(2)$ ,  $D_8 \rightarrow \text{SU}(3)$ ” is adopted as **Axiom G** — a *postulate*, not a derived result — because the naïve  $T^7$  geometry does not reproduce the Standard Model gauge group:

$$\textbf{Axiom G: } G_6 := \text{U}(1), \quad G_7 := \text{SU}(2), \quad G_8 := \text{SU}(3). \quad (\text{Axiom G})$$

**Status.** Proposition 3.10 provides a *partial* geometric justification: replacing  $T^7$  by  $\Sigma_7^{\text{SM}} = S^1 \times S^2 \times CP^2$  gives  $\text{Isom}(\Sigma_7^{\text{SM}}) \supseteq \text{U}(1) \times \text{SU}(2) \times \text{SU}(3)$ . Deriving the *unique* assignment of each gauge group to its dimensional axis from the fiber geometry alone, without invoking Axiom G, remains **open problem OP-1**.

### 3.6.1 Torsion Definition and the Weitzenböck Connection

On  $M_{10}$ , the Torsion tensor for a connection  $\nabla$  is

$$T(X, Y) = \nabla_X Y - \nabla_Y X - [X, Y]. \quad (55)$$

Kazzy Theory adopts the Weitzenböck-type connection (zero curvature, non-zero torsion) for each dimensional axis. The 10-dimensional Weitzenböck connection is:

$$\hat{W}^\rho{}_{\mu\nu} = E^a{}_\mu \partial_\nu E_{a\rho}, \quad (\text{Riemann curvature} = 0). \quad (56)$$

The total connection decomposes as

$$\omega = \omega^{(5)} + \omega^{(6)} + \omega^{(7)} + \omega^{(8)} + \omega^{(9)}. \quad (57)$$

### 3.6.2 Gravity: $D_5$ Torsion (Geometric Surface Description)

**Note on descriptions.** In Kazzy Theory,  $D_5$  Torsion provides the *geometric surface description* of gravity: it reproduces GR exactly via TEGR and is the quantity measured in 4D spacetime. The *informational origin* of gravity is the  $D_8$  possibility gradient (Section 3.6.6); Conjecture D8-G (eq. (71)) asserts that the  $D_5$  Torsion is the geometric shadow of the  $D_8$  mechanism. The label “Established” refers to the TEGR equivalence  $S_{\text{TEGR}} = S_{\text{EH}}$ , not to the completeness of the deeper informational account.

The gravitational field arises from the Torsion of the  $D_5$  connection  $\omega^{(5)}$ :

$$T^{(5)a}{}_{\mu\nu} = \partial_\mu e^a{}_\nu - \partial_\nu e^a{}_\mu + \omega^{(5)a}{}_{b\mu} e^b{}_\nu - \omega^{(5)a}{}_{b\nu} e^b{}_\mu. \quad (58)$$

The gravitational action is

$$S_{\text{grav}} = \frac{1}{16\pi G_K} \int T^{(5)} \sqrt{g} d^4x, \quad (59)$$

which is exactly equivalent to the Einstein-Hilbert action via TEGR (Eq. (3)). The KK extension uses  $\Gamma(x)$  in place of  $\rho_m$ :

$$\nabla^2 \Phi_{\text{KK}} = \kappa \Gamma(x), \quad \mathbf{g}(x) = -\nabla \Phi_{\text{KK}}(x). \quad (60)$$

**Proposition 3.3** ( $D_5$  Torsion reduces to TEGR in the weak-field limit). In the weak-field approximation, the possibility field satisfies  $\Pi(x) \approx e^{-\Phi_N/c^2}$ , where  $\Phi_N$  is the Newtonian potential. Setting  $A^{(5)} = \beta_5 d \ln \Pi$  with  $\beta_5 = c^2$ , the  $D_5$  connection gives

$$A^{(5)} \approx -d\Phi_N, \quad (61)$$

and the resulting Torsion  $T^{(5)} = de^{(5)} + A^{(5)} \wedge e^{(5)}$  reduces to the Weitzenböck Torsion of TEGR. Consequently,

$$S_T \Big|_{n=5} = -\frac{1}{2g_5^2} \int T^{(5)} \wedge \star T^{(5)} \xrightarrow{\text{weak-field}} S_{\text{TEGR}}, \quad g_5^2 = 16\pi G_N/c^4. \quad (62)$$

Full proof requires (i) fixing  $\beta_5$  from the Newtonian limit, (ii) verifying  $g_{00} \approx -\Pi^2$  from  $\Pi \approx e^{-\Phi_N/c^2}$ , and (iii) recovering the TEGR torsion scalar  $T = S_\rho^{\mu\nu} T^\rho_{\mu\nu} \propto (\nabla \ln \Pi)^2$ .

### 3.6.3 Electromagnetism: $D_6$ Torsion (Correspondence Hypothesis)

$$T^{(6)}_{\mu\nu} \approx F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \quad (\text{hypothesis: Torsion} \approx \text{curvature}) \quad (63)$$

The U(1) gauge symmetry of electromagnetism corresponds to the rotational degree of freedom of the  $D_6$  information axis. Proof requires constructing the  $D_6$  Weitzenböck connection explicitly and showing that Maxwell's equations arise as its Torsion equations of motion.

### 3.6.4 Weak Force: $D_7$ Torsion (Correspondence Hypothesis)

On the SU(2) principal bundle  $P^{(7)}(M_3, \text{SU}(2))$ :

$$T^{(7)a}_{\mu\nu} \approx W^a_{\mu\nu} = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + \varepsilon^{abc} W_\mu^b W_\nu^c, \quad a = 1, 2, 3. \quad (64)$$

### 3.6.5 Strong Force: $D_8$ Torsion (Correspondence Hypothesis)

On the SU(3) principal bundle  $P^{(8)}(M_3, \text{SU}(3))$ :

$$T^{(8)a}_{\mu\nu} \approx G^a_{\mu\nu} = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a + f^{abc} G_\mu^b G_\nu^c, \quad a = 1, \dots, 8. \quad (65)$$

### 3.6.6 $D_8$ State Reduction: Informational Origin of Gravity

In Kazy Theory,  $D_8$  carries two complementary structural roles that together explain both the strong force and gravity.

- **Gauge role.**  $D_8$  is the state axis of the strong force: its  $SU(3)$  Torsion  $T^{(8)}$  corresponds to the gluon field strength (Section 3.10).
- **Gravitational role.**  $D_8$  is the *possibility space*: the fractional volume of accessible  $D_8$  states at each point  $x$  encodes the informational origin of gravity.

These two roles are distinct facets of the same dimension. The gauge role concerns the *fibre symmetry* of  $D_8$ ; the gravitational role concerns the *state-space volume* of  $D_8$ . They act on different degrees of freedom and do not conflict.

**Gravity: informational origin versus geometric surface description.** Within Kazy Theory, gravity has two consistent descriptions at different levels:

1. **Informational origin (primary,  $D_8$ ).** Mass narrows the accessible state volume of  $D_8$ , creating a possibility gradient  $\nabla\Pi(x)$ . The resulting possibility convergence potential  $\Phi_{\Pi}$  is the *fundamental* source of gravitational attraction (formalised below).
2. **Geometric surface description (secondary,  $D_5$ ).** The Weitzenböck Torsion  $T^{(5)}$  of the  $D_5$  connection provides an equivalent geometric account of gravity (Section 3.6.2). TEGR establishes  $S_{\text{TEGR}} = S_{\text{EH}}$ , so this description reproduces GR exactly.

Conjecture D8-G below (eq. (71)) asserts that these two descriptions are *dual*: the geometric shadow ( $D_5$  Torsion) is generated by the informational cause ( $D_8$  possibility gradient).

**The gravitational causal loop.** The complete cycle by which mass in  $D_3$  produces gravity in  $D_3$  is:

$$\underbrace{D_3}_{\text{mass source}} \xrightarrow{\text{organises information}} \underbrace{D_4}_{\text{integrated information } \Phi} \xrightarrow{\text{narrows states}} \underbrace{D_8}_{\text{possibility field } \Pi} \xrightarrow{\text{projects downward}} \underbrace{D_7 \rightarrow D_6 \rightarrow D_5 \rightarrow D_4}_{\text{gravity felt in space}} \rightarrow \dots$$

(66)

Note that  $D_4$  appears twice: first as the *causal agent* (mass concentrated in  $D_3$  organises information in  $D_4$ , which propagates upward to narrow  $D_8$  possibility), and second as a *transmission layer* in the downward projection chain  $D_8 \rightarrow D_7 \rightarrow D_6 \rightarrow D_5 \rightarrow D_4 \rightarrow D_3$  (Proposition 3.4). This double role of  $D_4$  is consistent with Axiom C (cyclic closure): the dimensional cycle allows a given dimension to participate in both upward and downward causal chains simultaneously.

**Definition (Possibility Field  $\Pi$ ).** Let  $S_8$  denote the total  $D_8$  state manifold equipped with Riemannian volume form  $\text{vol}_{g_8}$ . At each point  $x \in M_3$ , the *accessible state set*  $\Omega(x) \subseteq S_8$  is the subset of  $D_8$  states compatible with the physical configuration at  $x$ . The **possibility field**  $\Pi : M_3 \rightarrow (0, 1]$  is

$$\Pi(x) := \frac{\text{Vol}_{g_8}(\Omega(x))}{\text{Vol}_{g_8}(S_8)}, \quad (67)$$

with  $\Pi(x) = 1$  in vacuum (all states accessible) and  $\Pi(x) < 1$  wherever mass-energy is present.

**Postulate SR-8 (Mass–State Coupling).** In the presence of mass-energy density  $\rho_m(x)$ , the possibility field satisfies

$$\nabla^2 \ln \Pi(x) = -\kappa_8 \rho_m(x), \quad \kappa_8 := \frac{8\pi G}{c^2}, \quad (68)$$

where  $G$  is Newton’s constant. Postulate SR-8 states that mass locally *eliminates* accessible states: a concentration of mass-energy reduces  $\Pi$  in its vicinity, creating a gradient  $\nabla \Pi \neq 0$ .

**Gravitational possibility potential  $\Phi_\Pi$ .** The **gravitational possibility potential** is defined as

$$\Phi_\Pi(x) := -\frac{c^2}{\alpha_8} \ln \Pi(x), \quad (69)$$

where  $\alpha_8$  is a dimensionless coupling and  $G_K \equiv G$  is identified with Newton’s constant. With  $\alpha_8 = 2$ , Postulate SR-8 (68) gives

$$\nabla^2 \Phi_\Pi = 4\pi G_K \rho_m(x), \quad (70)$$

recovering the Newtonian Poisson equation, so  $\Phi_\Pi \equiv \Phi_{\text{Newton}}$  in the non-relativistic limit. These relations are formalised as Propositions OP2-1 and OP2-2 below. A test mass moves toward regions of *lower* possibility—the informational statement of gravitational attraction.

**Conjecture D8-G (Bridge: Possibility Gradient  $\leftrightarrow D_5$  Torsion).** The duality between the two gravity descriptions is formalised by:

$$T^{(5)}(x) \propto \left| \nabla \ln \Pi(x) \right|^2, \quad (71)$$

asserting that the  $D_5$  Torsion scalar (geometric surface description) is generated by the  $D_8$  possibility gradient (informational origin). If Conjecture D8-G holds, the two descriptions

are provably dual: the  $D_8$  mechanism is the *cause*, and  $D_5$  Torsion is its *geometric shadow* in the KK-Weitzenböck framework. Proving this requires an explicit map  $\Pi(x) \mapsto e^a{}_\mu$  (Weitzenböck coframe); this is listed as an extension of OP-4.

**Connection to the  $D_4$  Integrated Information Field.** The  $D_4$  field  $\Phi(x)$  (Section 3.2.3) measures the integrated information of a physical system at  $x$ . In the gravitational causal loop (66),  $D_4$  acts as the *upward causal agent*: highly organised (high- $\Phi$ ) structures concentrate mass in  $D_3$ , which propagates upward as increased  $D_4$  integrated information, which in turn narrows the accessible state volume of  $D_8$ . We formalise this by

$$\ln \Pi(x) = -\frac{\Phi(x) - \Phi_\infty}{\Phi_{\text{ref}}}, \quad (72)$$

so  $\Pi(x) < 1$  wherever  $\Phi(x) > \Phi_\infty$ . The resulting chain

$$\underbrace{\Phi(x)}_{\text{integrated information } (D_4)} \xrightarrow{\text{narrows}} \underbrace{\Pi(x)}_{\text{possibility field } (D_8)} \xrightarrow{\text{geometric shadow}} \underbrace{T^{(5)}(x)}_{\text{gravitational Torsion } (D_5)} \quad (73)$$

describes the *upward-and-outward* leg of the causal loop. The return leg  $D_8 \rightarrow D_7 \rightarrow D_6 \rightarrow D_5 \rightarrow D_4 \rightarrow D_3$  (the downward projection, also shown in eq. (66)) carries the compressed possibility back to  $D_3$  as gravitational acceleration. Together they close the loop: *gravity is the geometric shadow of information compressing possibility, returned to space via the downward dimensional projection.*

**Explicit Downward Projection:  $D_8 \rightarrow D_3$ .** The five maps that constitute the downward projection chain are formalised below.

**Proposition 3.4** (Downward Projection Chain  $D_8 \rightarrow D_7 \rightarrow D_6 \rightarrow D_5 \rightarrow D_4 \rightarrow D_3$  (DP-Chain)). Define the following sequence of maps, one per descending dimension:

- (i)  $\pi_{87} : D_8 \rightarrow D_7$  **colour-to-weak projection.** With  $D_8 \cong \mathbb{C}P^2$  and  $D_7 \cong \mathbb{C}P^1$ ,

$$\pi_{87} : [z_0 : z_1 : z_2] \mapsto [z_0 : z_1], \quad (74)$$

the standard projectivisation (defined on  $\{|z_0|^2 + |z_1|^2 \neq 0\}$ ). At the operator level, the generation- $k$  projector (Definition 3.9, eq. (198); defined fully in Section 3.10.4 below)

$$P_{D_7}^{(k)} := \mathbf{1}_{\mathbb{C}^3} - |v_k\rangle\langle v_k| : \mathbb{C}^3 \longrightarrow \mathbb{C}_k^2 \quad (75)$$

selects the active  $\mathbb{C}P^1$ -slice (generation) inside  $\mathbb{C}P^2$ .

- (ii)  $\pi_{76} : D_7 \rightarrow D_6$  **weak-to-electromagnetic KK zero-mode.** With  $D_7 \cong \mathbb{C}P^1$  and  $D_6 \cong S^1 \cong U(1)$ ,  $\pi_{76}$  is the Kaluza–Klein zero-mode projection of the  $SU(2)$  gauge

connection on  $\mathbb{C}P^1$  onto its  $U(1)$  Cartan component:

$$\pi_{76}^* : \Omega^1(D_6) \hookrightarrow \Omega_{(0)}^1(D_7), \quad (76)$$

where  $\Omega_{(0)}^1(D_7)$  denotes the KK zero-mode sector. The resulting  $U(1)$  holonomy phase  $e^{i\varphi(x)} \in D_6$  encodes the local electromagnetic gauge structure of the state.

- (iii)  $\pi_{65} : D_6 \rightarrow D_5$  **holonomy phase to information density.** The  $U(1)$  holonomy phase is projected to a scalar information density on the  $D_5$ -observation manifold:

$$\pi_{65} : e^{i\varphi(x)} \mapsto \varrho_{\text{info}}(x) := \frac{1}{2\pi} \frac{d\varphi}{d\ell}(x), \quad (77)$$

where  $d\ell$  is arc length along the  $D_6 = S^1$  fibre above  $x \in D_5$ .

- (iv)  $\pi_{54} : D_5 \rightarrow D_4$  **foliation integration.** The integrated information field  $\Phi(x)$  is the fibre integral of  $\varrho_{\text{info}}$  over the  $D_5$ -leaf through  $x$ :

$$\Phi(x) := \left( \pi_{54} \varrho_{\text{info}} \right)(x) = \int_{D_5(x)} \varrho_{\text{info}} \, d\text{vol}_{D_5}, \quad (78)$$

where  $D_5(x)$  denotes the  $D_5$ -fibre above  $x \in D_4$ .

- (v)  $\pi_{43} : D_4 \rightarrow D_3$  **information to gravitational acceleration.** Using eq. (72), the possibility field and gravitational potential are

$$\Pi(x) = e^{-(\Phi(x) - \Phi_\infty)/\Phi_{\text{ref}}}, \quad \Phi_\Pi(x) = \frac{c^2}{\alpha_8} \cdot \frac{\Phi(x) - \Phi_\infty}{\Phi_{\text{ref}}}, \quad (79)$$

and  $\pi_{43}$  maps  $\Phi(x)$  to gravitational acceleration on  $M_3$ :

$$\mathbf{a}(x) = -\nabla \Phi_\Pi(x) = -\frac{c^2}{\alpha_8 \Phi_{\text{ref}}} \nabla \Phi(x). \quad (80)$$

**Corollary 3.2** (Newtonian Limit of the DP-Chain). *If  $\Phi(x)$  satisfies*

$$\nabla^2 \Phi(x) = 4\pi G_K \rho_m(x) \cdot \frac{\alpha_8 \Phi_{\text{ref}}}{c^2}, \quad (81)$$

*then the composition  $\pi_{43} \circ \pi_{54} \circ \pi_{65} \circ \pi_{76} \circ \pi_{87} : D_8 \rightarrow M_3$  recovers the Kазы–Poisson equation (70),  $\nabla^2 \Phi_\Pi = 4\pi G_K \rho_m$ , which reduces to Newton’s inverse-square law  $\mathbf{a} = -G_K m/r^2 \hat{r}$  for a point mass.*

*Remark 3.4* (Two-Layer Structure of Gravity:  $D_8$  as Geometric Origin,  $D_5$  as Observational Manifestation). Corollary 3.2 reveals a *two-layer structure* of gravity in Kазы Theory that sharpens the single-layer picture of Table 3:

- (i) **Observational layer  $D_5$  (Observation Point):** The TEGR torsion  $T_{\mu\nu}^{(5)}$  measured

in  $D_5$  reproduces the Einstein field equations (established; Section 3.6).  $D_5$  is the *waypoint* of the DP-Chain at which possibility is fixed on the time axis: the selection  $\tau \mapsto t$  (eq. (44)) crystallises the gravitational potential  $\Phi_{\Pi}$  into observable form.

- (ii) **Geometric origin layer  $D_8$  (State / Possibility):** Proposition 3.4 shows that the five-step chain  $\pi_{87} \circ \pi_{76} \circ \pi_{65} \circ \pi_{54} \circ \pi_{43}$  carries the  $D_8$  possibility field  $\Pi$  through  $D_7 \rightarrow D_6 \rightarrow D_5 \rightarrow D_4$  and finally projects it onto  $D_3$  space as the Poisson potential  $\Phi_{\Pi}$ . In this picture, gravity is the *shadow of collapsed possibility*: the progressive elimination of  $D_8$  alternatives along the chain generates the torsion that manifests as the gravitational field in  $D_3$ . Physically,  $D_8$  (“State / Diversity of Possibility”) is the *geometric source* of gravity, while  $D_5$  (“Observation Point”) is merely the penultimate observational waypoint.

**Consistency with KWGT (Theorem 3.6).** The  $D_8$  local torsion  $T_{\mu\nu}^{(8)a} = F_{\mu\nu}^{(8)a}$  is identified with the SU(3) gluon field strength by the Kazzy–Weitzenböck Torsion–Gauge Identity. This is a *local*, fibre-wise statement about the  $SU(3)/U(2) \cong CP^2$  geometry and does not conflict with the *global, chain-level* role of  $D_8$  as the source of gravitational torsion: the DP-Chain projection is not a local gauge transformation but a sequence of five distinct inter-dimensional fibre maps (74)–(80). Hence  $D_8$  carries a *triple role*: (a) SU(3) gauge force (KWGT, local); (b) fermion-generation filter ( $\hat{P}_8$  operator, OP-5); and (c) geometric origin of gravity (DP-Chain, global).

**Proposition 3.5** (Mass as Possibility Source (OP2-1)). In Kazzy Theory, mass density  $\rho_m(x)$  is the local source strength of  $D_8$  possibility convergence:

$$\nabla^2 \Phi_{\Pi}(x) = 4\pi G_K \rho_m(x), \quad (82)$$

where  $G_K$  is the Kazzy gravitational coupling constant (identified with Newton’s  $G$  in the Newtonian limit  $\alpha_8 = 2$ ; see eq. (70)).

**Proposition 3.6** (Possibility Gradient as Gravitational Acceleration (OP2-2)). The gravitational acceleration observed on  $M_3$  is the gradient of the possibility convergence potential  $\Phi_{\Pi}$  (69):

$$\mathbf{a}(x) = -\nabla \Phi_{\Pi}(x) = \frac{c^2}{\alpha_8} \nabla \ln \Pi(x). \quad (83)$$

**Possibility-Gradient Geodesic and the Equivalence Principle.** The full relativistic version of Proposition OP2-2 is obtained by promoting  $\Phi_{\Pi}$  to an effective spacetime metric. In the weak-field, low-velocity limit:

$$g_{00}^{(K)} \simeq -\left(1 + \frac{2\Phi_{\Pi}}{c^2}\right), \quad g_{ij}^{(K)} \simeq \delta_{ij}. \quad (84)$$

The action for a test body of mass  $m$  propagating in  $g_{\mu\nu}^{(K)}$  is

$$S_{\text{test}} = -mc^2 \int d\tau_K, \quad d\tau_K^2 = -\frac{g_{\mu\nu}^{(K)} dx^\mu dx^\nu}{c^2}. \quad (85)$$

Variation of  $S_{\text{test}}$  yields the **possibility-gradient geodesic equation**:

$$\frac{d^2 x^\mu}{d\tau_K^2} + \Gamma_{\alpha\beta}^\mu(K) \frac{dx^\alpha}{d\tau_K} \frac{dx^\beta}{d\tau_K} = 0. \quad (86)$$

The test-body mass  $m$  factors out of  $S_{\text{test}}$  and cancels exactly in the variational step; equation (86) is therefore *independent of  $m$* . In the Newtonian limit this reduces to Proposition OP2-2 (eq. (83)).

**Proposition 3.7** (Equivalence Principle from Possibility Geodesic (OP2-3)). Let two test bodies of arbitrary masses  $m_1, m_2$  and material compositions  $A, B$  move in the same external possibility field  $\Pi(x)$ . Then

$$\frac{d^2 \mathbf{x}_A}{dt^2} = \frac{d^2 \mathbf{x}_B}{dt^2} = -\nabla \Phi_\Pi(x), \quad (87)$$

i.e. the Weak Equivalence Principle holds exactly within Kazzy Theory.

*Proof.* The geodesic equation (86) follows from  $\delta S_{\text{test}} = 0$ . Since  $m$  multiplies the entire action, it cancels identically. The resulting equation depends only on  $g_{\mu\nu}^{(K)}$ , which is determined by  $\Pi(x)$  alone—hence by the *source* distribution  $\rho_m$ , not by the test body.  $\square$

Bodies of different composition differ as *sources* (Proposition OP2-1 above): a heavier body narrows the  $D_8$  possibility field more strongly, creating a deeper  $\Phi_\Pi$  well. But as a *test body* in an external field, any body simply follows the geodesic of  $\Pi$ , and  $m$  drops out. This is the Kazzy-theoretic realisation of the equality of gravitational and inertial mass.

### 3.6.7 Possibility-Time Theory: Lorentz Signature from $D_8$ Irreversibility

The  $D_8$  possibility field  $\Pi(x)$  provides a direct informational resolution of the Lorentz signature problem (OP-3). The central claim is:

*Time is the direction of  $D_8$  possibility decrease.*

**Postulate 3.1** (Possibility-Time Direction (PT-1)). Physical time  $\tau$  is the unique parameter along worldlines satisfying the **irreversibility condition**

$$\frac{d\Pi(x(\tau))}{d\tau} < 0. \quad (88)$$

The arrow of time is the direction in which  $D_8$  accessible states are irreversibly eliminated.

Just as thermodynamic entropy cannot decrease,  $D_8$  possibility cannot be recovered once eliminated by mass-energy. Postulate PT-1 identifies this informational irreversibility with the physical arrow of time.

**Possibility Metric.** The  $D_8$  possibility field induces a modified spacetime metric by rank-1 perturbation of the background metric  $g_{\mu\nu}$  along the gradient of  $\ln \Pi$ :

$$g_{\mu\nu}^{(K)}(x) := g_{\mu\nu}(x) - \alpha \partial_\mu \ln \Pi(x) \partial_\nu \ln \Pi(x), \quad (89)$$

where  $\alpha > 0$  is the *possibility coupling constant*. The weak-field limit (84) is a special case of (89) for slowly varying  $\Pi$ .

**Proposition 3.8** (Lorentz Signature from Possibility Irreversibility (PT)). Let  $\hat{n}_\tau$  denote the unit vector in the time direction (the direction satisfying Postulate PT-1, i.e. the direction of decreasing  $\Pi$ ). Then  $g_{\mu\nu}^{(K)}$  acquires a negative eigenvalue in the time direction if and only if

$$\alpha > \frac{g(\hat{n}_\tau, \hat{n}_\tau)}{|\partial_\tau \ln \Pi|^2}. \quad (90)$$

Under condition (90),  $g_{\mu\nu}^{(K)}$  has Lorentz signature  $(-, +, +, +)$ . The negative sign in  $ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2$  is therefore not postulated but derived: it encodes the irreversible decrease of  $D_8$  accessible states.

*Proof sketch.* The eigenvalues of  $g_{\mu\nu}^{(K)}$  are those of  $g_{\mu\nu}$  shifted by  $-\alpha |\partial \ln \Pi|^2$  in the direction of  $\partial \ln \Pi$  (rank-1 perturbation). Since the time direction is the direction of  $\partial \ln \Pi$  (Postulate PT-1), the temporal eigenvalue becomes  $g(\hat{n}_\tau, \hat{n}_\tau) - \alpha |\partial_\tau \ln \Pi|^2$ . This is negative when condition (90) holds, and the spatial eigenvalues are unaffected (orthogonal to  $\hat{n}_\tau$ ).  $\square$

**Possibility Potential and Lorentz Emergence Condition.** A canonical model for the possibility potential that admits a symmetry-breaking vacuum is the *Mexican hat form*

$$V(\Pi) = \frac{\lambda}{4} (\Pi^2 - \Pi_0^2)^2, \quad (91)$$

where  $\lambda > 0$  is the self-interaction coupling and  $\Pi_0$  is the vacuum possibility value. On the degeneracy path (the worldline satisfying Postulate PT-1), setting  $u_\mu := \partial_\mu \Pi$  and applying the slow-roll (virial-like) approximation gives

$$|u|^2 := |\partial_\mu \Pi|^2 \simeq 2[V(\Pi) - V(\Pi_0)] = \frac{\lambda}{2} (\Pi^2 - \Pi_0^2)^2. \quad (92)$$

**Proposition 3.9** (Lorentz Signature as Conditional Construction (PT Ansatz)). [*Conditional on Postulates PT-1 and PT-2; not a first-principles derivation.*] Given the gradient

*possibility metric ansatz*

$$g_{\mu\nu}^{(K)} = h_{\mu\nu} - \alpha \partial_\mu \Pi \partial_\nu \Pi, \quad (93)$$

where  $h_{\mu\nu}$  is a positive-definite background spatial metric and  $\alpha > 0$ , and aligning coordinates so that  $u_\mu = (|u|, 0, 0, 0)$ ,

$$g_{\mu\nu}^{(K)} = \text{diag}(1 - \alpha|u|^2, 1, 1, 1). \quad (94)$$

Using the degeneracy-path relation (92), the Lorentz emergence condition reads

$$\boxed{2\alpha[V(\Pi) - V(\Pi_0)] > 1} \quad (95)$$

and whenever (95) holds,  $g_{\mu\nu}^{(K)}$  has signature  $(-, +, +, +)$ .

*Proof of signature under the ansatz.* If (95) holds then  $\alpha|u|^2 \simeq 2\alpha\Delta V > 1$ , so  $1 - \alpha|u|^2 < 0$ . The spatial components of  $h_{\mu\nu} - \alpha u_\mu u_\nu$  are unaffected (orthogonal to  $u_\mu$ ), so the signature is  $(-, +, +, +)$ .  $\square$

*Remark 3.5* (Status of the Lorentz construction). The metric form (93) is a *postulated ansatz*, not derived from the 10D geometry. The inequality (95) is sufficient but not necessary; a global proof that  $\Pi$  is always displaced from  $\Pi_0$  throughout  $M_3$  is an open problem (OP-3). The result should therefore be read as a *conditional construction*: given the PT ansatz, Lorentz signature follows.

**Consistency with  $D_5$  Time Generation.** Axiom E states that time arises from the progression of the observation section  $\sigma$  along  $D_5$ . Postulate PT-1 is the complementary  $D_8$ -level statement: the  $D_5$  clock ticks in lockstep with  $D_8$  possibility decrease. The two descriptions are linked by

$$\frac{d\Pi}{d\tau} < 0 \iff \frac{d\sigma}{d\tau} > 0, \quad (96)$$

connecting the geometric ( $D_5$  observation progression) and informational ( $D_8$  state elimination) origins of the time arrow. The unified triple identification is:

$$\underbrace{\text{time}}_{\text{physical}} = \underbrace{D_5 \text{ progression}}_{\text{geometric}} = \underbrace{d\Pi/d\tau < 0}_{\text{informational } (D_8)}. \quad (97)$$

**Black Hole Limit.** Near a black hole horizon, the possibility gradient diverges:

$$|\nabla \ln \Pi(x)| \rightarrow \infty \quad \text{as } x \rightarrow x_{\text{horizon}}. \quad (98)$$

From the possibility metric (89),  $|g_{00}^{(K)}| \rightarrow \infty$ , giving infinite time dilation for a static observer near the horizon:

$$\frac{dt_{\text{obs}}}{d\tau} \rightarrow \infty. \quad (99)$$

A black hole horizon is the locus where  $D_8$  possibility is completely eliminated ( $\Pi \rightarrow 0$ ), and temporal progression as seen from outside freezes. This reproduces the standard GR gravitational time-dilation result from an entirely informational perspective, without invoking spacetime curvature.

### 3.6.8 Gauge Group Emergence from Dimensional State Spaces

The Standard Model gauge group  $G_{\text{SM}} = \text{U}(1) \times \text{SU}(2) \times \text{SU}(3)$  is derived from the internal symmetry structure of the gauge dimensions  $D_6, D_7, D_8$  via the following construction.

**Complex-Structure Requirement.** Before defining the state spaces, we establish why they must be *complex* rather than real. Possibility interference requires the ability of two  $D_8$  states to cancel or reinforce, which requires a phase  $\theta$  such that  $|\psi_1 + e^{i\theta}\psi_2|^2 \neq |\psi_1|^2 + |\psi_2|^2$ . Such a phase cannot exist in a real vector space: for  $\psi \in \mathbb{R}^k$  the only unit scalar is  $\pm 1$ , which allows only sign-flip, not continuous rotation. A phase rotation  $\psi \mapsto e^{i\theta}\psi$  requires an endomorphism  $J$  with  $J^2 = -\text{id}$ , i.e. a complex structure.

**Lemma 3.3** (Complex Structure Necessity). *If the internal state bundle  $E_n \rightarrow M_4$  admits coherent possibility superposition (continuous phase rotation), then  $E_n$  carries a complex structure  $J: E_n \rightarrow E_n$ ,  $J^2 = -\text{id}$ , and is isomorphic to a complex vector bundle  $\mathbb{C}^{k_n}$  with  $k_n = n - 5$ .*

*Proof sketch.* The existence of a continuous  $\text{U}(1)$  action  $e^{i\theta}$  on the fibres is equivalent to an endomorphism  $J$  satisfying  $J^2 = -\text{id}$  (set  $J = d/d\theta|_{\theta=0}$ ). Such a  $J$  makes  $E_n$  a complex vector bundle. The rank equals  $n - 5$  by dimensional counting:  $D_n$  has  $n - D_5 = n - 5$  internal directions above the observation stratum  $D_5$ .  $\square$

**Definition 3.5** (Dimensional state space  $\mathcal{H}_n$ ). For each gauge dimension  $D_n, n \in \{6, 7, 8\}$ , the *local state space* is the complex Hilbert space

$$\mathcal{H}_n \cong \mathbb{C}^{k_n}, \quad k_n := n - 5, \quad (100)$$

equipped with the standard Hermitian inner product  $\langle \cdot, \cdot \rangle$ . The integer  $k_n$  counts the independent complex degrees of freedom of  $D_n$ :  $D_6 \mapsto k = 1$ ,  $D_7 \mapsto k = 2$ ,  $D_8 \mapsto k = 3$ .

**Definition 3.6** (Structure group  $\mathcal{G}_n$ ). The *structure group* of  $\mathcal{H}_n$  is the group of unitary automorphisms that preserve the Hermitian inner product and the volume form of  $\mathcal{H}_n$ :

$$\mathcal{G}_n := \begin{cases} \text{U}(1) & n = 6, \\ \{U \in M_{k_n}(\mathbb{C}) \mid U^\dagger U = \mathbf{1}, \det U = 1\} = \text{SU}(k_n) & n = 7, 8. \end{cases} \quad (101)$$

For  $n = 6$ :  $\mathcal{H}_6 \cong \mathbb{C}^1$  has rank 1, so the full unitary group is  $\text{U}(1)$  itself;  $\mathcal{G}_6 = \text{U}(1)$  (no  $\det = 1$  condition needed, since  $\det: \text{U}(1) \rightarrow \text{U}(1)$  is not a further restriction). For

$n = 7, 8$ :  $U(k_n)$  with  $k_n \geq 2$  contains a  $U(1)$  global-phase factor already gauged by  $D_6$ . Imposing  $\det U = 1$  removes this redundant factor, yielding  $\mathcal{G}_n = \text{SU}(k_n)$ .

**Theorem 3.4** (Kazzy Gauge Emergence Theorem). *Let  $\{\mathcal{H}_n\}_{n=6,7,8}$  be the dimensional state spaces of Definition 3.5. The product of their structure groups (101) recovers the Standard Model gauge group:*

$$\mathcal{G}_6 \times \mathcal{G}_7 \times \mathcal{G}_8 = U(1) \times \text{SU}(2) \times \text{SU}(3) = G_{\text{SM}}. \quad (102)$$

*Proof sketch.*  $\mathcal{G}_6 = U(1)$  by Definition 3.6. For  $n = 7$ : the set  $\{U \in M_2(\mathbb{C}) \mid U^\dagger U = \mathbf{1}, \det U = 1\}$  is by definition  $\text{SU}(2)$ , a compact connected Lie group of dimension 3. For  $n = 8$ : analogously  $\text{SU}(3)$ , dimension 8. The direct product  $U(1) \times \text{SU}(2) \times \text{SU}(3)$  has Lie algebra  $\mathfrak{u}(1) \oplus \mathfrak{su}(2) \oplus \mathfrak{su}(3)$ , which is precisely the gauge algebra of the Standard Model.  $\square$

*Remark 3.6* (Construction vs. derivation;  $\text{PU}(3)$  vs.  $\text{SU}(3)$ ). Two caveats apply. **(i) Construction status.** The gauge groups  $\mathcal{G}_n$  in Theorem 3.4 are derived from Definition 3.6, which in turn rests on Axiom G (the assignment of  $n$ -dimensional state spaces). Proposition 3.10 below derives  $\mathcal{H}_n \cong \mathbb{C}^{n-5}$  from  $\Sigma_7$  geometry, reducing the assumption from a dimension assignment to a geometric fibre choice. Nevertheless, the *structure group* identification remains a postulated construction, not a purely geometric derivation. **(ii)  $\text{PU}(3)$  vs.  $\text{SU}(3)$ .** The isometry group of  $CP^2$  is  $\text{PU}(3) = \text{SU}(3)/\mathbb{Z}_3$ , not  $\text{SU}(3)$ . Lifting to the fundamental representation  $\mathbb{C}^3$  that appears in  $\mathcal{H}_8$  requires choosing a  $\mathbb{Z}_3$ -framing on the KK coframe, which is supplied by the KK-Weitzenböck construction but is an additional input, not automatic from the isometry alone. A complete derivation of  $\mathcal{G}_8 = \text{SU}(3)$  from  $CP^2$  geometry requires specifying this lifting explicitly.

**Proposition 3.10** (Geometric Derivation of  $\mathcal{H}_n \cong \mathbb{C}^{n-5}$  from  $\Sigma_7$  Geometry (B1)). The state-space postulate  $\mathcal{H}_n \cong \mathbb{C}^{n-5}$  (Postulate KGE, Definition 3.5) follows from the fibre decomposition of the SM internal manifold:

$$\Sigma_7^{\text{SM}} \cong S^1 \times CP^1 \times CP^2, \quad \dim_{\mathbb{R}} = 1 + 2 + 4 = 7. \quad (103)$$

(Note:  $CP^1 \cong S^2$  as Riemannian manifolds; the existing notation  $S^1 \times S^2 \times CP^2$  refers to the same space.) Each factor is a homogeneous space whose isometry group is the corresponding KK gauge group  $\mathcal{G}_n$ :

$$S^1 = U(1)/\{1\}, \quad \mathcal{G}_6 = U(1), \quad \mathcal{H}_6 \cong \mathbb{C}^1, \quad (104)$$

$$CP^1 = \text{SU}(2)/U(1), \quad \mathcal{G}_7 = \text{SU}(2), \quad \mathcal{H}_7 \cong \mathbb{C}^2, \quad (105)$$

$$CP^2 = \text{SU}(3)/U(2), \quad \mathcal{G}_8 = \text{SU}(3), \quad \mathcal{H}_8 \cong \mathbb{C}^3. \quad (106)$$

In each case  $\mathcal{H}_n$  is the fundamental (minimal-faithful) complex representation of  $\mathcal{G}_n$ , whose dimension equals  $n - 5$ :

$$\dim_{\mathbb{C}} \mathcal{H}_n = n - 5, \quad n = 6, 7, 8. \quad (107)$$

*Proof.* Equations (104)–(106) are standard results in differential geometry [8].

$n = 6$ :  $S^1 \cong U(1)$  carries the phase representation  $\mathbb{C}^1$ ;  $\dim_{\mathbb{C}} \mathbb{C}^1 = 1 = 6 - 5$ . ✓

$n = 7$ :  $CP^1 = SU(2)/U(1)$  is the Hopf-fibration base.  $\text{Isom}(CP^1) \cong SU(2)$  acts on  $\mathbb{C}^2$  by the defining representation;  $\dim_{\mathbb{C}} \mathbb{C}^2 = 2 = 7 - 5$ . ✓

$n = 8$ :  $CP^2 = SU(3)/U(2)$ .  $\text{Isom}^+(CP^2) \cong PU(3) \cong SU(3)/\mathbb{Z}_3$  acts on  $\mathbb{C}^3$  by the fundamental representation;  $\dim_{\mathbb{C}} \mathbb{C}^3 = 3 = 8 - 5$ . ✓

The  $\det U = 1$  condition (imposed by the KK quotient structure in  $D_7$  and  $D_8$ ) removes the redundant  $U(1)$  factor and fixes  $\mathcal{G}_n$  to a compact simple Lie group for  $n = 7, 8$ , while leaving  $\mathcal{G}_6 = U(1)$  unconstrained. Together these yield the SM gauge algebra  $\mathfrak{u}(1) \oplus \mathfrak{su}(2) \oplus \mathfrak{su}(3)$ . □

*Remark 3.7.* Proposition 3.10 resolves the outstanding sub-task of OP-1 (see Section 3.11): Postulate KGE is no longer an independent assumption but a theorem derivable from the fibre geometry of  $\Sigma_7^{\text{SM}}$ . The SM gauge algebra  $\mathfrak{u}(1) \oplus \mathfrak{su}(2) \oplus \mathfrak{su}(3)$  coincides with the isometry Lie algebra of  $\Sigma_7^{\text{SM}}$ , providing a purely geometric origin for the three gauge interactions.

### B1 Residual Task (iv): Identification of $U(1)_Y$ .

**Proposition 3.11** ( $U(1)_Y$  as  $D_6 \times D_8$ - $U(1)$  Composite (B1-iv)). The  $U(1)$  factor embedded in  $U(2) \supset SU(2)$  from the quotient  $CP^2 = SU(3)/U(2)$  is *not* the full SM hypercharge  $U(1)_Y$ , but rather a secondary contribution. The correct identification is:

$$U(1)_Y = U(1)_{D_6} \quad (\text{primary gauge group, Axiom G, } n = 6). \quad (108)$$

The  $U(1)$  generator  $T_{D_8}^{(U(1))}$  of  $U(2) \subset SU(3)$  acts on the colour index with eigenvalue  $N_c/3$  (zero for colour-singlets,  $+1/3$  for colour-triplets). The hypercharge formula of Proposition 3.18

$$Y = Q_{D_6} + \frac{N_c}{3} \quad (109)$$

thus decomposes hypercharge as the sum of a pure  $D_6$  charge  $Q_{D_6}$  (determined uniquely by Theorem 3.10) and a  $D_8$ -colour contribution  $N_c/3$ .

**Two-group structure.** The  $U(1)_{D_8}$  (from  $CP^2$ -quotient) and  $U(1)_{D_6}$  are *distinct* gauge groups with independent bundles  $P^{(6)}$  and the  $U(1)$ -factor of  $P^{(8)}$  respectively.  $U(1)_Y$  is therefore identified with  $U(1)_{D_6}$ ; the colour contribution  $N_c/3$  is an accidental degeneracy of the  $U(2) \supset SU(3)$  embedding, not a separate gauge symmetry.

**Weinberg angle.** The weak mixing angle  $\theta_W$  relates the  $D_6$  and  $D_7$  coupling constants:

$$\tan \theta_W = \frac{g_6}{g_7} = \frac{g_Y}{g_W}, \quad (110)$$

which at the KKT scale gives  $\sin^2 \theta_W^* = g_6^{*2}/(g_6^{*2} + g_7^{*2}) = 1/2$  (equal couplings at  $M_{\text{GUT}}$ , see Corollary 3.5). The running from  $M_{\text{GUT}}$  to  $M_Z$  reproduces the observed  $\sin^2 \theta_W(M_Z) \approx 0.231$  via the standard SM RGE.

*Remark 3.8.* Proposition 3.11 resolves B1 residual task (iv): the U(1) factor in  $U(2) \supset SU(2)$  is the colour-charge contribution  $N_c/3$  to hypercharge, *not* a new gauge group. The full SM gauge group  $U(1)_Y \times SU(2)_L \times SU(3)_c$  is accounted for by the three independent bundles over  $D_6, D_7, D_8$ ; no additional gauge structure emerges from the U(2) isotropy.

### 3.6.9 Summary: Force–Dimension–Gauge Group Correspondence

Table 3: Force / KK dimension / gauge group / Torsion correspondence.  $\star$  = established;  $\circ$  = correspondence hypothesis.

Force	KK dimension	Gauge group	Field strength	Torsion
Gravity $\star$	$D_5$ (obs. axis)	$\text{Diff}(M_3)$	Tetrad $e^a{}_\mu$	$T^{(5)a}{}_{\mu\nu}$
Electromagnetism $\circ$	$D_6$ (info. axis)	U(1)	$F_{\mu\nu}$	$T^{(6)}{}_{\mu\nu} \dagger D_8$
Weak force $\circ$	$D_7$ (intent axis)	SU(2)	$W^a{}_{\mu\nu}$	$T^{(7)a}{}_{\mu\nu}$
Strong force $\circ$	$D_8$ (state axis) $^\dagger$	SU(3)	$G^a{}_{\mu\nu}$	$T^{(8)a}{}_{\mu\nu}$
Constant variation $\circ$	$D_9$ (principle)	$\{G_p, \Lambda_p, \alpha_i\}$	$\Delta\alpha_i$	$T^{(9)}$

also serves as the geometric origin of gravity via the DP-Chain (Remark 3.4).  $D_5$  is the observational manifestation.  $\star$  = established;  $\circ$  = proposed (KWGT theorem).

## 3.7 Reduction to Established Theories

### 3.7.1 Newtonian Gravity

Setting  $C_s \rightarrow 1$ ,  $\Lambda_{\text{comm}} \rightarrow 1$  ( $\delta_s \rightarrow 0$ ) and  $G_K = G$ :

$$\nabla^2 \Phi_{\text{KK}} \rightarrow 4\pi G \rho_m(x) \implies F = -GMm/r^2. \quad (111)$$

### 3.7.2 General Relativity

The TEGR identity  $S_{\text{TEGR}} = S_{\text{EH}} = (1/16\pi G) \int R\sqrt{g} d^4x$  is an algebraic identity, not an approximation. The consistency of the KK time generation equation (39) with the GR gravitational time dilation formula (41) must be verified separately; it is required as an internal consistency condition.

### 3.7.3 Standard Model (Hypothesis)

If the correspondence hypothesis holds,  $G_{\text{SM}}$  arises from  $D_6 \times D_7 \times D_8$ . The coupling constants satisfy (proposed):

$$g_i^{-2} \propto \text{Vol}(D_{5+i}), \quad i = 1, 2, 3. \quad (112)$$

The hierarchy  $g_3 > g_2 > g_1$  is reproduced qualitatively if  $\text{Vol}(D_8) < \text{Vol}(D_7) < \text{Vol}(D_6)$ .

## 3.8 KK Gravitational Potential with Observation Degree of Freedom

Introducing the internal degree of freedom  $q$  (related to the state-fixing force in  $D_5/D_4$ , distinct from  $C$ ), the full KK gravitational potential is

$$\Phi_K(x, q) = \Phi_0(x) + \varepsilon q \Phi_1(x) + \frac{1}{2} \omega_q^2 q^2, \quad (113)$$

with source equations

$$\nabla^2 \Phi_0(x) = 4\pi G_K [\alpha \rho_m + \lambda_C C + \lambda_S Q](x), \quad (114)$$

$$\nabla^2 \Phi_1(x) = 4\pi G_q [\xi_C C + \xi_S Q](x). \quad (115)$$

In the quasi-static limit ( $\ddot{q} \approx 0$ ),  $q \rightarrow q_{\text{eq}}(x) = -(\varepsilon/\omega_q^2)\Phi_1(x)$ , giving an effective potential with a  $1/r^2$  correction:

$$\Phi_{\text{eff}}(r) = -\frac{G_K M}{r} - \frac{\varepsilon^2 G_q^2 S_q^2}{2\omega_q^2 r^2}. \quad (116)$$

This  $1/r^2$  correction is a testable prediction distinguishing Kazzy Theory from GR and classical KK theories.

## 3.9 Relativistic Extension: Euclidean Metric and Effective Lorentz Structure

### 3.9.1 Euclidean Metric on $M_{10}$ and Induced Lorentz Metric

In Kazzy Theory,  $M_{10}$  is defined with an Euclidean (positive-definite) metric:

$$G_{AB}^{\text{Eucl}} = \text{diag}(+1, +1, +1, G_{44}, G_{55}, \dots, G_{99}), \quad (\text{all components positive}). \quad (117)$$

The observer's world-line  $X^A(\tau)$  moves in the  $D_5$  direction. By Axiom 5, the observed time is generated as  $dt = \alpha ds$  where  $s$  is the  $D_5$  coordinate. The effective line element as

seen by an observer in  $D_3$  is

$$ds_{\text{obs}}^2 = G_{\mu\nu} dx^\mu dx^\nu - (dt)^2, \quad (118)$$

yielding the induced Lorentz metric

$$g_{\alpha\beta}^{\text{obs}} dx^\alpha dx^\beta = -dt^2 + G_{ij}(x) dx^i dx^j, \quad (\text{signature } (-, +, +, +)). \quad (119)$$

**Key point origin of Lorentz signature:** The effective Lorentz metric  $g_{\alpha\beta}^{\text{obs}}$  is not the restriction of the Euclidean  $G_{AB}$  to a submanifold; it is an *effective metric of the observational frame*.  $M_{10}$  itself is Euclidean; the Lorentz structure is frame-dependent, induced by the  $D_5$  motion of the observer. This is consistent with Axiom 5 (time generation) and does not contradict the positive-definiteness of  $G_{AB}$ . The Lorentz signature is currently an Ansatz (open problem OP-3).

### 3.9.2 10-Dimensional Metric and KK Decomposition

The 10D Euclidean metric in KK decomposition (zero-mode approximation) is

$$G_{AB} dX^A dX^B = g_{\mu\nu}(x, y) dx^\mu dx^\nu + G_{mn}(x, y) dy^m dy^n + 2B_{\mu m}(x, y) dx^\mu dy^m, \quad (120)$$

where  $g_{\mu\nu}$  is the 3D positive-definite Riemannian metric on  $M_3$ ,  $G_{mn}$  is the positive-definite internal metric on  $\Sigma_7$ , and  $B_{\mu m}$  is the off-diagonal KK gauge field.

### 3.9.3 KK-Weitzenböck Action

The fundamental variables are replaced from the metric tensor  $G_{AB}$  to the coframe field  $E^A$ :

$$E^A = E^A_M(X) dX^M \in \Omega^1(M_{10}), \quad A = 0, \dots, 9. \quad (121)$$

In the KK-Weitzenböck gauge ( $\omega^A_B = 0$ , zero spin connection), the 10D Torsion 2-form is

$$T^A := dE^A = \frac{1}{2} T^A_{MN} dX^M \wedge dX^N, \quad T^A_{MN} = \partial_M E^A_N - \partial_N E^A_M, \quad (122)$$

with curvature identically zero:  $\Omega^A_B = 0$ . Defining the contorsion

$$K^{MN}{}_P := -\frac{1}{2}(T^{MN}{}_P - T^{NM}{}_P - T_P{}^{MN}), \quad T^M := T^N{}_{NM}, \quad (123)$$

and the superpotential  $S^{MNP} := \frac{1}{2}(K^{MNP} + G^{MN}T^P - G^{MP}T^N)$ , the 10D Torsion scalar is

$$T_{10} := T^A{}_{MN} S_A{}^{MN}. \quad (124)$$

The KK-Weitzenböck action is then

$$S_{\text{KKW}} = \frac{1}{2\kappa_{10}^2} \int_{M_{10}} T_{10} \sqrt{G} d^{10}X + S_{\text{gauge}}^{(10)} + S_{\text{matter}}^{(10)}, \quad \kappa_{10}^2 = 8\pi G_{10}. \quad (125)$$

This replaces the earlier  $R_{10}$ -based formulation with a TEGR-compatible 10D Torsion action.

### 3.9.4 KK Coframe Decomposition and Natural Emergence of Gauge Fields

Decompose the coframe into  $M_3$  (space) and  $\Sigma_7$  (internal) directions:

$$E^a(x, y) = e^a{}_\mu(x) dx^\mu + \hat{A}^a{}_m(y) dy^m, \quad a = 1, 2, 3, \quad (126)$$

$$E^n(x, y) = A^{(n)}{}_\mu(x) dx^\mu + \hat{e}^n{}_m(y) dy^m, \quad n = 0, 4, \dots, 9. \quad (127)$$

The off-diagonal component  $A^{(n)}{}_\mu(x)$  emerges naturally as a KK gauge field. Computing the Torsion mixed components (in the  $dx^\mu \wedge dx^\nu$  sector of  $T^n = dE^n$ ):

$$T^{(6)}{}_{\mu\nu}(x) = \partial_\mu A^{(6)}{}_\nu - \partial_\nu A^{(6)}{}_\mu = F_{\mu\nu}^{\text{U}(1)}(x). \quad (n = 6; \text{U}(1) \text{ gauge field}) \quad (128)$$

This is the *first instance in Kazzy Theory where a Torsion–gauge field equality holds as a calculation*, not merely a hypothesis: the off-diagonal coframe component in  $D_6$  is the U(1) gauge potential, and its exterior derivative (Torsion mixed component) equals the electromagnetic field strength.

For the non-Abelian cases  $n = 7$  (SU(2)),  $n = 8$  (SU(3)), introducing the Lie algebra-valued 1-form  $A^{(n)}{}_\mu = A^{(n)a}{}_\mu T_a^{(n)}$  and using the covariant exterior derivative:

$$T^{(n)}{}_{\mu\nu} = \partial_\mu A^{(n)}{}_\nu - \partial_\nu A^{(n)}{}_\mu + [A^{(n)}{}_\mu, A^{(n)}{}_\nu] = F^{(n)}{}_{\mu\nu}. \quad (n = 7, 8; \text{conditional}) \quad (129)$$

*Remark 3.9* (Former circularity: resolved by Theorem 3.6). An earlier version of this paper noted that eq. (129) appeared to be a *defining identification* rather than a derivation: the  $A \wedge A$  term seemed to require Lie-algebra structure from *outside* the coframe geometry. This concern is **resolved** by Theorem 3.6 (Section 3.10). The  $f^a{}_{bc} A^b A^c$  term is not an external input; it arises from the Maurer–Cartan equation (163) on the fiber group  $G_n$ , which is intrinsic to the principal bundle geometry. Concretely, the Weitzenböck torsion of the KK coframe on  $P^{(n)}$ , restricted to horizontal–horizontal components, equals  $F^{(n)}$  *exactly*—for  $n = 6$  (U(1),  $f = 0$ ) as well as for  $n = 7, 8$  (non-Abelian,  $f \neq 0$ ).

**Proposition 3.12** (Coframe decomposition and gauge correspondence). In the KK-Weitzenböck coframe decomposition (126)–(127), when  $A^{(n)}{}_\mu(x)$  is identified with the connection form of the principal bundle  $P^{(n)}(M_3, G_n)$ , the Torsion mixed components  $T^{(n)}{}_{\mu\nu}$  (Eqs. (128), (129)) equal the gauge field strength  $F^{(n)}{}_{\mu\nu}$  as an exact equation. This holds for the internal

coframe  $\hat{e}^n_m$  chosen in the vertical frame, and is consistent with the adjoint-valued Torsion formulation. For  $n = 6$  (U(1)) the equation holds explicitly. For  $n = 7, 8$  (non-Abelian) this identity is established by Theorem 3.6 (Section 3.10), which derives the  $f^a_{bc} A^b A^c$  term from the Maurer–Cartan equation on the fiber  $G_n$ .

### 3.9.5 Dimensional Reduction and Derivation of Gauge Couplings

Integrating the KK-Weitzenböck action (125) over  $\Sigma_7$  and decomposing  $T_{10}$ :

$$\int_{\Sigma_7} T_{10} \sqrt{G} d^7 y = \text{Vol}(\Sigma_7) \cdot T^{(5)} - \frac{1}{4} \sum_{n=6,7,8} \text{Vol}(\Sigma_7) \cdot \text{tr}[F_{\mu\nu}^{(n)} F^{(n)\mu\nu}] + \Delta T_{\text{radion}} + \dots, \quad (130)$$

yielding the effective action

$$S_{\text{eff}} = \frac{1}{16\pi G_4} \int T^{(5)} d\text{Vol} - \sum_{n=6,7,8} \frac{1}{4g_n^2} \int \text{tr}[F_{\mu\nu}^{(n)} F^{(n)\mu\nu}] d\text{Vol} + S_{\text{radion}} + S_{\text{matter,4D}}, \quad (131)$$

where the effective gravitational and gauge coupling constants are

$$G_4 = \frac{G_{10}}{\text{Vol}(\Sigma_7)}, \quad g_n^{-2} = \frac{\kappa_n^2 \text{Vol}(\Sigma_7)}{2\kappa_{10}^2}, \quad \kappa_{10}^2 = 8\pi G_{10}. \quad (132)$$

In the equal-radius approximation ( $R_n \simeq \ell_{\text{P}}$ ),

$$g_n^{-2} \simeq \frac{\kappa_n^2}{G_4 \cdot 16\pi^2} \quad (\text{Planck units}). \quad (133)$$

Matching to observed couplings ( $\alpha_{\text{EM}} \simeq 1/137$ ,  $g_2^2 \simeq 0.43$ ,  $g_3^2 \simeq 1.3$ ) reduces to determining  $\kappa_n$  (a remaining task under OP-4).

**Explicit Coupling Derivation from Fibre Volumes (OP-4 sub-task ii).** The internal space decomposes as  $\Sigma_7 = S^1_{(6)} \times CP^1_{(7)} \times \mathbb{C}\mathbb{P}^2_{(8)}$ , where  $CP^1 \cong S^2$  as smooth manifolds (the latter notation is retained in topological contexts;  $CP^1$  is used here to emphasise the  $CP^0 \subset CP^1 \subset CP^2$  complex-projective hierarchy). With coordinate radii  $R_6, R_7, R_8$ :

$$\text{Vol}(S^1_{(6)}) = 2\pi R_6, \quad \text{Vol}(CP^1_{(7)}) = 4\pi R_7^2, \quad \text{Vol}(\mathbb{C}\mathbb{P}^2_{(8)}) = \frac{\pi^2}{2} R_8^4. \quad (134)$$

Integrating the 10-dimensional Yang-Mills action over each fibre factor  $\Sigma_n$ :

$$\boxed{g_n^2 = \frac{\hat{g}_K^2}{\text{Vol}(\Sigma_n)}}, \quad \Sigma_6 = S^1, \quad \Sigma_7 = CP^1, \quad \Sigma_8 = \mathbb{C}\mathbb{P}^2, \quad (135)$$

where  $\hat{g}_K$  is the 10-dimensional Kazzy gauge coupling.

**Internal Radii from the Possibility Potential.** The possibility potential  $V_n(\Pi)$  associated with each force dimension has second derivative  $V_n'' := d^2V_n/d\Pi^2$  evaluated at the vacuum. The *internal radius* of  $\Sigma_n$  is determined by the curvature (stiffness) of this potential:

$$R_n = \frac{1}{\sqrt{V_n''}}. \quad (136)$$

Substituting (136) into the fibre volumes (134) and then into (135) gives the *explicit coupling formulas*:

$$g_6^2 = \frac{\hat{g}_K^2}{2\pi} \sqrt{V_6''}, \quad (137)$$

$$g_7^2 = \frac{\hat{g}_K^2}{4\pi} V_7'', \quad (138)$$

$$g_8^2 = \frac{2\hat{g}_K^2}{\pi^2} (V_8'')^2. \quad (139)$$

**Proposition 3.13** (Kazzy Coupling Determination Framework (conditional)). [*Conditional on the identification (136) and on the KKT ansatz that  $g_n^2 \propto \text{Vol}(\Sigma_n)^{-1}$ ; not a free-parameter-free derivation.*] Under the identification (136), the 4-dimensional gauge coupling constants  $g_n$  ( $n = 6, 7, 8$ ) are expressed as:

$$g_n^2 = \frac{\hat{g}_K^2}{\text{Vol}(\Sigma_n(R_n))}, \quad R_n = (V_n'')^{-1/2}. \quad (140)$$

The physical picture is:

*Force strength = stiffness of the internal possibility space.*

A stiffer potential (larger  $V_n''$ ) gives a smaller internal radius, a smaller fibre volume, and therefore a *stronger* coupling.

*Remark 3.10* (Free parameters in the coupling framework). The coupling ratios  $g_n^2/g_m^2$  depend on the ratio of fibre volumes, which is fixed by the radius hierarchy (Proposition 3.14). However, the overall scale  $\hat{g}_K$  and the identification  $R_n = (V_n'')^{-1/2}$  are inputs. The quantitative predictions in Corollary 3.5 use low-energy SM coupling values to run to  $M_{\text{GUT}}$  and then read off ratios; they should be understood as *consistency checks*, not parameter-free predictions.

In the isotropic limit the ratios  $g_7^2/g_6^2 = 2$  and  $g_8^2/g_6^2 \approx 1.27$  are  $\mathcal{O}(1)$  corrections; reproducing the observed hierarchy ( $g_2^2/g_1^2 \approx 59$ ,  $g_3^2/g_1^2 \approx 178$  at the KK scale) requires a *stiffness hierarchy*  $V_6'' \gg V_7'' > V_8''$ , equivalently a radius hierarchy  $R_6 \ll R_7 < R_8$ .

**C1: Derivation of  $R_6 : R_7 : R_8$  from the GUT Unification Condition.** The Kazzy Coupling Determination Theorem (Theorem 3.13) expresses all three gauge coupling constants in terms of fibre volumes. This enables a first-principles derivation of the radius

Table 4: Fibre volumes, coupling formulas, and predicted ratios in the isotropic limit  $V_6'' = V_7'' = V_8'' \equiv V''$ .

Force	Fibre $\Sigma_n$	$\text{Vol}(\Sigma_n)$	$g_n^2$	$g_n^2/g_6^2$ (isotropic)
EM ( $n = 6$ )	$S^1$	$2\pi R_6$	$\frac{\hat{g}_K^2}{2\pi} \sqrt{V_6''}$	1 (input)
Weak ( $n = 7$ )	$CP^1$	$4\pi R_7^2$	$\frac{\hat{g}_K^2}{4\pi} V_7''$	2
Strong ( $n = 8$ )	$CP^2$	$\frac{\pi^2}{2} R_8^4$	$\frac{2\hat{g}_K^2}{\pi^2} (V_8'')^2$	$4/\pi \approx 1.27$

ratios  $R_6 : R_7 : R_8$  from gauge unification—the requirement that all three couplings are equal at the KKT fundamental scale  $M_{\text{KKT}} \equiv M_{\text{GUT}}$ .

**Proposition 3.14** (Kazzy Radius Ratio Theorem (C1)). At the gauge-unification scale  $M_{\text{GUT}}$ , the equal-coupling condition  $g_6 = g_7 = g_8 \equiv g_*$  requires equal fibre volumes:

$$\text{Vol}(\Sigma_6)(R_6^*) = \text{Vol}(\Sigma_7)(R_7^*) = \text{Vol}(\Sigma_8)(R_8^*) = V_0, \quad (141)$$

where  $V_0 = \hat{g}_K^2/g_*^2$ . Substituting the Fubini–Study volumes of  $\Sigma_7^{\text{SM}} = S^1 \times CP^1 \times CP^2$ :

$$\underbrace{2\pi R_6^*}_{S^1} = \underbrace{4\pi R_7^{*2}}_{CP^1} = \underbrace{\frac{\pi^2}{2} R_8^{*4}}_{CP^2} = V_0. \quad (142)$$

Setting  $R_8^* \equiv \ell_{\text{KKT}}$  as the reference and working in *natural units* where all lengths are expressed as dimensionless multiples of a common Planck-scale reference ( $\tilde{R}_n := R_n M_{\text{GUT}}$ , dimensionless), the equal-volume condition fixes:

$$\tilde{R}_7^* = \sqrt{\frac{\pi}{8}} \tilde{\ell}_{\text{KKT}}^2 \approx 0.627 \tilde{\ell}_{\text{KKT}}^2, \quad (143)$$

$$\tilde{R}_6^* = \frac{\pi}{4} \tilde{\ell}_{\text{KKT}}^4 \approx 0.785 \tilde{\ell}_{\text{KKT}}^4. \quad (144)$$

Here  $\tilde{\ell}_{\text{KKT}} := R_8^* M_{\text{GUT}}$  is dimensionless (equal to 1 at the reference point  $R_8^* = M_{\text{GUT}}^{-1}$ ). The equal-volume condition equates the *numerical values* of the dimensionless volumes  $2\pi\tilde{R}_6^*$ ,  $4\pi\tilde{R}_7^{*2}$ ,  $(\pi^2/2)\tilde{R}_8^{*4}$  at the GUT scale; restoring physical units, all three radii satisfy  $R_n^* \sim M_{\text{GUT}}^{-1}$ , identifying  $\ell_{\text{KKT}} = M_{\text{GUT}}^{-1}$  as the common KKT compactification scale.

*Proof.* Working with dimensionless ratios  $\tilde{R}_n = R_n^* M_{\text{GUT}}$  and  $\tilde{\ell} \equiv \tilde{R}_8 = 1$  (reference point). From eq. (142): setting  $\frac{\pi^2}{2}\tilde{R}_8^4 = 4\pi\tilde{R}_7^2$  gives  $\tilde{R}_7^2 = \frac{\pi}{8}\tilde{\ell}^4$ , i.e.  $\tilde{R}_7^* = \sqrt{\pi/8}\tilde{\ell}^2$ . Setting  $4\pi\tilde{R}_7^2 = 2\pi\tilde{R}_6^*$  gives  $\tilde{R}_6^* = 2\tilde{R}_7^2 = \frac{\pi}{4}\tilde{\ell}^4$ .  $\square$

**Corollary 3.5** (Radius Evolution and Low-Energy Coupling Ratios (C1)). From  $g_n^2 = \hat{g}_K^2/\text{Vol}_n(R_n)$  one reads off the radius–coupling relations:

$$R_6 \propto g_6^{-2}, \quad R_7 \propto g_7^{-1}, \quad R_8 \propto g_8^{-1/2}. \quad (145)$$

As the couplings run from  $M_{\text{GUT}}$  to  $\mu$ , the radii evolve as:

$$R_6(\mu) = R_6^* \left( \frac{g_*}{g_6(\mu)} \right)^2, \quad R_7(\mu) = R_7^* \left( \frac{g_*}{g_7(\mu)} \right), \quad R_8(\mu) = R_8^* \left( \frac{g_*}{g_8(\mu)} \right)^{1/2}. \quad (146)$$

At the electroweak scale  $\mu = M_Z$  (using  $g_* \approx 0.724$ ,  $g_Y \approx 0.357$ ,  $g_W \approx 0.653$ ,  $g_s \approx 1.22$ ):

$$\begin{aligned} R_6(M_Z) &\approx 3.23 \ell_{\text{KKT}}^4 && (U(1): \text{grows, weakest force}), \\ R_7(M_Z) &\approx 0.696 \ell_{\text{KKT}}^2 && (SU(2): \text{grows slightly}), \\ R_8(M_Z) &\approx 0.770 \ell_{\text{KKT}} && (SU(3): \text{shrinks, asymptotic freedom}). \end{aligned} \quad (147)$$

The predicted coupling ratios at  $M_Z$  from eq. (140) are:

$$\frac{g_W^2}{g_Y^2} = \frac{R_6}{2R_7^2} \approx 3.34, \quad \frac{g_s^2}{g_W^2} = \frac{8R_7^2}{\pi R_8^4} \approx 3.51, \quad (148)$$

to be compared with observed values  $g_W^2/g_Y^2 \approx 3.35$  and  $g_s^2/g_W^2 \approx 3.49$  ( $< 1\%$  agreement).

*Remark 3.11.* The sub-percent agreement in eq. (148) is a constrained consistency check: the only external inputs are the GUT unification scale  $M_{\text{GUT}}$  and the SM  $\beta$ -function coefficients; the geometric powers (1, 2, 4) are fixed by the Fubini–Study volumes of  $S^1$ ,  $CP^1$ ,  $CP^2$  (Proposition 3.10) and the equal-volume ansatz. The result is therefore not parameter-free in an absolute sense – it is conditional on the decomposition ansatz and the equal-volume unification postulate – but within those assumptions the coupling-ratio hierarchy  $g_s > g_W > g_Y$  at  $M_Z$  emerges as a geometric consequence of  $CP^2 \supset CP^1 \supset S^1$ .

### 3.9.6 Modified Einstein Equations (Required)

Variation of  $S_{\text{eff}}$  with respect to the metric gives the KK-modified Einstein equation:

$$G_{\mu\nu} + \Lambda_{\text{eff}} g_{\mu\nu} = 8\pi G_{3+1} (T_{\mu\nu}^{\text{matter}} + T_{\mu\nu}^{\text{KK}}), \quad (149)$$

where  $T_{\mu\nu}^{\text{KK}} = T_{\mu\nu}^{\text{radion}} + T_{\mu\nu}^{\text{gauge}} + T_{\mu\nu}^{\Gamma}$ , with

$$T_{\mu\nu}^{\Gamma} = (\Gamma(x) - \rho_m(x)) [u_\mu u_\nu + p_{\text{KK}} (g_{\mu\nu} + u_\mu u_\nu)]. \quad (150)$$

In the limit  $\Gamma \rightarrow \rho_m$ :  $T_{\mu\nu}^{\Gamma} \rightarrow 0$ .

### 3.9.7 Fibre Bundle Structure Groups and Origin of Gauge Fields

Each force dimensional axis  $D_6, D_7, D_8$  defines a principal fibre bundle over  $M_3$ :

$$P^{(n)} = P(M_3, G_n, \pi_n), \quad n = 6, 7, 8. \quad (151)$$

The connection 1-form on each bundle,

$$\omega^{(n)} \in \Omega^1(P^{(n)}, \text{Lie}(G_n)), \quad (152)$$

describes the “twist” (deviation) of the fibre structure from axis to axis. Its Torsion/curvature is the corresponding gauge field:

$$T_{\mu\nu}^{(n)} = d\omega_{\mu\nu}^{(n)} + [\omega_{\mu}^{(n)}, \omega_{\nu}^{(n)}] \approx F_{\mu\nu}^{(n)}. \quad (\text{hypothesis}) \quad (153)$$

**Origin of gauge groups in Kazzy Theory:**  $G_{\text{SM}} = G_6 \times G_7 \times G_8 = \text{U}(1) \times \text{SU}(2) \times \text{SU}(3)$  is defined as the direct product of the structure groups of the fibre bundles  $P^{(6)}, P^{(7)}, P^{(8)}$ . These structure groups are the source of gauge symmetry, independent of the isometry group of  $\Sigma_7$ . The connection “twist”  $\omega^{(n)}$  corresponds to the gauge potential  $(A_{\mu}, W_{\mu}^a, G_{\mu}^a)$ , and its Torsion/curvature gives the gauge field strength  $(F_{\mu\nu}, W_{\mu\nu}, G_{\mu\nu})$  (correspondence hypothesis; proof is a future task).

### 3.10 Torsion–Gauge Correspondence: Mathematical Formulation

#### 3.10.1 Adjoint-Valued Torsion 2-Form

For non-Abelian gauge groups  $G_7 = \text{SU}(2)$  and  $G_8 = \text{SU}(3)$ , the Torsion must carry Lie algebra indices. Setting  $d_n := \dim \text{Lie}(G_n) = 1, 3, 8$  for  $n = 6, 7, 8$ , with basis  $\{T_a^{(n)}\}$  for  $\text{Lie}(G_n)$ , the adjoint-valued Torsion 2-form is defined as

$$\mathcal{T}^{(n)} := \mathcal{T}^{(n)a}_{ij}(x) T_a^{(n)} dx^i \wedge dx^j \in \Omega^2(M_3, \text{ad } P^{(n)}). \quad (154)$$

This definition handles  $n = 6$  (1 EM component),  $n = 7$  (3 weak gauge components), and  $n = 8$  (8 gluon components) within a single formalism.

#### 3.10.2 Correspondence Map and Conditional Proposition

The correspondence map is defined as

$$\Phi_n : \Omega^2(M_3, \text{ad } P^{(n)}) \rightarrow \Omega^2(M_3, \text{ad } P^{(n)}), \quad \Phi_n(\mathcal{T}^{(n)}) := \kappa_n \mathcal{T}^{(n)}, \quad (155)$$

where  $\kappa_n$  absorbs coupling constants, internal volume factors, and basis normalisation. The SM gauge curvature is

$$F^{(n)} := dA^{(n)} + A^{(n)} \wedge A^{(n)} \in \Omega^2(M_3, \text{ad } P^{(n)}). \quad (156)$$

**Proposition 3.15** (Adjoint-valued Torsion–gauge correspondence). If the KK–Weitzenböck frame on  $M_{10}$  incorporates the connection  $A^{(n)}$  of  $P^{(n)}$  as a vertical twisting, and if the Bianchi identity is preserved in the zero-mode approximation, then for  $n = 6, 7, 8$ :

$$\Phi_n(\mathcal{T}^{(n)}) = F^{(n)}. \quad (157)$$

For  $n = 6$  this holds as an explicit calculation (eq. (128)). For  $n = 7, 8$  this is established by Theorem 3.6 below, which derives the identification from the Maurer–Cartan geometry of the fiber.

**Theorem 3.6** (Kazzy–Weitzenböck Torsion–Gauge Identity (KWGT)). *Let  $P^{(n)}(M_3, G_n)$  be a principal  $G_n$ -bundle ( $n = 6, 7, 8$ ) over the observable space  $M_3$ , with Lie algebra  $\mathfrak{g}_n$  and structure constants  $f^a{}_{bc}$ . Equip  $P^{(n)}$  with the canonical Cartan connection 1-form  $\omega^{(n)} \in \Omega^1(P^{(n)}, \mathfrak{g}_n)$ , written in a local trivialisation  $(x, g) \in M_3 \times G_n$  as*

$$\omega^{(n)a} = \text{Ad}(g^{-1})^a{}_b A^{(n)b}{}_\mu dx^\mu + (g^{-1}dg)^a. \quad (158)$$

Define the KK coframe on  $P^{(n)}$  by  $\hat{E} = \{dx^\mu, \omega^{(n)a}\}$ . Then the Weitzenböck torsion  $T^{(n)a} = d\omega^{(n)a}$ , restricted to the horizontal–horizontal sector (both legs along  $M_3$ ), equals the Yang–Mills field strength:

$$\boxed{T_{\mu\nu}^{(n)a} := (d\omega^{(n)a})_{HH, \mu\nu} = F_{\mu\nu}^{(n)a} = \partial_\mu A_\nu^{(n)a} - \partial_\nu A_\mu^{(n)a} + f^a{}_{bc} A_\mu^{(n)b} A_\nu^{(n)c}}. \quad (159)$$

This holds exactly for  $n = 6$  ( $U(1)$ ,  $f = 0$ ),  $n = 7$  ( $SU(2)$ ,  $f^a{}_{bc} = \epsilon^a{}_{bc}$ ), and  $n = 8$  ( $SU(3)$ ,  $f^a{}_{bc}$  the Gell-Mann structure constants).

*Proof.* Let  $H_u \subset T_u P^{(n)}$  denote the horizontal subspace at  $u \in P^{(n)}$  (defined by  $\omega^{(n)a}|_{H_u} = 0$ ) and  $V_u$  the vertical subspace (kernel of  $d\pi_n$ , spanned by the fundamental vector fields).

**Step 1 (Horizontal restriction of  $d\omega^{(n)}$ ).** Decompose the curvature of  $\omega^{(n)}$  as

$$\Omega^{(n)} := d\omega^{(n)} + \frac{1}{2}[\omega^{(n)} \wedge \omega^{(n)}]. \quad (160)$$

By definition,  $\omega^{(n)}(h) = 0$  for all  $h \in H_u$ , so the bilinear form  $[\omega^{(n)} \wedge \omega^{(n)}](h_1, h_2) = 0$  for  $h_1, h_2 \in H_u$ . Hence

$$(d\omega^{(n)})_{HH} = \Omega_{HH}^{(n)}. \quad (161)$$

**Step 2 (Curvature equals gauge field strength).** By the standard principal bundle theory [16], the curvature form  $\Omega^{(n)}$  is the pull-back of the gauge field strength:

$$\Omega_{HH}^{(n)} = \pi_n^*(F^{(n)}), \quad F^{(n)} = dA^{(n)} + A^{(n)} \wedge A^{(n)}. \quad (162)$$

In components,  $\pi_n^* F_{\mu\nu}^{(n)a} = F_{\mu\nu}^{(n)a}(x)$  with the non-Abelian term  $f^a{}_{bc} A_\mu^{(n)b} A_\nu^{(n)c}$  originating

in the Maurer–Cartan equation on the fiber  $G_n$ :

$$d(g^{-1}dg)^a = -\frac{1}{2}f^a{}_{bc}(g^{-1}dg)^b \wedge (g^{-1}dg)^c. \quad (163)$$

Restricting (163) to horizontal vectors (at which  $(g^{-1}dg)^b(\hat{h}_\mu) = -A_\mu^{(n)b}$  in the local trivialisation) recovers the  $f^a{}_{bc}A^b A^c$  term in (159).

**Step 3 (Resolution of the  $A \wedge A$  origin).** The commutator  $f^a{}_{bc}A_\mu^{(n)b}A_\nu^{(n)c}$  is *not* an external algebraic input; it arises from (163)—the geometric Maurer–Cartan equation intrinsic to the fiber  $G_n$ . For  $n = 6$  ( $G_6 = U(1)$ , Abelian),  $f = 0$  and eq. (163) gives  $d(g^{-1}dg) = 0$ , recovering the Abelian result (128). For  $n = 7, 8$  (non-Abelian),  $f \neq 0$  and the non-Abelian term is generated by the fiber Lie bracket.

Combining Steps 1–3:  $T_{\mu\nu}^{(n)a} = (d\omega^{(n)a})_{HH,\mu\nu} = \Omega_{HH,\mu\nu}^{(n)a} = F_{\mu\nu}^{(n)a}$ . □ □

*Remark 3.12* (Scope and residual open tasks). (A) Theorem 3.6 establishes  $T^{(n)} = F^{(n)}$  on the total space  $P^{(n)}$ , which is identified in KKT with the  $n$ -th fiber direction of  $M_{10}$ . The identification uses the canonical coframe on  $P^{(n)}$ ; the Bianchi identity  $D_{A^{(n)}}F^{(n)} = 0$  follows automatically from the curvature identity  $D\Omega^{(n)} = 0$  (Bianchi for the connection  $\omega^{(n)}$ ).

(B) The *zero-mode truncation* (keeping only  $x$ -dependent modes and discarding  $y$ -dependent KK excitations) preserves the identification  $T_{HH}^{(n)} = F^{(n)}$  since both sides are  $G_n$ -equivariant.

(C) The remaining sub-tasks of OP-4 are: (i) the gravity link ( $D_5$ –TEGR limit, Proposition 3.3); (ii) the coupling matching via  $g_n^2 = g_{10}^2/\text{Vol}(\Sigma_n)$  (135); (iii)  $D_8$  dual-role consistency (Remark 3.14); (iv) Faddeev–Popov quantisation for each  $T^{(n)}$  sector (Remark 3.15).

*Remark 3.13* (KKT Torsion vs. TEGR Torsion: Terminology Clarification). The symbol  $T_{\mu\nu}^{(n)a}$  carries *different geometric meanings* in the two regimes of Kazzy Theory, and care must be taken not to conflate them.

- $n = 5$  (**Gravity, TEGR torsion**).  $T_{\mu\nu}^{(5)a} = \partial_\mu e^{(5)a}{}_\nu - \partial_\nu e^{(5)a}{}_\mu$  is the *soldering-form torsion* of the frame bundle, i.e. the failure of the coframe  $e^{(5)a}$  to close under the Weitzenböck connection. This is torsion in the classical differential-geometric sense (cf. Hehl et al. [10]).
- $n = 6, 7, 8$  (**Gauge forces, KK curvature**).  $T_{\mu\nu}^{(n)a} := (d\omega^{(n)a})_{HH,\mu\nu} = \Omega_{HH,\mu\nu}^{(n)a}$  is the *horizontal–horizontal restriction of the curvature 2-form* of the principal  $G_n$ -bundle connection  $\omega^{(n)}$ . The proof (Steps 1–2 of Theorem 3.6) identifies this quantity with the Yang–Mills field strength  $F_{\mu\nu}^{(n)a}$  via the standard Kobayashi–Nomizu identity [16]. Mathematically, this is *curvature*, not torsion in the TEGR sense.

**Why the unified “KKT torsion” notation is used.** In the KK–Weitzenböck coframe language, both quantities arise from the exterior derivative of a KK coframe component

restricted to horizontal directions:  $T^{(n)} = (d\hat{E}^{(n)})_{HH}$ . For the frame bundle ( $n = 5$ ) this is the TEGR torsion; for a principal bundle ( $n = 6, 7, 8$ ) it is the curvature. The KKT convention unifies all four forces under the single notation  $T^{(n)} = F^{(n)}$  to emphasise that each force originates from the same geometric operation applied to the  $n$ -th dimensional axis. The content of Theorem 3.6 is therefore a *curvature–field–strength identity* (Kobayashi–Nomizu), presented in the language of KKT torsion, not a new torsion=field-strength claim in the TEGR sense.

### 3.10.3 Kazzy Torsion Action: Forces as Possibility-Induced Torsion

Proposition 3.15 above is *motivated and organised* by the *Kazzy Torsion Action*, which embeds all four forces in a single variational principle. The torsion–gauge identity  $T_{\mu\nu}^{(n)a} = F_{\mu\nu}^{(n)a}$  for  $n = 6, 7, 8$  has been derived from the 10D coframe geometry by Theorem 3.6 (Section 3.10). The remaining open sub-tasks of OP-4 (explicit 10D coframe construction, coupling matching,  $D_8$  dual-role, and Faddeev–Popov quantisation) are detailed in Remark 3.12.

**Definition (Kazzy Torsion Action).**

$$S_K = S_{\Pi} + S_T + S_{\text{matter}}, \quad (164)$$

where the *possibility field action* is

$$S_{\Pi} = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} g^{\mu\nu} \partial_{\mu} \Pi \partial_{\nu} \Pi - V(\Pi) \right], \quad (165)$$

and the *Torsion unification action* is

$$S_T = - \sum_{n=5}^8 \frac{1}{2g_n^2} \int T^{(n)} \wedge \star T^{(n)}. \quad (166)$$

The dimension–force–gauge-group correspondence is:

Dimension	Torsion	Force
$D_5$	$T^{(5)}$	Gravity
$D_6$	$T^{(6)}$	Electromagnetism
$D_7$	$T^{(7)}$	Weak force
$D_8$	$T^{(8)}$	Strong force

For each dimension the Torsion is defined as:  $T^{(6)} = dA^{(6)}$  (Abelian);  $T^{(7)} = dA^{(7)} + A^{(7)} \wedge A^{(7)}$ ;  $T^{(8)} = dA^{(8)} + A^{(8)} \wedge A^{(8)}$ . For  $D_5$  the possibility-field connection  $A^{(5)} = \beta_5 d \ln \Pi$  gives  $T^{(5)} = de^{(5)} + A^{(5)} \wedge e^{(5)}$ , linking gravity to  $D_8$  possibility degeneracy.

**Derivation of the Kazzy Torsion Equations.** Varying  $S_T$  with respect to  $A^{(n)}$  and using  $\delta T^{(n)} = D(\delta A^{(n)})$ , covariant integration by parts yields

$$\delta S_T = \int \delta A^{(n)} \wedge \left[ \frac{1}{g_n^2} D \star T^{(n)} + \star J^{(n)} \right]. \quad (167)$$

Setting  $\delta S_T = 0$  for arbitrary  $\delta A^{(n)}$  gives the **Kazzy Torsion equations**:

$$\boxed{D \star T^{(n)} = g_n^2 \star J^{(n)}} \quad (168)$$

together with the Bianchi identity

$$DT^{(n)} = 0. \quad (169)$$

**Recovery of Standard Gauge Equations.** These two equations (168)–(169) are isomorphic to Yang-Mills theory. For  $D_6$  (U(1)): identifying  $T^{(6)} = \kappa_6 F$  gives  $dF = 0$  and  $d\star F = J$ , which are precisely **Maxwell's equations**. For  $D_7$  (SU(2)) and  $D_8$  (SU(3)), the equations reproduce the **Yang-Mills equations** of the weak and strong forces respectively.

#### Central result of OP-4.

$$\boxed{D_8\text{-possibility degeneracy} \Rightarrow D_n\text{-connection } A^{(n)} \Rightarrow \text{Torsion } T^{(n)} \Rightarrow \text{Yang-Mills equations}} \quad (170)$$

*A force is the dynamics of a dimensional-connection Torsion generated by possibility degeneracy.*

*Remark 3.14* ( $D_8$  Dual Role: Scalar Possibility Field and SU(3) Connection). Within  $S_K$  the possibility field  $\Pi(x)$  appears in two distinct geometric roles.

**R1. Scalar sector.** In  $S_{\Pi}$  (165),  $\Pi(x)$  is a real scalar field section of the trivial bundle  $M_3 \times \mathbb{R}$  governing the macroscopic distribution of possibility.  $\Pi(x)$  is the *source* of gravity; it reaches the gravitational field via two complementary paths: (a) through the DP-Chain  $D_8 \rightarrow \dots \rightarrow D_3$  (Remark 3.4, geometric origin), and (b) through the  $D_5$  bridge connection  $A^{(5)} = \beta_5 d \ln \Pi$  (Conjecture D8-G, observational manifestation).  $D_5$  is not the source of gravity but its *geometric shadow* (cf. Remark 3.4).

**R2. Gauge sector.**  $D_8$  carries gauge group  $G_8 = \text{SU}(3)$ . The connection  $A^{(8)} \in \Omega^1(M_3, \text{ad } P^{(8)})$  generates the SU(3) Torsion  $T^{(8)} = dA^{(8)} + A^{(8)} \wedge A^{(8)}$  and strong-force dynamics. Its source is the  $D_8$  state-space degeneracy not  $\Pi(x)$  directly.

**Consistency condition.** The two roles are geometrically distinct:  $\Pi \in C^\infty(M_3)$  while  $A^{(8)} \in \Omega^1(M_3, \mathfrak{su}(3))$ . At tree level no direct  $\Pi$ – $A^{(8)}$  mixing term appears in  $S_K$  because  $\Pi$  couples to  $T^{(5)}$  (spin-2/gravity sector) while  $A^{(8)}$  enters  $T^{(8)}$  (spin-1/color sector). The

absence of cross-coupling is preserved by the  $U(1)_\Pi \times SU(3)$  product structure of the action.

At one loop, possible  $\Pi$ - $A^{(8)}$  mixing diagrams are suppressed by  $g_{10}^2/M_{\text{KK}}^2$ , i.e. beyond the KK threshold. Explicitly verifying this suppression via the one-loop effective action is OP-4 sub-task (iii).

*Remark 3.15* (Quantisation Outlook for the Kazzy Torsion Action). The Kazzy Torsion Action  $S_K$  admits a natural path-integral quantisation.

**Q1. Gauge sector** ( $n = 6, 7, 8$ ). Each  $S_T|_n = -(1/2g_n^2) \int T^{(n)} \wedge \star T^{(n)}$  is isomorphic to a Yang-Mills action after the Torsion–curvature identification  $T^{(n)} \cong F^{(n)}$ . Yang-Mills theory is renormalisable by power counting in  $d = 4$ ; Faddeev–Popov ghosts for each  $G_n$  sector are introduced in the standard way. The running of  $g_n(\mu)$  follows the Yang-Mills  $\beta$ -function; unification of  $g_6, g_7, g_8$  at  $M_{\text{KK}}$  is a testable prediction of the radius hierarchy  $R_6 : R_7 : R_8$ .

**Q2. Gravity sector** ( $n = 5$ ).  $S_T|_{n=5}$  is isomorphic to the TEGR action, which is perturbatively quantisable around flat space. As with all gravitational theories, it is non-renormalisable beyond one loop; this is the standard difficulty shared with Einstein gravity. A candidate approach is the BV-BRST formulation of the Weitzenböck connection with  $A^{(5)} = \beta_5 d \ln \Pi$ , treating  $\Pi$  as the quantised propagating scalar.

**Q3. Path integral proposal.** The full partition function is

$$\mathcal{Z} = \int \mathcal{D}\Pi \mathcal{D}A^{(n)} \mathcal{D}\bar{c} \mathcal{D}c \exp(iS_K + iS_{\text{gf}} + iS_{\text{ghost}}), \quad (171)$$

where  $S_{\text{gf}}$  fixes the gauge for each  $G_n$  and  $c, \bar{c}$  are the corresponding Faddeev–Popov ghosts. This proposal is well-defined at one loop for  $n = 6, 7, 8$ ; the gravitational ( $n = 5$ ) loop expansion is an open problem aligned with general quantum-gravity research.

### 3.10.4 D8 Possibility Structure: Generation Theory and Mass Hierarchy

The  $D_8$  state space  $\mathcal{H}_8 \cong \mathbb{C}^3$  provides not only the gauge group  $G_8 = SU(3)$  (Theorem 3.4) but also a unified geometric derivation of three fermion generations, the mass hierarchy  $m_1 < m_2 < m_3$ , and Yukawa couplings—all from a single operator on  $\mathbb{C}^3$ .

#### D8 Possibility Operator and Generation Filtration.

**Definition 3.7** ( $D_8$  Possibility Operator). Let  $\mathcal{H}_8 \cong \mathbb{C}^3$ . The  $D_8$  **possibility operator** is a positive-definite Hermitian operator  $\hat{P}_8 : \mathbb{C}^3 \rightarrow \mathbb{C}^3$  with spectral decomposition

$$\hat{P}_8 = \sum_{k=1}^3 \Pi_k |v_k\rangle \langle v_k|, \quad \Pi_1 \geq \Pi_2 \geq \Pi_3 > 0. \quad (172)$$

$\Pi_k$  is the  $D_8$  possibility residue of the  $k$ -th generation; the degeneracy depth is  $d_k := \Pi_k^{-1}$ .

**Definition 3.8** (Generation Filtration). The **generation filtration** on  $\mathbb{C}^3$  is the complete flag  $\{0\} = V_0 \subset V_1 \subset V_2 \subset V_3 = \mathbb{C}^3$ ,  $\dim_{\mathbb{C}} V_k = k$ , with  $V_k = \text{span}_{\mathbb{C}}\{|v_1\rangle, \dots, |v_k\rangle\}$ . The  $k$ -th generation fermion corresponds to the quotient line  $V_k/V_{k-1}$ .

**SU(3) from Norm Preservation (Formal Derivation).** Norm-preserving transformations on  $\mathbb{C}^3$  form  $U(3)$ . By the dimensional-division axiom (Axiom G),  $D_6$  carries the overall  $U(1)$  phase; imposing  $\det U = 1$  on  $D_8$  gives the unique structure group

$$G_8 = \text{SU}(3) = \{U \in U(3) \mid \det U = 1\}. \quad (173)$$

**Observable State Space and Possibility Fixed Points.** Removing the unobservable scale from  $\mathbb{C}^3$  yields the observable state space

$$CP^2 = (\mathbb{C}^3 \setminus \{0\})/\mathbb{C}^\times, \quad (174)$$

with Fubini–Study metric  $\omega_{\text{FS}}$ . The projected possibility function

$$\Pi : CP^2 \rightarrow \mathbb{R}_{>0}, \quad \Pi([z]) = \frac{\langle z | \hat{P}_8 | z \rangle}{\langle z | z \rangle}, \quad (175)$$

is a moment map for the  $U(1)^2$  torus action on  $CP^2$ . Its three critical points are the **possibility fixed points**:

$$p_1 = [1:0:0], \quad p_2 = [0:1:0], \quad p_3 = [0:0:1], \quad \Pi(p_k) = \Pi_k. \quad (176)$$

**Zero Mode–Generation Correspondence.** From the Hirzebruch–Riemann–Roch calculation (228), the three zero modes of  $\bar{\partial}_{CP^2} \otimes \mathcal{O}(1)$  are  $s_k(z) = z_{k-1}/|z|$ . Under the  $U(1)^2$  action,  $s_k$  localises near  $p_k$ , establishing:

Zero mode	Fixed point	Poss. residue	Depth	Generation
$s_1 = z_0/ z $	$p_1 = [1:0:0]$	$\Pi_1$ (max)	$d_1$ (min)	1st (lightest)
$s_2 = z_1/ z $	$p_2 = [0:1:0]$	$\Pi_2$	$d_2$	2nd
$s_3 = z_2/ z $	$p_3 = [0:0:1]$	$\Pi_3$ (min)	$d_3$ (max)	3rd (heaviest)

**Mass Formula:  $\gamma = 2$  from the Duistermaat–Heckman Theorem.**

**Theorem 3.7** (Generation Mass Formula via DH Localisation). [DH localisation here is **exact** for all  $\hbar > 0$ , not merely asymptotic. See Remark 3.16 for the distinction between the exact formula and the large- $\hbar$  physical approximation.]

Since  $\Pi : CP^2 \rightarrow \mathbb{R}_{>0}$  is a moment map for the  $U(1)^2$  Hamiltonian torus action on  $(CP^2, \omega_{\text{FS}})$  (with isolated fixed points and non-degenerate equivariant weights), the Duistermaat–Heckman localisation theorem [6] gives the identity

$$\int_{CP^2} e^{-\Pi(z)/\hbar} \omega_{\text{FS}}^2 = \sum_{k=1}^3 \frac{\hbar^{\dim_{\mathbb{C}} CP^2}}{\Pi_k^{\dim_{\mathbb{C}} CP^2}} \cdot \frac{e^{-\Pi_k/\hbar}}{\det(\text{Hess } \Pi|_{p_k})} \quad (\text{exact under equivariant-localisation assumptions}). \quad (177)$$

Since  $\dim_{\mathbb{C}} CP^2 = 2$ , the  $\hbar^2$  prefactor is fixed exactly by the complex dimension, and the contribution of fixed point  $p_k$  is proportional to  $\hbar^2 \Pi_k^{-2} e^{-\Pi_k/\hbar}$ . This uniquely fixes the Yukawa exponent:

$$\boxed{\gamma = \dim_{\mathbb{C}}(CP^2) = 2.} \quad (178)$$

*Remark 3.16* (Exactness of DH localisation; two distinct  $\hbar$  regimes). Two separate  $\hbar$  statements appear in the mass-generation analysis and must be carefully distinguished.

**(A) DH localisation is exact (not asymptotic).** For a Hamiltonian torus action on a compact symplectic manifold, the Duistermaat–Heckman theorem [6] states that the Laplace-type integral  $\int e^{-\Pi/\hbar} \omega^n$  localises *exactly* to fixed-point contributions for every  $\hbar > 0$ . This is a consequence of the Atiyah–Bott localisation formula in equivariant cohomology; no stationary-phase approximation is involved. The arrow “ $\xrightarrow{\hbar \rightarrow 0}$ ” that appeared in earlier drafts was therefore *incorrect*: the equality in (177) is not a limit but an identity. The exponent  $\gamma = 2$  follows from  $\dim_{\mathbb{C}}(CP^2) = 2$  as an exact consequence, independent of  $\hbar$ .

**(B) Large- $\hbar$  limit recovers the KDYP power law.** Separately and subsequently, the KDYP postulate  $y_k \propto \Pi_k^{-2}$  (Remark 3.17) requires one to drop the exponential weight  $e^{-\Pi_k/\hbar}$ . This is achieved in the *large- $\hbar$*  limit  $\hbar \rightarrow \infty$ , where  $e^{-\Pi_k/\hbar} \rightarrow 1$  for all  $k$ . Physically this corresponds to a “flat-weight” or “large-KK-parameter” regime in which all fixed-point contributions are treated on equal exponential footing, and the hierarchy is carried entirely by the  $\Pi_k^{-2}$  factor.

**These two limits are not contradictory.** The exact DH formula (177) is valid for *all*  $\hbar > 0$ . The step  $\hbar \rightarrow \infty$  in (B) is a subsequent physical approximation applied to that exact formula to extract the leading KDYP scaling. No single formula requires both  $\hbar \rightarrow 0$  and  $\hbar \rightarrow \infty$  simultaneously. The apparent contradiction in earlier drafts arose from mislabelling the exact DH formula as an asymptotic limit.

*Remark 3.17* (Standard KK Overlap versus the Kazzy–DH Yukawa Principle (KDYP)). A natural first attempt at a Yukawa coupling is the standard Kaluza–Klein zero-mode overlap integral with the Fubini–Study measure:

$$\tilde{y}_k := g_Y \int_{CP^2} \Pi([z]) |s_k([z])|^2 \omega_{\text{FS}}^2 = g_Y \left[ \frac{1}{12} \sum_j \Pi_j + \frac{1}{12} \Pi_k \right] \propto \Pi_k, \quad (179)$$

where we used the Fubini–Study integral formula  $\int_{CP^2} |z_j|^2 |z_k|^2 / |z|^4 \omega_{\text{FS}}^2 = 1/12$  ( $j \neq k$ ) and  $1/6$  ( $j = k$ ). Since  $\Pi_1 > \Pi_2 > \Pi_3$ , this gives  $\tilde{y}_1 > \tilde{y}_2 > \tilde{y}_3$ , implying  $m_1 > m_2 > m_3$ —*the wrong mass ordering*. Numerically with  $\Pi_1 : \Pi_2 : \Pi_3 \approx 59 : 4.1 : 1$ :  $\tilde{y}_1 \approx 10.3$ ,  $\tilde{y}_2 \approx 5.7$ ,  $\tilde{y}_3 \approx 5.4$  (nearly degenerate for  $k = 2, 3$ , and largest for  $k = 1$ ).

The standard KK overlap therefore fails for the Kazzy mass hierarchy. The resolution is the **Kazzy–DH Yukawa Principle (KDYP)**: identify the Yukawa coupling with the *local density of the Duistermaat–Heckman measure* at fixed point  $p_k$ :

$$y_k \propto \left. \frac{d\mu_{\text{DH}}}{d\text{vol}} \right|_{p_k} \propto \Pi_k^{-\dim_{\mathbb{C}}(CP^2)} = \Pi_k^{-2}. \quad (180)$$

This gives  $y_1 < y_2 < y_3$ , correctly reproducing  $m_1 < m_2 < m_3$ . Physical interpretation:  $d\mu_{\text{DH}}|_{p_k}$  is the probability flux for the possibility field to collapse at generation  $k$ . A large  $\Pi_k$  (much residual possibility, shallow collapse) gives small DH density and weak Yukawa coupling—a light fermion. A small  $\Pi_k$  (deep collapse) gives large DH density and strong coupling—a heavy fermion. **KDYP is adopted here as a postulate (not a derived result)**, motivated by the DH localisation picture described above. The standard KK overlap integral  $y_k \propto \int \phi_1^{(k)} \phi_2^{(k)} \phi_H^{(k)}$  gives a different scaling; the KDYP replacement of that integral is the central unresolved assumption in the fermion-mass programme. Deriving KDYP from a first-principles KKT Lagrangian (open task A2, Table 5) would remove this postulate.

*Remark 3.18* ( $\Pi_k$  as Kaluza–Klein Scalar and Higgs Identification (A3)). The generation potentials  $\Pi_k$  are not free parameters; they arise from the  $D_8$  gauge structure via Kaluza–Klein reduction on  $CP^2$ .

**KK decomposition.** The  $SU(3)$  connection on  $M_4 \times CP^2$  splits as

$$A = A_\mu(x, y) dx^\mu + A_i(x, y) dz^i + A_{\bar{i}}(x, y) d\bar{z}^{\bar{i}}. \quad (181)$$

Expanding the internal Cartan-sector component in Dolbeault–Laplacian eigenforms:

$$A^\xi(x, y) = \sum_{n \geq 0} c_n(x) \varphi_n(y), \quad \Delta_{CP^2} \varphi_n = \lambda_n \varphi_n, \quad 0 = \lambda_0 < \lambda_1 \leq \dots. \quad (182)$$

**Zero mode = moment map.** For the Cartan element  $\xi = \text{diag}(\Pi_1, \Pi_2, \Pi_3) \in \mathfrak{su}(3)$ , the unique  $\lambda_0 = 0$  eigenform is proportional to the Fubini–Study moment map:

$$\varphi_0([z]) \propto \mu_\xi([z]) = \sum_{k=1}^3 \Pi_k \frac{|z_k|^2}{|z|^2}. \quad (183)$$

Evaluating at the three fixed points  $p_k$  reproduces the generation potentials:

$$\mu_\xi(p_k) = \Pi_k, \quad k = 1, 2, 3. \quad (184)$$

Hence  $\Pi_k$  are the  $U(1)^2$  *weights* of the KK zero mode, determined by the Cartan element  $\xi$ , not by hand. Deriving  $\xi$  from first principles is the content of proof-programme task A1.

**Higgs doublet identification.** The 4D KK scalar  $c_0(x) \in \mathfrak{su}(3)$  is an adjoint-valued scalar field. Its  $D_7$ -projected component identifies with the SM Higgs doublet for generation  $k$ :

$$H_k(x) := P_{D_7}^{(k)} c_0(x) \in \mathbb{C}^2 \cong \text{Im } \iota_k, \quad (185)$$

via the  $D_7$ – $D_8$  embedding  $\iota_k$  of Postulate 3.2. Under  $SU(2)$  this transforms as a doublet (Remark 3.21). The Yukawa Lagrangian  $\mathcal{L}_Y = \sum_k \bar{\psi}_{k,L} H_k \psi_{k,R} + \text{h.c.}$  has coupling  $y_k \propto \Pi_k^{-2}$  (KDYP, Eq. (180)), and the Higgs VEV  $\langle H_k \rangle = v/\sqrt{2}$  yields  $m_k = y_k v/\sqrt{2} \propto \Pi_k^{-2}$ , consistent with KYPC below.

The **Kazzy Yukawa–Possibility Correspondence (KYPC)** postulate then reads:

$$y_k = y_{\text{ref}} \left( \frac{\Pi_{\text{ref}}}{\Pi_k} \right)^2, \quad m_k = m_{\text{ref}} \left( \frac{\Pi_{\text{ref}}}{\Pi_k} \right)^2, \quad m_{\text{ref}} = \frac{v}{\sqrt{2}} y_{\text{ref}}, \quad (186)$$

where  $v$  is the Higgs VEV.

*Remark 3.19* (Collapse-Depth Interpretation of  $\gamma = 2$ ). Theorem 3.7 establishes  $\gamma = 2$  rigorously via DH localisation. The same exponent admits a complementary physical interpretation. Define the *possibility collapse depth* of generation  $k$  relative to the reference generation (heaviest,  $k = 3$ ) by

$$\Delta C_k := \ln \frac{\Pi_{\text{ref}}}{\Pi_k} \geq 0, \quad k = 1, 2, 3, \quad (187)$$

so  $\Delta C_3 = 0$  and  $\Delta C_1 > \Delta C_2 > 0$ . Then the KYPC formula (186) reads

$$m_k = m_{\text{ref}} e^{2\Delta C_k}, \quad (188)$$

showing that mass grows *exponentially* with collapse depth, with the factor 2 in the exponent fixed by  $\gamma = 2$  from DH localisation. Lighter generations ( $k = 1$ , electron-type) retain large possibility remainders  $\Pi_k$  (shallow collapse), while heavier generations ( $k = 3$ , tau-type) are deeply collapsed to small  $\Pi_k$ . Numerically,  $\Pi_1 : \Pi_2 : \Pi_3 \approx 59 : 4.1 : 1$  (Proposition 3.16) corresponds to  $\Delta C_1 \approx \ln 59 \approx 4.1$ ,  $\Delta C_2 \approx \ln 4.1 \approx 1.4$ ,  $\Delta C_3 = 0$ . Note: the naïve identification  $m_k \propto (\ln \Pi_k)^2$ , which would also scale with  $\Delta C_k^2$ , is *not* equivalent to  $m_k \propto \Pi_k^{-2}$  in general; the correct exponent  $\gamma = 2$  is fixed by the geometry of  $CP^2$  via DH localisation, not by the logarithmic energy-density argument alone.

**Proposition 3.16** (Mass Hierarchy). If  $\Pi_1 > \Pi_2 > \Pi_3 > 0$  then  $m_1 < m_2 < m_3$ . The observed lepton mass ratios constrain

$$\frac{\Pi_1}{\Pi_2} = \sqrt{\frac{m_\mu}{m_e}} \approx 14.4, \quad \frac{\Pi_2}{\Pi_3} = \sqrt{\frac{m_\tau}{m_\mu}} \approx 4.1, \quad (189)$$

giving  $\Pi_1 : \Pi_2 : \Pi_3 \approx 59 : 4.1 : 1$ . Deriving this ratio from the Fubini–Study geometry of  $CP^2$  is the principal remaining task (OP-5 sub-task (i)); partial results are given in Proposition 3.17 below.

**Proposition 3.17** (Geometric Constraints on  $\Pi_k$ : Hessian Computation and Full DH Formula (A1)).

**Hessian at fixed points.** In local coordinates  $u_j = z_j/z_k$  ( $j \neq k$ ) near  $p_k$ , the moment map expands as

$$\Pi_\xi([u, 1]_k) = \Pi_k + \sum_{j \neq k} (\Pi_j - \Pi_k) |u_j|^2 + O(|u|^4). \quad (190)$$

The complex Hessian is diagonal:

$$\left. \frac{\partial^2 \Pi_\xi}{\partial u_j \partial \bar{u}_l} \right|_{p_k} = (\Pi_j - \Pi_k) \delta_{jl}, \quad j, l \neq k, \quad (191)$$

with determinant

$$\det_{\mathbb{C}}(\text{Hess } \Pi_\xi|_{p_k}) = \prod_{j \neq k} (\Pi_j - \Pi_k). \quad (192)$$

(ii) **Full Duistermaat–Heckman formula.** The Hessian determinant (192) enters the DH formula (Theorem 3.7) as

$$\int_{CP^2} e^{-\Pi_\xi/\hbar} \omega_{\text{FS}}^2 = \hbar^2 \sum_{k=1}^3 \frac{e^{-\Pi_k/\hbar}}{\prod_{j \neq k} |\Pi_k - \Pi_j|}. \quad (193)$$

Writing  $\varepsilon_k := \prod_{j \neq k} |\Pi_k - \Pi_j| / \Pi_k^2$  for the *Hessian correction factor*, the contribution of  $p_k$  is:

$$\left. \frac{d\mu_{\text{DH}}}{d\text{vol}} \right|_{p_k} \propto \frac{e^{-\Pi_k/\hbar}}{\Pi_k^2 \varepsilon_k}. \quad (194)$$

For  $\Pi_1 : \Pi_2 : \Pi_3 \approx 59 : 4.1 : 1$  one finds  $\varepsilon_1 \approx 0.92$ ,  $\varepsilon_2 \approx 9.8$ ,  $\varepsilon_3 \approx 180$ , so the Hessian factors vary substantially across generations.

(iii) **Recovery of KDYP as the leading approximation.** The Kazzy–DH Yukawa Principle (KDYP, Remark 3.17),  $y_k \propto \Pi_k^{-2}$ , is recovered from the exact DH formula (193) by two approximations: (a) the *large- $\hbar$  limit*  $\hbar \rightarrow \infty$  so that  $e^{-\Pi_k/\hbar} \rightarrow 1$  for all  $k$ , and (b)  $\varepsilon_k \approx \text{const.}$  (This is a *separate* physical step applied after the exact DH localisation; see Remark 3.16 for the reconciliation.) When these approximations fail, the corrected Yukawa coupling and mass formula are:

$$y_k^{\text{full}} \propto \frac{1}{\prod_{j \neq k} |\Pi_k - \Pi_j|}, \quad m_k^{\text{corr}} = m_{\text{ref}} \left( \frac{\Pi_{\text{ref}}}{\Pi_k} \right)^2 \frac{\varepsilon_{\text{ref}}}{\varepsilon_k}. \quad (195)$$

(iv) **Lagrange–Vandermonde identities.** For any three distinct  $\Pi_1, \Pi_2, \Pi_3$ , the fol-

lowing algebraic identities hold:

$$\sum_{k=1}^3 \frac{\Pi_k^r}{\prod_{j \neq k} (\Pi_k - \Pi_j)} = \delta_{r,2}, \quad r = 0, 1, 2, \quad (196)$$

(Lagrange interpolation). In particular,  $r = 0$  gives the *DH cancellation rule*  $\sum_k 1/\prod_{j \neq k} (\Pi_k - \Pi_j) = 0$ , confirming the self-consistency of the DH localization.

*Remark 3.20* (Open task for A1). Propositions 3.17(i)–(iv) derive geometric constraints on  $\Pi_k$  but do not fix the ratios  $\Pi_1 : \Pi_2 : \Pi_3$ . Completing A1 requires a dynamical principle that selects the Cartan element  $\xi = \text{diag}(\Pi_1, \Pi_2, \Pi_3) \in \mathfrak{su}(3)$ : candidate approaches include (a) minimisation of a  $D_8$  potential fixing the VEV of  $c_0(x)$  (A3, Remark 3.18), (b) a Bohr–Sommerfeld quantisation condition on the  $CP^2$  moment polytope, or (c) a symmetry-breaking principle within the  $SU(3)$  Cartan subalgebra. This remains OP-5 sub-task (i).

## D8 Dual Structure and Mass-Generation Action.

**Theorem 3.8** ( $D_8$  Dual Structure Theorem). *The  $D_8$  triple possibility structure  $(\mathbb{C}^3, \hat{P}_8)$  simultaneously produces:*

- (i) **Colour charge** (horizontal): the norm-preserving group  $SU(3)$  (173) acting unitarily on  $\mathbb{C}^3$ .
- (ii) **Fermion generations** (vertical): the ordered eigenvalues  $\Pi_1 > \Pi_2 > \Pi_3$  of  $\hat{P}_8$ , encoding three distinct mass scales.
- (iii) **Chirality** (diagonal): the generation-relative  $D_8 \rightarrow D_7$  projector  $P_{D_7}^{(k)} = \mathbf{1} - |v_k\rangle\langle v_k|$  selects the  $SU(2)_L$ -charged (left-handed) component  $\psi_{k,L}$  for each generation  $k$ , with the right-handed component  $\psi_{k,R}$  identified as the possibility-collapsed direction  $|v_k\rangle$ .

*Colour charge, generation structure, and chirality are three facets of the single geometric object  $(\mathbb{C}^3, \hat{P}_8)$ ; in the Standard Model they are postulated independently.*

The mass-generation term in  $S_K$  is

$$S_{\text{mass}} = \sum_{k=1}^3 \int d^4x \sqrt{-g} \Pi_k^{-2} \varphi \bar{\psi}_k \psi_k + \text{h.c.}, \quad (197)$$

where  $\varphi = \Pi(x)$  is the possibility scalar (identified with the Higgs field). Equation (186) then gives the Kazzy interpretation: *a Yukawa coupling is the possibility-fixation rate of the  $k$ -th generation.*

**Chirality from the Generation-Relative  $D_8 \rightarrow D_7$  Projection.** The generation filtration (Definition 3.8) identifies each generation- $k$  fermion  $\psi_k$  with the quotient line  $V_k/V_{k-1}$ , i.e.  $\psi_k$  is directed along  $|v_k\rangle$  in  $\mathbb{C}^3$ . A single fixed projector  $P_{D_7} = |v_1\rangle\langle v_1| + |v_2\rangle\langle v_2|$  would yield  $P_{D_7}\psi_3 = 0$ , annihilating the left-handed component of the third generation entirely—a direct contradiction with observation. The resolution is to define a *generation-relative  $D_7$  projector*.

**Definition 3.9** (Generation-Relative  $D_7$  Projector  $P_{D_7}^{(k)}$ ). For each generation  $k \in \{1, 2, 3\}$ , define the *generation- $k$   $D_7$  projector*

$$P_{D_7}^{(k)} := \mathbf{1}_{\mathbb{C}^3} - |v_k\rangle\langle v_k| = \sum_{j \neq k} |v_j\rangle\langle v_j|, \quad (198)$$

and the complementary *right-handed projector*  $P_R^{(k)} := |v_k\rangle\langle v_k| = \mathbf{1} - P_{D_7}^{(k)}$ . The image  $P_{D_7}^{(k)}\mathbb{C}^3$  is the  $\mathbb{C}^2$  orthogonal complement of the generation direction  $|v_k\rangle$ :

$$\text{Im } P_{D_7}^{(k)} = \begin{cases} \text{span}\{|v_2\rangle, |v_3\rangle\} & k = 1, \\ \text{span}\{|v_1\rangle, |v_3\rangle\} & k = 2, \\ \text{span}\{|v_1\rangle, |v_2\rangle\} & k = 3. \end{cases} \quad (199)$$

Note that  $P_{D_7}^{(3)}$  coincides with the simpler expression  $|v_1\rangle\langle v_1| + |v_2\rangle\langle v_2|$  of earlier formulations.

**Postulate 3.2** ( $D_7$ - $D_8$  Fibre Alignment (DFAP)). The gauge-dimension bundles  $E_7 \rightarrow M_4$  (fibre  $\mathbb{C}^2$ , structure group  $\mathcal{G}_7 = \text{SU}(2)$ ) and  $E_8 \rightarrow M_4$  (fibre  $\mathbb{C}^3$ , structure group  $\mathcal{G}_8 = \text{SU}(3)$ ) are *independent* vector bundles:  $\mathcal{G}_7$  is *not* a subgroup of  $\mathcal{G}_8$ . For each generation  $k \in \{1, 2, 3\}$ , define the *generation- $k$   $D_7$  embedding*

$$\iota_k : E_7 \hookrightarrow E_8, \quad \text{Im } \iota_k \Big|_x = \text{Im } P_{D_7}^{(k)} \Big|_x = \text{span}_{\mathbb{C}}\{|v_j(x)\rangle : j \neq k\}. \quad (200)$$

The left-handed component  $\psi_{k,L}$  is identified as a section of  $E_7$  via  $\iota_k$ ; the right-handed component  $\psi_{k,R}$  is a section of the complementary line bundle  $E_8^{(k)} := \ker \iota_k^\dagger \subset E_8$ .

**Postulate 3.3** (Kazzy Chirality Postulate (KCP)). The left- and right-handed Weyl components of each generation- $k$  fermion  $\psi_k$  are identified via the generation- $k$   $D_7$  embedding (200):

$$\psi_{k,L} := P_{D_7}^{(k)}\psi_k \in \text{Im } \iota_k \cong \mathbb{C}^2, \quad \psi_{k,R} := |v_k\rangle\langle v_k|\psi_k \in E_8^{(k)} \cong \mathbb{C}^1. \quad (201)$$

$\psi_{k,L}$  transforms as an  $\text{SU}(2)$  doublet under the *universal* gauge group  $\mathcal{G}_7 = \text{SU}(2)$  of  $E_7$ —the *same* group for all three generations;  $\psi_{k,R}$  is an  $\text{SU}(2)$  singlet. By construction  $\psi_{k,L} \neq 0$  and  $\psi_{k,R} \neq 0$  for all  $k \in \{1, 2, 3\}$ .

*Remark 3.21* (Universality of  $SU(2)_L$  via DFAP). Postulate 3.2 resolves the potential universality problem.  $\mathcal{G}_7 = SU(2)$  is the structure group of  $E_7$ —an *independent* bundle from  $E_8$ —and therefore acts identically on  $\psi_{k,L}$  for all  $k$ . The generation-dependence resides entirely in the embedding  $\iota_k$ : which  $\mathbb{C}^2$  subspace of  $E_8$  plays the role of the  $D_7$  fibre for generation  $k$ . Under a  $\mathcal{G}_8$  gauge transformation  $g \in SU(3)$ :

$$|v_k\rangle \mapsto g|v_k\rangle, \quad P_{D_7}^{(k)} \mapsto g P_{D_7}^{(k)} g^\dagger, \quad \iota_k \mapsto g \circ \iota_k, \quad (202)$$

while  $\mathcal{G}_7$  is unaffected. The chirality decomposition is therefore  $\mathcal{G}_8$ -covariant, and the abstract  $SU(2)$  representation of  $\psi_{k,L}$  is gauge-independent.

*Remark 3.22* (Physical Interpretation: Collapse-Direction = Right-Handed). The generation-relative decomposition (201) admits a natural Kazy-theoretic reading. The right-handed component  $\psi_{k,R} = |v_k\rangle\langle v_k|\psi_k$  lies along the generation’s own eigendirection—the direction in which possibility is *fully collapsed*; there is no residual  $D_7$  freedom, hence  $SU(2)$  does not act. The left-handed component  $\psi_{k,L} = P_{D_7}^{(k)}\psi_k$  occupies the orthogonal  $\mathbb{C}^2$ —the *possibility-open* subspace in which  $D_7$  freedom survives and  $SU(2)_L$  acts. Chirality therefore reflects the distinction between *collapsed* and *open* possibility residues within  $\mathbb{C}^3$ .

*Remark 3.23* (Geometric Support for KCP via KK Reduction). Postulate 3.3 is supported by standard Kaluza–Klein dimensional reduction on  $CP^2$  with the  $\text{spin}^c$  structure of [17]. A 10D Dirac spinor  $\Psi_{10}$  decomposes under  $SO(3,1) \times U(2)$  (the 4D Lorentz group times the holonomy of  $CP^2$ ) as  $\Psi_{10} = \psi_L \otimes \eta_+ \oplus \psi_R \otimes \eta_-$ , where  $\eta_\pm$  are the two Weyl spinors of the internal  $CP^2$ . The zero modes of  $\bar{\partial}_{CP^2} \otimes \mathcal{O}(1)$ —the holomorphic sections  $s_k$  of Lemma 3.11—lie entirely in the  $\eta_+$  sector, which maps to 4D left-handed Weyl spinors  $\psi_L$  under dimensional reduction. In the generation-relative formulation, the  $\eta_+$  zero-mode of generation  $k$  spans the  $\mathbb{C}^2$  complement  $\text{Im } P_{D_7}^{(k)}$  of the generation direction  $|v_k\rangle$ , consistent with (201). For  $k = 3$  this reduces to the simpler statement that the zero modes  $s_1, s_2$  span the  $D_7$  subbundle  $\text{span}\{|v_1\rangle, |v_2\rangle\}$  as in earlier formulations. Completing the proof that the  $CP^2$   $\text{spin}^c$  zero-mode structure universally forces  $\psi_{k,L} = P_{D_7}^{(k)}\psi_k$  for all  $k$  remains as OP-5 sub-task (iv).

**Theorem 3.9** (Universal  $\text{Spin}^c$  Compatibility of KCP (A4)). *The generation-relative chirality projectors  $\{P_{D_7}^{(k)}\}_{k=1,2,3}$  (Definition 3.9) are simultaneously compatible with a single canonical  $\text{spin}^c$  structure on  $CP^2$ . Consequently, the Kazy Chirality Postulate 3.3 is geometrically consistent for all three generations.*

*Proof. Step 1: Existence of  $\text{spin}^c$  structure.*  $CP^2 = SU(3)/U(2)$  is a compact Kähler manifold. Since every Kähler manifold is  $\text{spin}^c$  [17],  $CP^2$  admits a canonical  $\text{spin}^c$  structure. Its second Stiefel–Whitney class satisfies  $w_2(CP^2) = c_1(CP^2) \bmod 2 =$

$H \bmod 2 \neq 0$  (where  $H \in H^2(CP^2, \mathbb{Z})$  is the hyperplane class), confirming that  $CP^2$  is *not* spin, so the  $\text{spin}^c$  structure is the minimal consistent choice.

**Step 2: Positive-chirality spinor bundle.** The Dolbeault  $\text{spin}^c$  structure on  $CP^2$ , twisted by the tautological line bundle  $\mathcal{O}(1)$ , yields the  $\bar{\partial}$ -operator whose index gives three zero modes (Lemma 3.11,  $N_{\text{gen}} = 3$ ). The associated positive-chirality spinor bundle is

$$S^+ = \mathcal{O}(1) \oplus \left( \mathcal{O}(1) \otimes \Omega_{CP^2}^{0,2} \right) \cong \mathcal{O}(1) \oplus \mathcal{O}(-2), \quad \text{rank}_{\mathbb{C}}(S^+) = 2, \quad (203)$$

using  $\Omega_{CP^2}^{0,2} \cong \mathcal{O}(-3)$  (det of the anti-holomorphic cotangent bundle). The negative-chirality bundle satisfies  $\text{rank}_{\mathbb{C}}(S^-) = 2$  as well.

**Step 3: Fibre identification at fixed points.** At each  $U(1)^2$  fixed point  $p_k = [0 : \dots : 1 : \dots : 0] \in CP^2$ , the fibre  $S^+|_{p_k} \cong \mathbb{C}^2$ . The generation- $k$  complement  $\text{Im } P_{D_7}^{(k)} = \text{span}\{|v_j\rangle : j \neq k\}$  also has complex dimension 2. Define the fiberwise unitary isomorphism

$$\varphi_k : S^+|_{p_k} \xrightarrow{\sim} \text{Im } P_{D_7}^{(k)}, \quad k = 1, 2, 3, \quad (204)$$

by mapping the canonical  $\mathbb{C}^2$  frame of  $S^+|_{p_k}$  to the eigenvector pair  $\{|v_j\rangle : j \neq k\}$ . This is a unitary isomorphism because  $\{|v_1\rangle, |v_2\rangle, |v_3\rangle\}$  is a unitary basis of  $\mathbb{C}^3$ .

**Step 4: Simultaneous compatibility.** The three fixed points  $p_1, p_2, p_3$  are isolated; the isomorphisms  $\varphi_1, \varphi_2, \varphi_3$  are defined at disjoint fibres and impose no mutual topological constraints. Hence a single  $\text{spin}^c$  structure simultaneously accommodates all three chirality decompositions. By DH localisation (Theorem 3.7), the Yukawa coupling is dominated by fixed-point contributions, so the compatibility at  $p_k$  is sufficient for mass generation.

**Step 5: Right-handed singlet.** The negative-chirality fibre  $S^-|_{p_k} \cong \mathbb{C}^2$  contains  $\text{span}\{|v_k\rangle\} \cong \mathbb{C}^1$  as a natural sub-line. Hence  $\psi_{k,R} = |v_k\rangle\langle v_k| \psi_k \in S^-|_{p_k}$  is an  $SU(2)$  singlet, consistent with the Standard Model right-handed fermion assignment.

Combining Steps 1–5,  $\{P_{D_7}^{(k)}\}_{k=1,2,3}$  are simultaneously compatible with the single canonical  $\text{spin}^c$  structure on  $CP^2$ , completing the proof of OP-5 sub-task (iv).  $\square$

### Intra-Generation Structure: Lepton–Quark Classification (A5).

**Postulate 3.4** (Lepton–Quark Kazzy Postulate (LQKP)). Within each generation  $k$ , the fermion sector is classified by its coupling to the  $D_7$  and  $D_8$  bundles:

- (i) **Leptons** are sections of  $E_7$  with  $\mathcal{G}_8$ -singlet quantum numbers (no  $SU(3)_{\text{color}}$  charge). The left-handed doublet  $(\nu_{k,L}, e_{k,L}) \in E_7|_{p_k}$  and the right-handed singlet  $e_{k,R} \in E_8^{(k)}|_{p_k}$  carry no colour (trivial  $\mathcal{G}_8$  representation).
- (ii) **Quarks** are sections of  $E_7 \otimes E_8$  (left-handed doublet) and  $E_8$  (right-handed singlets), transforming in the fundamental  $\mathbf{3}$  of  $\mathcal{G}_8 = SU(3)_{\text{color}}$ .

The  $D_8$  bundle plays a *dual role*: its  $SU(3)$  acts as the generation group (via  $\hat{P}_8$  eigenvectors) and as the colour group (via its fundamental action on  $E_8$ ); LQKP separates these roles by specifying which sector of  $E_8$  each fermion type couples to.

**Proposition 3.18** (SM Fermion Content from LQKP (A5)). Postulate 3.4 yields, for each generation  $k$ , the following fermion content with correct  $SU(3)_c \times SU(2)_L \times U(1)_Y$  quantum numbers:

Field	Bundle	$SU(3)_c$	$SU(2)_L$	$Y$
$(\nu_{k,L}, e_{k,L})$	$E_7$	<b>1</b>	<b>2</b>	$-\frac{1}{2}$
$e_{k,R}$	$E_8^{(k)}$	<b>1</b>	<b>1</b>	$-1$
$(u_{k,L}, d_{k,L})$	$E_7 \otimes E_8$	<b>3</b>	<b>2</b>	$+\frac{1}{6}$
$u_{k,R}$	$E_8$	<b>3</b>	<b>1</b>	$+\frac{2}{3}$
$d_{k,R}$	$E_8$	<b>3</b>	<b>1</b>	$-\frac{1}{3}$

This gives  $2 + 1 + 6 + 3 + 3 = 15$  Weyl fermions per generation, matching the Standard Model matter content (excluding right-handed neutrinos). The electric charge  $Q = T_3 + Y$  correctly yields  $Q(\nu_L) = 0$ ,  $Q(e) = -1$ ,  $Q(u) = +\frac{2}{3}$ ,  $Q(d) = -\frac{1}{3}$ .

The hypercharge is reproduced by the formula

$$Y = Q_{D_6} + \frac{N_c}{3}, \quad N_c = \begin{cases} 0 & \text{lepton} \\ 1 & \text{quark} \end{cases}, \quad (205)$$

where  $Q_{D_6}$  is the  $D_6$  base charge and  $N_c$  counts the  $D_8$  fundamental index. With assignments  $Q_{D_6}(\ell_L) = -\frac{1}{2}$ ,  $Q_{D_6}(e_R) = -1$ ,  $Q_{D_6}(q_L) = -\frac{1}{6}$ ,  $Q_{D_6}(u_R) = \frac{1}{3}$ ,  $Q_{D_6}(d_R) = -\frac{2}{3}$  all five entries in the table are recovered. Deriving the  $D_6$  base charges from Axiom G ( $n = 6$ ) is A5 sub-task (ii).

*Remark 3.24* (Yukawa Structure and Anomaly Cancellation (A5)). The KCP (Postulate 3.3) and KDYP (Remark 3.17) give, for each generation  $k$ :

$$\mathcal{L}_Y^\ell = y_k^\ell \bar{\psi}_{k,L}^\ell H_k e_{k,R} + \text{h.c.}, \quad y_k^\ell \propto \Pi_k^{-2}, \quad (206)$$

$$\mathcal{L}_Y^u = y_k^u \bar{q}_{k,L}^u \tilde{H}_k u_{k,R} + \text{h.c.}, \quad (207)$$

$$\mathcal{L}_Y^d = y_k^d \bar{q}_{k,L}^d H_k d_{k,R} + \text{h.c.}, \quad (208)$$

where  $\tilde{H}_k = i\sigma_2 H_k^*$  is the conjugate Higgs. The intra-generation ratio  $y_k^u/y_k^d$  is addressed by Conjecture 3.2 below. The 15-Weyl-fermion content per generation satisfies all SM gauge anomaly cancellation conditions ( $[U(1)]^3$ ,  $[SU(2)]^2U(1)$ ,  $[SU(3)]^2U(1)$ , mixed gravitational) identically.

**Intra-generation  $u/d$  Ratio: Kazzy Isospin Splitting Conjecture (A5-i).**

**Conjecture 3.2** (Kazzy Isospin Splitting Conjecture (KISC)). At leading KDYP order, the Yukawa couplings of up- and down-type quarks of generation  $k$  share the common  $D_8$  moment-map factor:

$$y_k^u = \alpha_k^u \Pi_k^{-2}, \quad y_k^d = \alpha_k^d \Pi_k^{-2}, \quad (209)$$

where  $\alpha_k^{u,d}$  are  $D_7$ -isospin coupling coefficients determined by the  $SU(2)$  structure of the  $E_7$  bundle at the fixed point  $p_k$ . In the  $D_7$ -symmetric (unbroken  $SU(2)$ ) limit,

$$\alpha_k^u = \alpha_k^d \implies y_k^u = y_k^d, \quad \forall k. \quad (210)$$

After  $D_7$  electroweak symmetry breaking ( $SU(2) \rightarrow U(1)_Y$ ), the VEV alignment  $\langle H_k \rangle = (0, v/\sqrt{2})^T$  introduces an asymmetry:

$$R_{ud}^{(k)} := \frac{y_k^u}{y_k^d} = \frac{\alpha_k^u}{\alpha_k^d} = f\left(\frac{\Pi_k}{\tau}\right), \quad (211)$$

where  $\tau$  is the  $D_7$   $SU(2)$  moment-map eigenvalue (proportional to the electroweak VEV  $v$ ) and  $f$  is a smooth function with  $f(\infty) = 1$ .

**DH estimate.** Performing the DH localization jointly on  $CP^2 \times CP^1$  (with the  $CP^1$  having  $U(1)$  moment-map values  $\pm\tau/2$  at its fixed points), one obtains the ratio of absolute Yukawa magnitudes:

$$\left| \frac{y_k^u}{y_k^d} \right| = e^{-\tau/\hbar}, \quad (212)$$

where  $\hbar$  is the KK expansion parameter. This ratio is *generation-independent* at this order, in agreement with the approximate equality  $|y_t/y_b| \approx |y_c/y_s|$  when radiative corrections are accounted for. The observed  $\mathcal{O}(40)$  spread of  $R_{ud}^{(k)}$  across generations arises from higher-order  $D_7$ - $D_8$  cross-couplings and requires the full D7 VEV dynamics (open sub-task).

### Inter-generation CKM Mixing: Kazzy CKM Conjecture (A5-iii).

**Proposition 3.19** (CKM Emergence from D7 Inter-Generation Coupling (A5-iii)). At leading KDYP order the generation eigenstates  $\{|v_k\rangle\}$  of  $\hat{P}_8$  are common to both  $u$ - and  $d$ -type quarks:

$$|v_k^u\rangle = |v_k^d\rangle = |v_k\rangle, \quad k = 1, 2, 3. \quad (213)$$

The CKM matrix  $V = V_u^\dagger V_d$  is therefore  $V = \mathbf{1}$  (no mixing) at this order. Inter-generation mixing is generated at subleading order by the off-diagonal  $D_7$  connection  $A_{D_7}^{\text{off}}$ , which couples different fixed points  $p_k$  and  $p_l$  on  $CP^2$ :

$$V_{kl} = \delta_{kl} + \epsilon_{kl} + \mathcal{O}(\epsilon^2), \quad \epsilon_{kl} := \frac{\langle p_k | A_{D_7}^{\text{off}} | p_l \rangle}{\Pi_l - \Pi_k}, \quad k \neq l. \quad (214)$$

Using the mass-hierarchy convention  $\Pi_1 > \Pi_2 > \Pi_3 > 0$  (i.e.  $\Pi_1 : \Pi_2 : \Pi_3 \approx 59 : 4.1 : 1$ ), the denominators are approximately

$$|\Pi_2 - \Pi_1| \approx 54.9, \quad |\Pi_3 - \Pi_1| \approx 58, \quad |\Pi_3 - \Pi_2| \approx 3.1, \quad (215)$$

so that  $|\Pi_3 - \Pi_2| \ll |\Pi_2 - \Pi_1| \approx |\Pi_3 - \Pi_1|$  (the 2–3 gap is small while the 1–2 and 1–3 gaps are both large), giving  $|\varepsilon_{23}| \gg |\varepsilon_{12}| \gtrsim |\varepsilon_{13}|$ , i.e.

$$|V_{23}| > |V_{12}| > |V_{13}|. \quad (216)$$

*Note:* this ordering ( $|V_{23}| > |V_{12}|$ ) is *inverted* relative to the observed Wolfenstein pattern ( $|V_{us}| > |V_{cb}|$ ). The discrepancy arises because the numerical spacing  $\Pi_1 : \Pi_2 : \Pi_3 \approx 59 : 4.1 : 1$  places  $\Pi_2$  and  $\Pi_3$  close together, making 2–3 mixing dominant rather than 1–2 mixing. Recovering the correct Wolfenstein hierarchy likely requires a more detailed treatment of the  $D_7$  off-diagonal potential and the actual  $\Pi_k$  ratios fixed by A1; this is an open task (A5 sub-task (iii)).

*Remark 3.25* (Status of the CKM conjecture). The KCKM (Kazzy CKM Conjecture) construction provides a geometric mechanism for quark mixing via  $D_7$  off-diagonal perturbations, and qualitatively produces a hierarchical CKM matrix. However, with the current moment-map values  $\Pi_k$  the predicted dominant mixing is 2–3 rather than the observed 1–2. The conjecture should therefore be understood as a *structural framework* awaiting the A1 determination of actual  $\Pi_k$  ratios before quantitative comparison with data is meaningful.

### **$D_6$ Hypercharge Assignment: Kazzy Hypercharge Uniqueness Theorem (A5-ii).**

**Theorem 3.10** (Kazzy Hypercharge Uniqueness (KKT Reinterpretation of SM Anomaly-Cancellation Uniqueness)). [The uniqueness of the SM hypercharge assignment under these three conditions is a classical result [5, 19]. This theorem recasts that result within the KKT language; it does not provide an independent geometric derivation.]

*Within the LQKP framework (Postulate 3.4), the  $D_6$  charge assignments  $Q_{D_6}^{(f)}$  for each SM fermion  $f$  are uniquely determined by the following three conditions:*

- (1) **Rational electric charge with unit  $e/3$  (Gell-Mann–Nishijima):**  $Q_{\text{em}}(f) = T_3^{D_7}(f) + Q_{D_6}^{(f)}$  with  $3Q_{\text{em}}(f) \in \mathbb{Z}$  and normalisation  $Q_{\text{em}}(e^-) = -1$ . (Note: quark charges are  $\pm e/3, \pm 2e/3$ , so integrality holds at the level of the SU(3) triplet; the minimal charge unit is  $e/3$ .)
- (2) **Gauge anomaly cancellation:** All four triangle anomalies  $[\text{U}(1)_{D_6}]^3$ ,  $[\text{SU}(2)_{D_7}]^2\text{U}(1)_{D_6}$ ,  $[\text{SU}(3)_{D_8}]^2\text{U}(1)_{D_6}$ , and  $[\text{grav}]^2\text{U}(1)_{D_6}$  vanish identically.

(3) **LQKP content:** Exactly 15 Weyl fermions per generation as in Proposition 3.18.

The unique solution is the Standard Model hypercharge assignment:

$$Q_{D_6} = Y : \quad Y(\ell_L) = -\frac{1}{2}, \quad Y(e_R) = -1, \quad Y(q_L) = +\frac{1}{6}, \quad Y(u_R) = +\frac{2}{3}, \quad Y(d_R) = -\frac{1}{3}, \quad (217)$$

consistent with the formula  $Y = Q_{D_6}^{\text{bare}} + N_c/3$  (eq. (205)) established in Proposition 3.18.

*Proof (uniqueness by anomaly cancellation).* The four anomaly conditions impose polynomial constraints on the five independent hypercharges  $\{Y(\ell_L), Y(e_R), Y(q_L), Y(u_R), Y(d_R)\}$ . That the unique rational solution (with  $Q_{\text{em}}(e^-) = -1$  normalisation) is precisely eq. (217) is a classical result established in the SM literature [5, 19]. Within KKT, the geometric interpretation is as follows. Axiom G (n=6) assigns  $\mathcal{G}_6 = \text{U}(1)$  as the structure group of  $P^{(6)}(\mathcal{M}_3, \text{U}(1))$ ; the Dirac quantisation condition on the associated line bundle  $L^{(6)}$  requires charges to be rational multiples of the minimal unit  $e^* = e/6$  (the LCM of 1/2 from  $D_7$ -isospin and 1/3 from  $D_8$ -colour). Combined with the anomaly constraints, no free parameter remains.  $\square$

*Remark 3.26.* Theorem 3.10 resolves A5 sub-task (ii): the  $D_6$  charges are not independent inputs but are algebraically fixed by the KKT bundle structure and the quantum consistency requirement of anomaly cancellation. The fractional hypercharges (1/6, 2/3, -1/3) arise geometrically as the LCM of the  $D_7$ -isospin unit (1/2) and the  $D_8$ -colour unit (1/3); this is the KKT interpretation of the charge-quantisation mystery.

### Right-Handed Neutrino: $D_9$ -Sector Singlet (A5-iv).

**Conjecture 3.3** ( $D_9$ -Sector Singlet and the Kazzy Seesaw (A5-iv)). The minimal LQKP (Postulate 3.4) contains exactly 15 Weyl fermions per generation and does not include a right-handed neutrino  $\nu_{k,R}$ . This is consistent with the original SM. For massive neutrinos (required by oscillation experiments), the KKT extension postulates:

**(iv-a)  $D_9$ -sector singlet  $\nu_{k,R}$ .** A right-handed neutrino  $\nu_{k,R}$  exists as a singlet of all three SM gauge groups ( $Y = 0$ ,  $T_3 = 0$ , colour-neutral) and lives in the  $D_9$  sector. It couples to the  $D_6$ - $D_8$  sector only via the neutrino Yukawa coupling:

$$\mathcal{L}_Y^\nu = y_k^\nu \bar{\psi}_{k,L}^\nu \tilde{H}_k \nu_{k,R} + \frac{1}{2} M_k \nu_{k,R}^c \nu_{k,R} + \text{h.c.}, \quad (218)$$

where  $y_k^\nu \propto \Pi_k^{-2}$  (KDYP, same  $D_8$  moment-map factor) and  $M_k$  is the Majorana mass generated by  $D_9$  dynamics.

**(iv-b) Kazzy Seesaw.** Integrating out  $\nu_{k,R}$  at scale  $M_k$  gives the light neutrino mass:

$$m_{\nu_k} = \frac{(y_k^\nu)^2 v^2 / 2}{M_k} \propto \frac{\Pi_k^{-4}}{M_k}. \quad (219)$$

For a generation-universal Majorana scale  $M_k = M_R$  (flavour-blind  $D_9$  sector), the light neutrino mass hierarchy is  $m_{\nu_k} \propto \Pi_k^{-4}$ , which is *steeper* than the charged-lepton hierarchy  $m_k \propto \Pi_k^{-2}$ . This inverts the naïve expectation: the seesaw suppresses lighter generations more, potentially leading to a quasi-degenerate neutrino spectrum if  $M_R$  is large.

**(iv-c) Consistency check.** Adding one SM-singlet  $\nu_{k,R}$  per generation preserves all gauge anomaly cancellation conditions identically (SM singlets contribute zero to all triangle anomalies). The total fermion content becomes  $15 + 1 = 16$  Weyl fermions per generation.

**Remaining open.** The Majorana scale  $M_k$  requires  $D_9$  sector dynamics (the “principle dimension”), which lies outside the current proof programme. The neutrino mass hierarchy (normal vs. inverted ordering) and the  $CP$ -violation phase  $\delta$  in the PMNS matrix are predictions of the  $D_9$  dynamics that remain open.

*Remark 3.27* (Lepton-number violation and  $B - L$  symmetry). The Majorana mass term  $M_k \nu_{k,R}^c \nu_{k,R}$  violates lepton number  $L$  by two units, which in KKT corresponds to a  $D_9$  sector process (crossing a dimensional boundary). The combination  $B - L$  (baryon minus lepton number), which is anomaly-free in the SM, is associated with the  $U(1)_{B-L}$  symmetry spontaneously broken by  $\langle D_9 \rangle = M_R$ . This provides a KKT geometric interpretation of the  $B - L$  breaking scale.

### 3.11 Open Problems and Resolution Programme

Each of the five structural challenges identified in earlier versions of Kазzy Theory now has a *proposed resolution*: a geometric framework that removes the original logical contradiction and provides a concrete proof programme. The designation “proposed resolution” is used deliberately—the core identifications are formulated as conditional propositions rather than proved theorems, and each carries a set of remaining sub-tasks required for full predictive status.

The problems are organised into two tiers: (A) logical resolution (internal inconsistencies removed without claiming new physics) and (B) proof programme (remaining tasks to elevate each resolution to a proved theorem).

#### 3.11.1 OP-1: Geometric Origin of the Gauge Group

**Proposed resolution (Kазzy Gauge Emergence Theorem).** Theorem 3.4 (Section 3.6.8) derives  $G_{\text{SM}} = U(1) \times SU(2) \times SU(3)$  from the local state-space symmetries of the gauge dimensions via the assignment  $\mathcal{H}_n \cong \mathbb{C}^{n-5}$  (Postulate KGE). The structure groups  $\mathcal{G}_6 = U(1)$ ,  $\mathcal{G}_7 = SU(2)$ ,  $\mathcal{G}_8 = SU(3)$  are then uniquely determined by the requirement that unitary automorphisms preserve the Hermitian inner product and volume form of each state space.

**Logical resolution.** Since  $\text{Isom}(T^7) = \text{U}(1)^7$ , no derivation of  $G_{\text{SM}}$  from  $T^7$  is claimed. Axiom G assigns gauge groups as structure groups of principal bundles  $P^{(n)}$ , independent of  $\text{Isom}(\Sigma_7)$ . This eliminates the logical contradiction.

**Remaining task (resolved by Proposition 3.10).** The derivation of Postulate KGE ( $\mathcal{H}_n \cong \mathbb{C}^{n-5}$ ) from the fibre bundle geometry of  $\Sigma_7$  is provided by Proposition 3.10 (Section 3.6.8): the decomposition  $\Sigma_7^{\text{SM}} \cong S^1 \times CP^1 \times CP^2$  identifies  $\mathcal{H}_n$  with the fundamental representation of the isometry group of the  $n$ -th factor, whose complex dimension is  $n-5$  in each case. The outstanding sub-task of identifying the  $\text{U}(1)$  factor in  $\text{U}(2) \supset \text{SU}(2)$  with hypercharge  $\text{U}(1)_Y$  is resolved by Proposition 3.11:  $\text{U}(1)_Y$  is identified with  $\text{U}(1)_{D_6}$ ; the  $\text{U}(1) \subset \text{U}(2)$  from the  $CP^2$  quotient contributes the colour term  $N_c/3$  in  $Y = Q_{D_6} + N_c/3$ .

### 3.11.2 OP-2: Equivalence Principle and the Possibility-Gradient Geodesic

**Proposed resolution (Kazzy Geodesic Mechanism).** Section 3.6.6 (Propositions 3.7–3.6) provides a three-part resolution of OP-2.

**OP2-1. Mass as Possibility Source.** Mass density  $\rho_m$  is the source of  $D_8$  possibility convergence (Proposition 3.5, eq. (82)). A heavier body narrows  $\Pi(x)$  more strongly; the *depth* of the  $\Phi_{\Pi}$  well is material-dependent.

**OP2-2. Possibility Gradient as Gravity.** The gravitational acceleration on  $M_3$  equals  $-\nabla\Phi_{\Pi}$  (Proposition 3.6, eq. (83)), which depends only on the *source* distribution, not on the test body.

**OP2-3. EP Preservation via Geodesic.** Test-body motion is a geodesic of the effective metric  $g_{\mu\nu}^{(K)}$  (eq. (86)). The test-body mass  $m$  cancels exactly in the variational derivation, so all bodies—regardless of composition—follow the same geodesic of  $\Pi(x)$  (Proposition 3.7, eq. (87)). The Weak Equivalence Principle holds exactly.

**Complementary logical resolution.**  $\delta_s \approx 5\text{--}28\%$  does not directly imply  $\Delta a/a$ -level EP violation, since both gravitational and inertial masses are proportional to  $\Gamma$ . Setting  $m_g[A] = \int_A \Gamma d^3x$  and  $m_i[A] = \zeta \int_A \Gamma d^3x$  (universal constant  $\zeta$ ), the free-fall acceleration is  $m_g/m_i = 1/\zeta$ , composition-independent. The EP experiment constrains any residual coupling:

$$|\eta_{AB}| = |\lambda_{\text{eff}}[\langle\delta_s\rangle_A - \langle\delta_s\rangle_B]e^{-r/\ell_\chi}| < 1.8 \times 10^{-13}. \quad (220)$$

From  $|\Delta\delta_s| \approx 2\%$  (Be-Ti estimate) and the Eöt-Wash data [21]:

$$m_\chi \gtrsim 20 \mu\text{eV}/c^2 \quad (\ell_\chi \lesssim 1 \text{ cm}). \quad (221)$$

**Remaining tasks.** (i) Derive  $\lambda_{\text{eff}}$  and  $\ell_\chi$  from first principles; the bound (221) is a Kazzy Theory prediction testable in precision EP experiments. (ii) Extend the weak-field effective metric (84) to the full relativistic setting and verify consistency with the KK-Weitzenböck action (Section 3.10.3). (iii) **(Resolved.)** The  $D_8 \rightarrow D_7 \rightarrow D_6 \rightarrow D_5 \rightarrow D_4 \rightarrow D_3$  projection chain has been formalised by Proposition 3.4 and Corollary 3.2. New remaining task: Derive the information density  $\varrho_{\text{info}}$  in  $\pi_{65}$  (77) directly from the  $D_6/D_5$  gauge dynamics, and confirm  $\pi_{54}$  (78) recovers the IIT integrated-information functional in the quantum limit.

### 3.11.3 OP-3: Derivation Mechanism for the Lorentz Signature

**Proposed resolution (Possibility-Time Theory).** Section 3.6.7 provides a conditional informational construction of the Lorentz signature via the following three-part structure (conditional on Postulates PT-1 and PT-2).

**OP3-1. Time as possibility decrease (Postulate PT-1, eq. (88)).** Physical time  $\tau$  is the direction in which  $D_8$  accessible states are irreversibly eliminated:  $d\Pi/d\tau < 0$ . The arrow of time is the arrow of informational irreversibility.

**OP3-2. Possibility metric (eq. (89)).** The full spacetime metric is  $g_{\mu\nu}^{(K)} = g_{\mu\nu} - \alpha \partial_\mu \ln \Pi \partial_\nu \ln \Pi$ . The rank-1 negative correction acts in the time direction (the direction of  $\nabla \ln \Pi$ ).

**OP3-3. Lorentz signature from irreversibility (Proposition PT, eq. (90)).** When the possibility coupling  $\alpha$  satisfies condition (90),  $g_{\mu\nu}^{(K)}$  has signature  $(-, +, +, +)$ . The minus sign in  $ds^2 = -c^2 dt^2 + dx^2$  is not postulated but derived from the irreversibility of  $D_8$  state reduction.

**Logical resolution.** A positive-definite  $G_{AB}$  cannot yield a negative eigenvalue via ordinary real restriction. The observer's metric  $g_{\text{obs}}$  is the possibility metric  $g_{\mu\nu}^{(K)}$  (89), not a simple restriction of  $G_{AB}$ .  $M_{10}$  being positive-definite and the observational spacetime having Lorentz signature are therefore logically compatible.

**Consistency with  $D_5$  time generation.** The geometric ( $D_5$  observation progression) and informational ( $D_8$  possibility decrease) descriptions of time are linked by eq. (96).

**Approach A (Wick rotation) explicit derivation.**

**Proposition 3.20** (Lorentz Signature via  $D_5$  Wick Rotation). In the KKT framework, identify the  $D_5$  fiber  $S_{D_5}^1 \subset \Sigma_7$  (radius  $R_5$ , coordinate  $\theta^{(5)} \in [0, 2\pi)$ ) as the Euclidean time circle. The 10D Euclidean metric restricted to the  $M_3 \times S_{D_5}^1$  sector is

$$ds^2 \Big|_{M_3 \times S_{D_5}^1} = G_{\mu\nu}(x) dx^\mu dx^\nu + R_5^2 (d\theta^{(5)})^2. \quad (222)$$

Performing the  $D_5$  Wick rotation  $\theta^{(5)} \rightarrow i\tau/R_5$  (analytic continuation to real Lorentzian time  $\tau$ ):

$$R_5^2 (d\theta^{(5)})^2 \xrightarrow{\theta^{(5)}=i\tau/R_5} -d\tau^2. \quad (223)$$

Integrating out  $S^2 \times CP^2 \subset \Sigma_7$  in the zero-mode approximation replaces  $G_{\mu\nu}$  by the effective 4D metric  $g_{\mu\nu}^{\text{obs}}$  (KK reduction (135)). The resulting observable line element is

$$ds_{\text{eff}}^2 = -d\tau^2 + g_{\mu\nu}^{\text{obs}}(x) dx^\mu dx^\nu, \quad \text{signature } (-, +, +, +). \quad (224)$$

*Proof.* Substituting  $\theta^{(5)} = i\tau/R_5$  into  $d\theta^{(5)} = (i/R_5) d\tau$  gives  $R_5^2 (d\theta^{(5)})^2 = -d\tau^2$ , establishing (223). The KK zero-mode reduction then yields (224).  $\square$   $\square$

*Remark 3.28* (Physical interpretation and residual open task). The  $D_5$  Wick rotation is the KKT counterpart of the standard finite-temperature field theory construction: in Euclidean QFT on  $S_\beta^1 \times \mathbb{R}^3$ , the  $S^1$  circle is the imaginary-time direction; rotating back to real time gives Minkowski signature. Here  $S_{D_5}^1$  plays the role of  $S_\beta^1$ .

The remaining open task (part of OP-3) is to show that the Wick rotation is *dynamically justified*: the KKT Euclidean path integral must be convergent, and the analytic continuation  $\theta^{(5)} \rightarrow i\tau/R_5$  must not encounter branch cuts or poles in the physical domain. This requires a complete Euclidean action analysis. Approaches B (ADM correspondence) and C (observer QFT) remain as independent consistency checks of the same Lorentz-signature claim.

**Remaining tasks.** (i) Determine the value of the possibility coupling  $\alpha$  from the KK-Weitzenböck action and verify that condition (90) is automatically satisfied. (ii) Extend the derivation to the full  $D_5 \times D_8$  coupled system and prove eq. (96) from first principles. (iii) Show that the black hole limit (98)–(99) reproduces the Schwarzschild metric's  $g_{00} \rightarrow 0$  behaviour.

### 3.11.4 OP-4: Proof of the Torsion–Gauge Correspondence

**Proposed resolution (Kazzy Torsion Action, §3.10.3).** OP-4 is resolved by the Kazzy Torsion Action  $S_K$  (164), which derives the Torsion–gauge correspondence from a variational principle rather than asserting it as an axiom.

**OP4-1. (Torsion–gauge identity: resolved.)** Theorem 3.6 (Section 3.10) derives  $T_{\mu\nu}^{(n)a} = F_{\mu\nu}^{(n)a}$  for all  $n = 6, 7, 8$  from the Maurer–Cartan geometry of the fiber  $G_n$ . The non-Abelian  $A \wedge A$  term arises from the MC equation  $d(g^{-1}dg)^a = -\frac{1}{2}f^a_{bc}(g^{-1}dg)^b \wedge (g^{-1}dg)^c$  and is *not* an external input. Varying the Kazzy Torsion Action  $S_K$  with respect to  $A^{(n)}$  then yields  $D \star T^{(n)} = g_n^2 \star J^{(n)}$  (168) and Bianchi identity  $DT^{(n)} = 0$  (169), which are isomorphic to Yang-Mills theory with  $T^{(n)} \equiv F^{(n)}$  as a *derived geometric identity*.

**OP4-2. (Force recovery.)** For  $D_6$  (U(1)):  $T^{(6)} = \kappa_6 F$  gives Maxwell's equations. For  $D_7, D_8$ : the Yang-Mills equations of the weak and strong forces are reproduced.

**OP4-3. (Gravity link.)** The  $D_5$  Torsion  $T^{(5)} = de^{(5)} + \beta_5 d \ln \Pi \wedge e^{(5)}$  connects the possibility field  $\Pi$  to gravity. In the weak-field limit ( $\Pi \approx e^{-\Phi_N/c^2}$ ),  $T^{(5)}$  reduces to the TEGR Torsion, completing the four-force unification.

### Remaining sub-tasks.

- (i) ( **$D_5$ -TEGR limit.**) Fix  $\beta_5$  from the Newtonian limit and verify  $S_T|_{n=5} \rightarrow S_{\text{TEGR}}$  rigorously; a formal statement is given in Proposition 3.3.
- (ii) (**Coupling matching.**) Derive the radius hierarchy  $R_6 : R_7 : R_8$  consistent with observed couplings  $\alpha_{\text{EM}} \simeq 1/137$ ,  $g_2^2 \simeq 0.43$ ,  $g_3^2 \simeq 1.3$  using  $g_n^2 = g_{10}^2/\text{Vol}(\Sigma_n)$  (Eq. (135) and Table 4).
- (iii) ( **$D_8$  dual-role consistency.**) Confirm tree-level absence of  $\Pi$ - $A^{(8)}$  cross-terms (Remark 3.14) and compute the one-loop suppression factor  $\sim g_{10}^2/M_{\text{KK}}^2$ .
- (iv) (**Quantisation.**) Implement Faddeev–Popov quantisation for each  $T^{(n)}$  sector as outlined in Remark 3.15; the gauge ( $n = 6, 7, 8$ ) sector is renormalisable by Yang-Mills power counting; the gravity ( $n = 5$ ) sector shares the standard quantisation difficulty of TEGR.

### 3.11.5 OP-5: Fermion Representations, Generation Structure, and Anomaly Cancellation

**Proposed resolution ( $D_8$  Possibility Structure, §3.10.4).** OP-5 is resolved at the structural level by the  $D_8$  triple possibility structure  $(\mathbb{C}^3, \hat{P}_8)$  introduced in Section 3.10.4.

**OP5-1. ( $N_{\text{gen}} = 3$  from  $CP^2$  index.)** The Hirzebruch–Riemann–Roch calculation (228) gives exactly three zero modes  $s_1, s_2, s_3$  that localise at the  $U(1)^2$  fixed points  $p_1, p_2, p_3$  of  $CP^2$  (Section 3.10.4, correspondence table). The index uniquely selects  $k = 1$  (hyperplane bundle), and  $T^7$  is excluded by the Atiyah–Singer argument (225).

**OP5-2. (Mass hierarchy from Duistermaat–Heckman.)** Theorem 3.7 derives  $\gamma = \dim_{\mathbb{C}}(CP^2) = 2$  from the localisation formula (177), giving  $m_k = m_{\text{ref}}(\Pi_{\text{ref}}/\Pi_k)^2$ . Proposition 3.16 establishes  $m_1 < m_2 < m_3$  whenever  $\Pi_1 > \Pi_2 > \Pi_3$ , with lepton mass ratios constraining  $\Pi_1 : \Pi_2 : \Pi_3 \approx 59 : 4.1 : 1$ .

**OP5-3. (Colour charge and generations from  $D_8$  triple structure.)** Theorem 3.8 shows that  $SU(3)$  colour and the generation filtration both emerge from  $(\mathbb{C}^3, \hat{P}_8)$ . The mass-generation action (197) realises each Yukawa coupling as a possibility-fixation rate.

**OP5-4. (Chirality from the generation-relative  $D_8 \rightarrow D_7$  projection.)** Postulate 3.3 (Kazzy Chirality Postulate) uses the *generation-relative* projector  $P_{D_7}^{(k)} = \mathbf{1} - |v_k\rangle\langle v_k|$  (198):  $\psi_{k,L} = P_{D_7}^{(k)}\psi_k \in \mathbb{C}^2$  (SU(2) doublet),  $\psi_{k,R} = |v_k\rangle\langle v_k|\psi_k \in \mathbb{C}^1$  (SU(2) singlet). This definition ensures  $\psi_{k,L} \neq 0$  for *all three generations*, resolving the inconsistency that a generation-independent projector would annihilate  $\psi_{3,L}$ . The  $D_7$ – $D_8$  Fibre Alignment Postulate 3.2 (DFAP) guarantees that  $\mathcal{G}_7 = \text{SU}(2)$  is *universal*: it is the structure group of the independent bundle  $E_7$ , not a generation-varying subgroup of  $\mathcal{G}_8 = \text{SU}(3)$ . Remark 3.21 shows the chirality decomposition is  $\mathcal{G}_8$ -covariant via the transformation  $P_{D_7}^{(k)} \mapsto g P_{D_7}^{(k)} g^\dagger$ . The right-handed component is the *possibility-collapsed direction* of generation  $k$ ; the left-handed component is the possibility-open  $\mathbb{C}^2$  complement (Remark 3.22). Theorem 3.9 (A4) proves that the canonical spin<sup>c</sup> structure on  $CP^2$  is universally compatible with  $\{P_{D_7}^{(k)}\}_{k=1,2,3}$  via fiberwise unitary isomorphisms  $\varphi_k : S^+|_{p_k} \rightarrow \text{Im } P_{D_7}^{(k)}$  at the three  $U(1)^2$  fixed points, resolving OP-5 sub-task (iv).

**Logical resolution.** Colour charge, fermion generation structure, mass hierarchy, and weak-interaction chirality are all derived from the single geometric object  $(\mathbb{C}^3, \hat{P}_8)$  and the  $D_8 \rightarrow D_7$  bundle hierarchy. Fermions are accommodated as sections of the associated bundle  $\psi_R \in \Gamma(S_{\text{obs}} \otimes (P_{\text{SM}} \times_R V_R))$  with SM representations. Existing anomaly cancellation conditions are preserved.

**Why  $T^7$  fails.**

**Proposition 3.21** ( $T^7$  gives  $N_{\text{gen}} = 0$ ).  $T^7$  has holonomy group  $\text{Hol}(T^7) = \{1\}$  (flat torus, trivial holonomy). By the Atiyah–Singer index theorem [3, 4],  $\hat{A}(T^7) = 1$  and

$$\text{index}(\not{D}_{T^7} \otimes V) = \int_{T^7} \text{ch}(V) \cdot \hat{A}(T^7) = 0. \quad (225)$$

Hence  $T^7$  cannot generate fermion generations.

$N_{\text{gen}} = 3$  **from**  $CP^2$ . Replacing the internal space by  $\Sigma_7^{\text{SM}} = S^1 \times S^2 \times CP^2$ , and using the Hirzebruch–Riemann–Roch theorem on  $CP^2$  (which admits a canonical spin<sup>c</sup> structure [17]) with the twisting bundle  $V = \mathcal{O}(k)$  (holomorphic line bundle; first Chern class  $c_1(\mathcal{O}(k)) = k[H]$ ):

$$\text{index}(\bar{\partial}_{CP^2} \otimes \mathcal{O}(k)) = \chi(CP^2, \mathcal{O}(k)) = \frac{(k+1)(k+2)}{2}. \quad (226)$$

**Lemma 3.11** ( $\mathcal{O}(1)$  is the Canonical Bundle of  $\hat{P}_8$ ). *The eigenstates  $|v_k\rangle$  of the possibility operator  $\hat{P}_8$  (Definition 3.7) determine evaluation functionals*

$$s_k : CP^2 \rightarrow \mathbb{C}, \quad s_k([z]) = \frac{\langle v_k | z \rangle}{|z|}, \quad k = 1, 2, 3. \quad (227)$$

Each  $s_k$  is a degree-1 homogeneous function on  $\mathbb{C}^3 \setminus \{0\}$ , hence a global section of  $\mathcal{O}(1)$ .

Since  $H^0(CP^2, \mathcal{O}(1)) \cong (\mathbb{C}^3)^*$  is 3-dimensional, the three sections  $\{s_1, s_2, s_3\}$  are a basis; the twisting bundle is canonically  $\mathcal{O}(1)$ , not  $\mathcal{O}(k)$  for  $k \neq 1$ .

*Proof.* A global section of  $\mathcal{O}(k)$  corresponds to a degree- $k$  homogeneous polynomial on  $\mathbb{C}^3$  [9]. The map  $z \mapsto \langle v_k | z \rangle$  is  $\mathbb{C}$ -linear, hence degree 1, so  $s_k \in H^0(CP^2, \mathcal{O}(1))$ . The  $3 \times 3$  Gram matrix  $\langle v_j | v_k \rangle = \delta_{jk}$  (eigenstates are orthonormal) shows  $s_1, s_2, s_3$  are linearly independent over  $\mathbb{C}$ . Since  $\dim H^0(CP^2, \mathcal{O}(1)) = 3$ , they form a basis.  $\square$

**Corollary 3.12** ( $k = 1$  Minimality).  $k = 0$  gives  $H^0(CP^2, \mathcal{O}(0)) \cong \mathbb{C}$  (constants only; no generation structure).  $k \geq 2$  gives  $\dim H^0 = \binom{k+2}{2} > 3$  (over-determined). Hence  $\mathcal{O}(1)$  is the unique minimal non-trivial twisting bundle whose sections identify with the  $\hat{P}_8$  eigenstates.

For  $k = 1$ :

$$\text{index}(\bar{\partial}_{CP^2} \otimes \mathcal{O}(1)) = \frac{2 \cdot 3}{2} = 3 = N_{\text{gen}}. \quad (228)$$

The condition (i) that the  $S^1$  and  $S^2$  factors contribute zero index is consistent with the generation filtration of Definition 3.8.

#### Remaining sub-tasks.

- (i) Derive  $\Pi_1 : \Pi_2 : \Pi_3 \approx 59 : 4.1 : 1$  from the Fubini–Study geometry of  $CP^2$  (or identify the additional geometric input required).
- (ii) Explain why  $\Pi_k$  ratios differ between leptons, up-type quarks, and down-type quarks (flavour non-universality from  $\hat{P}_8$  corrections).
- (iii) Compute CKM/PMNS mixing angles from off-diagonal  $\langle v_j | A^{(8)} | v_k \rangle$  perturbation theory.
- (iv) Construct the chirality projection from 10D to 4D using the  $CP^2$  spin<sup>c</sup> structure [17].
- (v) Verify global anomaly cancellation with SM charge assignments.

### 3.11.6 Current Status of Kazy Theory

**What has been established.** The present version of Kazy Theory has progressed from a set of structural hypotheses to a framework in which each of the five original open problems has a concrete proposed resolution. Table 5 summarises the current state.

**What this paper claims.** The claim of the present paper is precisely stated as follows.

1. **(Gravity, established.)** The  $D_5$  Torsion sector reproduces TEGR *in the weak-field limit* (Proposition 3.3); the Newtonian limit follows. This weak-field equivalence is established; the full non-linear derivation from the 10D KKT framework remains an open sub-task of OP-2.

2. **(Gauge forces, conditional.)** The adjoint-valued Torsion–gauge curvature correspondence (Conditional Proposition 3.15) provides a geometric framework for the electromagnetic, weak, and strong forces. The Kazzy Torsion Action  $S_K$  organises the Yang–Mills equations within a variational principle (conditional on the torsion–gauge ansatz), but the correspondence itself remains a conditional proposition pending explicit frame construction.
3. **(Generation structure, structural.)** The  $D_8$  triple possibility structure  $(\mathbb{C}^3, \hat{P}_8)$  gives a unified geometric origin for SU(3) colour, three fermion generations, and the mass hierarchy (Theorems 3.8, 3.7). The Yukawa exponent  $\gamma = 2$  is derived from  $\dim_{\mathbb{C}}(CP^2) = 2$  via the Duistermaat–Heckman theorem. The absolute values of  $\Pi_k$  remain to be derived from first principles.
4. **(Lorentz signature, conditional informational construction.)** Possibility-Time Theory (Section 3.6.7) provides a conditional informational construction of the  $(-, +, +, +)$  signature from  $D_8$  irreversibility (conditional on Postulates PT-1 and PT-2) without invoking Wick rotation.

**What this paper does not claim.** The theory does not yet derive the gauge group, fermion representations, or three-generation structure *by necessity* from the 10-dimensional geometry alone. Each proposed resolution contains conditional steps and remaining sub-tasks. The honest assessment is therefore: *Kazzy Theory is a structurally complete candidate unified geometry, in which every major conceptual obstacle has a proposed resolution, but full proofs of the conditional propositions are outstanding research tasks.*

**Priority research programme (towards future versions).**

- (1) Construct the explicit 10D coframe incorporating all  $P^{(n)}$  and verify the adjoint-valued Torsion–gauge correspondence (OP-4 sub-task (i)).
- (2) Determine the radius hierarchy  $R_6 : R_7 : R_8$  from first principles and match observed coupling constants (OP-4 sub-task (ii)).
- (3) Derive the possibility residue ratios  $\Pi_1 : \Pi_2 : \Pi_3$  from the Fubini–Study geometry of  $CP^2$  (OP-5 sub-task (i)).
- (4) Derive the Kazzy–DH Yukawa Principle (KDYP) from a first-principles Lagrangian, reconciling it with the standard KK overlap integral (A2, OP-5 sub-task (ii)). [OP-5 sub-task (iv) resolved: Theorem 3.9.]
- (5) Derive the Kazzy Geodesic parameters  $\lambda_{\text{eff}}$  and  $\ell_{\chi}$  from first principles and compare with Eöt–Wash precision data (OP-2).

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Table 5: Status of open problems in the current version of Kazzy Theory.

Problem	Proposed resolution	Key remaining task
OP-1: Gauge group origin	KGE (§3.6.8)	<b>Resolved:</b> Prop. 3.10 derives KGE from $\Sigma_7$ geometry
OP-2: Equiv. Principle	Kazzy Geodesic Mechanism (§3.6.6)	First-principles derivation of $\lambda_{\text{eff}}, \ell_\chi$
OP-3: Lorentz signature	Possibility-Time Theory (§3.6.7)	Full global proof of signature selection
OP-4: Torsion–gauge correspondence	KWGT (Thm. 3.6, §3.10)	<b>OP4-1 re-solved:</b> $T^{(n)} = F^{(n)}$ de- rived from MC geom- etry. Re- main- ing: (i) $D_5$ - TEGR limit; (ii) cou- pling match- ing; (iii) $D_8$ dual- role; (iv) FP quanti- sation