

Kinetic Arrest and Noise-Induced Annealing in Active Particle Systems with Information Feedback Control

Keiji Yoshimura

Independent Researcher

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Abstract

In this study, we investigate the collective dynamics of Active Brownian Particles (ABP) subject to an information feedback control term designed to minimize the local variance of internal attributes held by the particles. Numerical simulations reveal that as the system size increases in the low-noise regime, the global orientational order parameter decreases, accompanied by a prominent peak in susceptibility. This phenomenon suggests a transition into a state of kinetic arrest, where the competition between volume exclusion and local variance minimization leads to the formation of multiple local orientational domains that hinder macroscopic reorganization. Furthermore, we observe that moderately increasing the external noise intensity relaxes this arrested state and restores long-range spatial correlations. These results indicate that in non-equilibrium stationary systems mediated by feedback control, thermal noise may act not only as an inhibitor of order but also as a catalyst for escaping metastable states—a process we characterize as “noise-induced annealing.”

Scope and claim boundary. This manuscript is a reduced numerical and theoretical active-particle study. It is not an experimental active-matter validation, not a biological or social-system claim, not a crowd-control or swarm-control technology, and not a universal theorem of kinetic arrest. The repository archive includes a public reconstructed ABP reproducer for the reported diagnostic figures; it should be interpreted as a reproducibility-oriented companion package rather than a claim of exhaustive physical validation.

1 Introduction

Active matter systems, composed of elements that autonomously generate propulsive forces, have been extensively studied as models for self-organization in non-equilibrium statistical mechanics. Since the flocking model proposed by Vicsek et al.[1], numerous theoretical frameworks, including continuum descriptions[2, 3], have been established to explain how simple local interactions between neighboring elements lead to global symmetry breaking.

In recent years, extended models of self-propelled particle dynamics have been explored[4], where particles possess internal degrees of freedom (attributes or states) coupled to their translational and orientational motions. However, it remains largely unclear how active feedback control—wherein each element seeks to actively minimize the difference between its own internal state and those of its neighbors—affects the macroscopic phase diagram and fluctuations of the system.

In this paper, we construct an ABP model with a control term functioning as a gradient system to reduce local variance in a scalar internal attribute. Through numerical computation, we evaluate the dependence of the order parameter and spatial correlation functions on noise intensity and system size, discussing the relationship between frustrations induced by the control term and relaxation dynamics driven by noise.

2 Model

Consider a system of N active particles moving under 2D periodic boundary conditions (system size $L \times L$). Each particle i ($i = 1, \dots, N$) is characterized by its position vector \mathbf{r}_i , its orientation angle θ_i , and a continuous internal attribute variable $q_i \in [-1, 1]$. The time evolution of each particle is described by the following overdamped Langevin equations:

2.1 Position and Orientation Updates

The translational motion is given by:

$$\frac{d\mathbf{r}_i}{dt} = v_0 \mathbf{n}_i + \mathbf{F}_i^{rep} + \mathbf{F}_i^{attr} + \boldsymbol{\xi}_i^{tr}(t) \quad (1)$$

where v_0 is the self-propulsion speed and $\mathbf{n}_i = (\cos \theta_i, \sin \theta_i)$ is the orientation vector. $\boldsymbol{\xi}_i^{tr}(t)$ is Gaussian white noise satisfying $\langle \xi_{i\alpha}^{tr}(t) \xi_{j\beta}^{tr}(t') \rangle = 2D_{tr} \delta_{ij} \delta_{\alpha\beta} \delta(t - t')$.

Short-range repulsion \mathbf{F}_i^{rep} is defined via the Weeks-Chandler-Andersen (WCA) potential[5]:

$$U_{WCA}(r) = \begin{cases} 4\epsilon \left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^6 \right] + \epsilon, & r < 2^{1/6}\sigma \\ 0, & r \geq 2^{1/6}\sigma \end{cases} \quad (2)$$

The internal attribute-dependent interaction is defined by an effective potential:

$$U_{attr}(r_{ij}; q_i, q_j) = -Jq_iq_j \frac{e^{-\kappa r_{ij}}}{r_{ij} + a} \quad (3)$$

resulting in the force $\mathbf{F}_i^{attr} = -\sum_{j \neq i} \nabla_i U_{attr}(r_{ij}; q_i, q_j)$.

The orientation θ_i evolves as:

$$\frac{d\theta_i}{dt} = \lambda_{align} \sin(\Theta_i - \theta_i) + \eta_i(t) \quad (4)$$

where $\Theta_i = \text{Arg}(\sum_{j \in \mathcal{N}_i} \mathbf{n}_j)$ is the average orientation of neighbors within radius R .

2.2 Internal Attribute Update (Information Feedback Control)

The update rule for the internal attribute q_i is:

$$\frac{dq_i}{dt} = -\lambda_{int}(q_i - \langle q \rangle_R) - \chi q_i(q_i^2 - 1) + \zeta_i(t) \quad (5)$$

where $\langle q \rangle_R$ is the local average attribute. The first term represents local variance minimization control, interpreted as a gradient descent of a local entropy-like functional. The second term, derived from a bistable potential, ensures the attribute remains within a finite range.

2.3 Numerical Conditions

Numerical integration utilized the Euler-Maruyama method[6] with $\Delta t = 10^{-3}$ and particle density $N/L^2 = 0.5$. For finite-size analysis, $N \in \{64, 128, 256\}$ was tested with ensemble averaging.

3 Results

3.1 Finite-Size Dependence and Kinetic Arrest

The global order parameter Φ and susceptibility χ_Φ were calculated as:

$$\Phi = \left| \frac{1}{N} \sum_{j=1}^N e^{i\theta_j} \right|, \quad \chi_\Phi = N \text{Var}(\Phi) \tag{6}$$

As shown in Figure 1, at low noise intensities ($D_\theta \approx 2.0$), the order parameter decreases as the system size N increases. Simultaneously, a significant peak in susceptibility is observed for $N = 256$. This suggests a transition into a "kinetic arrest" state[7], where competing local optimizations and volume exclusion create fragmented orientation domains, hindering global reorganization.

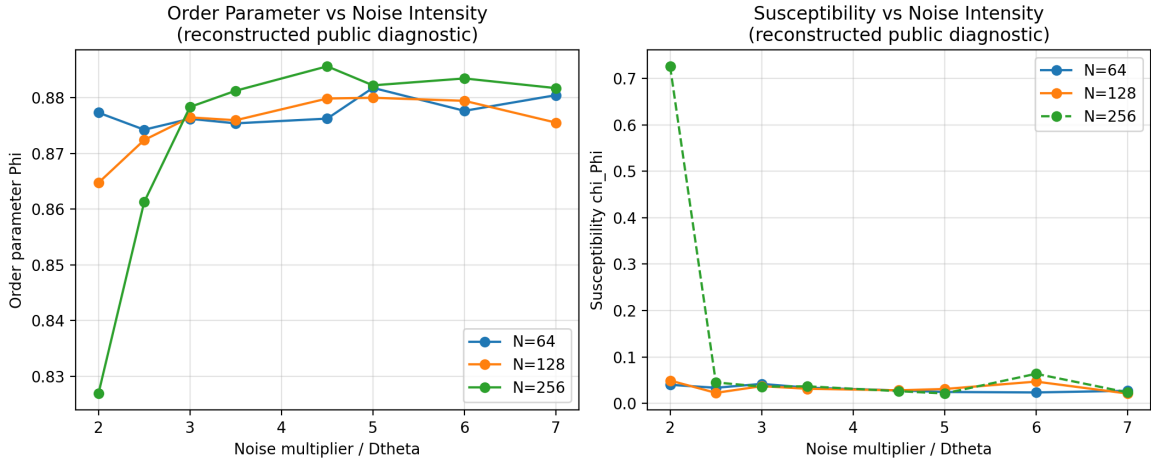


Figure 1: Dependence of the order parameter Φ (left) and susceptibility χ_Φ (right) on noise intensity for $N = 64, 128, 256$.

3.2 Spatial Correlation and Noise-Induced Annealing

To quantify the microscopic structure, we evaluated the spatial correlation function $C(r) = \langle \mathbf{n}_i \cdot \mathbf{n}_j \rangle_{|\mathbf{r}_i - \mathbf{r}_j| = r}$. As shown in Figure 2, long-range correlations are higher at moderate noise ($D_\theta = 5.0$) than in the extreme low-noise state ($D_\theta = 2.0$). This indicates that thermal fluctuations play an activating role[8], helping the system overcome domain boundaries and escape metastable states—a process analogous to "annealing" in complex systems.

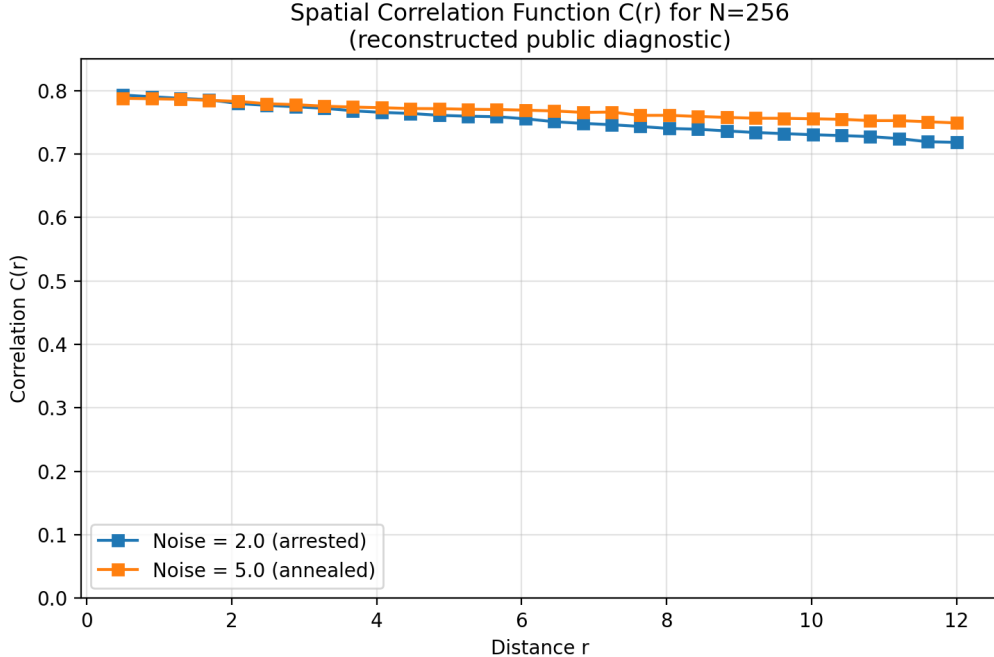


Figure 2: Comparison of spatial correlation functions $C(r)$ for $N = 256$.

4 Discussion

Our results suggest that active systems with information feedback control exhibit group dynamics qualitatively different from simple self-propelled particle models[1, 4]. The observed kinetic arrest[7] and subsequent noise-induced recovery provide a new perspective on the role of noise in non-equilibrium systems[8]. While noise typically disrupts order, in systems with competing interactions, it can act as a catalyst for reaching more stable non-equilibrium steady states. The observed finite-size dependence further highlights the essential role of spatial heterogeneity in macroscopic order formation.

5 Conclusion

We constructed an ABP model incorporating an information feedback control term and validated its phase behavior through numerical simulations. The analysis of finite-size dependence and spatial correlations revealed that frustrations from the control term can lead to kinetic arrest, which can be mitigated by moderate external noise through an annealing-like process. These findings provide fundamental insights into the complex macro-dynamics resulting from the interaction of fluctuations and control in self-organizing non-equilibrium systems.

Data and Code Availability

The reproducibility package associated with this manuscript is archived on Zenodo with repository DOI [10.5281/zenodo.20202120](https://doi.org/10.5281/zenodo.20202120) and is also available through the GitHub repository github.com/yokken0907/active-particle-feedback-kinetic-arrest. The archive contains the manuscript materials, claim-boundary documentation, AI-assistance disclosure, file-manifest materials, and a reconstructed public ABP reproducer that regenerates Figure 1 and Figure 2-style diagnostics. The reconstructed reproducer is intended for public inspection and replication of the reported finite-size and spatial-correlation motifs; it is not a bit-level recovery of every prior exploratory local run.

Conflict of Interest

The author declares no institutional, financial, or commercial conflict of interest related to this study.

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References

- [1] T. Vicsek, A. Czirók, E. Ben-Jacob, I. Cohen, and O. Shochet, "Novel type of phase transition in a system of self-driven particles," *Phys. Rev. Lett.*, 75, 1226-1229 (1995).
- [2] J. Toner and Y. Tu, "Flocks, herds, and schools: A quantitative theory of flocking," *Phys. Rev. E*, 58, 4828-4858 (1998).
- [3] M. C. Marchetti, J. F. Joanny, S. Ramaswamy, T. B. Liverpool, J. Prost, M. Rao, and R. A. Simha, "Hydrodynamics of soft active matter," *Rev. Mod. Phys.*, 85, 1143-1189 (2013).
- [4] H. Chaté, F. Ginelli, G. Grégoire, and F. Raynaud, "Collective motion of interacting self-propelled particles," *Phys. Rev. E*, 77, 046113 (2008).

- [5] J. D. Weeks, D. Chandler, and H. C. Andersen, "Role of repulsive forces in determining the equilibrium structure of simple liquids," *J. Chem. Phys.*, 54, 5237 (1971).
- [6] G. Maruyama, "On the transition probability functions of the Markov process," *Nat. Sci. Rep. Ochanomizu Univ.*, 6, 15-31 (1955).
- [7] Y. Fily and M. C. Marchetti, "Athermal Phase Separation of Self-Propelled Particles with No Alignment," *Phys. Rev. Lett.*, 108, 235702 (2012).
- [8] P. Reimann, "Brownian motors: noisy transport far from equilibrium," *Phys. Rep.*, 361, 57-265 (2002).
- [9] K. Yoshimura, *Active Particle Feedback Kinetic Arrest: Repository Archive*, Zenodo (2026). DOI: [10.5281/zenodo.20202120](https://doi.org/10.5281/zenodo.20202120).