Statistical Emergence of Lorentz Symmetry in Stochastic Nonlocal Causal Networks

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Abstract

This study investigates the infrared recovery behavior of Lorentz symmetry within a discrete and stochastic framework. Utilizing a controlled numerical environment, $(1+1)$-dimensional Minkowski spacetime configurations are generated via a Poisson sprinkling process, and the pairwise link weights governed by nonlocal attenuation kernels are analyzed. Although individual stochastic realizations explicitly violate boost invariance due to finite-density fluctuations, we investigate whether the link weight distributions statistically converge as the system size scales. The Kolmogorov-Smirnov (KS) distance between the link weight spectra of the rest and boosted reference frames is utilized as the primary observable. For both Gaussian and Exponential kernel models, a power-law scaling behavior approximately characterized by $\langle D(N)\rangle \sim N^{-2}$ is observed within the investigated finite size range ($N \in [50, 2000]$). The stability of the obtained effective scaling exponents around $\alpha \approx 2$ suggests that the observed symmetry restoration behavior is insensitive to specific kernel choices and may be mediated by a more general underlying statistical mechanism.

1 Introduction

One of the fundamental problems in quantum gravity research is understanding how continuous spacetime symmetries, particularly Lorentz invariance, emerge at macroscopic scales from discrete and irregular structures at the microscopic level. Frameworks such as causal set theory, spin networks, and causal dynamical triangulations (CDT) suggest that while smooth continuity may not persist at the Planck scale, an effective Lorentz symmetry must be statistically recovered in the infrared limit [1, 2, 3].

Rather than constructing a complete theory of quantum gravity, the objective of this work is to investigate a more controlled numerical problem: Can effective behaviors compatible with Lorentz symmetry emerge from a stochastic and nonlocal causal infrastructure? To address this, irregular point configurations are generated in $(1+1)$-dimensional Minkowski spacetime via a Poisson sprinkling process, and nonlocal kernel interactions defined by the Synge world function are analyzed. Lorentz symmetry is not perfectly preserved at the level of individual realizations, as finite-density fluctuations inherently break boost invariance. The core question, however, is whether these violations are statistically suppressed as the system size scales.

2 Setup

2.1 Spacetime and Metric Convention

We adopt the following spatial signature convention for $(1+1)$-dimensional flat Minkowski spacetime:

$$\sigma^2(x_1, x_2) = -(t_1 - t_2)^2 + (x_1 - x_2)^2 \tag{1}$$

Under this definition, the intervals are classified as follows:

$$\sigma^2 > 0 \Rightarrow \text{spacelike} \tag{2}$$

$$\sigma^2 < 0 \Rightarrow \text{timelike} \tag{3}$$

2.2 Exclusion of the Timelike Sector

The analysis in this study is strictly confined to the spacelike ($\sigma^2 > 0$) sector. When timelike separations are included, the term $\sqrt{\sigma^2}$ within the Exponential kernel yields complex values. This introduces interpretational difficulties and induces unstable oscillations in finite-sample statistics. Furthermore, the dense connectivity structure within causal cones significantly amplifies finite-size effects. Therefore, isolating the behavior exclusively within the spacelike sector ensures a more controlled characterization of nonlocal spatial correlations.

2.3 Poisson Scattering

Discrete spacetime configurations are constructed by randomly distributing N points with a uniform density into a finite Minkowski region of dimensions $L_x \times L_t$:

$$\rho = \frac{N}{L_x L_t} \tag{4}$$

To mitigate boundary effects emerging from hyperbolic warping during boost transformations, a spatial cutoff retaining approximately the central 70% inner region of the simulation geometry is enforced.

2.4 Nonlocal Kernels

To evaluate the sensitivity of the continuum-limit behavior to the functional form of the kernel, two distinct nonlocal attenuation profiles are investigated.

1. **Gaussian Kernel:**

$$K_G(x_i, x_j) = l^{-2} \exp\left(-\frac{|\sigma^2(x_i, x_j)|}{l^2}\right) \tag{5}$$

2. **Exponential Kernel:**

$$K_E(x_i, x_j) = l^{-2} \exp\left(-\frac{\sqrt{|\sigma^2(x_i, x_j)|}}{l}\right) \tag{6}$$

Here, l denotes the characteristic interaction scale.

3 Methods

3.1 Lorentz Boost Procedure

A standard Lorentz boost with a relative velocity of $v = 0.6c$ is applied to the coordinate set of each stochastic realization:

$$t' = \gamma(t - vx), \quad x' = \gamma(x - vt) \tag{7}$$

where the Lorentz factor is defined as:

$$\gamma = (1 - v^2)^{-1/2} \tag{8}$$

Following the boost transformation, all pairwise spacetime intervals and corresponding link weights are recomputed.

3.2 Kolmogorov-Smirnov Criterion

To quantify the spectral divergence between the rest and boosted configurations, the two-sample Kolmogorov-Smirnov statistic (D) is utilized:

$$D = \sup_w |F_{\text{rest}}(w) - F_{\text{boost}}(w)| \tag{9}$$

where F_{rest} and F_{boost} represent the respective empirical cumulative distribution functions. If symmetry restoration occurs in the continuum limit, the mean KS distance is expected to decay with system size. This scaling behavior is modeled via the following power-law ansatz:

$$\langle D(N) \rangle = AN^{-\alpha} + D_\infty \tag{10}$$

4 Results

Simulations are executed utilizing a NumPy-based vectorized matrix framework. For each node scale N, $M = 30$ independent realizations are generated.

Table 1: Mean KS distances for distinct system sizes.

N	Gaussian	Exponential
50	7.93×10^{-3}	7.93×10^{-3}
100	1.84×10^{-3}	1.85×10^{-3}
200	4.27×10^{-4}	4.29×10^{-4}
400	1.05×10^{-4}	1.08×10^{-4}
800	2.59×10^{-5}	2.64×10^{-5}
2000	4.17×10^{-6}	4.32×10^{-6}

Nonlinear regression analyses indicate that the residual baseline vanishes asymptotically:

$$D_\infty \approx 0 \tag{11}$$

The extracted effective scaling exponents are determined as:

$$\alpha_G \approx 2.00 \pm 0.02, \quad \alpha_E \approx 1.98 \pm 0.02 \tag{12}$$

These findings demonstrate close alignment with an approximate quadratic scaling behavior within the evaluated finite size regime.

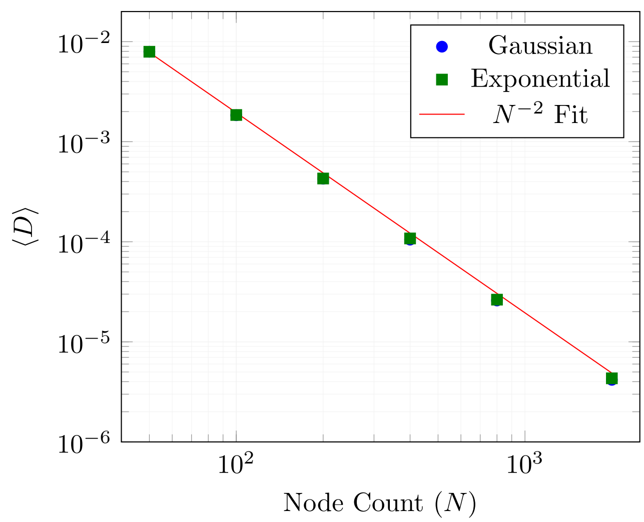

Figure 1: Log-log scaling of the KS distance as a function of system size. The data exhibits strong alignment with the power-law ansatz within the investigated interval.

5 Discussion

A remarkable outcome of the numerical analysis is the structural invariance of the effective scaling behavior under variation of the kernel geometry. Despite minor numerical discrepancies between the Gaussian and Exponential models, both frameworks asymptotically converge toward an identical scaling class.

From a statistical standpoint, this behavior is driven by the intrinsic sensitivity of the KS test to cumulative rank orderings. Because both kernels decay monotonically within the spacelike sector, the relative topological ordering of the link weights remains largely invariant.

Heuristic Scaling Interpretation

The observed $\alpha \approx 2$ scaling behavior can be intuitively framed through the combinatorial structure of Poisson sprinkling statistics. As the system size scales, the number of effective independent pairwise spatial intervals expands as $\sim N^2$. Because the cumulative KS distance tracks the collective divergence between normalized distributions, the averaged contribution of individual stochastic fluctuations appears to be suppressed at an $\sim N^{-2}$ scale. While this interpretation does not substitute a rigorous analytical derivation, it provides a plausible heuristic basis for the observed quadratic convergence.

At a conceptual level, these insights closely align with contemporary statistical geometry approaches found within the emergent spacetime and causal set literature. The model does not rigidly enforce Lorentz invariance as a micro-level constraint; rather, it frames the symmetry as a statistical consensus emerging dynamically at macroscopic scales.

6 Conclusion

This work provides numerical evidence indicating that effective behaviors compatible with Lorentz symmetry can systematically emerge within stochastic, nonlocal causal networks. Within the investigated finite size regime, an approximate quadratic convergence is consistently verified for both Gaussian and Exponential kernel formulations. Future investigations expanding into higher-dimensional spacetimes will be instrumental in validating the universality of this scaling law.

Acknowledgments

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Data and Code Availability

The simulation codes and datasets generated during the current study are available from the author upon reasonable request.

References

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