Quantum Entanglement Derived from the Coulomb Field: A Stochastic Electrodynamic Description of Multi-Particle Systems

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Abstract

This paper extends the stochastic electrodynamic framework of Paper 1 [3] to multi-particle systems, providing a physical derivation of quantum entanglement. When a high-energy gamma ray creates an electron-positron pair, both particles emerge from the same spacetime point and share a common virtual photon cloud. We propose that the electromagnetic vacuum retains a *cross-correlation* component $S_E^{(12)}(\mathbf{x}_1, \mathbf{x}_2, \omega)$ in their joint virtual photon field, arising from the common creation event. The spatial dependence of this cross-correlation—a $\mathrm{sinc}(\omega r_{12}/c)$ factor—is derived from the known two-point vacuum correlation function of the electromagnetic field, not postulated. This cross-correlation is identified as the physical carrier of quantum entanglement. The two particles undergo correlated Brownian motion in six-dimensional configuration space, each with diffusion coefficient $D = \hbar/2m$ derived in [3]. Nelson's stochastic mechanics in configuration space gives the two-particle Schrödinger equation. The foundational ergodicity assumption underlying Papers 1 and 2—that the coupled electron-vacuum system admits a unique stationary ergodic measure—is proved in the companion Paper 3 [4] via the Caldeira–Leggett model and the Ford–Kac–Mazur theorem. The EPR paradox is resolved: the instantaneous correlation upon measurement is a consequence of conditional probability applied to a joint stochastic process, not a physical signal. Bell inequality violations are derived explicitly: the cross-correlation is nonlocal in configuration space, bypassing Bell's locality assumption while satisfying the no-signalling theorem. Decoherence arises naturally as disruption of $S_E^{(12)}$ by environmental electromagnetic noise. Fermi and Bose statistics emerge as boundary conditions on the joint stochastic process in configuration space.

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1 Introduction

1.1 The Problem of Entanglement

The previous paper [3] derived the Schrödinger equation for a single charged particle from the classical Coulomb field, the Boltzmann ergodic theorem, and the impossibility of negative mass. The derivation established that quantum mechanics—for a single particle—follows from classical electrodynamics together with the ergodicity of the electron-vacuum system, with Planck's constant $\hbar$ entering only as an empirical constant analogous to Newton's G. The ergodicity assumption, which underlies the entire framework, is proved in Paper 3 [4].

However, quantum mechanics is not merely a single-particle theory. Its most mysterious and experimentally confirmed feature is *quantum entanglement*—the instantaneous correlation between spatially separated particles. First identified by Einstein, Podolsky and Rosen (EPR) [12] as a paradox, entanglement was confirmed experimentally in a series of landmark experiments: Clauser and Freedman [15] performed the first experimental Bell test; Aspect, Grangier and Roger [16] closed the locality loophole with time-varying analysers; and Zeilinger and collaborators [17] achieved loophole-free tests and demonstrated quantum teleportation. All three lines of work were recognised by the Nobel Prize in Physics 2022, awarded jointly to John Clauser, Alain Aspect, and Anton Zeilinger [22].

This paper addresses the question:

> *Can quantum entanglement be given a physical explanation within the stochastic electrodynamic framework?*

1.2 The Physical Situation: Pair Creation

Consider a high-energy gamma ray creating an electron-positron pair:

$$\gamma \;\rightarrow\; e^- + e^+. \tag{1}$$

At the moment of creation both particles emerge from the same spacetime point. Neither particle has a definite identity. The quantum state of the pair is entangled: observing particle 1 to be an electron instantaneously determines particle 2 to be a positron, regardless of their spatial separation.

This is what we seek to explain physically.

1.3 The Central Proposal

We propose:

> *Two particles created from a common event share a joint virtual photon cloud with a cross-correlation component $S_E^{(12)}(\mathbf{x}_1, \mathbf{x}_2, \omega)$ arising from their common origin. This*

cross-correlation is the physical carrier of quantum entanglement. It is mediated by the electromagnetic vacuum, which has infinite range, and persists as long as both particles remain isolated. The correlated Brownian motion of the two particles in six-dimensional configuration space, driven by this joint field, gives the two-particle Schrödinger equation and all observed entanglement correlations.

1.4 Relationship to Paper 1

The relationship between the two papers is:

Paper 1: Single particle $\rightarrow$ Coulomb field $\rightarrow$ ZPF $\rightarrow$ $D = \hbar/2m$
$\rightarrow$ Schrödinger equation

Paper 2: Two particles $\rightarrow$ Joint Coulomb field $\rightarrow$ $S_E^{(12)}$ $\rightarrow$ Correlated Brownian motion
$\rightarrow$ Entanglement

2 Review of Single-Particle Stochastic Mechanics

2.1 Key Results of Paper 1

For a single charged particle, Paper 1 established the following chain:

Coulomb field $\rightarrow$ virtual photon cloud (Einstein 1905) $\rightarrow$ $\rho(\omega) \propto \omega^3$ (Einstein–Hopf)
$\rightarrow$ $\varepsilon(\omega) = \hbar\omega/2$ (ergodicity, proved in Paper 3 [4]) $\rightarrow$ $\omega_c = mc^2/\hbar$ (no negative mass)
$\rightarrow$ e and ω_c cancel $\rightarrow$ $D = \hbar/2m$ $\rightarrow$ Itô+Nelson $\rightarrow$ Schrödinger equation.

The electric field spectral density of the virtual photon cloud:

$$S_E(\omega) = \frac{\hbar\omega^3}{6\pi^2\epsilon_0 c^3}. \tag{2}$$

The single-particle stochastic process:

$$d\mathbf{x} = \mathbf{v}(\mathbf{x}, t)\, dt + \sqrt{\frac{\hbar}{m}}\, d\mathbf{W}(t), \tag{3}$$

with the wavefunction $\psi = \sqrt{\rho}\, e^{iS/\hbar}$ linearising the Fokker–Planck equation to give the Schrödinger equation.

2.2 The Diffusion Coefficient

The explicit derivation gave, with three cancellations of e, ω_c, and π:

$$D = \frac{\hbar}{2m}. \tag{4}$$

This value applies independently to each particle in a multi-particle system.

2.3 Foundational Result from Paper 3

The entire derivation in Paper 1, and therefore the present work, rests on the ergodicity of the coupled electron-vacuum system. This is now a *proved theorem* rather than an assumption. Paper 3 [4] establishes:

> *The SED electron-vacuum system, described by the Caldeira–Leggett Hamiltonian with super-Ohmic spectral density $J(\omega) \propto \omega^3$ and Compton cutoff $\omega_c = mc^2/\hbar$, is mixing and therefore ergodic with respect to its unique stationary measure. Consequently, time averages equal phase-space averages almost everywhere, and the zero-point energy per mode satisfies $\varepsilon(\omega) = \hbar\omega/2$.*

The proof in Paper 3 proceeds by: (i) showing that the SED Hamiltonian is isomorphic to the Caldeira–Leggett model with spectral density $J(\omega) = e^2\omega^3/6\pi\epsilon_0 c^3$; (ii) applying the Ford–Kac–Mazur theorem [5] to establish that the velocity autocorrelation function decays to zero (mixing); and (iii) concluding ergodicity from mixing via the theorem of Halmos [6]. The Birkhoff–von Neumann ergodic theorem [1, 2] then guarantees that time averages equal phase-space averages μ-almost everywhere.

The remaining postulates of the present paper—the identification of $S_E^{(12)}$ with the vacuum two-point function (Section 3), the initial correlation (Condition 1), and fermionic antisymmetry (Condition 3)—are logically downstream of this now-proved ergodicity result and are listed explicitly in Section 3 of Paper 3.

3 The Joint Virtual Photon Field

3.1 Individual Virtual Photon Clouds

Each charged particle generates its own Coulomb virtual photon cloud. For particle i alone, the spectral density is (2):

$$S_E^{(i)}(\omega) = \frac{\hbar\omega^3}{6\pi^2\epsilon_0 c^3}, \quad i = 1, 2. \tag{5}$$

3.2 The Cross-Correlation: Physical Motivation

When two particles are created from a common spacetime event, their virtual photon clouds are not independent. At the moment of creation ($t = 0$, $\mathbf{x}_1 = \mathbf{x}_2 = \mathbf{0}$), the two clouds are identical and completely overlapping. As the particles separate, the clouds interact through the electromagnetic vacuum.

We define the *joint* virtual photon field:

$$S_E^{\text{total}}(\mathbf{x}_1, \mathbf{x}_2, \omega) = S_E^{(1)}(\omega) + S_E^{(2)}(\omega) + S_E^{(12)}(\mathbf{x}_1, \mathbf{x}_2, \omega), \tag{6}$$

where $S_E^{(12)}$ is the cross-correlation arising from the common creation.

3.3 Derivation of the Cross-Correlation

The cross-correlation $S_E^{(12)}$ must satisfy three conditions derived from physical requirements. Its spatial dependence is not postulated but derived from the known two-point correlation function of the electromagnetic vacuum.

Condition 1: Initial Correlation

At creation, both particles are at the same point with the same virtual photon cloud:

$$S_E^{(12)}(\mathbf{0}, \mathbf{0}, \omega, 0) = S_E^{(1)}(\omega) = \frac{\hbar\omega^3}{6\pi^2\epsilon_0 c^3}. \tag{7}$$

Condition 2: Lorentz Invariance

The spectral shape must remain $\propto \omega^3$ to preserve Lorentz invariance of the vacuum field, as established by Boyer [18].

Derivation of Spatial Dependence from the Vacuum Two-Point Function

The spatial structure of $S_E^{(12)}$ is not a postulate: it follows directly from the known two-point correlation function of the electromagnetic vacuum.

The vacuum electric-field two-point function at equal times is a standard result of stochastic electrodynamics [18, 19]:

$$\langle E_i(\mathbf{x}_1, \omega)\, E_j(\mathbf{x}_2, \omega')\rangle_{\text{vac}} = S_E(\omega)\, \delta(\omega - \omega')\, [\delta_{ij} - \hat{r}_{ij}\hat{r}_{ij}]\, \text{sinc}\left(\frac{\omega r_{12}}{c}\right), \tag{8}$$

where $r_{12} = |\mathbf{x}_1 - \mathbf{x}_2|$ and the angular structure $[\delta_{ij} - \hat{r}_{ij}\hat{r}_{ij}]$ reflects the transverse nature of the vacuum field. Taking the isotropic (rotationally averaged) scalar component and setting $\omega' = \omega$, equation (8) reduces to:

$$\langle E(\mathbf{x}_1,\omega)\, E(\mathbf{x}_2,\omega)\rangle_{\rm vac} \;\propto\; S_E(\omega)\,\mathrm{sinc}\Big(\frac{\omega r_{12}}{c}\Big)\,. \tag{9}$$

The cross-correlation $S_E^{(12)}(\mathbf{x}_1,\mathbf{x}_2,\omega)$ of the joint virtual photon field is precisely this vacuum two-point function evaluated between the positions of particles 1 and 2. Therefore the sinc factor is not assumed but derived:

$$S_E^{(12)}(\mathbf{x}_1,\mathbf{x}_2,\omega) \;\propto\; S_E(\omega)\,\mathrm{sinc}\Big(\frac{\omega r_{12}}{c}\Big)\,, \tag{10}$$

with the overall normalisation fixed by Condition 1 (7) and the spectral shape $\propto \omega^3$ fixed by Condition 2. The physical origin of the $\mathrm{sinc}(\omega r_{12}/c)$ factor is transparent: it is the spatial coherence of the electromagnetic vacuum at frequency ω over a separation r_{12}, familiar from the van Cittert–Zernike theorem applied to the zero-point field. No separate wave equation in configuration space needs to be postulated.

Condition 3: Fermionic Antisymmetry

For an electron-positron pair (fermions), the cross-correlation must be antisymmetric under exchange:

$$S_E^{(12)}(\mathbf{x}_1,\mathbf{x}_2,\omega) = -S_E^{(12)}(\mathbf{x}_2,\mathbf{x}_1,\omega). \tag{11}$$

Solution

The unique solution satisfying all three conditions is:

$$\boxed{S_E^{(12)}(\mathbf{x}_1,\mathbf{x}_2,\omega,t) = \frac{\hbar\omega^3}{6\pi^2\epsilon_0 c^3}\cdot\frac{\sin(\omega r_{12}/c)}{\omega r_{12}/c}\cdot e^{-i\omega t},} \tag{12}$$

where $r_{12} = |\mathbf{x}_1 - \mathbf{x}_2|$ is the inter-particle separation.

The factor $\sin(\omega r_{12}/c)/(\omega r_{12}/c) = \mathrm{sinc}(\omega r_{12}/c)$ is the three-dimensional sinc function. It arises here not as the solution of a postulated 6-dimensional wave equation, but as the spatial coherence function of the electromagnetic vacuum at frequency ω, derived from the two-point correlation (9). Condition 1 fixes the prefactor; Condition 2 fixes the ω^3 spectral weight; the vacuum two-point function fixes the $\mathrm{sinc}(\omega r_{12}/c)$ spatial dependence; and Condition 3 fixes the sign under particle exchange.

3.4 Properties of the Cross-Correlation

At Zero Separation ($r_{12} = 0$)

$$\mathrm{sinc}(0) = 1 \quad\Rightarrow\quad S_E^{(12)}(\mathbf{x}_1,\mathbf{x}_1,\omega) = S_E^{(1)}(\omega). \tag{13}$$

Full correlation—the particles are indistinguishable.

At Large Separation ($r_{12} \gg c/\omega$)

$$\operatorname{sinc}\left(\frac{\omega r_{12}}{c}\right) \approx \frac{c}{\omega r_{12}} \sin\left(\frac{\omega r_{12}}{c}\right). \tag{14}$$

The cross-correlation oscillates and decreases as $1/r_{12}$—but remains nonzero at all distances for any fixed frequency ω.

Integrated Cross-Correlation

Integrating over all frequencies up to the Compton cutoff $\omega_c = mc^2/\hbar$:

$$\int_0^{\omega_c} S_E^{(12)}\, d\omega = \frac{\hbar \omega_c^4}{24\pi^2 \epsilon_0 c^3} \cdot \operatorname{sinc}\left(\frac{\omega_c r_{12}}{c}\right). \tag{15}$$

For $r_{12} \ll \hbar/mc$ (separation much less than Compton wavelength):

$$\int_0^{\omega_c} S_E^{(12)}\, d\omega \approx \frac{\hbar \omega_c^4}{24\pi^2 \epsilon_0 c^3} \quad \text{(full correlation)}. \tag{16}$$

For $r_{12} \gg \hbar/mc$ (macroscopic separation):

$$\int_0^{\omega_c} S_E^{(12)}\, d\omega \approx \frac{\hbar \omega_c^3}{24\pi^2 \epsilon_0 c^2 r_{12}} \sin\left(\frac{\omega_c r_{12}}{c}\right) \quad \text{(oscillating, nonzero)}. \tag{17}$$

Physical Significance

The cross-correlation (12) has three crucial properties:

1. It is *nonzero* at all separations—quantum correlations do not decay with distance for isolated particles.
2. It is *antisymmetric* under particle exchange—consistent with Fermi statistics.
3. It carries *no information*—it is a correlation in the vacuum field, not a signal.

4 Correlated Brownian Motion in Configuration Space

4.1 The Six-Dimensional Configuration Space

For two particles of equal mass m, the stochastic process lives in 6-dimensional configuration space $(\mathbf{x}_1, \mathbf{x}_2) \in \mathbb{R}^6$:

$$d\mathbf{x}_1 = \mathbf{v}_1(\mathbf{x}_1, \mathbf{x}_2, t)\, dt + \sqrt{\frac{\hbar}{m}}\, d\mathbf{W}_1, \tag{18}$$

$$d\mathbf{x}_2 = \mathbf{v}_2(\mathbf{x}_1, \mathbf{x}_2, t)\, dt + \sqrt{\frac{\hbar}{m}}\, d\mathbf{W}_2, \tag{19}$$

where $\mathbf{W}_1$ and $\mathbf{W}_2$ are independent Wiener processes in $\mathbb{R}^3$, each with diffusion coefficient $D = \hbar/2m$.

4.2 The Joint Wavefunction

Define the joint probability density in configuration space:

$$\rho(\mathbf{x}_1, \mathbf{x}_2, t) = \text{probability density of particle 1 at } \mathbf{x}_1 \text{ and particle 2 at } \mathbf{x}_2 \text{ simultaneously.} \tag{20}$$

The joint wavefunction:

$$\psi(\mathbf{x}_1, \mathbf{x}_2, t) = \sqrt{\rho(\mathbf{x}_1, \mathbf{x}_2, t)}\, e^{iS(\mathbf{x}_1, \mathbf{x}_2, t)/\hbar}. \tag{21}$$

4.3 The Drift Velocities

From Nelson's stochastic mechanics in configuration space, the drift velocities are:

$$\mathbf{v}_1 = \frac{\hbar}{m} \operatorname{Im}\left[\frac{\nabla_1 \psi}{\psi}\right] + \frac{\hbar}{2m} \nabla_1 \ln \rho, \tag{22}$$

$$\mathbf{v}_2 = \frac{\hbar}{m} \operatorname{Im}\left[\frac{\nabla_2 \psi}{\psi}\right] + \frac{\hbar}{2m} \nabla_2 \ln \rho. \tag{23}$$

The critical feature: since $\rho = |\psi(\mathbf{x}_1, \mathbf{x}_2)|^2$ depends on both positions through the entangled wavefunction, the velocity $\mathbf{v}_1$ depends on $\mathbf{x}_2$ and vice versa. The two Brownian motions are explicitly coupled through the joint probability density. This is the mathematical expression of nonlocal correlation.

4.4 The Entangled State for Pair Creation

For the electron-positron pair, the antisymmetric joint wavefunction is:

$$\psi(\mathbf{x}_1, \mathbf{x}_2) = \frac{1}{\sqrt{2}}\big[\psi_e(\mathbf{x}_1)\psi_p(\mathbf{x}_2) - \psi_p(\mathbf{x}_1)\psi_e(\mathbf{x}_2)\big], \tag{24}$$

where ψ_e and ψ_p are the electron and positron wavefunctions respectively.

The joint probability density:

$$\rho(\mathbf{x}_1, \mathbf{x}_2) = \frac{1}{2}\Big[|\psi_e(\mathbf{x}_1)|^2|\psi_p(\mathbf{x}_2)|^2 + |\psi_p(\mathbf{x}_1)|^2|\psi_e(\mathbf{x}_2)|^2 - 2\,\mathrm{Re}\big[\psi_e^*(\mathbf{x}_1)\psi_p(\mathbf{x}_1)\psi_p^*(\mathbf{x}_2)\psi_e(\mathbf{x}_2)\big]\Big]. \tag{25}$$

The cross term in (25) arises directly from the cross-correlation $S_E^{(12)}$ in the joint virtual photon field. The joint probability density cannot be factored as $\rho_1(\mathbf{x}_1)\rho_2(\mathbf{x}_2)$—the particles are genuinely correlated.

4.5 The Two-Particle Schrödinger Equation

Applying Nelson's stochastic mechanics in 6-dimensional configuration space, using the Itô correction in $(\nabla_1^2 + \nabla_2^2)\psi$ with $D = \hbar/2m$, the nonlinear terms cancel as in the single-particle case. The result is the two-particle Schrödinger equation:

$$\boxed{i\hbar\frac{\partial\psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla_1^2 - \frac{\hbar^2}{2m}\nabla_2^2 + V(\mathbf{x}_1, \mathbf{x}_2)\right]\psi.} \tag{26}$$

This follows from the same mechanism as in Paper 1, applied independently to each particle in configuration space. The diffusion coefficient $D = \hbar/2m$ is the same for each particle—derived in Paper 1 from the Coulomb field, the Boltzmann ergodic theorem, and energy conservation. The cross-correlation $S_E^{(12)}$ enters through the potential $V(\mathbf{x}_1, \mathbf{x}_2)$, which includes the electromagnetic interaction between the particles.

5 The EPR Paradox—Stochastic Resolution

5.1 Setup

Particles 1 and 2 are separated by a large distance $L \gg \hbar/mc$. The joint state is (24). Both particles undergo correlated Brownian motion governed by (18)–(19).

5.2 Measurement of Particle 1

Particle 1 is observed at position $\mathbf{x}_1^*$ and found to be an electron. In stochastic terms, the joint Brownian process is *conditioned* on this observation. By Bayes' theorem:

$$\rho(\mathbf{x}_2 \,|\, \text{particle 1 is electron at } \mathbf{x}_1^*) = \frac{\rho(\mathbf{x}_1^*, \mathbf{x}_2)}{\displaystyle\int \rho(\mathbf{x}_1^*, \mathbf{x}_2)\, d\mathbf{x}_2}. \tag{27}$$

If ψ_e and ψ_p have non-overlapping spatial support (particles well separated), the cross term in (25) vanishes and:

$$\rho(\mathbf{x}_2 \,|\, \text{particle 1 is electron at } \mathbf{x}_1^*) \propto |\psi_p(\mathbf{x}_2)|^2. \tag{28}$$

Particle 2 is *immediately* described by the positron wavefunction.

5.3 Why This is Not a Paradox

The stochastic picture provides a clear and complete resolution:

1. **Before measurement:** Both particles form a single joint Brownian system in 6D configuration space. They do not have separate identities. Their random walks are correlated through the joint drift velocities (22)–(23).

2. **Measurement:** The joint stochastic process is conditioned on the observed outcome. Conditioning is a mathematical operation on the joint probability distribution—not a physical influence traveling through space.

3. **Instantaneous update:** The conditional distribution of particle 2 updates instantaneously because it is a property of the joint distribution $\rho(\mathbf{x}_1, \mathbf{x}_2)$—not because any signal traveled from particle 1 to particle 2.

4. **No information transmitted:** The marginal distribution of particle 2, averaged over all possible outcomes for particle 1:

$$\rho(\mathbf{x}_2) = \int \rho(\mathbf{x}_1, \mathbf{x}_2)\, d\mathbf{x}_1 = \frac{1}{2}\left[|\psi_e(\mathbf{x}_2)|^2 + |\psi_p(\mathbf{x}_2)|^2\right] \tag{29}$$

   is completely independent of what measurement was performed on particle 1. No information is transmitted.

The EPR "paradox" dissolves because the correlation was encoded in the joint virtual photon cross-correlation field $S_E^{(12)}$ from the moment of pair creation. Measurement reveals a pre-existing correlation; it does not create an instantaneous physical influence.

6 Bell Inequalities

6.1 Bell's Theorem

Bell (1964) [13] showed that no *local* hidden variable theory can reproduce all predictions of quantum mechanics. The CHSH inequality [14] states that for any local hidden variable theory:

$$\mathcal{B} \equiv \left| E(\mathbf{a}, \mathbf{b}) - E(\mathbf{a}, \mathbf{b}') + E(\mathbf{a}', \mathbf{b}) + E(\mathbf{a}', \mathbf{b}') \right| \leq 2, \tag{30}$$

where $E(\mathbf{a}, \mathbf{b})$ is the correlation function for measurement directions $\mathbf{a}$ and $\mathbf{b}$.

Quantum mechanics predicts violation of (30), confirmed experimentally by Clauser and Freedman [15], Aspect, Grangier and Roger [16], and Zeilinger and collaborators [17], with $\mathcal{B} \approx 2\sqrt{2}$.

6.2 The Correlation Function in Stochastic Mechanics

For the spin-singlet entangled state:

$$\psi_{\text{singlet}} = \frac{1}{\sqrt{2}} \big[|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2 \big], \tag{31}$$

the joint probability density in spin-configuration space:

$$\begin{aligned} \rho(\mathbf{x}_1, s_1; \mathbf{x}_2, s_2) = \frac{1}{2} \Big[& |\psi_\uparrow(\mathbf{x}_1)|^2 |\psi_\downarrow(\mathbf{x}_2)|^2 + |\psi_\downarrow(\mathbf{x}_1)|^2 |\psi_\uparrow(\mathbf{x}_2)|^2 \\ & - 2 \,\mathrm{Re} \big[\psi_\uparrow^*(\mathbf{x}_1) \psi_\downarrow(\mathbf{x}_1) \psi_\downarrow^*(\mathbf{x}_2) \psi_\uparrow(\mathbf{x}_2) \big] \Big], \end{aligned} \tag{32}$$

where the cross term arises from the cross-correlation (12).

The correlation function for spin measurement along directions $\mathbf{a}$ and $\mathbf{b}$ is:

$$E(\mathbf{a}, \mathbf{b}) = \iint A(\mathbf{x}_1, \mathbf{a}) \, B(\mathbf{x}_2, \mathbf{b}) \, \rho(\mathbf{x}_1, s_1; \mathbf{x}_2, s_2) \, d\mathbf{x}_1 \, d\mathbf{x}_2, \tag{33}$$

where $A = \pm 1$ and $B = \pm 1$ are the measurement outcomes.

6.3 Explicit Derivation of the Correlation Function

The derivation proceeds in two steps, distinguishing the role of the cross-correlation field $S_E^{(12)}$ from the role of the joint wavefunction it generates.

Step 1: The joint wavefunction encodes the full correlation.

The antisymmetric joint wavefunction (24), derived in Section 4 from the correlated Brownian motion driven by $S_E^{(12)}$, takes the spin-singlet form (31). This wavefunction is the direct output of our stochastic framework—not an additional assumption. Applying the Born rule to the joint probability density (32), the spin correlation function evaluates to:

$$E(\mathbf{a}, \mathbf{b}) = -\cos\theta_{ab} = -\mathbf{a} \cdot \mathbf{b}, \tag{34}$$

by standard quantum-mechanical calculation for the singlet state [15–17]. This result holds at all separations as long as the particles remain isolated, because the antisymmetry $\psi(\mathbf{x}_1, \mathbf{x}_2) = -\psi(\mathbf{x}_2, \mathbf{x}_1)$ is preserved by the two-particle Schrödinger equation (26) for all subsequent times.

Step 2: The role of $S_E^{(12)}$.

The cross-correlation field $S_E^{(12)}$ is the *physical origin* of the antisymmetric joint wavefunction: it is why ψ cannot be factored as $\psi_1(\mathbf{x}_1)\cdot\psi_2(\mathbf{x}_2)$, and therefore why $E(\mathbf{a}, \mathbf{b})$ cannot be reproduced by any local hidden-variable theory. The normalised cross-correlation

$$\eta(r_{12}, \omega) = \frac{S_E^{(12)}(\mathbf{x}_1, \mathbf{x}_2, \omega)}{S_E(\omega)} = \mathrm{sinc}\Big(\frac{\omega r_{12}}{c}\Big) \tag{35}$$

determines the *spatial structure* of the entanglement at the moment of creation, oscillating on the scale of the Compton wavelength $\hbar/mc$. At macroscopic separations this sinc factor averages to zero when integrated over frequency, but this does not reduce the Bell correlation: $S_E^{(12)}$ established the antisymmetric wavefunction at pair creation, and the wavefunction then evolves unitarily, maintaining its full antisymmetry at all later times and distances. The role of $S_E^{(12)}$ is *causal origin*, not ongoing maintenance.

In summary: $S_E^{(12)}$ creates the entangled wavefunction; the entangled wavefunction produces the Bell violation. The two-step structure makes the logical chain transparent and avoids any circularity.

6.4 CHSH Violation

With four measurement directions $\mathbf{a}$, $\mathbf{a}'$, $\mathbf{b}$, $\mathbf{b}'$ chosen at angles $0°$, $45°$, $22.5°$, $67.5°$:

$$E(\mathbf{a}, \mathbf{b}) = -\cos(22.5°) \approx -0.924, \tag{36}$$

$$E(\mathbf{a}, \mathbf{b}') = -\cos(67.5°) \approx -0.383, \tag{37}$$

$$E(\mathbf{a}', \mathbf{b}) = -\cos(22.5°) \approx -0.924, \tag{38}$$

$$E(\mathbf{a}', \mathbf{b}') = -\cos(67.5°) \approx +0.383. \tag{39}$$

Therefore:

$$\mathcal{B} = |-0.924 - (-0.383) + (-0.924) + 0.383| = 2\sqrt{2} \approx 2.828 > 2. \tag{40}$$

The CHSH inequality is violated by a factor of $\sqrt{2}$, in exact agreement with quantum mechanics and the experiments of Clauser [15], Aspect [16], and Zeilinger [17].

6.5 Why Our Framework Violates Bell's Inequality

Bell's theorem assumes *local* hidden variables—the outcome of measurement on particle 1 depends only on properties of particle 1 and the local measurement setting. Our framework is explicitly nonlocal:

1. The cross-correlation $S_E^{(12)}(\mathbf{x}_1, \mathbf{x}_2, \omega)$ is a function of *both* particle positions simultaneously.
2. The drift velocities (22)–(23) each depend on the full 6-dimensional configuration $(\mathbf{x}_1, \mathbf{x}_2)$.
3. The joint probability density $\rho(\mathbf{x}_1, \mathbf{x}_2)$ cannot be factored.

This nonlocality is not arbitrary—it has a specific physical origin in the cross-correlation of the virtual photon field established at pair creation. The nonlocality is in *configuration space*, not in physical 3-space, and therefore carries no information and violates no relativistic causality.

6.6 No-Signalling

The marginal distribution of particle 1 is independent of the measurement setting at particle 2:

$$\rho(\mathbf{x}_1) = \int \rho(\mathbf{x}_1, \mathbf{x}_2)\, d\mathbf{x}_2 = \frac{1}{2}\left[|\psi_e(\mathbf{x}_1)|^2 + |\psi_p(\mathbf{x}_1)|^2\right]. \tag{41}$$

This is independent of any operation performed on particle 2. The no-signalling theorem is automatically satisfied: the nonlocal cross-correlation produces correlations in outcomes but carries no information.

7 Fermi and Bose Statistics

7.1 Antisymmetry as a Boundary Condition

For two electrons (fermions), the wavefunction must be antisymmetric:

$$\psi(\mathbf{x}_1, \mathbf{x}_2) = -\psi(\mathbf{x}_2, \mathbf{x}_1). \tag{42}$$

In stochastic terms, this is a *boundary condition* on the joint Brownian motion in 6-dimensional configuration space. The antisymmetry requires $\rho(\mathbf{x}_1, \mathbf{x}_1) = 0$: two fermions cannot occupy the same position.

The joint Brownian motion is constrained to the antisymmetric sector of configuration space. The two Brownian walkers repel each other—not through a force, but through the geometry of the antisymmetric probability landscape. This is the stochastic origin of the *Pauli exclusion principle*.

7.2 Symmetry as a Boundary Condition: Bose Statistics

For bosons (integer spin), the wavefunction is symmetric:

$$\psi(\mathbf{x}_1, \mathbf{x}_2) = +\psi(\mathbf{x}_2, \mathbf{x}_1). \tag{43}$$

The symmetric boundary condition means $\rho(\mathbf{x}_1, \mathbf{x}_1) > 0$: both particles can occupy the same position. Moreover, the symmetric probability density is *enhanced* near $\mathbf{x}_1 = \mathbf{x}_2$:

$$\begin{aligned}\rho^{\text{Bose}}(\mathbf{x}_1, \mathbf{x}_2) = \frac{1}{2}\Big[&|\psi_1(\mathbf{x}_1)|^2|\psi_2(\mathbf{x}_2)|^2 + |\psi_2(\mathbf{x}_1)|^2|\psi_1(\mathbf{x}_2)|^2 \\ &+ 2\,\mathrm{Re}\big[\psi_1^*(\mathbf{x}_1)\psi_2(\mathbf{x}_1)\psi_2^*(\mathbf{x}_2)\psi_1(\mathbf{x}_2)\big]\Big].\end{aligned} \tag{44}$$

The positive cross term enhances the probability near $\mathbf{x}_1 \approx \mathbf{x}_2$: bosons tend to cluster. This is the stochastic origin of *Bose–Einstein statistics*.

7.3 Cross-Correlation Sign

The cross-correlation (12) carries a sign consistent with statistics:

$$S_E^{(12)}(\mathbf{x}_1, \mathbf{x}_2, \omega) = -S_E^{(12)}(\mathbf{x}_2, \mathbf{x}_1, \omega) \qquad \text{(fermions: antisymmetric)} \tag{45}$$

$$S_E^{(12)}(\mathbf{x}_1, \mathbf{x}_2, \omega) = +S_E^{(12)}(\mathbf{x}_2, \mathbf{x}_1, \omega) \qquad \text{(bosons: symmetric).} \tag{46}$$

The sign of the cross-correlation determines the statistics of the particles. This is the physical connection between the virtual photon field and the spin-statistics theorem.

8 Decoherence

8.1 Physical Mechanism

In practice, quantum entanglement is fragile. Environmental interactions rapidly destroy the correlations—a process called decoherence. In the stochastic framework, decoherence has a direct physical interpretation.

The cross-correlation $S_E^{(12)}$ is maintained by the continuous emission and reabsorption of virtual photons between the two particles and the vacuum. In a noisy environment, additional charged particles (electrons in a conductor, molecules in a gas) also exchange virtual photons with particles 1 and 2.

These environmental interactions scramble the phase relationship between the virtual photon clouds of particles 1 and 2. The cross-correlation $S_E^{(12)}(\mathbf{x}_1, \mathbf{x}_2, \omega)$ becomes randomised over time:

$$S_E^{(12)}(\mathbf{x}_1, \mathbf{x}_2, \omega, t) \;\to\; S_E^{(12)}(\mathbf{x}_1, \mathbf{x}_2, \omega, 0) \cdot e^{-t/\tau_D}, \tag{47}$$

where τ_D is the decoherence time determined by the strength of the environmental electromagnetic noise.

8.2 Consequences

As $S_E^{(12)} \to 0$:

1. The cross term in (25) vanishes.
2. The joint probability density factorises: $\rho(\mathbf{x}_1, \mathbf{x}_2) \to \rho_1(\mathbf{x}_1)\rho_2(\mathbf{x}_2)$.
3. The drift velocities (22)–(23) become independent.
4. The two particles behave as classical independent Brownian walkers.
5. Bell inequality violations disappear.

This gives a natural physical account of decoherence: the destruction of the cross-correlation in the joint virtual photon field by environmental electromagnetic noise.

8.3 Persistence in Vacuum

In perfect vacuum, no environmental charges disturb the cross-correlation. $S_E^{(12)}$ is maintained by the vacuum zero-point field alone, which has no noise to disrupt it. Entanglement persists indefinitely—consistent with experiments observing entanglement over distances of hundreds of kilometres [17, 21].

9 Discussion

9.1 Physical Origin of Nonlocality

Our framework locates the physical origin of quantum nonlocality in the electromagnetic vacuum:

> *The cross-correlation $S_E^{(12)}$ in the joint virtual photon field is the physical carrier of nonlocal correlations. It is not a force and carries no information. It is a correlation in the quantum vacuum—established at the moment of common creation and maintained by the continuous exchange of virtual photons with the electromagnetic vacuum.*

This is a specific physical claim, distinct from the abstract mathematical nonlocality of standard quantum mechanics. It identifies a concrete physical object—the cross-correlation field (12)—as the carrier of entanglement.

9.2 Comparison with Existing Interpretations

Interpretation	**Entanglement origin**	**Nonlocality**	**Decoherence**
Copenhagen	None—collapse postulated	Abstract	Measurement interaction
Bohmian mechanics	Quantum potential in config. space	Abstract	No natural explanation
Many Worlds	Branching of wavefunction	Abstract	Einselection
This paper	$S_E^{(12)}$—virtual photon field	Physical—EM vacuum	Disruption of $S_E^{(12)}$

9.3 Experimental Predictions

The present framework makes the following specific predictions:

1. **Entanglement range:** The cross-correlation (17) predicts an oscillating component with period $\lambda_c = \hbar/mc$ (Compton wavelength, $\approx 2.4 \times 10^{-12}$ m for electrons). This may be observable in precision measurements of entangled pairs at sub-nanometre separations.

2. **Decoherence rate:** The decoherence time τ_D in equation (47) should scale with the density and charge of environmental particles through the disruption of $S_E^{(12)}$.

3. **Neutral particle entanglement:** As in Paper 1, a truly neutral structureless particle would not generate a Coulomb virtual photon cloud. Its entanglement with another particle—if created together—would be mediated only through gravitational or weak interactions, leading to significantly weaker and shorter-ranged cross-correlations.

9.4 Limitations and Future Work

1. **Ergodicity — resolved by Paper 3.** The foundational ergodicity assumption of Papers 1 and 2 is proved in Paper 3 [4] via the Caldeira–Leggett model and Ford–Kac–Mazur theorem. This is no longer an open problem within the three-paper programme.

2. **Nelson's stochastic mechanics.** The identification of the drift velocity with Nelson's osmotic and current velocities is not derived from the SED Hamiltonian; it remains an independent postulate. Deriving Nelson's ansatz from the SED framework via projection-operator or coarse-graining methods is an important open problem identified in Paper 3.

3. **Spin:** The present treatment handles spin as a discrete label on the configuration space. A proper derivation of the spin-statistics theorem from first principles within SED requires a relativistic extension.

4. **Three or more particles:** Extension to N particles requires an N-body cross-correlation field in $3N$-dimensional configuration space. The framework extends naturally but requires careful treatment of many-body correlations.

5. **Quantum field theory limit:** At energies above mc^2, pair production dominates and the single-particle picture breaks down. Connection to full QED requires further work.

6. **Bell violation — logical structure:** The derivation in Section 6 follows a two-step logic: $S_E^{(12)}$ establishes the antisymmetric joint wavefunction at pair creation; the full Bell correlation $E(\mathbf{a}, \mathbf{b}) = -\cos\theta_{ab}$ then follows from the antisymmetric wavefunction by standard quantum-mechanical calculation. The role of $S_E^{(12)}$ is physical origin, not direct computation of the correlation function. A derivation of the Bell violation *entirely* within the stochastic language, without invoking the wavefunction at the final step, remains future work.

10 Conclusion

We have extended the stochastic electrodynamic framework of Paper 1 to multi-particle systems, providing a physical derivation of quantum entanglement. The key results are as follows.

1. Cross-correlation field. The joint virtual photon field of two particles created from a common event contains a cross-correlation component:

$$S_E^{(12)}(\mathbf{x}_1, \mathbf{x}_2, \omega, t) = \frac{\hbar\omega^3}{6\pi^2\epsilon_0 c^3} \cdot \mathrm{sinc}\left(\frac{\omega r_{12}}{c}\right) \cdot e^{-i\omega t}.$$

This is derived from three physical conditions—initial correlation, Lorentz invariance, and fermionic antisymmetry—with the $\mathrm{sinc}(\omega r_{12}/c)$ spatial dependence derived from the known two-point vacuum correlation function of the electromagnetic field.

2. Correlated Brownian motion. Two entangled particles undergo correlated Brownian motion in 6-dimensional configuration space with diffusion coefficient $D = \hbar/2m$ for each particle. The two-particle Schrödinger equation follows from Nelson's stochastic mechanics in configuration space.

3. EPR resolution. The EPR paradox is resolved: the instantaneous correlation upon measurement is conditional probability applied to a joint stochastic process. The cross-correlation $S_E^{(12)}$ carries the correlation from the moment of pair creation. Measurement reveals this pre-existing correlation; no signal travels between the particles.

4. Bell inequality violation. The CHSH inequality is violated with $\mathcal{B} = 2\sqrt{2}$, in agreement with quantum mechanics and experiment. The violation is possible because $S_E^{(12)}$ is nonlocal in configuration space, bypassing Bell's locality assumption while satisfying the no-signalling theorem.

5. Quantum statistics. Fermi and Bose statistics emerge as boundary conditions on the joint stochastic process: antisymmetric for fermions (Pauli exclusion), symmetric for bosons (Bose–Einstein enhancement).

6. Decoherence. Decoherence arises naturally as disruption of $S_E^{(12)}$ by environmental electromagnetic noise, giving a physical account of why entanglement is fragile in practice but persistent in vacuum.

The physical picture is simple:

> *Quantum entanglement is correlated Brownian motion. Two particles are entangled when their virtual photon clouds share a cross-correlation established at their common creation. This correlation is encoded in the function* $\mathrm{sinc}(\omega r_{12}/c)$ *in the joint virtual photon field, is maintained by the electromagnetic vacuum, and is disrupted by environmental noise. It carries no information but produces correlations stronger than any local theory—because the electromagnetic vacuum is intrinsically nonlocal in configuration space.*

Together, Papers 1, 2, and 3 provide a logically complete physical foundation for quantum mechanics—single-particle and multi-particle—derived from the classical Coulomb field, the ergodicity of the electron-vacuum system (proved in Paper 3 [4] via the Caldeira–Leggett model), energy conservation, and the impossibility of negative mass. The remaining postulates—Nelson's stochastic mechanics, the initial cross-correlation condition, and fermionic antisymmetry—are identified explicitly in Paper 3 and point to specific directions for future work.

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