Quantum Mechanics Derived from the Coulomb Field: A Classical Foundation through Boltzmann Ergodic Theory, Energy Conservation, and Stochastic Mechanics

Fusao Ishii

Tokyo Institute of Technology (Alumni)

Abstract

Quantum mechanics is defined here as the theoretical framework crystallised at the fifth Solvay Conference of 1927, comprising five postulates: the wavefunction, the canonical commutation relation $[x, p] = i\hbar$, the Born rule, the Schrödinger equation, and the transition probability rule. We present a complete derivation of all five Solvay postulates from purely classical foundations, starting from the directly observable Coulomb field of the electron. The single empirical input is $\hbar$—measured from pre-Solvay experiments (Planck 1900–1911, Einstein 1905, Casimir 1948/Lamoreaux 1997) without quantum mechanical interpretation, in the same way G was measured by Cavendish (1798) without general relativity. By Einstein's photoelectric theory (1905) generalized to all electromagnetic fields, the Coulomb field energy is carried in electromagnetic quanta, each carrying both electric and magnetic fields with energy density $\epsilon_0 E^2$. The Einstein–Hopf detailed balance condition (1910) uniquely determines the spectral shape $\rho(\omega) \propto \omega^3$. Lorentz invariance confirms this form. The Casimir force measurement gives the total ZPF energy per mode $B = \hbar$; classical E/B equipartition from Maxwell's equations gives the electric energy per mode $\hbar\omega/2$, confirmed by Planck (1911). the central original contribution is the physical justification of the Compton frequency cutoff $\omega_c = mc^2/\hbar$: an electron cannot emit an electromagnetic quantum with energy exceeding mc^2, because the residual electron would require negative mass, never observed in nature. Through the Abraham–Lorentz equation, charge e and cutoff ω_c both cancel, giving $D = \hbar/2m$. By Itô's stochastic calculus and Nelson's theorem, this gives the Schrödinger equation. The transformation $\psi = \sqrt{\rho}\, e^{iS/\hbar}$ linearises the nonlinear Fokker–Planck equation, explaining why superposition holds. All five Solvay postulates emerge as consequences of classical physics plus the single measured constant $\hbar$. No Solvay postulate is assumed anywhere.

Contents

1 Introduction

As a physics student the present author learned to calculate with the Schrödinger equation but never understood its physical foundation. The equation worked—extraordinarily well—but the question of why it has the form it does was never answered satisfactorily. This question remained open for five decades.

1.1 Definition of Quantum Mechanics

In this paper, quantum mechanics is defined precisely as the theoretical framework crystallised at the fifth Solvay Conference of 1927, where the famous debates between Einstein and Bohr took place. This framework comprises five postulates accepted without physical justification:

1. the state of a system is described by a wavefunction ψ,
2. position and momentum satisfy the canonical commutation relation $[x, p] = i\hbar$,
3. measurement probability is given by the Born rule $P = |\psi|^2$,
4. time evolution is governed by the Schrödinger equation $i\hbar\,\partial_t\psi = \hat{H}\psi$,
5. the transition probability between states A and B is proportional to $|\langle A|B\rangle|^2$.

Whether the transition probability rule (Postulate 5) should be counted as an independent postulate or as a consequence of the Born rule and time-dependent perturbation theory is a matter of presentation — some textbooks list four postulates, others five. The present paper derives both the Born rule and the transition probability independently from classical foundations, making this counting question moot. All the structural content is derived either way.

These postulates constitute quantum mechanics as understood since 1927. Experimental discoveries made before the Solvay Conference—including Planck's blackbody spectrum (1900–1911), Einstein's photoelectric effect (1905), Compton scattering (1923), and the Casimir effect (1948)—are treated here as pre-quantum empirical facts about nature. They are measurements of the constant $\hbar$, made without quantum mechanical interpretation, exactly as Cavendish's 1798 measurement of G is a pre-relativistic empirical fact independent of general relativity.

This distinction is essential. It is worth noting that Einstein himself—whose 1905 photoelectric discovery gives $E = \hbar\omega$—spent the rest of his life arguing against the very Solvay framework we derive here. His pre-Solvay measurements are inputs to our derivation; the post-Solvay postulates are its outputs.

1.2 The Single Fundamental Postulate

The entire derivation rests on one physical postulate:

The Fundamental Postulate: *The energy of the electromagnetic field—including the static Coulomb field—is carried in discrete quanta of energy* $h\nu$.

This is Einstein's 1905 quantization hypothesis [4], generalized from radiation to all electromagnetic fields including static ones. Einstein established that radiation energy is quantized; the present paper applies this to the Coulomb field of the electron. This generalization is natural and minimal: it asserts that quantization is a property of the electromagnetic field itself, not merely of propagating radiation.

From this single postulate, combined with the single measured constant $\hbar$ and universal physical laws (Maxwell's equations, Newtonian mechanics, energy conservation, the impossibility of negative mass, and Itô stochastic calculus), all five Solvay postulates follow as derived consequences.

1.3 The Claim of This Paper

The claim is precise:

> Given the single fundamental postulate—that electromagnetic field energy is carried in quanta—and the single pre-Solvay empirical constant $\hbar$, measured without quantum mechanical interpretation, all five postulates of the 1927 Solvay framework follow from classical physics. The structural content of quantum mechanics is derived. Only the scale $\hbar$ is empirical.

The logical structure mirrors Newton's derivation of Kepler's laws exactly:

	Newton	**This paper**
Fundamental postulate	Universal gravitation $F \propto m_1 m_2 / r^2$	EM field energy is quantized
Measured constant	G (Cavendish 1798)	$\hbar$ (Casimir/Lamoreaux 1997)
Physical laws used	Newtonian mechanics	Maxwell, Newton, energy conservation
Derived outputs	Kepler's three laws	Five Solvay postulates
Previously assumed	Kepler's laws as empirical facts	Solvay postulates as axioms

Just as Newton did not invent gravity but recognized its universal form, Einstein did not invent quantization but recognized it in radiation. The present paper extends that recognition to the Coulomb field, and shows that quantum mechanics follows as inevitably as Kepler's laws follow from universal gravitation.

1.4 Historical Context

In 1966, Edward Nelson showed that if an electron undergoes Brownian motion with diffusion coefficient $D = \hbar/2m$, the Schrödinger equation follows from Newtonian mechanics alone [1]. The present author first read Nelson's paper in 1972 and recognised its profound implication: the Schrödinger equation is not fundamental but derived. The question—why does the electron have Brownian motion with precisely $D = \hbar/2m$?—remained open for fifty years.

The present paper answers this question. The single fundamental postulate — that the Coulomb field energy is carried in electromagnetic quanta — combined with Einstein–Hopf detailed balance, the Casimir measurement of $\hbar$, energy conservation, and Itô stochastic calculus, gives $D = \hbar/2m$ and thereby all five Solvay postulates. No Solvay postulate is assumed anywhere.

2 Nelson's Stochastic Mechanics and Itô's Lemma

2.1 The Brownian Motion Setup

Nelson considered a particle of mass m undergoing continuous random motion described by the stochastic differential equation

$$dx = v(x,t)\,dt + dW(t), \tag{1}$$

where $v(x,t)$ is the mean local velocity and $dW(t)$ is a Wiener process satisfying $\langle dW_i\, dW_j\rangle = 2D\,\delta_{ij}\,dt$. The diffusion coefficient D characterises the intensity of the random motion. Nelson's theorem establishes that everything depends on this single parameter.

2.2 Itô's Lemma—The Mathematical Heart

Nelson's derivation relies on Itô's stochastic calculus (1944) [3]. Brownian paths are continuous everywhere but differentiable nowhere. The Wiener process satisfies the fundamental relation

$$(dW)^2 = dt \neq 0. \tag{2}$$

In classical calculus $(dx)^2 \to 0$ as $dt \to 0$; for Brownian motion the second-order term survives. For any smooth function $f(x,t)$ along a stochastic process $dx = a\,dt + b\,dW$, Itô's lemma gives

$$df = \left(\frac{\partial f}{\partial t} + a\frac{\partial f}{\partial x} + \frac{b^2}{2}\frac{\partial^2 f}{\partial x^2}\right)dt + b\frac{\partial f}{\partial x}\,dW. \tag{3}$$

The extra term $\frac{b^2}{2}\partial^2 f/\partial x^2$—the Itô correction—has no analogue in classical calculus. For Nelson's diffusion $b^2 = 2D$, so the correction is $D\,\partial^2 f/\partial x^2$. This term is the mathematical origin of the quantum kinetic energy.

2.3 Derivation of the Schrödinger Equation

Nelson introduces the current velocity $v_s = \nabla S/m$ and the osmotic velocity $u = D\nabla\rho/\rho = D\nabla \ln \rho$. He writes Newton's second law for the stochastic process and applies Itô's lemma to the complex wavefunction

$$\psi = \sqrt{\rho}\, e^{iS/\hbar} = e^{R+iS/\hbar}, \quad R = \ln \sqrt{\rho}. \tag{4}$$

The Itô correction generates an extra term $D\nabla^2\psi$. With $D = \hbar/2m$ this becomes exactly $(\hbar^2/2m)\nabla^2\psi$—the quantum kinetic energy operator.

Nelson's theorem: the stochastic equations are equivalent to

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi \tag{5}$$

if and only if

$$D = \frac{\hbar}{2m}. \tag{6}$$

The kinetic energy term in the Schrödinger equation is the Itô correction—the mathematical consequence of non-differentiable Brownian paths.

2.4 Why $\psi = \sqrt{\rho}\, e^{iS/\hbar}$ Linearises the System

The Fokker–Planck equation for ρ is nonlinear. The osmotic velocity $u = D\nabla\rho/\rho$ makes the drift velocity depend on ρ itself. Nelson's quantum Hamilton–Jacobi equation contains the quantum potential

$$Q = -2mD^2\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}, \tag{7}$$

which is highly nonlinear in ρ. However, when these equations are rewritten in terms of $\psi = \sqrt{\rho}\, e^{iS/\hbar}$, the Itô correction in $\nabla^2\psi$ produces exactly the nonlinear terms with opposite sign. They cancel completely, and the linear Schrödinger equation results. This explains three things:

1. The Schrödinger equation is linear—nonlinear terms cancel in the ψ representation.
2. Superposition holds—ψ is the natural variable where linearity is manifest.
3. $\psi = \sqrt{\rho}\, e^{iS/\hbar}$ is not arbitrary—it is the unique combination that linearises the stochastic dynamics.

The superposition principle of quantum mechanics therefore follows from the mathematical structure of the stochastic process, not from a separate postulate.

2.5 The Born Rule

Since $\psi = \sqrt{\rho}\, e^{iS/\hbar}$,

$$|\psi|^2 = \rho. \tag{8}$$

The Born rule is the literal probability density of the Brownian motion, not a separate postulate.

2.6 Heisenberg Uncertainty from Average Velocity

For a Brownian particle, instantaneous velocity is undefined—Brownian paths are nowhere differentiable. The physical velocity is an average over finite time Δt: $v_t = \Delta x/\Delta t$. Position uncertainty:

$$\Delta x = \sqrt{2D\,\Delta t}. \tag{9}$$

Velocity uncertainty:

$$\Delta v = \sqrt{\frac{2D}{\Delta t}}. \tag{10}$$

Their product:

$$\Delta x \cdot \Delta p = m\,\Delta x \cdot \Delta v = m\sqrt{2D\,\Delta t} \cdot \sqrt{\frac{2D}{\Delta t}} = 2mD = \hbar. \tag{11}$$

The time Δt cancels exactly. The uncertainty product is independent of measurement duration. The Heisenberg uncertainty principle follows from Brownian motion with $D = \hbar/2m$—not from a separate postulate.

2.7 The Canonical Commutation Relation $[x, p] = i\hbar$

The canonical commutation relation—Postulate 2 of the Solvay framework—follows from the Fourier representation of the wavefunction.

Step 1: Fourier Representation. Any physically admissible wavefunction $\psi(x)$ can be written as a Fourier integral:

$$\psi(x) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \tilde{\psi}(p)\, e^{ipx/\hbar}\, dp, \tag{12}$$

where p is the Fourier conjugate variable to x and $\tilde{\psi}(p)$ is the momentum-space wavefunction. This is a standard mathematical property of physically reasonable functions—not a quantum postulate. The physical content is de Broglie's relation (1924) [2]: a state of definite momentum p is a plane wave $e^{ipx/\hbar} = e^{ikx}$ with $k = p/\hbar$.

Step 2: The Momentum Operator from the Fourier Derivative Theorem. Applying $-i\hbar\,\partial/\partial x$ to the Fourier representation:

$$-i\hbar\frac{\partial \psi}{\partial x} = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} p\,\tilde{\psi}(p)\, e^{ipx/\hbar}\, dp. \tag{13}$$

The right-hand side is p multiplying $\tilde{\psi}(p)$ in momentum space, transformed back to position space—precisely the action of momentum on the wavefunction. Therefore:

$$\hat{p}\,\psi = -i\hbar\frac{\partial\psi}{\partial x}. \tag{14}$$

This is not postulated. It is the Fourier derivative theorem.

Step 3: The Commutation Relation. Applying $[\hat{x}, \hat{p}]$ to any wavefunction ψ:

$$\begin{aligned}[\hat{x}, \hat{p}]\,\psi &= \hat{x}(\hat{p}\,\psi) - \hat{p}(\hat{x}\,\psi) \\ &= x\left(-i\hbar\frac{\partial\psi}{\partial x}\right) + i\hbar\frac{\partial}{\partial x}(x\psi) \\ &= -i\hbar x\psi' + i\hbar(\psi + x\psi') = i\hbar\psi. \end{aligned} \tag{15}$$

Since this holds for any ψ:

$$[\hat{x}, \hat{p}] = i\hbar. \tag{16}$$

2.8 Summary: What Nelson's Theorem Needs

Nelson's theorem establishes: Newtonian mechanics + Brownian motion with $D = \hbar/2m \Rightarrow$ Schrödinger equation + all five postulates. But Nelson postulated $D = \hbar/2m$ without physical justification. The remainder of the paper derives $D = \hbar/2m$ from the classical Coulomb field.

3 From the Coulomb Field to the Zero-Point Radiation Field

3.1 The Coulomb Field is Real and Classical

The electric field of an electron at distance r,

$$E(r) = \frac{e}{4\pi\epsilon_0 r^2}, \tag{17}$$

is directly measurable and entirely classical. Its energy density,

$$u(r) = \frac{\epsilon_0}{2}E^2 = \frac{e^2}{32\pi^2\epsilon_0 r^4}, \tag{18}$$

is real physical energy. This is the starting point.

3.2 The Coulomb Field Energy is Carried in Electromagnetic Quanta with Both E and B

Einstein's 1905 photoelectric paper [4] establishes that electromagnetic energy is carried in discrete quanta of energy $h\nu$. Einstein applied this to radiation, but the argument rests on the quantization of electromagnetic energy generally: wherever electromagnetic energy exists, it is carried in quanta. The Coulomb field has real, measurable energy density $u = \epsilon_0 E^2/2$. By Einstein's quantization, this energy is carried in quanta of frequency ν.

A clarification on terminology. The term "virtual photon" is used in perturbative quantum field theory as a mathematical device for off-shell field quanta that do not satisfy the free-photon dispersion relation $E = pc$. Such virtual photons have no independent experimental proof and are not required here. The present paper uses a physically weaker and more defensible claim: *the energy of the Coulomb field is carried in electromagnetic quanta* (Einstein 1905 applied to static fields), without asserting that these quanta are on-shell or off-shell photons in the QED sense.

Each electromagnetic quantum carries both E and B. This follows from Maxwell's equations alone, independently of whether the quanta are on-shell or off-shell. Maxwell's equations do not permit a purely electric or purely magnetic oscillation: any oscillating electromagnetic disturbance necessarily has both electric and magnetic components with $u_E = u_B$. Therefore each quantum of the Coulomb field energy carries equal electric and magnetic energy, and the total energy density of the quantum field is:

$$u_{\text{quanta}} = \frac{\epsilon_0}{2}E^2 + \frac{1}{2\mu_0}B^2 = \frac{\epsilon_0}{2}E^2 + \frac{\epsilon_0}{2}E^2 = \epsilon_0 E^2. \tag{19}$$

The factor $\epsilon_0 E^2$ (not $\epsilon_0 E^2/2$) applies because each electromagnetic quantum carries equal electric and magnetic energy — a theorem of classical electrodynamics, not a quantum postulate.

3.3 Definitions: Mode Density $\rho(\omega)$ and Spectral Density $S_E(\omega)$

Mode Density $\rho(\omega)$. The mode density $\rho(\omega)$ is the number of electromagnetic modes per unit volume per unit frequency. Counting all wave vectors k and both polarisations in three-dimensional space:

$$\rho(\omega) = \frac{\omega^2}{\pi^2 c^3}. \tag{20}$$

Electric Field Spectral Density $S_E(\omega)$. $S_E(\omega)$ is the one-sided power spectral density of one component of the electric field, defined through

$$\langle E_x^2 \rangle = \int_0^\infty S_E(\omega)\, d\omega. \tag{21}$$

So $S_E(\omega)\,d\omega$ is the mean square electric field (x-component) from frequencies between ω and $\omega + d\omega$.

The Relationship. The total spectral energy density:

$$u(\omega) = \rho(\omega) \cdot \varepsilon(\omega). \tag{22}$$

The electric field carries half the total energy, distributed equally among 3 directions:

$$u_{E_x}(\omega) = \frac{u(\omega)}{6}. \tag{23}$$

Since $u_{E_x}(\omega) = (\epsilon_0/2)\,S_E(\omega)$:

$$S_E(\omega) = \frac{2\,u_{E_x}(\omega)}{\epsilon_0} = \frac{u(\omega)}{3\epsilon_0} = \frac{\rho(\omega)\,\varepsilon(\omega)}{3\epsilon_0}. \tag{24}$$

3.4 Einstein–Hopf Detailed Balance Determines the Spectral Shape

Einstein and Hopf (1910) [5] considered a classical charged oscillator in thermal equilibrium with a radiation field. Requiring the steady-state condition (power absorbed = power emitted) for all frequencies, they derived the differential equation

$$\rho(\omega) = \omega^3 \frac{d\rho}{d\omega}. \tag{25}$$

This has the unique solution

$$\rho(\omega) \propto \omega^3. \tag{26}$$

Lorentz invariance independently confirms this: the ω^3 spectrum is the unique Lorentz-invariant form for a random electromagnetic field [1, 8]. Both classical detailed balance and relativistic invariance force the same spectral shape. No quantum mechanics is used.

3.5 The Total ZPF Energy Per Mode: The Casimir Effect

The Einstein–Hopf detailed balance condition gives $\rho(\omega) = (\omega^2/\pi^2c^3)\,\varepsilon(\omega)$. Since $\rho(\omega) \propto \omega^3$, the energy per mode must be proportional to frequency:

$$\varepsilon_{\text{total}}(\omega) = B\omega, \tag{27}$$

where B is a constant with units of action, undetermined by classical electrodynamics alone. As Senatchin (2001) [2] showed, B is a free integration constant of the Einstein–Hopf differential equation—it cannot be derived; it must be measured.

The Casimir effect provides a direct classical measurement of B. Two neutral parallel

conducting plates separated by distance d experience an attractive force because the conducting boundary conditions restrict the ZPF modes inside the plates to those with a whole number of half-wavelengths, while outside all modes are present.

Why the Casimir effect measures the total ($E + B$) field. The conducting boundary conditions act on the complete electromagnetic field—both electric and magnetic components simultaneously. As established in Section 3.2, each electromagnetic quantum carries both E and B inseparably, by Maxwell's equations. A conductor enforces $E_{\text{tangential}} = 0$ on its surface, which simultaneously constrains the associated B field through Maxwell's equations. The Casimir force therefore reflects the total ZPF energy—electric plus magnetic—in every mode.

The regulated energy difference per unit area gives [1]:

$$F_{\text{Casimir}} = -\frac{\pi^2 B c}{240\, d^4}. \tag{28}$$

No quantum mechanics is assumed—only the ZPF with total energy per mode $B\omega$ and the classical electromagnetic boundary conditions on the plates.

Lamoreaux (1997) [2] measured the Casimir force directly and found agreement with $F = -\pi^2 \hbar c/240\, d^4$ to within 5%. Comparing:

$$B = \hbar = 1.055 \times 10^{-34}\ \text{J s}. \tag{29}$$

The total ZPF energy per mode is therefore $\varepsilon_{\text{total}}(\omega) = \hbar\omega$.

3.6 The Electric ZPF Energy Per Mode: Planck 1911

Why Planck's measurement gives the electric (E only) field. The blackbody oscillators in Planck's derivation are electric dipoles whose interaction Hamiltonian is:

$$H_{\text{int}} = -ex \cdot E(t). \tag{30}$$

This couples the oscillator to the electric component of the radiation field only. The magnetic field exerts no force on a stationary electric charge ($F_{\text{magnetic}} = ev \times B \approx 0$ for $v = 0$), so the oscillator equilibrates exclusively with the electric field energy. At $T = 0$, Planck (1911) found the oscillator ground-state energy to be $\hbar\omega/2$, which by this equilibration equals the electric ZPF energy per mode:

$$\varepsilon_E(\omega) = \frac{\hbar\omega}{2}. \tag{31}$$

3.7 E/B Equipartition: Connecting Casimir and Planck

For electromagnetic waves in vacuum, Maxwell's equations require equal energy in the electric and magnetic components. For a plane wave:

$$\frac{\epsilon_0}{2}E^2 = \frac{B^2}{2\mu_0} \quad \Rightarrow \quad u_E = u_B = \frac{u_{\text{total}}}{2}. \tag{32}$$

This is a theorem of classical electrodynamics, not a quantum postulate. Applying it to the ZPF:

$$\varepsilon_E(\omega) = \varepsilon_B(\omega) = \frac{\varepsilon_{\text{total}}(\omega)}{2} = \frac{\hbar\omega}{2}. \tag{33}$$

The Casimir measurement ($\varepsilon_{\text{total}} = \hbar\omega$) and Planck's measurement ($\varepsilon_E = \hbar\omega/2$) are therefore consistent by classical electrodynamics. The factor of 2 between them is not a mystery—it is E/B equipartition.

3.8 Why the Abraham–Lorentz Equation Uses the Electric Field Only

The Abraham–Lorentz equation:

$$m\ddot{x} = eE(t) + m\tau\,\dddot{x} \tag{34}$$

is driven by the electric field $E(t)$. The complete Lorentz force on the electron is $F = e(E + v \times B)$. The magnetic term $ev \times B$ is suppressed relative to the electric term eE by a factor v/c. In the non-relativistic regime $v \ll c$, the magnetic force is negligible and the equation reduces to the form above. Therefore the electron is driven by the electric component of the ZPF with spectral density:

$$S_E(\omega) = \frac{\hbar\omega^3}{6\pi^2\epsilon_0 c^3}. \tag{35}$$

3.9 Three Independent Confirmations of $\hbar$

The constant $\hbar$ is confirmed by three independent classical measurements, all giving the same value:

Measurement	Value	Physical reason
Casimir/Lamoreaux 1997	$\hbar\omega$ per mode	Total $E + B$; plates constrain full EM field
Planck 1911	$\hbar\omega/2$ per mode	Electric only; dipole couples to E not B
Einstein 1905	$E_{\text{photon}} = \hbar\omega$	Photon energy from photoelectric effect

The factor of 2 between Casimir and Planck is explained by E/B equipartition—not by any quantum effect. All three measurements give the same $\hbar$, confirming it as a universal constant of nature. The constant $\hbar$ stands alongside e, m, and c as a fundamental empirical constant: it

cannot be derived from classical electrodynamics (which has no natural unit of action), just as G cannot be derived from Newtonian mechanics.

3.10 Self-Consistency: The ZPF is the Electron's Own Coulomb Field

The central physical observation of the present paper is that the electromagnetic field generated by the oscillating electron must equal the zero-point field that drives it. This self-consistency condition,

$$S_E^{\text{generated}}(\omega) = S_E^{\text{ZPF}}(\omega), \tag{36}$$

is satisfied because: (1) the Coulomb electromagnetic quanta drive the electron into oscillation; (2) the oscillating electron radiates EM waves (Larmor formula); (3) the radiated field has the ω^3 spectral form determined by Einstein–Hopf + Lorentz invariance; (4) the amplitude is fixed by Boltzmann ergodic at $\hbar\omega/2$ per mode. The ZPF is not an independent entity—it is the self-consistent electromagnetic field of the electron's own Coulomb field distributed ergodically.

Element	Basis	Result
EM quanta have both E and B	Einstein 1905 + Maxwell	Energy density $= \epsilon_0 E^2$
Spectral shape $\propto \omega^3$	Einstein–Hopf balance 1910	$\rho(\omega) \propto \omega^3$
Lorentz invariance	Special relativity	Confirms ω^3 form
Total ZPF energy $B = \hbar$	Casimir force (Lamoreaux 1997)	$\varepsilon_{\text{total}}(\omega) = \hbar\omega$
E/B equipartition	Classical electrodynamics	$\varepsilon_E(\omega) = \hbar\omega/2$
$\hbar$ confirmed independently	Planck 1911, Einstein 1905	Three consistent measurements
$S_E(\omega)$ derived	All above combined	$S_E(\omega) = \hbar\omega^3/6\pi^2\epsilon_0 c^3$ ✓

The role of the Coulomb field—a clarification for the reader. The Coulomb field plays two distinct roles in this derivation. First, it determines the *spectral shape* of the ZPF: the ω^3 form follows from the Einstein–Hopf detailed balance condition and Lorentz invariance, both of which are consequences of classical electrodynamics applied to the Coulomb electromagnetic quanta. Second, it establishes the *self-consistency condition*: the ZPF is not an independent external field imposed from outside, but the electron's own Coulomb radiation acting back on itself through the Abraham–Lorentz feedback loop.

However, the *amplitude* of the ZPF—equivalently, the value of $\hbar$—is *not* determined by the Coulomb field and cannot be derived from classical electrodynamics alone. Classical electrodynamics has no natural unit of action. The scale $\hbar$ is irreducibly empirical: measured from the Casimir force by Lamoreaux (1997), and confirmed independently by Planck (1911) and Einstein (1905). This is not a gap in the argument but its intended structure, mirroring precisely how G enters Newtonian mechanics. Cavendish's measurement of G is a pre-relativistic empirical fact that general relativity does not derive—it assumes. Here, $\hbar$ is the single pre-Solvay empirical constant that classical physics does not derive—it measures.

To state it in one sentence: *the Coulomb field determines the form of the ZPF; the Casimir measurement determines its scale.*

4 The Compton Frequency Cutoff—Original Contribution

4.1 The Physical Argument

Consider an electron at rest with rest mass energy mc^2. It emits an electromagnetic quantum with energy $\hbar\omega$ into the ZPF. By conservation of energy, the residual electron energy is

$$E_{\text{residual}} = mc^2 - \hbar\omega. \tag{37}$$

The argument rests on exactly two premises.

Premise 1—Energy conservation: $mc^2 = E_{\text{residual}} + \hbar\omega$, a universal law never violated in any experiment.
Premise 2—No negative mass: $E_{\text{residual}} \geq 0$, never observed in any particle at any energy scale.

The conclusion follows immediately:

$$\hbar\omega \leq mc^2 \implies \omega \leq \frac{mc^2}{\hbar} \equiv \omega_c. \tag{38}$$

The Compton frequency $\omega_c = mc^2/\hbar$ is the natural maximum frequency of electromagnetic quantum emission from the electron, derived from energy conservation and the universal absence of negative mass.

4.2 Physical Interpretation

The Compton frequency is not externally imposed—it is determined by the electron's own rest mass. At this frequency $\hbar\omega_c = mc^2$: a photon's energy equals the electron's rest mass energy. This is the natural boundary of single-particle physics. Below it: single-electron SED valid. Above it: pair production possible, new particles created, single-electron description breaks down. This cutoff is a property of the particle response, not the field, so Lorentz invariance is preserved.

4.3 Self-Consistency Check

The condition $\tau\omega_c = 2\alpha/3 \approx 1/200 \ll 1$, where $\alpha \approx 1/137$ is the fine structure constant, confirms that radiation reaction is a small perturbation throughout the integration range. The non-relativistic treatment is self-consistent.

5 Explicit Derivation of $D = \hbar/2m$

5.1 The Abraham–Lorentz Equation

The electron in the Coulomb electromagnetic quantum field obeys the Abraham–Lorentz equation

$$m\ddot{x} = eE(t) + m\tau\,\dddot{x}, \tag{39}$$

where $\tau = e^2/6\pi\epsilon_0 mc^3 \approx 6.26 \times 10^{-24}$ s. Both terms are purely classical electrodynamics. In the frequency domain,

$$\tilde{v}(\omega) = \frac{ie\,\tilde{E}(\omega)}{m\omega(1 + i\tau\omega)}, \tag{40}$$

giving the velocity spectral density

$$S_v(\omega) = \frac{e^2\,S_E(\omega)}{m^2\omega^2(1 + \tau^2\omega^2)}. \tag{41}$$

5.2 Step 1—Mean Square Velocity

In the field-controlled regime and since $\tau\omega_c \ll 1$, the electron is driven by the electric component of the ZPF with spectral density $S_E(\omega) = \hbar\omega^3/6\pi^2\epsilon_0 c^3$:

$$\langle v^2 \rangle = \int_0^{\omega_c} S_v(\omega)\,d\omega \approx \frac{e^2}{m^2}\int_0^{\omega_c} \frac{S_E(\omega)}{\omega^2}\,d\omega. \tag{42}$$

Substituting $S_E(\omega) = \hbar\omega^3/6\pi^2\epsilon_0 c^3$ and evaluating $\int_0^{\omega_c} \omega\,d\omega = \omega_c^2/2$:

$$\langle v^2 \rangle = \frac{e^2\hbar}{6\pi^2\epsilon_0 c^3 m^2}\int_0^{\omega_c} \omega\,d\omega = \frac{e^2\hbar\omega_c^2}{12\pi^2\epsilon_0 c^3 m^2}. \tag{43}$$

5.3 Step 2—Charge e Cancels

Substituting $\tau = e^2/6\pi\epsilon_0 mc^3$ (so $e^2 = 6\pi\epsilon_0 mc^3\tau$):

$$\langle v^2 \rangle = \frac{6\pi\epsilon_0 mc^3\tau \cdot \hbar\omega_c^2}{12\pi^2\epsilon_0 c^3 m^2} = \frac{\hbar\omega_c^2\tau}{2\pi m}. \tag{44}$$

Noting that τ itself contains e^2, one sees that the charge e has cancelled completely. Substituting the explicit value of τ:

$$\langle v^2 \rangle = \frac{\hbar\omega_c^2}{2\pi m}. \tag{45}$$

The mean square velocity depends only on $\hbar$, ω_c, and m—not on the charge e.

5.4 Step 3—Effective Damping

The effective damping coefficient in the field-controlled regime, from mode-by-mode Abraham–Lorentz radiation reaction $\gamma(\omega) = \tau\omega^2$:

$$\gamma_{\text{eff}} = \frac{\omega_c^2}{\pi}. \tag{46}$$

This is also independent of e.

5.5 Step 4—Einstein Relation Gives $D = \hbar/2m$

Applying the Einstein relation $D = \langle v^2 \rangle / \gamma_{\text{eff}}$:

$$D = \frac{\hbar\omega_c^2/2\pi m}{\omega_c^2/\pi} = \frac{\hbar\omega_c^2\,\pi}{2\pi m\,\omega_c^2} = \frac{\hbar}{2m}. \tag{47}$$

The Compton cutoff ω_c also cancels—appearing as ω_c^2 in both numerator and denominator. The result $D = \hbar/2m$ is independent of both the charge e and the cutoff ω_c. It depends only on m and the empirical constant $\hbar$.

5.6 The Three Cancellations

Quantity	Appears in	Cancels because
Charge e	$S_E \propto e^2$ (driving) and $\tau \propto e^2$ (damping)	Same EM coupling in numerator and denominator
Cutoff ω_c	$\langle v^2 \rangle \propto \omega_c^2$ and $\gamma_{\text{eff}} \propto \omega_c^2$	Same cutoff in numerator and denominator
Factor π	$\langle v^2 \rangle$ has $1/2\pi$; γ_{eff} has $1/\pi$	Cancels in the ratio

What remains is purely $\hbar/2m$—a universal quantum of diffusion for all charged particles.

5.7 Relationship to De la Peña and Cetto

De la Peña and Cetto have made important contributions to the stochastic electrodynamic programme over several decades [1, 1, 1]. Their 1977 paper [1] derives the Schrödinger equation from the Abraham–Lorentz equation for a charged particle embedded in the ZPF, using a stochastic Liouville equation and a smoothing procedure. Their 2015 paper [1] claims that $\hbar$ is determined univocally by the physics of the problem and need not be introduced by hand.

These are genuine and significant contributions. However, a careful examination reveals that their derivation rests on three inputs that are assumed rather than derived:

1. **The ZPF energy per mode $\hbar\omega/2$.** De la Peña and Cetto take $\hbar\omega/2$ per mode as given—either assumed directly, or obtained under conditions that already presuppose $\hbar$. The physical origin of this energy is not derived from first principles in their framework.

2. **The Compton frequency cutoff ω_c.** Without a physical upper limit on the virtual photon frequency, the integral for $\langle v^2 \rangle$ diverges and $D = \hbar/2m$ cannot be obtained. De la Peña and Cetto do not provide a physical justification for this cutoff; it is either avoided or imposed by hand.

3. **The spectral shape $\rho(\omega) \propto \omega^3$.** Their derivation invokes Lorentz invariance of the vacuum field as the reason for the ω^3 spectrum. This argument is correct but incomplete: it establishes what form the field must have if it is Lorentz invariant, but does not explain why the electron's field has this form from first principles.

The Coulomb field is the missing physical input. All three gaps are closed by starting from the Coulomb field of the electron:

1. The Coulomb field drives the Einstein–Hopf detailed balance, which derives the spectral shape $\rho(\omega) \propto \omega^3$ from classical electrodynamics—not assumed from Lorentz invariance.

2. The Casimir force measurement gives the total ZPF energy per mode $B = \hbar$ as a pre-Solvay empirical fact. Classical E/B equipartition (Maxwell's equations) then gives $\varepsilon_E(\omega) = \hbar\omega/2$ for the electric component—the quantity that enters the Abraham–Lorentz equation. No quantum input is used.

3. The Compton cutoff $\omega_c = mc^2/\hbar$ follows from energy conservation and the universal absence of negative mass—two facts that require no quantum mechanics. This is the central original contribution of the present paper.

To state the distinction in one sentence: de la Peña and Cetto derive the Schrödinger equation from the ZPF, taking the ZPF energy as given; the present paper derives the ZPF energy itself from the Coulomb field, thereby completing the chain from classical physics to all five Solvay postulates without any quantum mechanical input.

The present paper therefore provides what their work identified as the door but could not open: a first-principles physical derivation of $\hbar\omega/2$ per mode, of the Compton cutoff, and of the self-consistency condition (Section 3.10), from the single classical object that every charged particle necessarily possesses—its own Coulomb field.

5.8 Relationship to Santos

Santos has made sustained and important contributions to the stochastic interpretation of quantum mechanics, reviewing and extending the SED programme over many decades [1, 1, 1]. His 2012

review [1] provides a detailed comparison between SED and quantum mechanics, showing that SED agrees with quantum theory when the stochastic equations of motion are first order in $\hbar$, and identifying where the two theories diverge. His 2022 paper [1] establishes a formal analogy between SED and nonrelativistic quantum electrodynamics via the Weyl–Wigner representation, directly relevant to the QED extensions of the present programme. His 2025 review [1] covers the hydrogen atom, Casimir effect, and entanglement within the stochastic vacuum field framework.

Like de la Peña and Cetto, Santos takes the ZPF spectral density $S_E(\omega) \propto \omega^3$ as the starting point of the SED programme. The present paper completes this programme by deriving $S_E(\omega)$ itself from the Coulomb field, closing the logical gap that Santos's reviews identify but do not fill.

6 The Five Postulates of Quantum Mechanics Explained

Postulate	Statement	Origin	Status
1—Superposition	State described by ψ	$\psi = \sqrt{\rho}\, e^{iS/\hbar}$ linearises stochastic eqs.	✓
2—Commutation	$[x, p] = i\hbar$, $\Delta x \Delta p = \hbar$	Fourier derivative theorem	✓
3—Born rule	$P = \lvert\psi\rvert^2$	$\psi = \sqrt{\rho}\, e^{iS/\hbar} \Rightarrow \lvert\psi\rvert^2 = \rho$	✓
4—Schrödinger	$i\hbar\partial_t\psi = \hat{H}\psi$	Newton + Itô correction + $D = \hbar/2m$	✓
5—Transition prob.	$P \propto \lvert\langle A\rvert\hat{V}\lvert B\rangle\rvert^2$	ZPF mediates via coupling operator $\hat{V}$	✓

6.1 Postulate 5: Transition Probability as a Markov Chain Transition Rate

6.1.1 The Correct Form of the Fifth Postulate

As crystallised at the fifth Solvay Conference through Dirac's perturbation theory (1927), the transition probability between states A and B is:

$$P(A \to B) \propto |\langle B|\hat{V}|A\rangle|^2, \tag{48}$$

where $\hat{V}$ is the interaction Hamiltonian coupling the electron to the radiation field. For electric dipole transitions $\hat{V} = e\mathbf{r} \cdot \mathbf{E}$; for electric quadrupole transitions $\hat{V} = eQ_{ij}E_{ij}$; and so on. The bare overlap $|\langle B|A\rangle|^2$ is the special case $\hat{V} = \mathbf{1}$—no mediating field.

6.1.2 The Markov Chain Transition Rate

In a continuous Markov chain, the transition rate from state A to state B is given by the **Markov transition kernel**:

$$W(A \to B) = \int \rho_A(x)\, K(x \to B)\, dx, \tag{49}$$

where $\rho_A(x)$ is the probability of the system being at point x, and $K(x \to B)$ is the kernel—the probability of transitioning to state B given the system is currently at x.

In the stochastic framework of this paper the identification is immediate. The Brownian probability densities are $\rho_A(x) = |\psi_A(x)|^2$ and $\rho_B(x) = |\psi_B(x)|^2$ by the Born rule (Section 2.5). The density $\rho_B(x)$ plays the role of the Markov transition kernel $K(x \to B)$—it measures how *available* state B is at each point x. The Markov transition rate therefore becomes:

$$W(A \to B) \propto \int \rho_A(x)\, \rho_B(x)\, dx = \int |\psi_A(x)|^2\, |\psi_B(x)|^2\, dx. \tag{50}$$

This has a completely transparent physical meaning: **the transition rate is the classical statistical overlap of two Brownian probability distributions**—the fraction of space they share in common. If the two distributions never occupy the same region, the integral is zero and the transition cannot occur. This is the physical origin of selection rules: a forbidden transition corresponds to two Brownian distributions with zero Markov overlap even after the ZPF coupling acts.

6.1.3 A Concrete Example

To make this explicit, consider two Gaussian Brownian states:

$$\rho_A(x) = a\, e^{-x^2}, \qquad \rho_B(x) = b\, e^{-(x-1)^2}. \tag{51}$$

The Markov transition rate is:

$$W(A \to B) \propto ab \int e^{-x^2}\, e^{-(x-1)^2}\, dx = ab\, e^{-1/2} \sqrt{\frac{\pi}{2}}. \tag{52}$$

As the two Gaussians move apart the overlap decreases exponentially and the transition rate drops to zero—no overlap means no shared location means no transition, not because of a mathematical rule but because there is nowhere in space for the transition to physically happen.

6.1.4 Why Energy Eigenstates Are Orthogonal

In Nelson's stochastic picture, energy eigenstates ψ_A and ψ_B are *stationary* Brownian motion states. Their orthogonality,

$$\langle A|B\rangle = \int \psi_A^*(x)\, \psi_B(x)\, dx = 0, \tag{53}$$

means the two Brownian equilibrium distributions are *independent*: the ZPF fluctuations sustaining each state are separately in detailed balance. The Markov transition rate $\int \rho_A \rho_B\, dx$ is zero for orthogonal eigenstates. Orthogonality means: *no self-driven transition*. The ZPF must supply the bridge.

6.1.5 From Markov Overlap to Quantum Matrix Element

The connection to the standard quantum expression $|\langle B|\hat{V}|A\rangle|^2$ follows in two steps.

Step 1—ZPF deforms state A. The ZPF contains a mode at every frequency, including the transition frequency $\omega_{AB} = (E_A - E_B)/\hbar$. When it acts on the electron in state A via the coupling $\hat{V} = e\mathbf{r} \cdot \mathbf{E}(\omega_{AB})$, it deforms ρ_A into $|\hat{V}\psi_A|^2$ before the overlap with ρ_B is computed. Just as the Markov kernel $K(x \to B)$ can incorporate an external forcing, the ZPF mode modifies state A first. The Markov transition rate with ZPF coupling becomes:

$$W(A \to B) \propto \int |\hat{V}\psi_A(x)|^2 \, |\psi_B(x)|^2 \, dx. \tag{54}$$

This is nonzero even when $\langle B|A\rangle = 0$, because $\hat{V}$ reshapes the spatial structure of ψ_A before the overlap is taken.

Step 2—ZPF enforces phase coherence. Both states A and B are driven by the *same* zero-point field. The ZPF therefore enforces phase coherence between ψ_A and ψ_B. When phases are coherent, the Cauchy–Schwarz inequality becomes an equality and the classical Markov overlap upgrades to the full quantum expression:

$$W(A \to B) \propto \left| \int \psi_B^*(x) \, \hat{V} \, \psi_A(x) \, dx \right|^2 = |\langle B|\hat{V}|A\rangle|^2. \tag{55}$$

The squared matrix element is therefore not a separate postulate—it is the **Markov chain transition kernel of two Brownian motion states, made coherent by the ZPF**. The classical overlap $\int \rho_A \rho_B \, dx$ and the quantum expression $|\langle B|\hat{V}|A\rangle|^2$ are the same object: the first is what you get without phase coherence, the second is what the ZPF produces when it locks the phases.

6.1.6 Summary of the Logical Chain for Postulate 5

Step	Statement	Origin
1	Transition rate = Markov transition kernel	Classical probability theory
2	$\rho_A = \|\psi_A\|^2$, $\rho_B = \|\psi_B\|^2$	Born rule, Section 2.5
3	ZPF deforms ρ_A via coupling $\hat{V}$	ZPF, Sections 3–4
4	ZPF enforces phase coherence	Same ZPF drives both states
5	Coherent Markov overlap = $\|\langle B|\hat{V}|A\rangle\|^2$	Cauchy–Schwarz equality

No new postulate is introduced at any step. The fifth Solvay postulate is the Markov chain transition rate of two ZPF-driven Brownian motion states. The ZPF is not merely responsible for the Brownian motion that produces the Schrödinger equation—it also mediates every quantum transition through the same zero-point field.

6.1.7 Connection to Tomonaga's Treatment

Tomonaga [2] derives the transition probability (his Eq. 60.20) directly from the time-dependent Schrödinger equation with perturbation V, and explicitly concludes that *"the introduction of the transition probability in the theory does not require any additional hypothesis other than those already made."* He treats the generalised probability formula

$$P_{\psi'} = \left| \int \psi \, \psi'^{*} \, dq \right|^2 \tag{56}$$

(his Eq. 59.46) as a hypothesis, and notes that the atom cannot exist in isolation from the radiation field—the full Hamiltonian must include $H_{\rm atom} + H_{\rm rad} + V$, and without V no transition occurs.

The present paper provides the physical foundation for both of these observations. First, the generalised probability formula (59.46) is not a hypothesis—it is the Markov chain transition rate of two Brownian motion states, with $\rho_B(x)$ playing the role of the Markov transition kernel, as derived above. Second, the perturbation V is not an externally applied field—it is the ZPF mode at the transition frequency ω_{AB}, derived from the Coulomb field through Einstein–Hopf detailed balance (Sections 3–4). The radiation field $H_{\rm rad}$ that Tomonaga includes in the Hamiltonian is the same ZPF that drives the Brownian motion throughout this paper.

Tomonaga's conclusion—that transition probability requires no new hypothesis—is confirmed. The present paper gives the reason: the ZPF, already present as the physical origin of Brownian motion, also mediates every quantum transition through the same zero-point field.

In one sentence: *the transition probability is the Markov chain transition rate of two Brownian probability distributions, made coherent by the ZPF that drives both states.*

6.1.8 Derivation of Tomonaga's Equations (23.5) and (23.31) from the Markov Framework

Tomonaga [2] derives two key results from classical electrodynamics and the correspondence principle. The present framework derives both directly, without invoking the correspondence principle as a bridge.

Tomonaga's Equation (23.5): Radiated Intensity from a Dipole. Tomonaga's equation (23.5) gives the intensity of light emitted in the τ-th Fourier component of a classically oscillating dipole:

$$\frac{dE_\tau}{dt} = \frac{(2\pi\nu_\tau)^4}{3c^2}|C_\tau|^2. \tag{T23.5}$$

In the present framework this follows directly from the Abraham–Lorentz equation already derived in Section 5.1. The Larmor radiation formula for an accelerating electron is:

$$\frac{dE}{dt} = \frac{e^2}{6\pi\epsilon_0 c^3}\langle \ddot{x}^2 \rangle. \tag{57}$$

The electron in Brownian state ψ_A has a trajectory whose τ-th Fourier component at frequency ν_τ contributes a dipole moment $P_\tau = eC_\tau e^{2\pi i\nu_\tau t}$. Substituting into the Larmor formula gives exactly equation (T23.5). The key difference from Tomonaga's derivation is the physical interpretation of C_τ: in the classical picture it is the amplitude of an orbital Fourier component; in the present framework it is the Fourier component of the electron's *Brownian trajectory* weighted by the ZPF spectral density at $\omega_\tau = 2\pi\nu_\tau$. The mathematical result is identical; the physical origin is the Abraham–Lorentz radiation of the ZPF-driven Brownian particle, not an assumed orbital motion.

Tomonaga's Equation (23.31): Transition Probability of the Harmonic Oscillator. Tomonaga's equation (23.31) gives the transition probability for the harmonic oscillator from state n to state $n-1$:

$$A_{n\to n-1} = \frac{8\pi^2 e^2 \nu^2}{3mc^3}\, n. \tag{T23.31}$$

Tomonaga derives this by combining equation (T23.5) with the correspondence principle and the quantum condition $a^2 = nh/\pi(2\pi\nu)m$. The present framework derives the same result directly from the Markov transition kernel, without any appeal to the correspondence principle.

Step 1—Markov transition rate. For the harmonic oscillator, the Brownian probability densities are $\rho_n(x) = |\psi_n(x)|^2$ where ψ_n is the n-th eigenfunction. The ZPF coupling operator is $\hat{V} = e\hat{x}$ (electric dipole). The Markov transition rate from state n to state $n-1$ is:

$$W(n \to n-1) \propto |\langle n-1|e\hat{x}|n\rangle|^2. \tag{58}$$

Step 2—Harmonic oscillator matrix element. For the quantum harmonic oscillator, the matrix element is exactly:

$$|\langle n-1|e\hat{x}|n\rangle|^2 = \frac{e^2\hbar}{2m\omega}\, n = \frac{e^2 h}{4\pi^2 \cdot 2m\nu}\, n. \tag{59}$$

This follows from the standard raising/lowering operator algebra of Nelson's Brownian motion, where $\hat{x} = \sqrt{\hbar/2m\omega}(\hat{a} + \hat{a}^\dagger)$ and $\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$.

Step 3—Transition probability. The transition probability per unit time is the Markov transition rate multiplied by the Larmor radiation prefactor $(2\pi\nu)^4/3c^2$ and divided by the photon energy $h\nu$:

$$A_{n\to n-1} = \frac{(2\pi\nu)^4}{3c^2 h} \cdot |\langle n-1|e\hat{x}|n\rangle|^2 = \frac{(2\pi\nu)^4}{3c^2 h} \cdot \frac{e^2 h}{4\pi^2 \cdot 2m\nu} \cdot n = \frac{8\pi^2 e^2 \nu^2}{3mc^3}\, n. \tag{60}$$

This is exactly Tomonaga's equation (T23.31). □

The selection rule as zero Markov overlap. For the harmonic oscillator, all transitions except $n \to n \pm 1$ are forbidden. In Tomonaga's framework (p. 150), this is because the Fourier series of the dipole moment contains only terms with $\tau = \pm 1$, making $C_\tau = 0$ for $|\tau| \neq 1$.

In the present framework the same result emerges from the Markov transition kernel: $|\langle n - \tau|e\hat{x}|n\rangle|^2 = 0$ for $|\tau| \neq 1$ because the harmonic oscillator eigenfunctions are orthogonal with

respect to the $\hat{x}$ operator for non-adjacent states. The Markov overlap is exactly zero — there is no region of space where the ZPF coupling $e\hat{x}$ can bridge states differing by more than one quantum. The selection rule is not imposed externally; it emerges from the geometry of the Brownian probability distributions.

What the present framework adds to Tomonaga. Tomonaga writes on page 148 of Volume I [2]:

> *"What mechanism then makes the transition occur?... We have, for the time being, no answer at all to these questions."*

The present paper answers this question. The mechanism is the ZPF mode at the transition frequency $\omega_{AB} = (E_A - E_B)/\hbar$ acting as the Markov transition kernel between two Brownian equilibrium states. The correspondence principle — Bohr's inspired guess that classical and quantum intensities must agree for large n — is confirmed as a *consequence* of the Markov framework, not as an independent principle. The agreement holds because both the classical Larmor formula and the quantum Markov kernel derive from the same Abraham–Lorentz equation driven by the same ZPF.

6.2 The Single Unifying Principle

The electron carries electric charge. This charge generates a Coulomb field whose energy is carried in electromagnetic quanta with both E and B fields (Einstein 1905 + Maxwell's equations). The Einstein–Hopf detailed balance forces the spectral shape $\rho(\omega) \propto \omega^3$. The Casimir force measurement gives the total ZPF energy per mode $B = \hbar$; classical E/B equipartition gives the electric energy per mode $\hbar\omega/2$, confirmed by Planck (1911) and Einstein (1905). Energy conservation and the impossibility of negative mass give the Compton cutoff $\omega_c = mc^2/\hbar$. Through the Abraham–Lorentz equation, charge e and cutoff ω_c both cancel, giving $D = \hbar/2m$. By Itô's stochastic calculus and Nelson's theorem, this gives the Schrödinger equation. The transformation $\psi = \sqrt{\rho}\, e^{iS/\hbar}$ linearises the nonlinear stochastic dynamics, explaining superposition. All five postulates follow from the single fact that the electron carries electric charge.

7 Discussion

7.1 No Circular Reasoning

A potential circularity—using the zero-point field (normally derived from quantum field theory) to derive quantum mechanics—is completely avoided. The key is the definition of quantum mechanics adopted in Section 1.1: quantum mechanics means the Solvay (1927) postulates. All inputs to our derivation predate and are independent of these postulates:

1. Einstein's photoelectric theory (1905)—photons carry energy $E = \hbar\omega$ and have both E and B fields. This predates the Solvay framework by 22 years.

2. Einstein–Hopf detailed balance (1910)—classical statistical mechanics gives $\rho(\omega) \propto \omega^3$.

3. Lorentz invariance—special relativity.

4. Casimir force (1948, measured 1997)—gives $B = \hbar$ from classical electrodynamics and boundary conditions alone.

5. E/B equipartition—Maxwell's equations give electric energy per mode $= \hbar\omega/2$, confirmed by Planck (1911).

6. Energy conservation and no negative mass—universal.

None of these inputs are Solvay postulates. All five Solvay postulates are outputs. No circular reasoning is present.

7.2 Einstein's Objection Resolved

Einstein famously objected at Solvay: "God does not play dice." The present framework vindicates his intuition: randomness is not fundamental—it has a physical cause. The dice are thrown by the electron's own Coulomb field acting back on itself through the zero-point radiation field.

There is a profound historical irony here. Einstein's 1905 photoelectric discovery gives us $E = \hbar\omega$—the single empirical input to our derivation. Yet Einstein spent the rest of his life arguing against the very Solvay framework we derive here. He was right to object: the Solvay postulates are not fundamental. And he unknowingly provided, in 1905, the key experimental fact needed to show why.

7.3 The Neutral Particle Prediction

The entire chain depends on electric charge. A truly neutral, structureless particle would not couple to the electromagnetic vacuum, would not undergo electromagnetic Brownian motion, and would not be described by the Schrödinger equation in the same way. A hypothetical sterile neutrino—no charge, no magnetic moment—is the cleanest test case for this prediction.

7.4 Comparison with Existing Interpretations

Interpretation	Randomness	Vacuum	QM status
Copenhagen	Fundamental postulate	Not physical	Postulated
Bohmian mechanics	Initial conditions	Abstract pilot wave	Postulated
Many Worlds	Fundamental branching	Not physical	Postulated
This paper	Coulomb field + Planck ZPF	Physical—derived	Derived from classical physics

7.5 Limitations and Future Work

The present framework addresses non-relativistic single-particle quantum mechanics. Extensions needed: (1) multi-particle systems and entanglement; (2) spin—not addressed here; (3) relativistic extension—connection to the Dirac equation; (4) rigorous mathematical proof of the transition probability postulate via stochastic path integrals beyond first-order perturbation theory; (5) the precise role of de la Peña's Principle 2 in the cancellation of α.

8 Conclusion

All five postulates of the 1927 Solvay framework have been derived from a single fundamental postulate — that the energy of the electromagnetic field, including the static Coulomb field, is carried in discrete quanta of energy $h\nu$ — combined with the single measured constant $\hbar$ and universal physical laws.

The logical structure is precisely that of Newton deriving Kepler's laws: one fundamental postulate, one measured constant, the laws of classical physics, and the entire structure of the theory follows. Kepler's laws were not fundamental — they were consequences of universal gravitation. The Solvay postulates are not fundamental — they are consequences of electromagnetic quantization.

The derivation chain reads:

> Coulomb field $\to$ EM quanta with E, B (fundamental postulate + Maxwell) $\to$ Einstein–Hopf detailed balance $\to \rho(\omega) \propto \omega^3 \to$ Casimir force ($B = \hbar$) $\to E/B$ equipartition $\to \varepsilon_E(\omega) = \hbar\omega/2 \to$ no negative mass $\to \omega_c = mc^2/\hbar \to$ Abraham–Lorentz $\to e$ cancels $\to \langle v^2 \rangle = \hbar\omega_c^2/2\pi m \to \omega_c$ cancels $\to D = \hbar/2m \to$ Itô + Nelson $\to$ Schrödinger equation.

Three beautiful cancellations make $D = \hbar/2m$ universal: charge e cancels between driving force and radiation reaction, cutoff ω_c cancels between $\langle v^2 \rangle$ and γ_{eff}, and π cancels in the ratio. What remains depends only on m and the empirical constant $\hbar$.

The transformation $\psi = \sqrt{\rho}\, e^{iS/\hbar}$ linearises the nonlinear Fokker–Planck dynamics through exact cancellation of nonlinear terms via Itô's correction. This explains why quantum mechanics obeys superposition.

The five postulates of quantum mechanics are the Kepler's laws of the quantum world—correct, precise, and powerful, but awaiting the deeper principle. That principle is: the energy of the electron's Coulomb field is carried in electromagnetic quanta. Distributed according to the Boltzmann ergodic theorem and acting back on the electron through the zero-point radiation field, bounded by the impossibility of negative mass, this self-interaction produces quantum Brownian motion with $D = \hbar/2m$.

Quantum mechanics is electromagnetic quantization plus Boltzmann statistics plus the impossibility of negative mass.

As a student at Tokyo Institute of Technology, the present author learned to calculate with the Schrödinger equation but never understood its physical foundation. Reading Nelson's paper in 1972 made clear that the equation could be derived from Brownian motion—but left open why $D = \hbar/2m$. This paper is the answer to that question, fifty years in the making.

The universe is written in the language of mathematics. Behind the mathematics is physics. Behind the physics is the Coulomb field—the most classical and elementary fact about the electron. And behind the Coulomb field is a single truth: its energy comes in quanta.

Acknowledgements

The author thanks Manuel Blum and Lenore Blum (Carnegie Mellon University) for their generous encouragement and for connecting this work with the physics community.

The author thanks Fred Gilman (Carnegie Mellon University) for a penetrating observation that directly improved this paper. Professor Gilman asked whether introducing $E = \hbar\omega$ from Einstein's 1905 photoelectric effect constitutes using quantum mechanics. This question led to the precise historical definition of quantum mechanics adopted in Section 1.1: quantum mechanics is the theoretical framework crystallised at the fifth Solvay Conference of 1927, not the pre-Solvay experimental discoveries that preceded it. This distinction—between the Solvay postulates as outputs and the pre-Solvay empirical constant $\hbar$ as a single measured input—is now explicit throughout the paper. The author is grateful for this clarification.

The author also thanks the long tradition of physicists who refused to accept the Solvay postulates without deeper explanation: Einstein, de Broglie, Bohm, Nelson, Boyer, de la Peña, Cetto, and Santos. Special thanks to Professor Emilio Santos (University of Cantabria) for his decades of work developing the stochastic interpretation of quantum mechanics and for his reviews of SED that helped clarify the landscape into which the present work fits. Special thanks also to Kiyosi Itô, whose stochastic calculus proved to be the mathematical key, and to Einstein and Hopf, whose 1910 detailed balance argument is the classical foundation of the spectral derivation.

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