The Collapse of Recursive Logic: A Critique of Actual Infinity via the Fixed-Point Singularity of Sequence S

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May 11, 2026

Abstract

This paper presents a logical critique of the concept of Actual Infinity by analyzing a specific recursive sequence S. By examining the algebraic properties of the limit as a potential member of the set, we demonstrate that the transition from a generative process to a "completed" state results in a fixed-point singularity. This singularity destroys the bidirectional logic of succession and traceability, suggesting that the notion of a completed infinite set is structurally incompatible with the recursive definitions that govern countable sets.

1 Introduction

In classical set theory, a countable infinite set is often treated as an "actually" completed totality. This paper challenges that assumption by examining the internal logical consistency of a sequence S generated by a specific recursive rule. We argue that the inclusion of a terminal "limit" point violates the fundamental principles of succession and traceability to the origin.

2 The Generative System of Sequence S

Consider a set S defined by a discrete recursive process. The system is governed by a starting seed and a unidirectional successor operation:

- **Initial Seed (b):** 0
- **Successor Operation $f(r)$:** $f(r) = \frac{r}{10} + \frac{1}{10}$
- **Predecessor Operation $f^{-1}(r)$:** $f^{-1}(r) = (r - 0.1) \times 10$

Under the framework of Potential Infinity, the set $S = \{0, 0.1, 0.11, 0.111, \dots\}$ is an open-ended collection where every element r_n is obtained via a finite number of iterations of f.

3 The Fixed-Point Singularity (The "Black Hole")

The doctrine of Actual Infinity posits that the sequence can reach a completed state, effectively attaining its limit:

$$\omega = \lim_{n\to\infty} r_n = \frac{1}{9}$$

If we assume $\omega \in S$, we encounter a stasis in the generative logic. By substituting ω into the successor function:

$$f\left(\frac{1}{9}\right) = \frac{1/9}{10} + \frac{1}{10} = \frac{1}{9}$$

At this terminal point, the successor function collapses into an **identity map**. The dynamic movement that defines the arrangement of the set ceases to exist.

4 The Failure of Traceability and Irreversibility

A rigorous definition of a countably arranged set requires that every element (except the origin) must possess a unique predecessor that leads back to the seed 0. However, if we apply the predecessor operation to the attained limit:

$$f^{-1}\left(\frac{1}{9}\right) = \left(\frac{1}{9} - \frac{1}{10}\right) \times 10 = \frac{1}{9}$$

The point 1/9 functions as a logical **"Black Hole"**. It is an element that possesses no distinct predecessor other than itself. Consequently, it is impossible to trace 1/9 back to the initial seed 0 through the inverse of the rules that supposedly generated the set.

To facilitate a more intuitive understanding of the aforementioned concept, one may imagine, from the perspective of *Actual Infinity*, that the sequence of natural numbers has been completely arranged. In this completed state, the terminal value would be infinity (∞). However, the operation ($\infty - 1$) remains ∞. Consequently, much like the fixed-point singularity in sequence S, this state also precludes any logical return to the initial seed 0.

5 Conclusion

The analysis of sequence S reveals that "attaining" infinity is not merely a quantitative extension but a qualitative logical rupture. The terminal point 1/9 is logically isolated from the origin 0. Because 1/9 cannot be reduced to the seed 0 via the predecessor operation, it cannot be a member of a sequence arranged *from* 0. We conclude that a set cannot be simultaneously "countably arranged" and "actually completed," as the latter state destroys the recursive logic required by the former. So we should give up the opinion of Actual Infinity.

Acknowledgements

The author would like to acknowledge the fruitful discussions with Gemini, which helped in shaping the logical framework and formalizing the arguments presented in this article.