CLASSP: The Continuum Limit of Apollonian–Soddy Sphere Packing Dynamics as the Origin of Weak-Field Gravity and Dirac Quantum Mechanics

Steven E. Elliott

May 2026

Abstract

The companion TFOFT paper proposes that the universe is a finite computational object defined by a dynamical Apollonian-Soddy sphere-packing fractal quine under Presburger arithmetic. This paper develops the first continuum-limit equations of that framework. The sphere radius is treated as a local clock field; its coarse-grained graph dynamics yields Newtonian gravity and the weak-field time component of general relativity. On the same tangency graph, a one-dimensional chiral transfer channel with a two-state boundary-spin register yields the 1+1 dimensional Dirac equation, with mass interpreted as the chirality-reversal rate per local radius-clock tick. The extension to full curved 3+1 dimensional spinor dynamics is stated as a frame-rotation ansatz, not a theorem. Electromagnetism is sketched as lower-layer callback momentum bookkeeping whose graph form is a candidate edge-phase holonomy limit. The paper also derives a testable Milky Way prediction: if the Fermi bubbles are the polar lobes of a scaled 3dz2 boundary state, then a weaker equatorial "Fermi donut" should appear at roughly 10 to 15 kpc, with an inner shoulder near 8 to 12 kpc and one-quarter the angular density contrast of the polar lobes. This places the key observational test in the Gaia DR3 outer-disk regime.

Contents

1 Motivation

The original TFOFT paper [1] proposes MATHICCS: every mathematical operation in a physical law must correspond to a persistently realizable physical procedure. Completed infinities, exact real continua, and singular points are therefore not ontological primitives. They may remain useful approximations, but only as coarse-grained descriptions of finite physical computation.

The candidate finite ontology is the dynamical Apollonian–Soddy Sphere Packing [3, 4, 5]. Primitive objects are not fields on a continuum, but spheres, curvatures, tangencies, and local update rules. A single sphere stores a state; two tangent spheres form the minimal law-generating unit, the Gyro Ball. Across fractal scales, the same object is conjectured to appear as electron/nucleus at atomic scale and star/core at galactic scale. Flat galactic rotation curves [12, 13] and atomic orbital quantization are then interpreted as opposite-side projections of one fractal boundary dynamics.

This paper asks a narrow technical question:

> If the ASSP ontology of [1] is correct, what continuum field equations appear when a bounded-resolution observer coarse-grains the finite tangency dynamics?

The answer developed here is that the two dominant continuum theories emerge from complementary limits of the same graph:

$$\begin{aligned} \text{radius-clock scalar } u = \ln(R/R_0) &\longrightarrow \text{Newtonian gravity and weak-field } g_{00}, \\ \text{directed tangency walk with Gyro mixing} &\longrightarrow \text{Dirac quantum matter}, \\ \text{lower-layer callback momentum and U(1) holonomies} &\longrightarrow \text{candidate Maxwell/gauge field limit.} \end{aligned}$$

The target is not to quantize general relativity. Under TFOFT, that target is category-mistaken: GR and QM are already effective views of the same finite fractal update, observed from different sides of the boundary.

2 The ASSP Tangency Graph

2.1 Graph structure

Let the sphere packing define a graph $G = (V, E)$, where each vertex $i \in V$ is a sphere with center $x_i \in \mathbb{R}^3$, radius $R_i > 0$, and curvature $b_i = 1/R_i$. An edge is a tangency:

$$(i, j) \in E \iff |x_i - x_j| = R_i + R_j. \tag{1}$$

The edge vector and length are

$$s_{ij} = x_j - x_i, \qquad \ell_{ij} = |s_{ij}| = R_i + R_j. \tag{2}$$

The packing is recursively constrained by the oriented three-dimensional Descartes–Soddy curvature relation [3, 5]. For five mutually tangent oriented spheres with signed curvatures $k_i = \pm R_i^{-1}$, where an enclosing sphere carries negative curvature,

$$\left(\sum_{i=1}^{5} k_i\right)^2 = 3\sum_{i=1}^{5} k_i^2. \tag{3}$$

Solving for a fifth curvature given $k_1, \ldots, k_4$, define

$$S = \sum_{i=1}^{4} k_i, \qquad Q = \sum_{i=1}^{4} k_i^2. \tag{4}$$

Then

$$k_5 = \frac{S \pm \sqrt{3S^2 - 6Q}}{2}. \tag{5}$$

Equation (5) gives the two possible fifth spheres completing one Descartes configuration. Recursive Apollonian generation is more naturally written as a *reflection* rule on an existing five-sphere Descartes configuration. Replacing sphere i while holding the other four fixed gives

$$k_i' = \sum_{j \neq i} k_j - k_i. \tag{6}$$

The same reflection applies to curvature-weighted centers. Writing $m_i^a = k_i x_i^a$ for $a = 1, 2, 3$,

$$(m_i^a)' = \sum_{j \neq i} m_j^a - m_i^a, \qquad (x_i^a)' = \frac{(m_i^a)'}{k_i'}. \tag{7}$$

Equations (6)–(7) are the microscopic update rules used by the oriented 3D ASSP generator. For a tetrahedral seed of four unit spheres, the virtual enclosing sphere has negative curvature

$$k_{\text{out}} = \frac{4 - \sqrt{24}}{2} < 0, \tag{8}$$

and the first reflected interior sphere has radius

$$R_{\text{in}} = \frac{1}{2 + \sqrt{6}} \approx 0.224745. \tag{9}$$

The explicit curvature law fixes the microscopic recursive generation of the packing. The continuum-limit arguments below do not require this recursion directly; they require the induced tangency graph, finite local second moments, and statistical balance/isotropy after coarse-graining.

2.2 Radius as local time

The defining physical identification is

$$u_i = \ln \frac{R_i}{R_0}, \tag{10}$$

where R_0 is the reference radius. In the Presburger-minimal TFOFT basis one takes $R_0 = \pi$ [1], but the continuum-limit derivations below do not depend on that choice. The local clock rate is

$$\mathrm{d}\tau_i = e^{u_i}\mathrm{d}t = \frac{R_i}{R_0}\mathrm{d}t. \tag{11}$$

A bounded-resolution observer describes this through the emergent metric

$$\mathrm{d}s^2 = -c^2 e^{2u(x)}\mathrm{d}t^2 + \mathrm{d}\ell_G^2, \tag{12}$$

where $\mathrm{d}\ell_G^2$ is the spatial metric induced by the packing. In the GR weak field, $\mathrm{d}\tau \approx (1+\Phi/c^2)\mathrm{d}t$; comparing with $\mathrm{d}\tau = e^u\mathrm{d}t \approx (1+u)\mathrm{d}t$ gives

$$\Phi = c^2 u = c^2 \ln \frac{R}{R_0}. \tag{13}$$

Gravity is the coarse-grained logarithmic gradient of the ASSP radius-clock field.

The metric form

$$ds^2 = -c^2 d\tau^2 + d\ell_G^2$$

is consistent with the TFOFT definition of time as a Gyro-Ball clock register. Kepler closure supplies a quadratic period relation,

$$T_{\text{orb}}^2 = \frac{4\pi^2}{GM} a^3,$$

so the continuum time coordinate naturally appears through a squared proper-time element. In CLASSP this becomes

$$d\tau = e^u dt, \qquad u = \ln(R/R_0),$$

and hence

$$ds^2 = -c^2 e^{2u} dt^2 + d\ell_G^2.$$

Thus the Lorentzian time term is the coarse-grained form of a squared Gyro-Ball orbital clock, rather than an independently postulated continuum time dimension.

2.3 Normalized graph Laplacian

For a field $f_i = f(x_i)$ slowly varying across the graph, define the weighted graph Laplacian

$$(\Delta_G f)_i = A_i \sum_{j\sim i} w_{ij}(f_j - f_i), \tag{14}$$

with symmetric tangency conductances $w_{ij} = w_{ji} > 0$. A natural radius-symmetric choice is

$$w_{ij} = \frac{4\pi}{R_i + R_j} \left(\frac{R_i R_j}{R_i + R_j} \right)^2, \tag{15}$$

though the continuum proof below requires only local symmetry and finite second moments. Define

$$M_i^{ab} = \sum_{j\sim i} w_{ij} s_{ij}^a s_{ij}^b, \qquad M_i = \sum_{j\sim i} w_{ij} |s_{ij}|^2. \tag{16}$$

We assume local coarse-grained balance and isotropy:

$$\sum_{j\sim i} w_{ij} s_{ij}^a = 0, \tag{17}$$

$$M_i^{ab} = \frac{M_i}{3} \delta^{ab}. \tag{18}$$

Condition (17) holds in idealized ASSP interiors by packing symmetry; condition (18) holds statistically under local coarse-graining over an ASSP cluster [6, 4]. Choose $A_i = 6/M_i$, giving

$$(\Delta_G f)_i = \frac{6}{\sum_{j\sim i} w_{ij} |s_{ij}|^2} \sum_{j\sim i} w_{ij}(f_j - f_i). \tag{19}$$

3 Weak-Field Gravity as the Radius-Clock Continuum Limit

3.1 Coarse-graining theorem

Theorem 1 (Laplacian limit)**.** *Let $f_i = f(x_i)$ with f smooth on scales large compared to the local sphere spacing $h_i \sim R_i$. Under conditions (17)–(18),*

$$(\Delta_G f)_i = \nabla^2 f(x_i) + O(h_i |\nabla^3 f|). \tag{20}$$

Proof. Taylor-expand $f_j = f(x_i + s_{ij})$:

$$f_j - f_i = s_{ij}^a \partial_a f + \frac{1}{2} s_{ij}^a s_{ij}^b \partial_a \partial_b f + O(|s_{ij}|^3 \nabla^3 f). \tag{21}$$

Substitute into (14). The first-order term vanishes by (17). The leading contribution is

$$\begin{aligned}(\Delta_G f)_i &= \frac{A_i}{2} \sum_{j\sim i} w_{ij} s_{ij}^a s_{ij}^b \partial_a \partial_b f + O(h_i |\nabla^3 f|) \\ &= \frac{A_i}{2} \cdot \frac{M_i}{3} \delta^{ab} \partial_a \partial_b f + O(h_i |\nabla^3 f|).\end{aligned}$$

Using $A_i = 6/M_i$ gives $(\Delta_G f)_i = \nabla^2 f + O(h_i |\nabla^3 f|)$. □

3.2 Poisson equation and Newtonian gravity

Postulate the ASSP radius-clock source equation

$$\Delta_G u_i = \frac{4\pi G}{c^2} \rho_i. \tag{22}$$

By Theorem 1, the continuum limit is $\nabla^2 u = (4\pi G/c^2)\rho$. Multiplying by c^2 and using $\Phi = c^2 u$ from (13):

$$\boxed{\nabla^2 \Phi = 4\pi G \rho.} \tag{23}$$

The slow-particle geodesic acceleration follows from metric (12):

$$\vec{a} = -\frac{c^2}{2} \nabla \ln\left(-\frac{g_{00}}{c^2}\right) = -c^2 \nabla u = -\nabla \Phi. \tag{24}$$

For a spherical mass M, $\Phi(r) = -GM/r$, giving

$$R(r) = R_0 \exp\left(-\frac{GM}{c^2 r}\right) \approx R_0 \left(1 - \frac{GM}{c^2 r}\right), \tag{25}$$

so local radius, local clock rate, and local coordinate light speed decrease in a potential well.

3.3 Weak-field GR slot

From $\Phi = c^2 u$ and metric (12):

$$g_{00} = -c^2 e^{2\Phi/c^2} \approx -c^2 \left(1 + \frac{2\Phi}{c^2}\right), \tag{26}$$

which is the weak-field GR time component. The ASSP radius-clock scalar u is the pre-continuum object whose smooth limit is the gravitational potential and whose weak-field metric form is the g_{00} sector of GR. The radius-clock argument alone does not derive the full Einstein tensor; the tensor sector is deferred to a later microscopic derivation based on spatial packing dynamics, frame transport, and two-ball boundary exchange.

3.4 Mass, inertia, and the deferred tensor sector

The radius-clock calculation above recovers the Newtonian potential and the weak-field time component of GR, but it does *not* derive the full Einstein tensor. Earlier heuristic visual-relocation language is deliberately avoided here because it introduces ambiguous displacement variables and stress-energy normalizations before the microscopic tensor-sector dynamics has been fixed.

The CLASSP interpretation also does not assume the Einstein Equivalence Principle as a primitive. Instead:

$$\text{mass} \equiv \text{a persistent angular-momentum register,} \tag{27}$$

$$\text{inertia} \equiv \text{the linear-momentum cost required to translate that angular-momentum register.} \tag{28}$$

Thus gravitational response is not postulated as universal free fall in a smooth spacetime. It is the continuum appearance of the translation cost of moving angular-momentum systems through a nonuniform radius-clock substrate. The scalar sector is captured by

$$\Phi = c^2 \ln(R/R_0), \qquad \vec{a} = -\nabla\Phi, \tag{29}$$

while the full tensor sector must come from the coarse-grained dynamics of spatial packing geometry, frame transport, and boundary exchange.

The future tensor-sector task is therefore to derive an effective equation of the schematic form

$$G^{\text{eff}}_{\mu\nu} = \frac{8\pi G}{c^4} T^{\text{eff}}_{\mu\nu} \tag{30}$$

from explicit ASSP update rules for curvature-weighted centers, tangency-frame transport, and two-ball boundary exchange. This paper does not claim that result. It claims only the controlled scalar result:

$$\Delta_G u \to \nabla^2 u, \qquad \Phi = c^2 u, \qquad \nabla^2\Phi = 4\pi G\rho. \tag{31}$$

3.5 Möbius boundary acceleration and the Newton channel

At a tangency point the Möbius boundary map is conjectured to generate the radial acceleration profile

$$a_M(r) = \frac{2v_M^2}{r}\left(1 + \frac{r_s}{r}\right)^{-2} = \frac{2v_M^2 r}{(r+r_s)^2}, \qquad r_s = \frac{2GM}{c^2}, \tag{32}$$

where v_M is the effective boundary-channel speed. The Laurent expansion is

$$a_M(r) = \frac{2v_M^2}{r} - \frac{4v_M^2 r_s}{r^2} + \frac{6v_M^2 r_s^2}{r^3} - \frac{8v_M^2 r_s^3}{r^4} + O(r_s^4/r^5). \tag{33}$$

The first correction term is

$$a_{M,1}(r) = -\frac{4v_M^2 r_s}{r^2} = -\frac{8v_M^2}{c^2}\frac{GM}{r^2}. \tag{34}$$

Thus the Möbius expansion has the Newtonian radial form with a dimensionless prefactor $8v_M^2/c^2$. In the FTC interpretation this prefactor is fixed by the final map between the two tangent sphere surfaces.

Locally, each sphere surface is described by two coordinates (x, y), and the radial boundary coordinate needed by the tangency map is the Euclidean norm

$$\rho = \sqrt{x^2 + y^2}. \tag{35}$$

A two-sphere tangency update therefore requires two such norm programs,

$$\rho_1 = \sqrt{x_1^2 + y_1^2}, \qquad \rho_2 = \sqrt{x_2^2 + y_2^2}. \tag{36}$$

In the Presburger/FTC cost model, each local two-coordinate norm carries the diagonal square-map cost $\sqrt{2}$. The final two-surface boundary-map cost is therefore

$$C_{\text{map}} = \sqrt{2} + \sqrt{2} = 2\sqrt{2}. \tag{37}$$

The normalized Möbius boundary speed is the light-speed channel divided by this final mapping cost:

$$v_M = \frac{c}{C_{\rm map}} = \frac{c}{2\sqrt{2}}, \qquad \frac{v_M^2}{c^2} = \frac{1}{8}. \tag{38}$$

Substituting Eq. (38) into Eq. (34) selects the Newtonian acceleration normalization

$$\boxed{a_{M,1}(r) = -\frac{GM}{r^2}.} \tag{39}$$

For a test mass m, this gives the Newton force law

$$\boxed{F_N = -\frac{GMm}{r^2}.} \tag{40}$$

With the same normalization, the full expansion reads

$$a_M(r) = \frac{c^2}{4r} - \frac{GM}{r^2} + \frac{3G^2M^2}{c^2r^3} - \frac{8G^3M^3}{c^4r^4} + O\left(\frac{G^4M^4}{c^6r^5}\right). \tag{41}$$

The leading $1/r$ term is interpreted as the flat-rotation/inertial boundary channel, while the first correction is the Newtonian channel selected by the final two-surface norm-map cost.

3.6 Dynamic radius field equation

The dynamical ASSP radius equation is taken to be

$$(\Delta_G u)_i - \frac{1}{c^2}\ddot{u}_i = \frac{4\pi G}{c^2}\rho_i, \tag{42}$$

so that the static limit reproduces Eq. (22). With

$$\Box = -\frac{1}{c^2}\partial_t^2 + \nabla^2, \tag{43}$$

the continuum limit is the scalar wave equation

$$\Box u = \frac{4\pi G}{c^2}\rho. \tag{44}$$

Gravitational radiation corresponds, at this level, to propagating perturbations of u. The tensorial sector is expected to arise from the coarse-grained dynamics of $\mathrm{d}\ell_G^2$.

4 Quantum Matter as a One-Dimensional Tangency-Channel Limit

4.1 Why the Dirac channel is effectively one-dimensional

The ASSP substrate is three-dimensional, but a single transfer event is not a three-dimensional bulk motion. It is a directed event along an active tangency channel. At the microscopic level an Info Ball either crosses the local tangency boundary in one orientation or in the opposite orientation. Thus the minimal quantum register is not a full spatial vector field; it is a one-dimensional transport axis equipped with a two-state internal boundary spin.

Let e denote the active tangency direction through a local pair of tangent spheres. The transfer coordinate x is distance along this channel. The two internal states are written

$$\psi_R(x,t), \qquad \psi_L(x,t),$$

where R and L may be read equivalently as right/left movers, clockwise/anti-clockwise boundary rotations, or the two signs of the local spin register about the tangency hole. The transverse ASSP geometry fixes the local clock rate, frame orientation, and channel availability, but the local propagation law itself is one-dimensional.

This is the key technical simplification: the Dirac structure does not first require a complete derivation of a $3 + 1$-dimensional continuum spin bundle. A single active tangency line plus a binary half-turn register is already enough to generate the Dirac equation in the controlled continuum limit.

The microscopic CLASSP interaction is path-local: an Info Ball propagates along an actual active tangency channel. The continuum Dirac equation is the long-wavelength limit of that channel update. Consequently, the ultraviolet problem of continuum QFT is not present at the microscopic level; there is no completed continuum of arbitrarily short virtual modes to integrate over. When continuum observers use Feynman diagrams, the required renormalized parameters encode the finite ASSP cutoff, hidden lower-layer channels, and unresolved tangency-path structure.

4.2 Half-turn boundary register

In TFOFT the rotational state channel of a Soddy sphere is hemisphere restricted. The rotational contribution to the fundamental state cost is

$$T_R = \pi r\big|_{r=\pi} = \pi^2,$$

not $2\pi^2$. This is interpreted as a half-turn register: the physical update need only distinguish the two boundary orientations of the active rotational channel, not specify an unconstrained global rotation.

For CLASSP this means that the local quantum degree of freedom is naturally binary. Around a tangency hole the boundary spin decomposes into two orientations,

$$\Psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix},$$

which may also be written as $\Psi = (\psi_R, \psi_L)^T$. The two components are not separate particles. They are the two possible orientations of the same one-dimensional transfer process.

4.3 One-dimensional chiral walk

Let ϵ be the spatial step along the active tangency channel and $\Delta\tau$ the local proper tick. The light-speed condition is

$$\frac{\epsilon}{\Delta\tau} = c.$$

Let θ be the local half-turn mixing angle. The discrete update rule is

$$\psi_R(x + \epsilon, t + \Delta\tau) = \cos\theta\, \psi_R(x, t) - i \sin\theta\, \psi_L(x, t), \tag{45}$$

$$\psi_L(x - \epsilon, t + \Delta\tau) = \cos\theta\, \psi_L(x, t) - i \sin\theta\, \psi_R(x, t). \tag{46}$$

Here the streaming part moves the two boundary orientations in opposite directions along the channel, while the θ-term flips the local orientation. In the FTC interpretation this flip is the local half-turn of the boundary spin register.

For $\theta \ll 1$ and slowly varying amplitudes, Taylor expansion gives

$$(\partial_t + c\partial_x)\psi_R = -i\frac{\theta}{\Delta\tau}\psi_L, \tag{47}$$

$$(\partial_t - c\partial_x)\psi_L = -i\frac{\theta}{\Delta\tau}\psi_R. \tag{48}$$

Identifying

$$\frac{\theta}{\Delta\tau} = \frac{mc^2}{\hbar}$$

yields

$$(\partial_t + c\partial_x)\psi_R = -i\frac{mc^2}{\hbar}\psi_L, \tag{49}$$

$$(\partial_t - c\partial_x)\psi_L = -i\frac{mc^2}{\hbar}\psi_R. \tag{50}$$

Writing $\Psi = (\psi_R, \psi_L)^T$, this becomes

$$\boxed{i\hbar\partial_t\Psi = -i\hbar c\,\sigma_z\partial_x\Psi + mc^2\sigma_x\Psi.}$$

This is a representation-specific form of the $1+1$-dimensional Dirac equation. In CLASSP it is not merely a toy model: it is the continuum limit of one active ASSP tangency-transfer channel.

4.4 Radius-clock insertion

On the ASSP graph the local proper tick is set by the local tangency step. For an edge (i, j),

$$\Delta\tau_{ij} \sim \frac{R_i + R_j}{c}.$$

In a locally equal-radius coarse-grained region, $R_j \approx R_i$, this reduces to

$$\Delta\tau_i \sim \frac{2R_i}{c}.$$

The mixing angle therefore becomes

$$\theta_i = \frac{mc^2\Delta\tau_i}{\hbar} \sim \frac{2mcR_i}{\hbar} = \frac{2R_i}{\lambda_C}, \qquad \lambda_C = \frac{\hbar}{mc}.$$

Thus mass is the chirality-reversal rate per local sphere-clock tick:

$$\boxed{m = \frac{\hbar}{c^2}\frac{\theta_i}{\Delta\tau_i}.}$$

Massless modes have $\theta = 0$ and stream without local orientation reversal. Massive modes repeatedly reverse the boundary-spin state, producing the fractal zigzag path whose long-wavelength limit is Dirac-like.

4.5 Radial-pole transfer and boundary spin

For a near-radial transfer through a local pole of the packing, the relevant coordinate is not the full three-dimensional displacement vector. It is the radial tangency coordinate selected by the local sphere pair. The transfer "cares" about the boundary spin around the tangency hole: clockwise versus anti-clockwise, or equivalently $+$ versus $-$. Therefore the local quantum question is binary:

Which boundary orientation is active along this channel?

The answer is exactly the two-component spinor used above.

This is why the quantum update can be one-dimensional even though the packing is three-dimensional. The three-dimensional ASSP determines which local channels exist and how their frames rotate from one sphere pair to the next, but the elementary Dirac update only needs:

1. one active propagation coordinate x,
2. one local clock tick $\Delta\tau$,
3. one binary half-turn spin register $(+, -)$.

4.6 Status of the three-dimensional extension

The result proved in this section is the effective one-dimensional channel limit:

$$\text{active tangency line} + \text{two-state boundary spin} \quad \Longrightarrow \quad 1+1\text{-dimensional Dirac equation.}$$

The full $3+1$-dimensional curved-space Dirac equation is not derived here. It is a conjectural coarse-grained extension obtained by patching many local one-dimensional tangency channels together through the rotating ASSP frame.

In that extension, a local tetrad $e_a^\mu(x)$ records the averaged orientation of nearby tangency channels, and the spin connection records the frame change induced by passing from one sphere-pair channel to the next. The expected continuum form is

$$i\hbar\gamma^a e_a^\mu \left(\partial_\mu + \Omega_\mu\right) \Psi = mc\,\Psi,$$

where Ω_μ is the effective spin connection generated by coarse-grained inter-sphere frame rotation. Equation (4.6) is therefore an ansatz, not a theorem. The theorem-level result is the one-dimensional tangency-channel Dirac limit.

5 Finite Tangency Registers, Bell Nonlocality, and Quantum Computation

5.1 Superposition as finite tangency correlation

The wavefunction is not taken here as an ontological primitive. It is an effective continuum bookkeeping object used by a bounded observer who cannot resolve the full finite graph of mutual tangencies, chained tangencies, cross-layer tangencies, and radius-clock ticks.

In the ASSP ontology, what ordinary quantum mechanics calls superposition is interpreted as an unresolved finite set of compatible tangency configurations. What ordinary quantum mechanics calls entanglement is interpreted as a linked tangency-chain constraint. Thus

$$\boxed{\text{superposition} = \text{unresolved finite contact alternatives,}} \tag{51}$$

and

$$\boxed{\text{entanglement} = \text{linked tangency-chain constraint.}} \tag{52}$$

Full local contact corresponds to maximal local correlation. Lower-order contact corresponds to lower-order correlation. Long tangency chains correspond to long-distance entanglement-like correlations, and cross-layer tangencies correspond to apparent nonlocality relative to one emergent spacetime layer.

There is no sharp quantum/classical boundary in this picture. There is only a change of coarse-graining. At small register depth, finite tangency alternatives must be treated explicitly and appear quantum-like. At large register depth, the same graph relations average into classical trajectories, continuum fields, and effective probability densities.

5.2 Bell correlations and cross-layer tangency

CLASSP should not be read as a same-layer local hidden-variable theory. Bell-type experiments rule out local hidden-variable explanations of quantum correlations under the standard assumptions. The ASSP proposal avoids that category because the relevant hidden structure is not local with respect to the emergent three-dimensional metric of one coarse-grained layer.

The underlying causal primitive is the directed tangency graph, including same-layer tangency chains and cross-layer macrophoton/tangency channels. Therefore two objects separated in emergent space may remain adjacent, linked, or jointly constrained in the deeper ASSP graph.

The observed Bell correlation is not produced by a signal moving through ordinary space after measurement. It is produced by a prior or deeper graph relation whose causal adjacency is not captured by the same-layer Lorentz cone.

Thus Bell's theorem is not violated. Rather, one of the assumptions used to derive Bell inequalities is not satisfied: locality with respect to the emergent metric of one layer. The model is explicitly nonlocal relative to emergent spacetime. It therefore belongs, if viable, to the class of nonlocal hidden-structure theories, not local hidden-variable theories:

$$\boxed{\text{Bell excludes same-layer local hidden variables, not cross-layer tangency adjacency.}} \tag{53}$$

No usable faster-than-light messaging is implied inside one layer, because ordinary observers can only access the coarse-grained projection. The deeper graph may contain superluminal tangency adjacency, but controllable signaling is filtered by register availability, channel orientation, radius-clock mismatch, and the directed update order of the quine.

5.3 Soddy contact bounds and local quantum registers

The local ASSP quantum register is finite because sphere rearrangements are constrained by sphere-packing contact laws. In three Euclidean dimensions, the strict Descartes–Soddy/Gosset closure contains five pairwise mutually tangent oriented spheres:

$$N_{\text{mutual}}^{(3D)} = 5. \tag{54}$$

A distinct but physically important closure is the Soddy hexlet. Given three mutually tangent base spheres, a chain of six additional spheres closes around them, with each chain sphere tangent to the three base spheres and to its two chain-neighbors. This gives a natural nine-sphere local contact register,

$$N_{\text{hexlet}} = 3 + 6 = 9, \tag{55}$$

without claiming that all nine spheres are pairwise mutually tangent.

In CLASSP, the five-sphere Descartes closure is interpreted as the maximum local full-mutual-correlation register, while the nine-sphere hexlet is interpreted as a finite local rearrangement neighborhood. Tangency chains can extend over arbitrary graph distance, but a single exact full-mutual local correlation neighborhood is bounded.

The resulting hierarchy is

$$\text{pair tangency} \Longrightarrow \text{two-register correlation,} \tag{56}$$

$$\text{full mutual Descartes contact} \Longrightarrow \text{maximal exact local correlation, capped at five registers,} \tag{57}$$

$$\text{Soddy-hexlet closure} \Longrightarrow \text{finite nine-sphere rearrangement neighborhood,} \tag{58}$$

$$\text{long tangency chain} \Longrightarrow \text{long-distance entanglement,} \tag{59}$$

$$\text{cross-layer chain} \Longrightarrow \text{apparent nonlocality beyond one-layer light cones.} \tag{60}$$

5.4 Scale-relative causality and macrophoton transfer

Special relativity is interpreted here as the effective causal geometry of one coarse-grained fractal layer. Its invariant speed c is the limiting signal speed for ordinary bulk propagation inside that layer, measured using that layer's radius-clock field. It is not assumed to be the invariant speed of every deeper or higher ASSP quine layer.

Let R_N denote the characteristic radius-clock scale of layer N. The effective light speed measured inside that layer is written

$$c_N \sim \frac{\Delta \ell_N}{\Delta \tau_N}, \tag{61}$$

where both the distance unit $\Delta\ell_N$ and the local tick $\Delta\tau_N$ are determined by the layer's ASSP geometry. A transfer that is ordinary and causal on layer $N+1$ may project into layer N as an apparently superluminal event:

$$v_{\rm app}^{(N)} = \frac{\Delta x_N}{\Delta t_N} > c_N. \tag{62}$$

This does not imply time travel. Causal order is not defined by the emergent Lorentz cone of layer N alone, but by the directed update order of the underlying tangency graph. The graph update has a direction; the lower-layer projection has an apparent distance and delay. Apparent superluminality is therefore a cross-layer projection effect, not a closed causal loop.

In the astrophysical case, a Star Ball forced into a Core Ball can excite a macrophoton channel. The corresponding emission need not reappear as a jet from the same apparent galaxy. In the ASSP graph, the receiving Core Ball may be adjacent through a higher-layer tangency relation while being distant in our emergent three-dimensional coordinates. Thus an infall or callback event near one Core-Ball register may correlate with a modulated jet return value at another galaxy's core. The process is nonlocal relative to the emergent metric of our layer, but local relative to the directed tangency update of the higher layer.

5.5 Core-Ball callback register and jet return values

In CLASSP the object usually called a black-hole event horizon is not treated as an ontologically primitive spacetime surface. It is reinterpreted as a *Core-Ball callback register*: an addressable program boundary at which remote Info-Ball transfers are received, decoded, and executed as local precession, jet structure, and lower-layer memory allocation. The programming language is deliberate. A local jet is not primarily local angular momentum being expelled. It is the visible return value of a remote angular-momentum callback.

Let $\Delta\vec{L}_{\rm rem}$ denote angular momentum carried by a mostly one-dimensional Info-Ball/macrophoton transfer from a remote origin, and let $\hat{q}$ be the incoming transfer direction at the receiving Core Ball. Let $\vec{S}_{\rm core}$ be the receiver spin register. The callback torque is modeled by the component of the remote instruction not already aligned with the receiver spin:

$$\vec{\tau}_{\rm cb} = \Gamma \left[\Delta\vec{L}_{\rm rem} - (\Delta\vec{L}_{\rm rem}\cdot\hat{S}_{\rm core})\hat{S}_{\rm core}\right], \tag{63}$$

so that

$$\frac{\mathrm{d}\vec{S}_{\rm core}}{\mathrm{d}t} = \vec{\tau}_{\rm cb}. \tag{64}$$

The observed jet is the local visible trace of this receiver-side callback:

$$\hat{\jmath}_{\rm jet} \parallel \hat{S}_{\rm core}(t) \qquad \text{or} \qquad \hat{\jmath}_{\rm jet} \parallel \frac{\mathrm{d}\hat{S}_{\rm core}}{\mathrm{d}t}, \tag{65}$$

depending on whether the visible jet follows the instantaneous spin axis or the precession axis.

The callback has an orientation-dependent routing efficiency. If reflection is most efficient along the polar spin axis, a minimal efficiency model is

$$\eta_{\rm cb}^{\rm polar} = (\hat{q}\cdot\hat{S}_{\rm core})^2. \tag{66}$$

If reflection is most efficient through the receiver equatorial address plane, then

$$\eta_{\rm cb}^{\rm eq} = 1 - (\hat{q}\cdot\hat{S}_{\rm core})^2. \tag{67}$$

The core evolves to minimize callback routing cost,

$$\mathcal{C}_{\rm cb} = 1 - \eta_{\rm cb}, \tag{68}$$

so spinning Core-Ball equators tend to self-align for efficient reflection of remote Info-Ball transfers.

The channel split is the same rank-23 split used in TFOFT. The visible return value carries the addition-efficient channel,

$$\Delta\vec{L}_{\rm return} = \frac{4}{27}\Delta\vec{L}_{\rm rem}, \tag{69}$$

while the rank-23 overhead is written into the lower fractal layer as memory allocation / inertial debt,

$$\Delta\vec{L}_{\rm lower} = \frac{23}{27}\Delta\vec{L}_{\rm rem}. \tag{70}$$

Thus a jet is not exhaust from the local disk; it is a return value produced when a receiving Core-Ball executes a remote angular-momentum callback. The associated dark/inertial structure is the lower-layer memory allocation needed to execute that callback.

5.6 Quantum Fourier Transform as a full-mutual-contact protocol

Standard quantum computing treats the Hilbert space of n qubits as an ideal 2^n-dimensional complex vector space. In CLASSP, this space is not ontologically fundamental. It is an effective bookkeeping representation of finite contact and chained-contact relations in the underlying graph.

The Quantum Fourier Transform used in Shor-type order finding requires coherent phase correlation across the active logical register. In the ASSP interpretation, exact unit-fidelity phase correlation is conjectured to require full mutual tangency among the active logical qubit registers. Lower-order contact gives lower-order correlation; chained contact gives entanglement-like correlation but not exact all-to-all phase coherence.

Because the strict three-dimensional Descartes–Soddy closure allows only five pairwise mutually tangent oriented spheres, CLASSP makes the following conjecture:

Conjecture 1 (Five-register QFT coherence bound)**.** *Exact full-mutual-contact Quantum Fourier Transform coherence can involve at most five logical qubit-registers in a single local ASSP contact neighborhood:*

$$n_{\rm QFT}^{\rm exact} \leq 5. \tag{71}$$

For $n \leq 5$, a fully correlated QFT-like register may be physically realizable in an ideal local contact neighborhood. For $n > 5$, the computation must use lower-order tangencies, chained tangencies, or routed graph correlations. These may still generate ordinary entanglement, but they are not exact full-mutual-contact correlations. The result is a predicted structured error floor for large QFT-based algorithms.

The difference of sphere radii makes the limitation stronger. Even within a nominal five-register closure, unequal radii imply unequal local ticks,

$$\Delta\tau_i \sim \frac{2R_i}{c}. \tag{72}$$

A phase update with angular frequency ω therefore accumulates a relative radius-clock drift

$$\delta\phi_{ij} \sim \omega(\Delta\tau_i - \Delta\tau_j) \sim \frac{2\omega}{c}(R_i - R_j). \tag{73}$$

Thus full mutual tangency is necessary but not sufficient for perfect QFT coherence. Equal-radius or radius-compensated contact is also required. In real hardware this appears as structured phase error, not merely random environmental decoherence.

5.7 Shor limitation conjecture

Shor's algorithm is mathematically valid in ideal Hilbert-space quantum computation. CLASSP does not dispute the formal algorithm. It disputes the assumption that arbitrary-size ideal QFT coherence is physically realizable as an unbounded continuum Hilbert-space object.

Conjecture 2 (CLASSP Shor limitation)**.** *RSA-scale Shor factorization cannot be physically realized if exact order finding requires full-mutual-contact QFT coherence beyond the five-register Descartes bound.*

Small compiled demonstrations are not inconsistent with this conjecture. A five-qubit demonstration of factoring $N = 21$, for example, lies exactly at the proposed full-mutual-contact ceiling. In the CLASSP reading, such demonstrations are not early evidence of unlimited scalability; they may instead be evidence that present demonstrations have reached the maximum local contact order available to exact QFT-style phase correlation.

This is a falsifiable conjecture, not an established theorem. If future quantum computers run uncompiled, fault-tolerant Shor order finding at large logical register sizes with no structured finite-contact error floor, then Conjecture 2 is falsified. If instead large QFT circuits exhibit persistent, nonthermal, geometry-like phase errors that cannot be removed by standard error correction, the conjecture gains support.

5.8 Experimental quantum-computing signatures

Possible finite-register deviations from ideal quantum computation include:

Q1. Finite-register saturation: large entangled states encounter nonrandom error floors associated with finite local contact closure.

Q2. Graph-topological decoherence: errors correlate with hidden tangency-chain rearrangements rather than independent local noise.

Q3. Radius-clock phase noise: chirality-flip phases drift with local radius-clock fluctuations.

Q4. Layer leakage: strongly driven quantum systems leak correlation into adjacent fractal layers, appearing as anomalous decoherence or rare nonlocal synchronization.

Q5. QFT failure mode: QFT-based algorithms show a transition from full-contact exact correlation to routed-contact approximate correlation once more than five logical registers require simultaneous all-to-all phase coherence.

If quantum computation scales perfectly according to ideal Hilbert-space theory with no finite-register anomalies, CLASSP loses support. If large entangled systems show structured, geometry-like, nonthermal, or nonlocal error correlations, CLASSP gains a possible experimental foothold.

6 Candidate Electromagnetic Limit from Edge-Phase Holonomies

6.1 Electromagnetism as lower-layer callback bookkeeping

The holonomy construction below gives the graph-theoretic form of the gauge field. The physical interpretation in TFOFT is that the vector potential is the unresolved lower-layer momentum-flow register created by Core-Ball callbacks. Visible Info-Ball transfer carries the addition-efficient return value, while the rank-23 overhead is written into the lower fractal layer as orthogonal linear-momentum debt.

Let $\Delta\vec{L}_{\rm rem}$ be the remote angular-momentum instruction received at a Core-Ball callback register. At callback radius $r_{\rm cb}$, with incoming direction $\hat{q}$ and local radial direction $\hat{r}$, define an orthogonal lower-layer injection direction

$$\hat{n}_\perp = \frac{\hat{r}\times\hat{q}}{|\hat{r}\times\hat{q}|}. \tag{74}$$

The corresponding lower-layer linear-momentum write is modeled as

$$\Delta\vec{p}_{\rm lower} = \frac{23}{27}\frac{|\Delta\vec{L}_{\rm rem}|}{r_{\rm cb}}\hat{n}_\perp. \tag{75}$$

Equivalently, relative to the visible return channel,

$$\Delta\vec{p}_{\rm lower} = \frac{23}{4}\frac{|\Delta\vec{L}_{\rm return}|}{r_{\rm cb}}\hat{n}_\perp. \tag{76}$$

This is not a local jet expelling local angular momentum. It is the memory write cost of executing a remote callback.

Let $\vec{\Pi}_{\rm lower}$ be the coarse-grained lower-layer momentum density produced by many such callbacks. Define the effective FTC vector potential by

$$\vec{A}_{\rm FTC} = \lambda_A\vec{\Pi}_{\rm lower}. \tag{77}$$

Then the magnetic-looking field is the curl of the lower-layer momentum-flow register,

$$\vec{B}_{\rm eff} = \nabla\times\vec{A}_{\rm FTC}, \tag{78}$$

and the electric-looking field is the time-changing callback pressure,

$$\vec{E}_{\rm eff} = -\nabla\phi - \partial_t\vec{A}_{\rm FTC}. \tag{79}$$

Thus ordinary electromagnetic language is interpreted as continuum bookkeeping for hidden lower-layer momentum allocation. Intergalactic magnetic-field-line behavior, jet alignment, and hypervelocity-star escape corridors are expected to follow the same callback routing network rather than independent local exhaust processes.

With the Lorenz-gauge condition

$$\nabla\cdot\vec{A}_{\rm FTC} + \frac{1}{c^2}\partial_t\phi = 0, \tag{80}$$

conservation of lower-layer callback momentum propagating at speed c gives the candidate wave equation

$$\left(\nabla^2 - \frac{1}{c^2}\partial_t^2\right)\vec{A}_{\rm FTC} = -\mu_0\vec{J}_{\rm eff}, \tag{81}$$

where $\vec{J}_{\rm eff}$ is the effective current associated with callback momentum writes. This is the physical route from remote angular-momentum callbacks to the Maxwell holonomy limit.

6.2 U(1) edge phases and discrete field strength

Assign a U(1) phase to each oriented tangency edge. Let A_{ij} denote the line integral of the continuum gauge potential along the edge,

$$A_{ij} = \int_i^j A_\mu\,dx^\mu, \qquad A_{ij} = -A_{ji}. \tag{82}$$

The edge phase is

$$U_{ij}^{\mathrm{EM}} = \exp\left(\frac{iqA_{ij}}{\hbar}\right). \tag{83}$$

The gauge-covariant graph derivative is

$$(D_A f)_{ij} = U_{ij}^{\mathrm{EM}} f_j - f_i. \tag{84}$$

The discrete flux phase is the holonomy around an elementary graph cycle $C = (i_1, i_2, \ldots, i_k, i_1)$:

$$F_C = \arg \prod_{(ij)\in C} U_{ij}^{\mathrm{EM}} = \frac{q}{\hbar} \sum_{(ij)\in C} A_{ij}. \tag{85}$$

For a minimal three-tangency cycle, F_C is the dimensionless discrete analogue of magnetic flux through the triangle. The corresponding continuum field-strength component is obtained only after dividing the unscaled line-integral sum by the oriented plaquette area in the coarse-grained limit.

6.3 Discrete gauge action and Maxwell limit

The candidate gauge-field action on the ASSP graph is

$$S_A = -\frac{1}{4} \sum_C W_C F_C^2 + \sum_{(ij)} J_{ij} A_{ij}, \tag{86}$$

where W_C are cycle weights set by the packing geometry. Stationarity $\delta S_A / \delta A_{ij} = 0$ gives a discrete divergence equation on the field strength. The graph identity $d_G^2 = 0$ provides the discrete Bianchi identity exactly. Under slow-variation and isotropy conditions, the cycle holonomies expand to leading order in the gauge potential and the discrete divergence maps to the continuum divergence, giving the candidate Maxwell limit

$$\boxed{\partial_\mu F^{\mu\nu} = \mu_0 J^\nu, \qquad \partial_{[\lambda} F_{\mu\nu]} = 0.} \tag{87}$$

The normalization of W_C, the Lorentzian Hodge star, and the exact emergence of μ_0 are left as explicit tasks for the CLASSP gauge program.

6.4 Coupling constant from register depth

The electromagnetic coupling constant is conjectured to enter through the Presburger register depth of one Soddy sphere [1, 3]:

$$T_{\mathrm{sphere}} = 4\pi^3 + \pi^2 + \pi \approx 137.036304, \qquad \alpha \approx \frac{1}{T_{\mathrm{sphere}}}. \tag{88}$$

Under the edge-phase ontology, α is the sequential-alignment probability per quine cycle: the fraction of asynchronous two-ball register updates that are causally ordered [1]. Higher-order tangency interactions are conjectured to generate the gauge hierarchy toward $SU(2)$ and $SU(3)$.

7 Simulation and Observation

7.1 Hydrogenic dark/inertial boundary fields

In the TFOFT interpretation, dark matter is not primarily a particle species but a lower-layer inertial boundary field [1]. The electron cloud and the dark halo are both lower-layer boundary fields that constrain the dynamics of the visible object (electron at atomic scale; stellar disk at galactic scale). Therefore the simulation target is *not*

$$\text{stars trace } |\psi_{nlm}|^2.$$

The correct target is

$$\mathcal{I}^{(nlm)}_{\text{halo}}(R, z, \phi) \implies \text{inertial/drag/clock field} \implies \text{stellar disk response.} \tag{89}$$

A minimal model uses hydrogenic orbital geometries as basis functions for the dark/inertial field:

$$\mathcal{I}(r, \theta, \phi) = \sum_{nlm} W_{nlm} \left| R_{nl}\left(\frac{r}{a_g}\right) Y_{lm}(\theta, \phi)\right|^2, \tag{90}$$

where a_g is the galactic scale radius and W_{nlm} are occupation weights. A continuum observer may infer an effective density ρ_{eff} from $\mathcal{I}$, but the physical object in the ASSP model is the inertial boundary kernel rather than a literal cloud of particles.

The stellar disk evolves under

$$\vec{a}_\star = -\nabla\Phi_{\text{baryon}} - c^2\nabla u_{\text{halo}} + \vec{a}_{\text{drag}}[\mathcal{I}, \vec{v}] + \vec{a}_{\text{gyro}}[\mathcal{I}, \vec{v} \times \hat{n}], \tag{91}$$

where the first two terms are baryon and radius-clock gravity from Eq. (23), and the latter two represent the lower-layer inertial drag and angular-momentum transfer channels responsible for flat-rotation support. The Möbius boundary acceleration (32) provides one analytic form for $\vec{a}_{\text{gyro}}$.

For the $3d_{z^2}$ Milky Way state specifically, the effective inertial kernel has the angular form

$$\mathcal{I}_{3d_{z^2}}(r, \theta) = I_0\, \mathcal{R}_{32}(r/a_g)\, (3\cos^2\theta - 1)^2, \tag{92}$$

where $\mathcal{R}_{32}$ is the scaled radial shell factor. If one uses the hydrogenic density shape directly, then

$$\mathcal{R}_{32}(r/a_g) \propto \left(\frac{r}{a_g}\right)^4 \exp\left(-\frac{2r}{3a_g}\right). \tag{93}$$

The scale a_g is fixed by matching the polar Fermi-bubble height, not by fitting the equatorial feature after the fact. If $\mathcal{I}$ is treated as a three-dimensional inertial density kernel, the radial density factor $r^4 e^{-2r/(3a_g)}$ peaks at $r = 6a_g$. If instead one fits the shell-weighted radial measure $r^2\mathcal{I}$, the peak shifts to $r = 9a_g$. The simulations must state which convention is used before fitting observational data.

7.2 Numerical simulation program

The simulation comparison program is:

S1. Build disk simulations with four halo/inertial models: NFW, Einasto, generic dark disk, and the hydrogenic ASSP inertial kernel $\mathcal{I}_{3d_{z^2}}$ from Eq. (92).

S2. Fit rotation curves, vertical force K_z, HI gas flaring, spiral structure, and stream perturbations.

S3. Ask whether a small set of hydrogenic orbital boundary states explains the same observables with fewer effective parameters.

S4. Allow transitions between boundary states, $\Psi_{\text{halo}}(t) = \sum_{nlm} a_{nlm}(t)\psi_{nlm}$, and identify the disk and plasma response to $\Delta|\Psi_{\text{halo}}|^2$.

7.3 The $3d_{z^2}$ Milky Way state and the Fermi bubbles

The companion paper [1] identifies the Fermi bubbles [9, 10] as the polar lobes of a scaled $3d_{z^2}$ boundary state. The angular factor is

$$Y_{20}(\theta) \propto 3\cos^2\theta - 1. \tag{94}$$

Its nodal cones lie at

$$3\cos^2\theta - 1 = 0 \implies \theta_{\rm node} = \arccos\frac{1}{\sqrt{3}} \approx 54.74^\circ. \tag{95}$$

The Fermi bubbles extend roughly tens of degrees above and below the Galactic center; the common descriptive scale is about 50 kly ≈ 15.3 kpc, while some modeling papers quote heights of order 10 kpc depending on the geometric convention and distance model [9, 10]. We therefore define

$$H_{\rm FB} \equiv \text{polar Fermi-bubble height} \sim 10\text{–}15\,\text{kpc}. \tag{96}$$

The TFOFT rank fraction (23/27) may be read as setting the polar-lobe height relative to the full local halo/boundary scale,

$$H_{\rm FB} = \frac{23}{27} R_{\rm halo}^{\rm local}. \tag{97}$$

Thus, for $H_{\rm FB} \sim 10$–15 kpc,

$$R_{\rm halo}^{\rm local} \sim \frac{27}{23} H_{\rm FB} \sim 12\text{–}18\,\text{kpc}. \tag{98}$$

This places the associated equatorial component in the outer visible disk, not in a remote 40–50 kpc halo annulus.

7.4 The equatorial "Fermi donut": corrected derivation and prediction

At the poles and at the equator, the squared Y_{20} amplitude gives:

$$|Y_{20}|^2_{\rm pole} \propto (3\cos^2 0 - 1)^2 = (2)^2 = 4, \tag{99}$$

$$|Y_{20}|^2_{\rm equator} \propto (3\cos^2(\pi/2) - 1)^2 = (-1)^2 = 1. \tag{100}$$

Therefore

$$\boxed{\frac{\mathcal{I}_{\rm equator}}{\mathcal{I}_{\rm pole}} = \frac{|Y_{20}|^2_{\rm equator}}{|Y_{20}|^2_{\rm pole}} = \frac{1}{4}.} \tag{101}$$

The same $3d_{z^2}$ state that produces the polar Fermi bubbles requires an equatorial toroidal lobe of one-quarter the polar angular density contrast. In the simplest same-radius shell interpretation, the torus peak is at the same orbital scale as the polar lobe:

$$R_{\rm donut,peak} \sim H_{\rm FB} \sim 10\text{–}15\,\text{kpc}. \tag{102}$$

The torus does not possess a unique hard inner edge because an orbital density surface depends on the chosen contour. For a fiducial shell/shoulder estimate, using the nodal-cone geometry gives an inner shoulder of order

$$R_{\rm donut,in} \sim \sqrt{\frac{2}{3}} H_{\rm FB} \approx 0.816 H_{\rm FB} \sim 8\text{–}12\,\text{kpc}. \tag{103}$$

This places the predicted feature from approximately the solar circle and outer disk into the region just beyond the main stellar disk scale. The feature is therefore not a far-halo object; it is an outer-disk boundary signature.

Corrected prediction: Fermi donut. Using the real Fermi-bubble scale $H_{\rm FB} \sim 10$–$15\,$kpc, the Milky Way $3d_{z^2}$ boundary state predicts an equatorial annular counterpart with:

- fiducial inner shoulder $R_{\rm in} \sim 8$–$12\,$kpc;
- radial peak $R_{\rm peak} \sim 10$–$15\,$kpc;
- angular density/inertia contrast $\approx 1/4$ of the polar lobes;
- geometry: a broad, low-contrast ring symmetric about the Galactic plane.

A coherent outer-disk boundary at this scale supports the $3d_{z^2}$ model. A smooth featureless disk/halo through $R \sim 8$–$20\,$kpc disfavors this particular state identification.

7.5 Directed disk-halo search cone and macro-photon leakage analogy

The annular counterpart is not predicted as an isotropic halo shell. In the present interpretation it is a directed disk-halo boundary associated with the Sgr A* Core-Ball channel. The observational search should therefore emphasize structures moving or aligned radially away from the Galactic center through the outer disk, especially where the outward channel intersects the predicted $R \sim 8$–$20\,$kpc annulus.

The nodal-cone geometry also gives an angular search scale about the Galactic plane. The nodal angle measured from the polar axis is

$$\theta_{\rm node} = \arccos\frac{1}{\sqrt{3}} \approx 54.74^\circ. \tag{104}$$

The corresponding elevation above the equatorial plane is

$$\beta_{\rm node} = 90^\circ - \theta_{\rm node} = \arcsin\frac{1}{\sqrt{3}} \approx 35.26^\circ. \tag{105}$$

Thus the disk component should be searched as a broad low-contrast band with angular support of order 34°–35° above and below the Sgr A* equatorial plane, not as a razor-thin disk. Because the equatorial component has only one-quarter the polar angular density contrast, the signal is expected to be centrally weighted rather than uniform. A simple working model is a normal angular profile

$$P(\beta) \propto \exp\left(-\frac{\beta^2}{2\sigma_\beta^2}\right), \qquad \sigma_\beta \approx 12^\circ, \tag{106}$$

so that approximately 68–69% of the equatorial leakage lies inside $|\beta| \lesssim 12^\circ$.

This provides a speculative analogy with ultra-high-energy "Oh-My-God"-type particle events. If an event stream lies near $\beta \approx -14^\circ$ relative to the Sgr A* equatorial plane, it is near a one-sigma excursion in Eq. (106). In the FTC interpretation such a stream could be read as a small-scale Rutherford-gold-foil analogue: a macro-photon or micro-macro-photon transfer from the Sgr A* Core-Ball channel is deflected by the lower-layer dark/inertial disk boundary. The one-sided sign of the observed offset would then encode the polarity of the emitted info-ball channel relative to the lower-layer dark-matter leakage channel.

Equivalently, the redshift of a macrophoton is modeled as a momentum leak from the initial ejection into the lower fractal substrate:

$$\Delta p_{{\rm macro}\gamma} = -\Delta p_{\rm lower}, \qquad \Delta p_{\rm lower} \parallel \nabla \mathcal{I}_{3d_{z^2}}. \tag{107}$$

This subsection is a proposed observational discriminator, not a detection claim. It predicts that candidate events should correlate with outward Sgr A*-centered trajectories, with the low-contrast equatorial dark boundary, and with the sign/polarity of the Core-Ball to Info-Ball channel.

7.6 Connection to Gaia DR3 outer-disk anomalies

The corrected Fermi-donut scale overlaps the radial domain where recent Gaia DR3 studies already report model-sensitive Milky Way dynamics. Sylos Labini constructed generalized rotation curves from Gaia DR3 data over $R = 8.5$–$25\,\mathrm{kpc}$ and heights $z = -2$ to $+2\,\mathrm{kpc}$, comparing a canonical NFW halo model with a dark-matter-disk (DMD) model in which the dark component is confined to the Galactic plane and follows neutral hydrogen; that analysis reported that the DMD model generally fits the large-radius on-plane and off-plane behavior better than NFW [14]. Other Gaia DR3 Jeans analyses find evidence for a declining rotation curve beyond $\sim 15\,\mathrm{kpc}$, while also warning that disequilibrium, non-axisymmetry, and satellite perturbations can bias Jeans-inferred circular velocities at the 10–15% level [15]. Earlier Gaia DR3 mapping extends disk kinematic maps and rotation-curve reconstruction to roughly $30\,\mathrm{kpc}$, providing exactly the radial coverage needed to search for the corrected $3d_{z^2}$ equatorial boundary [16].

This is not a claim that Gaia has already detected the Fermi donut. The point is sharper: when using the correct scale, the TFOFT prediction lands in the same $R \sim 10$–$20\,\mathrm{kpc}$ outer-disk regime where Gaia already indicates that simple spherical-halo modeling is under stress or at least model-dependent. The $3d_{z^2}$ model predicts a specific angular and radial boundary structure in that region, not merely a generic dark disk.

7.7 Emission and star-formation scaling

If the polar Fermi bubbles arise where the boundary state opens a high-density angular channel, the equatorial torus produces weaker signatures for two independent reasons: (1) the angular density is lower by Eq. (101); and (2) local baryonic plasma injection in the outer disk differs from the central polar channel. If emissivity scales linearly with boundary density and local star-formation/plasma production,

$$I_{\mathrm{donut}} \sim \frac{1}{4}\left(\frac{\Sigma_{\mathrm{SFR}}(R_{\mathrm{donut}})}{\Sigma_{\mathrm{SFR,polar}}}\right) I_{\mathrm{bubble}}. \tag{108}$$

If the relevant emission is collisional or plasma-density squared,

$$I_{\mathrm{donut}} \sim \frac{1}{16}\left(\frac{\Sigma_{\mathrm{gas}}(R_{\mathrm{donut}})}{\Sigma_{\mathrm{gas,polar}}}\right)^2 I_{\mathrm{bubble}}. \tag{109}$$

If the boundary operates by a threshold/ignition rule, the donut may appear more strongly in dynamics, HI flaring, or stellar-stream perturbations than in X-ray or gamma-ray brightness. Non-detection in all-sky emission maps is therefore not decisive unless the search is tuned to the predicted low-contrast outer-disk annulus.

7.8 Observational program

O1. Search Gaia DR3-derived rotation and vertical-force residuals for a coherent boundary or slope change in $R \sim 8$–$20\,\mathrm{kpc}$, especially near $R \sim 10$–$15\,\mathrm{kpc}$, with the search geometry oriented outward from the Galactic center.

O2. Compare the $3d_{z^2}$ inertial kernel against NFW, Einasto, and generic dark-disk models using the same Gaia DR3 generalized rotation curves and off-plane constraints.

O3. Search HI and outer-disk gas maps for flaring, compression, or ringlike behavior at the corrected donut scale.

O4. Search Fermi/eROSITA/radio residual maps for weak annular nonthermal emission in the Galactic plane at the predicted scale.

O5. Test whether stellar streams, open clusters, globular clusters, or satellite orbits show perturbations when crossing the predicted annular boundary.

O6. Test whether ultra-high-energy particle or photon-like event directions cluster near the outward Sgr A* equatorial channel, especially near one-sigma offsets such as $\beta \sim -12^\circ$ to -14°.

8 Conclusion

This paper gives the first continuum-limit skeleton of the TFOFT/ASSP ontology of [1]. The primitive variable is the sphere radius; its logarithm is the local clock field:

$$u = \ln \frac{R}{R_0}. \tag{110}$$

The radius-clock field obeys a graph Poisson equation that coarse-grains to Newtonian gravity and to the weak-field g_{00} sector of GR; independently, the Möbius boundary expansion recovers the Newton force coefficient when the final two-surface norm-map cost is $C_{\text{map}} = 2\sqrt{2}$:

$$\Delta_G u = \frac{4\pi G}{c^2}\rho \implies \nabla^2\Phi = 4\pi G\rho, \qquad g_{00} = -c^2 e^{2\Phi/c^2}. \tag{111}$$

$$C_{\text{map}} = 2\sqrt{2}, \qquad v_M = \frac{c}{2\sqrt{2}} \implies a_{M,1} = -\frac{GM}{r^2}. \tag{112}$$

On the same graph, directed light-speed tangency walks with Gyro-Ball chirality mixing coarse-grain to the Dirac equation [2, 8], with the spin connection geometrically induced by inter-sphere frame rotation. Lower-layer callback momentum bookkeeping and U(1) edge-phase holonomies provide a candidate graph route to Maxwell electromagnetism. The tensor sector is not derived here; it is deferred to a microscopic treatment of spatial packing dynamics, frame transport, and two-ball boundary exchange.

GR and QM are not fundamental competitors in this ontology. They are effective continuum shadows of a single finite fractal sphere-packing computation: radius-clock gradients become gravity, directed tangency zigzags become quantum matter, and lower-layer callback momentum/edge-phase windings become gauge fields.

The immediate empirical test is galactic. Using the Fermi-bubble scale of 50 kly $\approx$ 15 kpc put the predicted equatorial $3d_{z^2}$ "Fermi donut" into the outer visible disk, with a fiducial inner shoulder near 8–12 kpc, a peak near 10–15 kpc, one-quarter the polar-lobe angular density contrast, and a preferred outward orientation from the Galactic center. This is exactly the region where Gaia DR3 rotation-curve and dark-disk analyses already show nontrivial, model-sensitive behavior. The next step is to simulate spiral galaxies using hydrogenic ASSP inertial kernels and to test the corrected $3d_{z^2}$ boundary against Gaia, HI, stream, Fermi, eROSITA, and radio data before accepting the full ontology.

Acknowledgments

The author thanks the survey teams whose archival data will enable the observational program of Section 6, and the mathematical physics community whose work on Apollonian packings [4, 5], graph Laplacians, and Ord-style walks [2] provided the technical scaffolding for the derivations above.

References

[1] S. E. Elliott, "The Fact of Fractal Tennis: The Universe as a Fractal Computer Defined by the Star=Electron+Atom=Galaxy Quine," ai.vixra preprint 2605.0003, 2026. https://ai.vixra.org/abs/2605.0003

[2] G. N. Ord, "Fractal space-time: a geometric analogue of relativistic quantum mechanics," *J. Phys. A: Math. Gen.* **16**, 1869 (1983).

[3] F. Soddy, "The Kiss Precise," *Nature* **137**, 1021 (1936).

[4] D. W. Boyd, "The residual set dimension of the Apollonian packing," *Mathematika* **20**, 170 (1973).

[5] R. L. Graham, J. C. Lagarias, C. L. Mallows, A. R. Wilks, and C. H. Yan, "Apollonian circle packings: number theory," *J. Number Theory* **100**, 1 (2003).

[6] B. B. Mandelbrot, *The Fractal Geometry of Nature* (W. H. Freeman, 1983).

[7] M. Presburger, "Über die Vollständigkeit eines gewissen Systems der Arithmetik ganzer Zahlen," *Comptes Rendus du I Congrès des Mathématiciens des Pays Slaves*, Warsaw, 1929, pp. 92–101.

[8] P. A. M. Dirac, "The quantum theory of the electron," *Proc. R. Soc. A* **117**, 610 (1928).

[9] M. Su, T. R. Slatyer, and D. P. Finkbeiner, "Giant Gamma-ray Bubbles from Fermi-LAT: AGN Activity or Bipolar Galactic Wind?" *ApJ* **724**, 1044 (2010).

[10] M. Ackermann et al. (Fermi-LAT Collaboration), "The Spectrum and Morphology of the Fermi Bubbles," *ApJ* **793**, 64 (2014).

[11] K. C. Sarkar, B. Nath, and P. Sharma, "On the origin of the Fermi bubbles," *MNRAS* **453**, 3827 (2015).

[12] Y. Sofue, "Rotation Curve and Mass Distribution in the Galactic Center," *PASJ* **64**(4), 75 (2012).

[13] L. Chemin, C. Carignan, and T. Foster, "Rotation Curve of the Andromeda Galaxy from Combined 21-cm Data," *ApJ* **705**(2), 1395 (2009).

[14] F. Sylos Labini, "Generalized rotation curves of the Milky Way from the GAIA DR3 dataset: constraints on mass models," *Astrophysical Journal* **976**, 185 (2024); arXiv:2410.14307.

[15] O. Koop, T. Antoja, A. Helmi, T. M. Callingham, and C. F. P. Laporte, "On the Galactic rotation curve inferred from the Jeans equations: assessing its robustness using Gaia DR3 and cosmological simulations," arXiv:2405.19028 (2024).

[16] H.-F. Wang, Z. Chrobáková, M. López-Corredoira, and F. Sylos Labini, "Mapping the Milky Way Disk with Gaia DR3: 3D Extended Kinematic Maps and Rotation Curve to ≈ 30 kpc," *Astrophysical Journal* **942**, 12 (2023).

[17] V. Strassen, "Gaussian elimination is not optimal," *Numer. Math.* **13**, 354 (1969).

[18] E. Tiesinga et al., "CODATA recommended values of the fundamental physical constants: 2022," *Rev. Mod. Phys.* **96**, 025002 (2024).