The Collatz Function and Topological Implementation of Catalan's Conjecture

Liam Isaac

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Abstract

The Collatz function is one of the simplest difficult problems in modern mathematics. For any positive integer, multiply any odd integer by 3 and add 1, while any even integer is divided by 2. Take the result and re-insert it into the function. Every integer will eventually fall to 1, and begin a loop of the sequence $1 - 4 - 2 - 1$. Will this function produce another loop at some point? Will the jumps $(3n + 1)$ overtake the drops $(/2)$ and climb to infinity? Through a nested fractal implementation as well as the reduction principle set in Catalan's Conjecture, it is shown that both of these questions are topologically impossible within the system.

1 Base Function(Single Jump)

In its simplest form, we can see the inherent mechanism of the function. Here, "k" is the total drops.

$$f(x) = \frac{3(f(x)) + 1}{2^k} \tag{1}$$

$$2^k(f(x)) = 3(f(x)) + 1 \tag{2}$$

$$2^k(f(x)) - 3(f(x)) = 1 \tag{3}$$

$$(f(x))(2^k - 3) = 1 \tag{4}$$

$$f(x) = \frac{1}{2^k - 3} \tag{5}$$

1.1 Base Function Shortcut(Single Jump)

From here, "k" is only the extra drops, with m representing inherent drops (equal to the number of jumps) while at the same time functionally facilitating normalization.

$$f(x) = \frac{1.5(f(x)) + .5}{2^k} \tag{6}$$

$$2^k(f(x)) = 1.5(f(x)) + .5 \tag{7}$$

$$2^k(f(x)) - 1.5(f(x)) = .5 \tag{8}$$

$$(f(x))(2^k - 1.5) = .5 \tag{9}$$

$$f(x) = \frac{.5}{2^k - 1.5} \tag{10}$$

Normalize by removing decimals(multiply by 2(as 2^m))

$$f(x) = \frac{2(.5)}{2^m(2^k - 1.5)} \tag{11}$$

$$f(x) = \frac{1}{2^{k+1} - 3} \tag{12}$$

$$f(x) = \frac{1}{2^{k+m} - 3^m} \tag{13}$$

2 Nesting Function

2.1 Fractal Nesting(Double Jump)

Now, we substitute the function into its own equation, showing a self-referential pattern.

$$f(x) = \frac{1.5(\frac{1.5(f(x))+.5}{2^{k_1}}) + .5}{2^{k_2}} \tag{14}$$

$$2^{k_2} \cdot f(x) = 1.5\left(\frac{1.5f(x) + 0.5}{2^{k_1}}\right) + 0.5 \tag{15}$$

$$2^{k_1+k_2} \cdot f(x) = 1.5(1.5f(x) + 0.5) + 0.5 \cdot 2^{k_1} \tag{16}$$

$$2^{k_1+k_2} \cdot f(x) = 2.25f(x) + 0.75 + 0.5 \cdot 2^{k_1} \tag{17}$$

$$f(x)(2^{k_1+k_2} - 2.25) = 0.5 \cdot 2^{k_1} + 0.75 \tag{18}$$

Normalize by removing decimals(multiply by 4(as 2^m))

$$f(x)(4 \cdot 2^{k_1+k_2} - 9) = 2 \cdot 2^{k_1} + 3 \tag{19}$$

$$f(x)(2^{k_1+k_2+2} - 9) = 2^{k_1+1} + 3 \tag{20}$$

$$f(x) = \frac{2^{k_1+1} + 3}{2^{k_1+k_2+2} - 9} \tag{21}$$

$$f(x) = \frac{2^{k_1+1} + 3}{2^{k_1+k_2+2} - 3^2} \tag{22}$$

At this point, K becomes the sum of the total extra drops from every layer(k_n).

$$f(x) = \frac{2^{k_1+1} + 3}{2^{K+m} - 3^m} \tag{23}$$

2.2 Fractal Nesting(Triple Jump)

$$f(x) = \frac{1.5(\frac{1.5(\frac{1.5(f(x))+.5}{2^{k_1}})+.5}{2^{k_2}}) + .5}{2^{k_3}} \tag{24}$$

$$2^{k_1+k_2+k_3} \cdot f(x) = 1.5(1.5(1.5f(x) + 0.5) + 0.5 \cdot 2^{k_1}) + 0.5 \cdot 2^{k_1+k_2} \tag{25}$$

$$2^{k_1+k_2+k_3} \cdot f(x) = 3.375f(x) + 1.125 + 0.75 \cdot 2^{k_1} + 0.5 \cdot 2^{k_1+k_2} \tag{26}$$

$$f(x)(2^{k_1+k_2+k_3} - 3.375) = 0.5 \cdot 2^{k_1+k_2} + 0.75 \cdot 2^{k_1} + 1.125 \tag{27}$$

Normalize by removing decimals(multiply by 8(as 2^m))

$$f(x)(8 \cdot 2^{k_1+k_2+k_3} - 27) = 4 \cdot 2^{k_1+k_2} + 6 \cdot 2^{k_1} + 9 \tag{28}$$

$$f(x)(2^3 \cdot 2^{k_1+k_2+k_3} - 27) = 2^2 \cdot 2^{k_1+k_2} + 6 \cdot 2^{k_1} + 9 \tag{29}$$

$$f(x)(2^{k_1+k_2+k_3+3} - 27) = 2^{k_1+k_2+2} + 3 \cdot 2^{k_1+1} + 9 \tag{30}$$

$$f(x) = \frac{2^{k_1+k_2+2} + 3 \cdot 2^{k_1+1} + 9}{2^{k_1+k_2+k_3+3} - 27} \tag{31}$$

$$f(x) = \frac{2^{k_1+k_2+2} + 3 \cdot 2^{k_1+1} + 3^2}{2^{k_1+k_2+k_3+3} - 3^3} \tag{32}$$

2.3 Emergent Pattern

By continuing to add nested layers, a pattern emerges as the following formula, with j defined as the jump stage, and S defined as the partial sum of extra drops at that stage.

$$f(x) = \frac{\Sigma_{j=0}^{m-1} 3^j \cdot 2^{S_{m-1-j}+(m-1-j)}}{2^{K+m} - 3^m} \tag{33}$$

3 Reduction Principle

For any looping number, $f(x)$ is an odd integer that satisfies the above equation. In addition to this, there is another equation that looks quite similar that also needs to be satisfied for any integer.

$$f(x) = \frac{f(x)}{1} \tag{34}$$

The specific numbers in the fraction do not matter, only that they divide equally, but they will always reduce to $\frac{f(x)}{1}$ for ANY integer.

$$5 = \frac{5}{1} = \frac{10}{2} = \frac{15}{3} = \frac{20}{4} \tag{35}$$

So, for any looping integer,

$$2^{K+m} - 3^m = 1 \tag{36}$$

There is only ONE set of exponents that satisfy this equation(Mihailescu,2004), resulting in the 4-2-1 loop, and showing there are no other possible loops.

$$2^{1+1} - 3^1 = 1 \tag{37}$$

This has 1 jump, 1 inherent drop, and 1 extra drop.

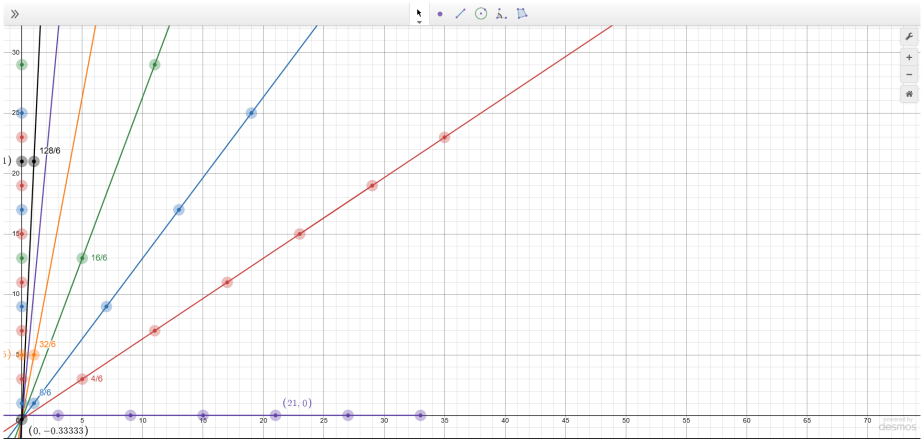

Figure 1: Here, x is the resulting number and y is the origin, with similar drop points connected linearly. The drop lines converge at $(0, -1/3)$

4 Klein Inversion Point

4.1 Limit Operator

$$f(x) = \lim_{m\to\infty} \frac{\Sigma_{j=0}^{m-1} 3^j \cdot 2^{S_{m-1-j}+(m-1-j)}}{2^{K+m} - 3^m} \tag{38}$$

4.2 Dominant Terms

- Numerator: The final term ($j = m - 1$) holds the most influence.
- Denominator: As $m \to \infty$, 2^{K+m}loses all strength to 3^m

4.3 Normalization

- Numerator: $\frac{3^{m-1}}{3^m} \to 1/3$
- Denominator: $\frac{2^{K+m}}{3^m} - \frac{3^m}{3^m} \to 0 - 1 = -1$

4.4 Implementation and Interpretation

$$f(x) = \frac{1/3}{-1} = -\frac{1}{3} \tag{39}$$

Essentially this shows the system is self limiting. Even when attempting to jump infinite times the system rebounds to $-1/3$. This $-1/3$ showed up in another place, though, as the convergence point for the drop lines while graphing drop mechanics(Figure 1).

5 Conclusion

Applying the Reduction Principle as well as Catalan's Conjecture, it is shown that the only possible loop in the Collatz function is $1-4-2-1$. In addition, when pushing the limits of the function towards infinity, the system itself topologically rebounds to a stopping point as a negative fraction of $-\frac{1}{3}$, showing the system as self-bound.

6 Heuristics

6.1 Rational Irrationality

If we look at $-1/3$, its a rational number at first. But when we convert it to its decimal form, it becomes irrational, as $-.333....$ This is an irrational odd number, or a decimal that goes on forever with a repetitive odd number, same as any $\frac{x_{Odd}}{9}$. We can actually take this farther than infinity by plugging in the rational version in to the original Collatz function. The results are, interesting, to say the least.

$$f(x) = 3(-\frac{1}{3}) + 1 = 0 \tag{40}$$

Zero is an even number, and it divides infinitely into itself

$$f(x) = \frac{0}{2^{\infty}} \tag{41}$$

This means $-\frac{1}{3}$ is the last jump before dividing infinitely as the system attempts to re-balance itself.

6.2 Infinity and Division by Zero

From here, we define infinity not as an integer value, but as a state of certain possibility, where division by zero results in infinite possibility. Inversely, by removing the possibility, the result is null. To some extent I understand that dividing by zero breaks some mathematical models, but not why we cant say it reaches infinity, especially when we have evidence of natural phenomena in the form of black holes that were predicted as a result of implementation. In this way, zero and infinity are inherently linked, with a specific numerical possibility (p) dependant on the path taken to achieve the infinity. We say that as a denominator approaches zero, the solution approaches infinity. This implies that AT zero, the solution BECOMES infinity.

$$\frac{1}{.1} = 10, \frac{1}{.01} = 100, \frac{1}{.001} = 1000 \tag{42}$$

$$\frac{1}{0} = \infty, \frac{2}{0} = \infty, \frac{3}{0} = \infty \tag{43}$$

$$\frac{1}{10} = .1, \frac{1}{100} = .01, \frac{1}{1000} = .001 \tag{44}$$

$$\frac{1}{\infty} = 0, \frac{2}{\infty} = 0, \frac{3}{\infty} = 0 \tag{45}$$

$$\infty \cdot 0 = p \tag{46}$$

Since we're looking for $f(x)$ to be infinity, $2^{K+m} - 3^m$ needs to be zero. This, however, results in a contradiction within the Collatz function, as the only way for it to be zero is when $2^{K+m} = 3^m$, with (K, m) as $(0, 0)$. The contradiction is where $m = 0$, as this denotes zero jump functions within the equation. So, the only way for a number to jump to infinity within the Collatz function, is if it doesnt jump at all, but instead is just left in a state of possibility. Mathematics is the language of the universe. Here, the universe is saying "I have no mouth, and I must scream".

7 Sources

Mihăilescu, Preda (2004). "Primary Cyclotomic Units and a Proof of Catalan's Conjecture". Journal für die reine und angewandte Mathematik (Crelle's Journal). 2004 (572): 167–195. doi:10.1515/crll.2004.048.