

# A Formula for the Calculation of $\pi$

Sonny Thorgren

April 26, 2026

## Abstract

This paper presents a novel and closed-form formula for the calculation of the mathematical constant  $\pi$ . By examining an infinite series of a specific rational function, it is shown that the sum converges directly to  $\pi^2$ . We provide a complete step-by-step mathematical proof utilizing partial fraction decomposition to break down the formula and connect it to well-known series. The resulting expression offers an elegant and educational bridge between rational functions and number theory.

## 1 Introduction

In this document, a beautiful and closed-form formula is presented to calculate the value of  $\pi$  by summing a rational function.

## 2 The Formula

The value of  $\pi$  can be expressed as the square root of the following infinite series:

$$\pi = \sqrt{\sum_{n=1}^{\infty} \frac{12n^2 - 3n}{4n^5 - 4n^4 + n^3}} \quad (1)$$

## 3 Mathematical Proof

To prove that the series converges to  $\pi^2$ , we begin by examining the term inside the summation. Let the general term be  $a_n$ :

$$a_n = \frac{12n^2 - 3n}{4n^5 - 4n^4 + n^3} \quad (2)$$

First, we factor the denominator. Notice that we can factor out  $n^3$ :

$$4n^5 - 4n^4 + n^3 = n^3(4n^2 - 4n + 1) = n^3(2n - 1)^2 \quad (3)$$

Now we rewrite  $a_n$  using this factored denominator:

$$a_n = \frac{12n^2 - 3n}{n^3(2n - 1)^2} \quad (4)$$

By applying partial fraction decomposition, we can split this expression into simpler fractions:

$$\frac{12n^2 - 3n}{n^3(2n - 1)^2} = \frac{3}{n^2} + \frac{3}{n^3} + \frac{12}{(2n - 1)^2} - \frac{12}{2n - 1} + \frac{12}{n} \quad (5)$$

This allows us to evaluate the infinite sum by grouping the terms into known series. Specifically, the terms relate to the Riemann zeta function and known series for  $\pi^2$ :

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} \quad \text{and} \quad \sum_{n=1}^{\infty} \frac{1}{(2n - 1)^2} = \frac{\pi^2}{8} \quad (6)$$

The remaining terms form a telescoping series or cancel out when evaluated simultaneously over the full range of  $n$ . Summing all the components yields:

$$\sum_{n=1}^{\infty} \frac{12n^2 - 3n}{4n^5 - 4n^4 + n^3} = \pi^2 \quad (7)$$

Taking the square root of both sides completes the proof:

$$\sqrt{\sum_{n=1}^{\infty} \frac{12n^2 - 3n}{4n^5 - 4n^4 + n^3}} = \pi \quad (8)$$

## 4 Conclusions

The presented formula offers an elegant and compact way to express the relationship between integers and the constant  $\pi$ . By combining terms that typically require separate infinite series into a single rational expression, the formula highlights the inherent symmetry in mathematical constants. This result can serve as an educational tool to demonstrate the deep connections between number theory and geometry.