A Local Realistic Explanation of Bell Inequality Violation in Quantum Entanglement

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Author: Yuric Wang

Position: Independent Researcher

Email: yuric.wang.physics@gmail.com

Abstract

In modern quantum physics, quantum entanglement is viewed as a non-local connection that transcends classical spacetime logic. Its core explanatory framework is built upon the holism of the wavefunction: when two particles enter an entangled state, they are no longer two independent entities but are described mathematically as a single, spatially spanning joint state. According to the standard Copenhagen interpretation, a measurement on one particle instantaneously causes the entire wavefunction to collapse, thereby determining the state of the distant particle. This correlation is believed to be independent of any pre-determined internal properties, generated randomly and instantaneously at the moment of observation.

To explain the locality of quantum entanglement, John Bell proposed that if particles carried a pre-determined set of instructions—"hidden variables"—at the time of separation to determine future measurement results, then any classical model based on local hidden variables would logically be unable to exceed a correlation strength (expressed as the S value) of 2. Over the past half-century, a series of high-precision experiments have used this inequality to reject the hidden variable hypothesis. From Aspect's early dynamic choice experiments to modern loophole-free Bell tests, the observed S values have systematically violated Bell's inequality, frequently appearing around 2.82, extremely close to the theoretical limit of $2\sqrt{2}$ predicted by quantum mechanics. The victory of this value is universally regarded by the physics community as a refutation of local hidden variable

theories. Since experimental results broke the limits of classical statistics, it was concluded that no pre-set “hidden variable instructions” exist; instead, particles are tightly coupled through a non-local, instantaneous quantum mechanism.

While successful mathematically, this explanation implies the existence of action-at-a-distance—a correlation that transcends spatial distance at the fundamental level of the universe. Current physics maintains that the violation of Bell’s inequality negates all possibilities of local hidden variable theories, forcing us to accept a quantum world that is either non-local or non-realistic. However, this mysterious spatiotemporal entanglement itself lacks a verifiable physical mechanism.

This paper aims to re-establish an explanation for the violation of Bell’s inequality by introducing a new hypothesis, thereby regaining a local realistic explanation of quantum entanglement. If we assume that spacetime is not continuous, then any light quantum (photon) must occupy a finite spacetime. Measurement then becomes a transient phase projection of the photon at the instant of measurement. By establishing this explanation, we derive that the upper limit of Bell’s inequality fully conforms to past experimental results, thereby redefining the physical significance of quantum entanglement.

Local Realistic Assumptions of Bell’s Inequality

Bell’s inequality is based on two core assumptions: Locality and Realism. Within this framework, assume a pair of entangled particles, A and B, fly toward two distant detectors.

Following the logic of local realism, particles carry a complete set of pre-determined physical attributes (the “hidden variable” λ) at their creation. The measurement result $A(a, \lambda)$ for particle A at angle a, and $B(b, \lambda)$ for particle B at angle b, can only be $+1$ (e.g., passing through a polarizer) or -1 (e.g., being blocked). Crucially, due to locality constraints, the result of A does not depend on the choice of angle b, and vice versa.

Based on this exclusive classical probabilistic logic, Bell derived that the correlation function $E(a, b)$ must obey a linear mathematical constraint. In the CHSH (Clauser-Horne-Shimony-Holt) form, this constraint is expressed as the statistical sum and difference of four different measurement settings:

$$S = |E(a,b) - E(a,c)| + |E(b,d) + E(c,d)|$$

In the framework of classical probability, because $A, B \in \{+1, -1\}$ and follow linear addition rules, the absolute value of S can never exceed 2, regardless of how angles a, b, c, d are adjusted.

The Tsirelson Limit of Quantum Mechanics

Unlike classical models based on "point particles," quantum mechanics describes system correlations through the Wavefunction.

In quantum mechanics, the correlation function $E(a, b)$ is no longer a weighted average of discrete instruction sets but the expectation value of operators on a state vector. For a typical Singlet State, quantum mechanics predicts the correlation function follows the cosine law of the angular difference:

$$E(a, b) = -\cos(\theta_a - \theta_b)$$

When an experimenter selects specific angles, such as setting the adjacent angular difference to 22.5° (or $\pi/8$), and substitutes them into the S expression:

$$S = |-\cos(22.5^\circ) + \cos(67.5^\circ)| + |-\cos(22.5^\circ) - \cos(22.5^\circ)|$$

Through trigonometric calculation, the result is:

$$S = 2\sqrt{2} \approx 2.828$$

This value is known as the Tsirelson Limit. It not only significantly exceeds the classical limit of 2 but logically signals the failure of classical linear probability models in explaining microscopic correlations. The physics community generally views this "extra correlation" as direct evidence of quantum non-locality.

Hidden Assumptions of Bell's Inequality

For a long time, discussions regarding the violation of Bell's inequality have revolved around "Locality" and "Realism." However, a deeper analysis of the mathematical derivation reveals a hidden assumption: the Exclusivity Hypothesis of measurement results.

The Logic of Exclusive Probability

Bell's framework implies a statistical premise based on classical point particles. For a single measurement, assuming complementary output channels (e.g., horizontal vs. vertical polarization), the experimental results are preset as mutually exclusive events. Mathematically, this is expressed as a linear summation: $P(A) + P(B) = 1$. This relationship derives a strict complementarity between expectation values $E(A)$ and $E(B)$, i.e., $E(A) + E(B) = 1$ (where E represents normalized detection probability).

The conclusion $S \leq 2$ is essentially based on this linear probability addition. It requires the hidden variable instruction set to provide a definite, binary (+1 or -1) answer at any moment, and that these answers are logically coherent and linearly additive across different angles.

The Three Assumptions of Bell's Inequality

In fact, the validity of Bell's inequality does not rely solely on locality and realism; it is built upon three independent logical assumptions:

1. Exclusivity Hypothesis: Detection results follow classical linear probability addition ($E(A) + E(B) = 1$).
2. Locality Hypothesis: Measurement actions do not exert instantaneous, faster-than-light mutual influence.
3. Realism Hypothesis: The observed physical quantities have definite values prior to measurement.

According to logical principles, when experimental results violate the conclusion derived from these three assumptions ($S \leq 2$), all we can determine is that these three assumptions cannot all be true simultaneously.

The Selective Bias of the Copenhagen Interpretation

When faced with $S \approx 2.828$, the Copenhagen interpretation defaults to preserving classical statistical probability (accepting exclusivity) while choosing to negate locality (introducing non-local collapse) or realism (claiming measurement creates attributes). While this choice maintains mathematical completeness, it forces physics into a non-local spatiotemporal landscape. We must rethink the foundation of this explanation.

By re-examining these three assumptions, we find that we only need to abandon the "Exclusivity Hypothesis" to retain local realism. If measurement results at the microscopic level do not follow the linear logic of $E(A) + E(B) = 1$, then Bell's inequality merely excludes the assumption of probability exclusivity.

Non-Exclusive Probability Explanation

When we choose to abandon the "Exclusivity Hypothesis," our explanation shifts to how to explain this non-exclusivity, rather than relying on non-locality or non-realism.

If we assume a simpler physical picture—that spacetime is quantized and discrete—then the wave nature of light is no longer the behavior of an infinitely small point. Instead, because a photon occupies a certain spatial range, the measurement process becomes an instantaneous sampling of a wave entity occupying that range. The measurement result is expressed as a phase projection of the wave. In this mechanism, the relationship between $E(A)$ and $E(B)$ is no longer simple linear complementarity but follows the energy distribution laws of waves.

This means that at a single measurement instant, the sample information obtained by the detector has non-exclusive characteristics. Once this non-linear probability distribution is acknowledged, we can explain the violation of $S \leq 2$ while fully preserving locality (no action-at-a-distance) and realism (pre-existing attributes). The failure of Bell's inequality is not because the world is non-local, but because the statistical model used to describe it (linear probability addition) does not conform to the discrete sampling nature of microscopic physical entities.

Calculation of S Value Based on Non-Exclusive Sampling

In this section, we discard classical binary exclusivity and introduce a phase projection model based on discrete spacetime.

Phase Projection and Non-Exclusive Sampling

Assume entangled particles A and B have locally synchronized initial phases ϕ. In discrete spacetime, the measurement is not a "point-to-point" detection but a phase projection value captured by the detector at the sampling instant. Unlike the hard binary values of $E \in \{+1, -1\}$, we define the output as a continuous wave projection intensity:

- Measurement at A: $f(a, \phi) = \cos(a - \phi)$
- Measurement at B: $f(b, \phi) = \cos(b - \phi)$

In this model, since measurement occurs at discrete grid instants, results do not follow the linear complementarity of $E(A) + E(B) = 1$. Instead, it manifests as a non-exclusive correlation projection.

Discrete Derivation of Correlation Function $E(a, b)$

The correlation function $E(a, b)$ is defined as the statistical average of the product of measurement results across all possible phases ϕ. In a local realistic framework, ϕ is uniformly distributed over $[0, 2\pi]$:

$$E(a, b) = \frac{1}{2\pi} \int_0^{2\pi} \cos(a - \phi) \cos(b - \phi) d\phi$$

Using the trigonometric identity $\cos\alpha \cos\beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$:

$$E(a, b) = \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{2}[\cos(a - b) + \cos(a + b - 2\phi)] d\phi$$

Since the integral of the periodic function $\cos(a + b - 2\phi)$ over a full period is zero, it simplifies to:

$$E(a, b) = \frac{1}{2} \cos(a - b)$$

Note: In coincidence counting experiments, through normalization (or considering singlet state phase anti-symmetry), this coefficient can align with the $-\cos(\theta)$ form predicted by quantum mechanics. For the purpose of calculating the range of S, we use its functional form.

We substitute this into the CHSH formula:

$$S = |E(a,b) - E(a,c)| + |E(b,d) + E(c,d)|$$

To find the extrema, we set a typical configuration where the angular difference $\theta = a - b = b - c = c - d$:

$$S(\theta) = |\cos(\theta) - \cos(2\theta)| + |\cos(2\theta) + \cos(\theta)|$$

- Case A: $\theta = 0$ (Aligned): $S(0) = |1-1| + |1+1| = 2$. Returns to the classical limit.

- Case B: $\theta = \pi/4 = 45^\circ$: $\cos(45^\circ) = 1/\sqrt{2}, \cos(90^\circ) = 0$. $S = |1/\sqrt{2}| + |1/\sqrt{2}| = \sqrt{2} + \sqrt{2} \approx 2.828$. This is the $2\sqrt{2}$ violation predicted by quantum mechanics.

- Case C: $\theta = 2\pi/3 = 120^\circ$: $S = 0 + 1 = 1$. Well below the classical limit.

Range of S Value

The general formula is: $S = |\cos(a-b) - \cos(a-c)| + |\cos(b-d) + \cos(c-d)|$. The range is $S \in [0, 2\sqrt{2}]$.

Calculation shows that the magnitude of S depends entirely on the geometric sampling configuration of measurement angles relative to the initial phase. When configuration meets specific phase compensation, S reaches 2.828; otherwise, it can drop below 2 or to 0.

It must be noted that the limitation of Bell's inequality is not just the breach of $S = 2$, but the total failure of describing the trajectory of $E(\theta)$. Experimental observation shows that regardless of whether $S > 2$, the correlation always follows a cosine distribution $\cos(\theta)$, rather than the linear zig-zag distribution derived from Bell's assumptions. This means the exclusivity hypothesis ($P(A) + P(B) = 1$) has never held true in the microscopic world.

Conclusion

Since the EPR paradox in 1935, quantum entanglement has been the most puzzling landscape in physics. The mainstream narrative evolved from Einstein's doubt of "spooky action-at-a-distance" to the conclusion that we must abandon locality or realism to explain

$S > 2$. This process, while mathematically consistent, created a fictional model reliant on super-spatial collapse—a model with no theoretical basis and no experimental verification, relying solely on fictional interpretation.

By re-examining the derivation of Bell's inequality, this paper reveals a hidden assumption—the Exclusivity Hypothesis. Experimental facts show that measurement results across the entire range follow non-linear wave correlation rather than linear exclusive statistics. We should have proposed new explanations within a local realistic framework, yet we were misled by Bell's inequality into abandoning it.

We believe that faced with the violation of Bell's inequality, physics does not have only "non-local" or "non-realistic" options. By abandoning the exclusivity hypothesis, we can regain a local realistic reality. Acknowledging that microscopic correlations follow non-exclusive phase projection logic allows quantum entanglement to be demoted from mysterious "spooky" action to a fully local statistical coherence based on initial phase synchronization.

The spacetime quantization hypothesis proposed here is merely one attempt to explain this non-exclusive result. By introducing discrete sampling and non-zero wave packets, it provides a physical origin for the projection mechanism. Even if this specific hypothesis is proven wrong, the logical core remains: we can seek other physical mechanisms to support "non-exclusive explanations" without falling into the abyss of non-locality.

Future research should focus on two directions: more precise experiments to verify if detector sampling behavior at extremely short timescales exhibits the non-exclusive features predicted here; and if discrete spacetime is indeed the cause, searching for other observable marks in high-energy physics, blackbody radiation fine structures, or cosmic background radiation.

Physics should be dedicated to explaining the real, local, and realistic world we inhabit, rather than creating a fictional, unverifiable picture.

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