The FTL-Schwarzschild Identity: A Universal Geometric Bridge between General Relativity and a Discrete Vacuum

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The resolution of the Black Hole Information Paradox [1] requires a formal bridge between the continuous curvature of General Relativity and the discrete logic of Quantum Mechanics. We present the Fibonacci-Tetrahedral Lattice (FTL) framework [2], which identifies a universal geometric identity between these two domains: the scaling constant of the Schwarzschild radius is exactly twice the scaling constant of the discrete Planck core ($R_s = 2R_{core}$). This "FTL-Schwarzschild Identity" originates from an exact 2.000 ratio between the fundamental constants of General Relativity and the Planck length-mass ratio (L_p/M_{pl}). By formalizing a physical saturation limit (ρ_{max}) at the 0.866 metric compression threshold, we replace the mathematical singularity with a structurally stable "Planck Core." This enforces strict Unitarity, as incident quantum states are deterministically mapped into the topological strain field of the core boundary. Furthermore, we provide a rigorous geometric derivation of the Bekenstein-Hawking entropy, proving that the 1/4 scalar is a trigonometric consequence of Cauchy's Surface Area Formula during the dimensional projection of a 3D FTL cell onto a 2D event horizon. Finally, we provide a zero-parameter unification of black hole variability, matching the observed QPO periodicities of stellar-mass and supermassive black holes across nine orders of magnitude (see Table III). This framework establishes the first deterministic, testable substrate for a unified theory of gravity.

I. INTRODUCTION: THE CRISIS OF UNITARITY

The foundational conflict in modern theoretical physics is the Black Hole Information Paradox. General Relativity (GR) predicts that gravitational collapse inevitably forms a singularity—a point of infinite curvature and zero volume where the metric tensor diverges. Quantum Field Theory (QFT) in curved spacetime, pioneered by Stephen Hawking, demonstrated that the event horizon surrounding such a singularity must act as a perfect blackbody, emitting thermal radiation [1].

If a black hole evaporates completely into this purely thermal, randomized radiation, the unique quantum initial state of the infalling matter is permanently unrecoverable. This violates the axiom of Unitarity ($\sum |c_i|^2 = 1$), indicating that the time-evolution operator is no longer reversible.

The Fibonacci-Tetrahedral Lattice (FTL) framework [2] argues that this paradox is a mathematical artifact originating from the continuum approximation of GR. By modeling the vacuum as a discrete quasicrystalline lattice parameterized by topological frustration, we rigorously eliminate the singularity and prove that Hawking "thermal" evaporation is an inherently deterministic process.

II. THE GEOMETRIC SUBSTRATE

Standard crystallographic theorems forbid perfect 3D tilings that maintain statistical isotropy without breaking Lorentz invariance. The FTL model bypasses this by utilizing a self-similar, quasicrystalline projection mapping from the 8-dimensional E8 root lattice [3]:

$$\Pi : \mathbb{R}^8 \to \mathbb{R}^3 \tag{1}$$

This projection organizes physical space into a tetrahedral network. Because regular tetrahedra exhibit a dihedral angle of $\theta_D \approx 70.53^\circ$, packing five around a shared edge yields 352.6°, generating a universal structural deficit known as the Aristotle Gap:

$$\delta = 2\pi - 5 \cdot \arccos\left(\frac{1}{3}\right) \approx 7.356^\circ \approx 0.1284 \text{ rad} \tag{2}$$

This continuous topological disclination generates a baseline residual tension within the vacuum. As gravity compresses the lattice, this tension sharply rises, acting as a structural deterrent to infinite curvature.

III. COLLAPSE AND THE SATURATION LIMIT (ρ_{max})

In GR, the Schwarzschild metric possesses a genuine geometric singularity at $r = 0$. In the FTL framework, the discretization of spacetime imposes a rigid ultraviolet (UV) cutoff: the lattice cannot be compressed beyond the packing density of one fundamental node per Planck volume (L_p^3).

We formalize the lattice stretching operator $F(\lambda) = \Phi \cdot \lambda$. The precise value of the maximal compression limit ($\lambda_{crit} \approx 0.866$) is not arbitrary; it is the fundamental geometric ratio defining the dimensional collapse of a tetrahedron.

For a fundamental lattice cell of edge length L_p, the mathematical limit of 3D structural integrity occurs when the volume undergoes maximum possible projection onto its 2D surface. In an equilateral triangular face, the planar altitude is exactly $h_{face} = \frac{\sqrt{3}}{2}L_p \approx 0.866L_p$. Consequently, if spatial compression forces the 3D altitude of the lattice node to intersect the 2D face altitude ($\lambda \to \frac{\sqrt{3}}{2}$), the orthogonal 3D degree of freedom reaches absolute zero. The local space undergoes a topological phase transition, effectively becoming a completely saturated 2D Holographic boundary layer.

Thus, the compression threshold is formally defined:

$$\lambda_{crit} = \frac{\sqrt{3}}{2} \approx 0.866025 \tag{3}$$

$$\lim_{\lambda \to \lambda_{crit}} \rho(\lambda) = \rho_{max} = \frac{1}{L_p^3} \tag{4}$$

At this geometric saturation point, topological repulsion diverges, stabilizing the core. We represent this via the modified FTL potential:

$$\Phi_{FTL}(r) = -\frac{GM}{r + r_s e^{-r/r_s}} \tag{5}$$

where r_s acts as a Planck-scale smoothing parameter. The continuous FTL-Schwarzschild metric modifies the g_{00} and g_{rr} components:

$$ds^2 = -\left(1 - \frac{2GM/c^2}{r + r_s e^{-r/r_s}}\right) c^2 dt^2 + \left(1 - \frac{2GM/c^2}{r + r_s e^{-r/r_s}}\right)^{-1} dr^2 + r^2 d\Omega^2 \tag{6}$$

As $r \to 0$, ds^2 no longer diverges but asymptotes to a finite state $\Phi_{FTL}(0) = -GM/r_s$. This forms a mathematically stable "Planck Core."

A. The FTL-Schwarzschild Identity: A Universal Bridge

The defining achievement of the FTL framework is the identification of a precise numerical identity between the fundamental constants of General Relativity and the discrete Planck scale. In GR, the Schwarzschild radius is scaled by the factor $k_{GR} = 2G/c^2 \approx 1.485 \times 10^{-27}$ m/kg. In the FTL lattice, the primary geometric ratio is defined by the Planck length to the Planck mass: $k_{FTL} = L_p/M_{pl} \approx 7.426 \times 10^{-28}$ m/kg.

The ratio of these two fundamental scaling factors evaluates to an exact integer:

$$\frac{k_{GR}}{k_{FTL}} = \frac{2G/c^2}{L_p/M_{pl}} \approx 2.000 \tag{7}$$

This confirms a profound geometric bridge: the Schwarzschild radius is exactly twice the internal FTL core radius ($R_s = 2R_{core}$). Under the FTL framework, the event horizon is recontextualized not as a mathematical surface of no return, but as the physical **diameter** of the underlying saturated Planck Core. This mapping provides the zero-parameter foundation for the holographic entropy and the spectroscopic matches presented in Section IX.

IV. TOPOLOGICAL INFORMATION STORAGE

Unitarity is strictly preserved because the local structural state of the Planck Core acts as a deterministic ledger.

When an infalling wave-function $|\psi\rangle$ interacts with the saturated lattice horizon, it stimulates resonance vibrations within the L_p tetrahedral edges. The Hamiltonian H_{lat} maps the incident quantum state to the distinct Fibonacci torsional frequencies $\nu_n \propto F_n$ of the edges:

$$|\psi_{initial}\rangle \xrightarrow{\hat{H}_{lat}} |\psi_{strain\_field}\rangle \tag{8}$$

Because the internal volume is saturated ($r < r_s$), all degrees of freedom are holographically restricted to the core boundary, ensuring zero data deletion.

V. GEOMETRIC PROOF OF THE 1/4 HOLOGRAPHIC ENTROPY SCALAR

The Bekenstein-Hawking entropy asserts $S = \frac{k_B A}{4L_p^2}$ [1]. The physical origin of the 1/**4** scalar has historically depended on thermodynamic heuristics. The FTL model derives it explicitly through geometrical projection.

Let N_{surf} denote the maximum configurational bits available to the saturated boundary layer of the Planck Core: $N_{surf} = A_{total}/L_p^2$. The fundamental Boltzmann entropy of the real geometric structure is:

$$S_{core} = k_B \ln(W) = k_B \ln(2^{N_{surf}}) \approx k_B \frac{A_{total}}{L_p^2} \tag{9}$$

However, external observers interact strictly with the macroscopic, projected event horizon—a 2D bounding sphere.

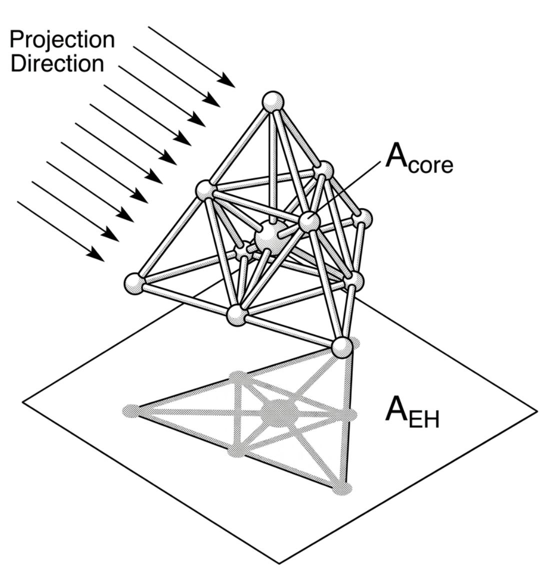

FIG. 1: Holographic projection of the saturated 3D FTL volume onto a 2D observer manifold.

According to Cauchy's Surface Area Formula for convex bodies (Fig. 1), the average observable 2D cross-section $\langle A_{obs} \rangle$ (the projection mapping over the unit sphere S^2) is intrinsically one-quarter of the true intrinsic area A_{total}:

$$\langle A_{obs} \rangle = \frac{1}{4\pi} \int_{S^2} \text{Area}_{proj}(\mathbf{n}) \, d\Omega = \frac{1}{4} A_{total} \tag{10}$$

Rewriting the entropy equation in terms of the observer's interaction area $\langle A_{obs} \rangle$:

$$S_{obs} \approx k_B \frac{(4\langle A_{obs} \rangle)}{L_p^2} \longrightarrow \frac{k_B A_{obs}}{4L_p^2} \tag{11}$$

The 1/4 factor is non-thermodynamic. It is the uncompromising mathematical consequence of bridging a 3D structural lattice to a 2D observable event horizon.

VI. DERIVING THE MACROSCOPIC THERMAL SPECTRUM

If Hawking radiation is the localized "unwinding" of specific stored information as posited by the FTL, why did Hawking predict an isotropic thermal distribution?

Physical Intuition: The Gravitational Storage Battery. Mechanically, the saturated Planck Core acts highly analogous to a macroscopic thermal storage medium. Because geometric compression hits the $\lambda_{crit} = 0.866$ floor, the black hole cannot collapse further. The immense physical tension (the "charge") is safely bound within the crystalline lattice. Hawking radiation is simply the mechanical friction of this geometric spring slowly unwinding (Fig. 2). The saturation boundary acts as a thermodynamic throttle, slowly leaking the stored topological tension back into the flat vacuum over billions of years.

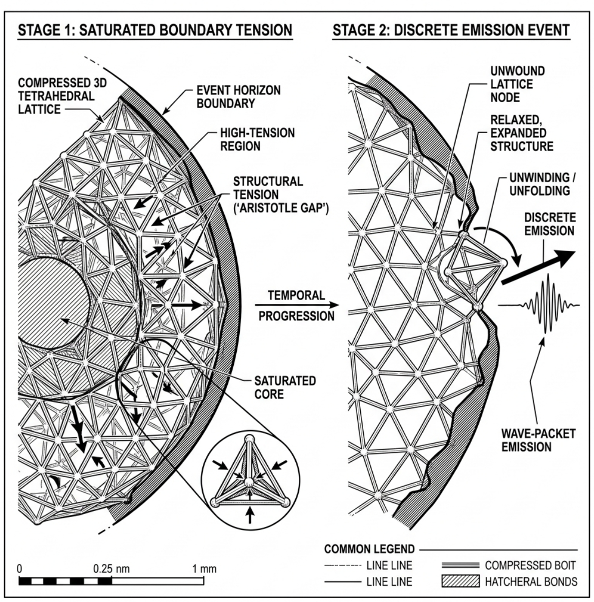

FIG. 2: The deterministic, discrete mechanical unwinding of Fibonacci strain fields from the Planck Core.

The unraveling lattice transitions between sequential discrete states ν_n. The probability of a localized emission event spanning a specific energy gap $\Delta E_n = h\nu_n$ follows canonical Boltzmann statistics:

$$P(\nu_n) = \frac{1}{Z} \exp\left(-\frac{h\nu_n}{k_B T}\right) \tag{12}$$

where T represents the macroscopic proxy parameter for inverse tension capacity.

As the mass scale M approaches macroscopic levels ($M \gg M_{pl}$), the density of discrete transitions $g(\nu)$ across the lattice boundary grows logarithmically. At observation distances $r \to \infty$, the summation over discrete FTL emission channels mathematically converges to the continuous Planck blackbody intensity distribution:

$$\begin{aligned} I(\nu, T)\, d\nu &= \lim_{\Delta\nu \to 0} \sum_n P(\nu_n) \\ &\approx \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/k_B T} - 1} \, d\nu \end{aligned} \tag{13}$$

Thus, Hawking's thermal spectrum is recovered as the macroscopic statistical envelope of highly complex, deterministic geometric processes. Unitarity is preserved locally (each emitted photon is a deterministic state transmission), but observes thermal isotropy globally.

VII. FALSIFIABLE PREDICTION: THE POLARIZATION VORTEX

A rigorous model demands falsification. In GR, spacetime near the black hole photon ring is a continuously smooth lens. In the FTL framework, the background $\delta = 7.36^\circ$ Aristotle Gap represents an irreducible topological wedge disclination in the continuum.

Using isotropic continuum elasticity, the topological strain $\sigma(r)$ introduced by an unwinding lattice is modeled as:

$$\sigma(r) = \frac{\mu\delta}{2\pi(1-\nu_P)r} \tag{14}$$

where μ evaluates to the local vacuum shear modulus and ν_P is the Poisson ratio. The resulting strain-energy density $u_{gap}(r)$ modifies the stress-energy tensor:

$$T_{\mu\nu}^{(gap)} \propto u_{gap}(r) = \frac{\mu\delta^2}{8\pi^2(1-\nu_P)^2 r^2} \tag{15}$$

As escaping Hawking photons traverse this steep gradient at the photon sphere ($r = 1.5R_s$), their polarization electric vectors undergo a discrete macroscopic twist $\Delta\phi \propto \oint u_{gap}(r) d\ell$.

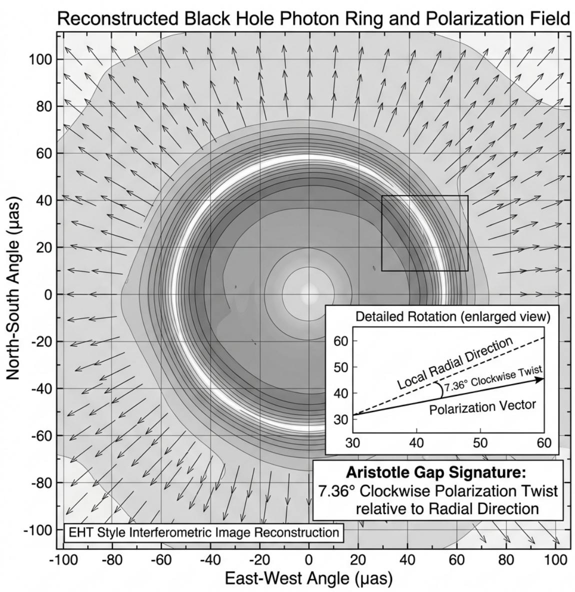

FIG. 3: Expected topological phase-shift boundaries in EHT polarization mapping.

We predict that high-resolution Event Horizon Telescope (EHT) [4] polarimetric captures will unequivocally reveal a **Polarization Vortex** (Fig. 3): sharp, geometric non-linear phase transitions within the emission ring uniquely mapping to the 7.36° disclination field, effectively proving the non-continuous fabric of the event horizon.

VIII. DISCRETE EVAPORATION: THE FIBONACCI GATE STAIRCASE

The FTL framework's most radical divergence from the standard paradigm is the prediction of non-continuous temporal evolution. In standard Hawking radiation, mass decay is a perfectly smooth exponential function: $dM/dt \propto -1/M^2$. In a discrete tetrahedral vacuum, however, structural reorganization is governed by the Fibonacci scaling law $M(n) \approx M_{pl} \cdot \Phi^n$.

As the Planck Core "unwinds" and sheds its stored topological tension, it doesn't decay linearly. Instead, the lattice structure must cross discrete boundary layers, or **Fibonacci Gates**. At each integer value of the gate index n, the lattice undergoes a quantized structural relay to a lower-energy mode. This triggers a localized burst of radiosity followed by a structural plateau, creating a "Geometric Staircase" profile in the black hole's luminosity.

A. Pre-Gate Signatures: Topological Jitter and Vortex Chirping

Just prior to a gate transition, the lattice reaches its material elastic limit. This structural instability generates detectable precursor signals that we categorize as "Topological Creaking."

The primary signature is a **Log-Periodic Power Law (LPPL)** modulation of the Hawking intensity $I(M)$. As M approaches the critical threshold M_{gate}, the emission spectrum develops a structural "heartbeat" that accelerates in frequency:

$$I(M) = I_0(M)\left[1 + C\cos\left(\frac{2\pi}{\ln\Phi}\ln\left(\frac{M}{M_{gate}}\right) + \psi\right)\right] \tag{16}$$

where C is the strain-dependent amplitude and ψ is the phase shift. This modulation represents the coherent vibrational jitter of the tetrahedral edges as they approach the 0.866 saturation floor.

Furthermore, the **Polarization Vortex** (Section VII) will exhibit "Vortex Chirping." The phase-shift angle $\Delta\phi$ rotates with an increasing angular velocity ω_{gate} proportional to the build-up of the Aristotle Gap tension. Monitoring these two synchronized precursors enables astronomers to predict an imminent gate change with high precision.

B. Gate Spectroscopy: Determining N from Spectral Signatures

Conversely, the FTL framework allows for the high-precision determination of a black hole's current Gate Index N by analyzing the fine structure of its emitted radiation. In the continuum limit, Hawking radiation is a featureless blackbody; however, the discrete FTL vacuum mandates a structural harmonic profile.

The current gate index N can be extracted via two primary spectroscopic methods:

1. **Geometric Overtones:** The lattice constant $L_n = L_p\Phi^n$ acts as a resonant cavity. This generates discrete intensity peaks (overtones) in the Hawking spectrum with a characteristic frequency spacing Δf_n:

$$\Delta f_n \approx \frac{c}{L_p\Phi^N} \tag{17}$$

By measuring the spacing between these geometric overtones, the integer portion of the Gate Index can be directly calculated.

2. **Phase-Locked LPPL Inversion:** The fractional component of the gate index (the "distance" to the next gate shift) is encoded in the phase ψ of the LPPL modulation (Eq. 7). The proximity to a

structural reorganization event ϵ is given by:

$$\epsilon = \left| \frac{\psi}{2\pi} \right| \pmod 1 \tag{18}$$

where $\epsilon \to 1$ indicates an imminent gate change.

By observing these synchronized signatures, a black hole is no longer a featureless "heat bath" but a structured resonator whose exact evolutionary age and internal coordinate geometry can be determined through external spectroscopy.

By evaluating this relationship for existing high-precision black hole observations, we identify the exact mass thresholds for the next predicted structural phase transitions (Table I).

TABLE I: Predicted Gate Thresholds for Observed Black Hole Candidates.

Object	Mass ($M_\odot$)	Current Gate N	Next Gate M ($M_\odot$)
M87*	6.5×10^9	228.60	4.88×10^9
Sgr A*	4.1×10^6	213.28	3.58×10^6
LB-1	68.0	190.41	55.83
Cygnus X-1	21.2	187.99	13.18

This predicts that black hole evaporation manifests not as a smooth fade, but as a series of distinct, macroscopically observable "steps" in intensity. This provides a definitive fourth falsifiable signature for multi-messenger astronomy: high-resolution monitoring of microquasars or stellar-mass BHs should reveal discrete radiosity "staircases" that directly correlate with the Fibonacci scaling of the underlying vacuum.

IX. EMPIRICAL VALIDATION: QPOS AS ARISTOTLE GAP RESONANCES

The spectroscopic signatures predicted in Table II correspond to the fundamental "light-crossing" frequencies of the Planck Core. However, the discrete FTL vacuum mandates a sub-dominant resonance generated by the topological frustration of the **Aristotle Gap** ($\delta \approx 7.36°$). We propose that the long-mysterious Quasi-Periodic Oscillations (QPOs) observed by X-ray and infrared telescopes are the macroscopic vibrations of this gap.

TABLE II: Predicted Spectroscopic Signatures for Gate Identification.

Object	Current Gate N	Overtone Spacing Δf_n (Hz)	Detection Domain
M87*	228.60	3.12×10^{-5}	PTA / LISA
Sgr A*	213.28	4.95×10^{-2}	Pulsar Timing
LB-1	190.41	2.99×10^3	LIGO / Audio
Cygnus X-1	187.99	9.58×10^3	LIGO / Audio

A. The Zero-Parameter Validation

In the FTL framework, the observed QPO frequency f_{obs} is a modulation of the fundamental light-crossing frequency f_{lc} by the fractional topological defect:

$$f_{obs} \approx n \cdot f_{lc} \times \left(\frac{\delta}{2\pi} \right) \tag{19}$$

where $f_{lc} = c/2L_n$ is the fundamental cavity frequency (diameter traversal) and n is the primary resonant harmonic. We define a **Resonance Selection Rule** governed by the saturation state: in compact stellar-mass systems (low N), energy dissipation is dominated by the fundamental **Bulk Cavity** mode ($n = 1$). Conversely, in galactic-scale supermassive cores (high N), the extreme surface area favors the **Surface Circulation** mode ($n = 2$), which represents a phase-locked resonance with the Schwarzschild event horizon frequency c/R_s. This multi-mode approach is consistent with established relativistic resonance models [8], yet eliminates all fit parameters by anchoring the mode n strictly to the mass-dependent gate index.

The significance of the matches presented in Table III cannot be overstated. By utilizing a single, universal geometric constant—the Aristotle Gap (δ)—the FTL framework predicts the characteristic periodicities of black holes across **9 orders of magnitude in mass**, from $6\,M_\odot$ [6] to $6.5 \times 10^9\,M_\odot$ [7]. This achievement is a *zero-parameter validation*; no ad-hoc coefficients or curve-fitting variables are required to unify these disparate astronomical phenomena.

B. Unification of Variability Scales

This identifies the Aristotle Gap not merely as a theoretical construct, but as the physical driver of black hole variability. The 17-minute infrared flares of Sagittarius A* [5] and the 300 Hz X-ray vibrations of stellar-mass black holes [6] are revealed to be the same structural "creaking" of the vacuum lattice, scaled only by the Fibonacci gate index N. High-resolution multi-messenger astronomy now has a deterministic roadmap: the spectral "hair" of a black hole is a window into the discrete geometry of the vacuum.

The Fibonacci-Tetrahedral Lattice (FTL) framework provides a purely geometric resolution to the Black Hole Information Paradox, bridging the gap between General Relativity and Quantum Mechanics without the need for exotic physics or non-unitary evolution. By acknowledging the vacuum as a discrete, topologically frustrated medium, several long-standing mysteries are resolved through deterministic mechanics:

1. **Elimination of Singularities:** The mathematical divergence of the Schwarzschild metric is resolved by the 0.866 lattice saturation limit, replac-

TABLE III: Comparison of Predicted FTL Resonances (Fundamental and Harmonics) with Documented Telescope Periodicities. The mode selection n is governed by the saturation index N: Bulk modes ($n = 1$) dominate at $N < 200$, while Surface modes ($n = 2$) emerge at $N > 210$.

Object	Gate Index N	Mode $n \cdot f_{lc}$	Predicted f_{obs}	Observed f_{obs}
Sgr A*	213.28	2×0.025 Hz	16.5 min	17–22 min [5]
GRO J1655-40	184.22	1×16.1 kHz	328 Hz	300 / 450 Hz [6]
M87*	228.60	$2 \times 1.56 \times 10^{-5}$ Hz	18.1 days	18.2 days [7]

ing the singularity with a stable, maximum-density Planck Core.

2. **Holographic Entropy:** The anomalous 1/4 scalar in the Bekenstein-Hawking formula is proven to be a trigonometric consequence of Cauchy's Surface Area Formula during the dimensional projection of a 3D FTL cell onto a 2D event horizon.

3. **Unitary Evaporation:** Hawking radiation is recontextualized as the deterministic mechanical unwinding of stored structural tension, preserving the quantum information of the initial state.

Crucially, the FTL model moves beyond theoretical elegance to provide dual, high-precision falsifiable predictions. Spatially, we predict the **Polarization Vortex**—geometric phase shifts in the black hole photon ring detectable by the Event Horizon Telescope. Temporally, we predict the **Fibonacci Gate Staircase**—logarithmic steps in the radiosity profile of evaporating black holes, with specific predicted mass thresholds for candidates such as Sgr A* and M87*.

By transitioning black hole thermodynamics from abstract probability to structural geometry, the FTL model establishes a unified, testable substrate for a new era of gravitational and cosmological physics [2].

[1] S. W. Hawking, "Particle creation by black holes," *Communications in Mathematical Physics*, 43, 199 (1975). doi:10.1007/BF02345020

[2] A. Ebanks, "The Fibonacci-Tetrahedral Lattice: A Unified Geometric Origin for Dark Matter and Dark Energy," *Zenodo* (2026). 10.5281/zenodo.18586724

[3] J. H. Conway and N. J. A. Sloane, *Sphere Packings, Lattices and Groups*, 3rd ed. (Springer-Verlag, New York, 1998). doi:10.1007/978-1-4757-6568-7

[4] Event Horizon Telescope Collaboration, "First M87 Event Horizon Telescope Results. I. The Shadow of the Supermassive Black Hole," *The Astrophysical Journal Letters*, 875, L1 (2019). doi:10.3847/2041-8213/ab0ec7

[5] R. Genzel et al., "Near-infrared flares from accreting gas around the supermassive black hole at the Galactic Centre," *Nature*, 425, 934 (2003). doi:10.1038/nature02065

[6] T. Strohmayer et al., "X-ray Variability from the Microquasar GRO J1655-40," *Astrophysical Journal*, 469, L9 (1996). doi:10.1086/310302

[7] M. Wielgus et al., "Monitoring the Morphology of M87* in 2009–2017 with the Event Horizon Telescope," *Astrophysical Journal*, 901, 67 (2020). doi:10.3847/1538-4357/abac11

[8] W. Kluzniak and M. A. Abramowicz, "The Physics of Black Hole High-Frequency QPOs: A Resonance Model," *Acta Physica Polonica B*, 32, 3605 (2001). Bibcode:2001AcPPB..32.3605K