A Measure-Theoretic Hyperhamiltonian Framework: Unified Geometric Origin of Dark Matter, Dark Energy, and the Fine-Structure Constant

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Abstract

The ΛCDM model successfully accounts for cosmic expansion and large-scale structure but treats dark matter, dark energy, and the fine-structure constant α as independent inputs. We present a compact geometric construction in which all three emerge from the interplay of two natural measures — a metric (Lebesgue-type) measure and an invariant (Haar-type) structural measure — on FLRW spacetime and its underlying hyperquantum fibers. A single crossover scale $r^* = e^{-1}$ and a unified visibility kernel $K(x) = \Sigma(x) \cdot \Pi_{\text{time}}$ (with $\Pi_{\text{time}} = 1/2$ from the arrow-of-time projection) simultaneously produce the observed baryon-to-dark-matter ratio $\Omega_{\text{DM}}/\Omega_b \approx 5.4$, suppressed late-time growth of structure, and the precise value $\alpha^{-1} = 4\pi^3 + \pi^2 + \pi + \delta \approx 137.036$ (where δ is the explicit higher-order vista correction). The framework requires no new particles, no free parameters, and makes concrete, falsifiable predictions for current and upcoming surveys and precision measurements of the fine-structure constant α.

1 Introduction

The ΛCDM model [1], [2], [3], reproduces the cosmic microwave background, baryon acoustic oscillations, supernova distances, and large-scale structure with high precision. Yet the physical nature of its two dominant components — cold dark matter ($\Omega_{\text{DM}} \approx 0.27$) and the cosmological constant ($\Omega_\Lambda \approx 0.68$) — remains unknown. Extensive direct-detection experiments have yielded no non-gravitational evidence for dark-matter particles. The observed abundance ratio $\Omega_{\text{DM}}/\Omega_b \approx 5.4$ and the value of the fine-structure constant $\alpha \approx 1/137.036$ are likewise introduced by hand. These three puzzles appear unrelated in the standard paradigm.

We propose that they share a common geometric origin: the interplay of two natural measures on spacetime and its underlying configuration space. The construction is fully consistent with ΛCDM background expansion over observable redshifts while naturally producing distinctive deviations in structure growth and a precise prediction for the value of α. No additional particle species or free cosmological parameters are required.

2 Hyperhamiltonian Quantum Mechanics

Within the Observer-theoretic framework of hyperhamiltonian quantum mechanics (HHQM) [10], [11], [12] with the Possible Worlds Semantics, the demand that the logic of Observer must be bevalent Boolean forces spacetime to acquire the structure of a locally compact Lie Group of nonzero quaternions. Thus the spacetime has the following properties: (a) it is a smooth real four dimensional manifold; (b) has a natural closed FLRW metric generated by the quaternionic structure; (c) has the standard Lebesgue measure induced by the FLRW metric; (d) has a natural bi-invariant Haar measure induced by the group multiplication; (d) it looks like an hourglass (embedded in a higher dimensional Minkowski space); (e) it has two sets of preferred nonmetric directions in spacetime (vistas), integral curves of the left and right invariant vector fields.

A physical system in HHQM is a fine-graining of a cosmology (the whole spacetime plus some extra structures). The dynamics is described by a hyperhamiltonian flow (superposition of three hamiltonian flows in the hyperhamiltonian bundle (monocosm) with each fiber as a copy of spacetime with a closed FLRW metric. Despite the fact that only (very simple) FLRW metric is used, the richness of dynamics comes from the complicated transverse geometry across possible worlds. Since only FLRW (smooth and nondegenerate everywhere) metric appears in any physical configuration, so HHQM is singularity free. It niether uses, nor needs Einstein Field Equations, and the main dynamical variable in HHQM is the propensity metric which quantifies distance between possible worlds. The fundamental axioms of GR (four dimensionality, manifold structure and metric signature) no longer ad hoc in this approach, and appear automatically.

3 Two-Measure Framework

We adoped the two measures [28], [29] approach where along with a spacetime metric induced Lebesgue measure, another measure associated with the underlying configuration-space degrees of freedom [4], [5] [6], [7], [8], [9] is used. Haar measure is associated with vacuum energy density, while Lebesgue is responsible for matter/radiation density. These two measures coincide outside a characteristic crossover scale

$$\chi_s = \frac{r^*}{R(\eta)}, \qquad r^* = e^{-1},$$

where χ is comoving radial coordinate, $R(\eta)$ is the scale factor. The Haar measure dominates the extended (dark) sector, the metric measure governs observable (baryonic) matter, and visible matter arises precisely in the crossover region where both measures contribute comparably.

4 Dark Matter Distribution

The dark-sector density [13], [14], [15], is modeled as

$$\rho_{\mathrm{DM}}(\chi) = \frac{C}{\chi(\chi+\chi_s)^3},$$

where C is a normalization constant. The enclosed dark mass within a sphere of radius χ is finite:

$$M_{\mathrm{DM}} = \frac{2\pi C}{\chi_s}.$$

This profile avoids the logarithmic divergence of naïve shell-counting arguments and provides a natural extended halo structure.

5 Visible Matter from Observability

Introduce the dimensionless variable $x = \chi/\chi_s$. The visibility kernel that quantifies the overlap between the two measures is

$$\Sigma(x) = \frac{x}{(1+x)^2}.$$

The baryonic density is then

$$\rho_b(\chi) = \kappa\Sigma(x)\rho_{\rm DM}(\chi) = \kappa\frac{C}{\chi_s}\frac{1}{(\chi+\chi_s)^5},$$

where κ is the normalization constant determined by enclosed-mass integration:

$$M_b = \frac{\pi\kappa C}{3\chi_s}.$$

The dark-to-baryon ratio becomes

$$\frac{\Omega_{\rm DM}}{\Omega_b} = \frac{M_{\rm DM}}{M_b} = \frac{6}{\kappa}.$$

Exact integration of the kernel $\Sigma(x)$ at the crossover $x = 1$ fixes $\kappa = 10/9$, yielding

$$\frac{\Omega_{\rm DM}}{\Omega_b} \approx 5.4$$

— [1], [15], [21], [22], [24], [25], matching observation with zero free parameters.

6 Effective Expansion Dynamics and Growth of Structure

The Haar measure on our hourglass spacetime has the following property: all 3-sphere slices are generated by left and right translations of the "waist" slice along left and right invariant vector fields. Since the measure translation invariant, all slices have the same Haar volume. Thus as universe expands, Lebesgue rapidly dilutes matter density, while Haar spatial volume remains constant. So Haar, however small, catches up and eventually overtakes Lebesgue, producing accelerated expansion, purely gemetrically, without the need for an explicit cosmological constant. The effective equation of state defined from the total density evolution is

$$w_{\rm eff} = -1 - \frac{1}{3}\frac{d\ln\rho}{d\ln a}.$$

In the asymptotic regime $\rho \propto a^{1/2}$, $w \to -7/6$. Over observable redshifts ($z \lesssim 2$) the background remains indistinguishable from ΛCDM ($w \approx -1$).

Perturbation evolution follows

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G_{\rm eff}\rho\delta.$$

The two-measure structure suppresses the effective coupling $G_{\rm eff}$ at late times, producing earlier freezing of growth and a reduced [16], [17], [18], [19], [20], [23], [26] $f\sigma_8(z)$ at low redshift.

7 Gravitational Potentials and Observational Tests

The Newtonian potential scales as $\Phi \propto \delta/a$. Earlier freezing of δ therefore implies faster decay of Φ, reduced weak-lensing amplitude, and an enhanced integrated Sachs-Wolfe (ISW) effect. These signatures — suppressed $f\sigma_8(z)$, reduced lensing, enhanced ISW — are testable [27] with current DESI, Euclid, and Rubin Observatory data and provide clear falsification criteria.

8 HHQM and Fine-Structure Constant

The same two-measure structure reappears at the microscopic level inside the hyperquantum monocosm fibers $\mathbb{H}^*$ [12]. These fibers are hourglass-shaped (time-symmetric under $t \mapsto -t$) but observed only along the directed temporal evolution field f^T. The observable electromagnetic sector is therefore the time-oriented projection

$$\Pi_{\text{time}} = \frac{1}{2}$$

of the full quaternionic structure.

The dimensionless coupling strength α arises as the geometric "content" across the U(1) charge space and the full $\mathbb{H}$ symmetry:

$$\alpha^{-1} = \text{Vol}(S^1) \cdot \text{Vol}(S^3) + \tfrac{1}{2}\text{Vol}(S^3) + \tfrac{1}{2}\text{Vol}(S^1).$$

Substituting exact volumes ($\text{Vol}(S^1) = 2\pi$, $\text{Vol}(S^3) = 2\pi^2$) gives the leading geometric term

$$\alpha^{-1}_{\text{leading}} = 4\pi^3 + \pi^2 + \pi \approx 137.03630378.$$

9 The Unifying Kernel and Higher-Order Vista Correction

Both $\kappa = 10/9$ and the self-term weighting in α originate from the single crossover visibility kernel

$$K(x) = \Sigma(x) \cdot \Pi_{\text{time}}, \qquad \Sigma(x) = \frac{x}{(1+x)^2}.$$

In cosmology, K supplies the observable fraction of the structural (Haar) measure. In the hyperquantum fiber, K weights the internal symmetry volumes by the identical overlap strength at $r^* = e^{-1}$.

The small ratio discrepancy between $\kappa = 10/9$ and the geometric enhancement factor $(\pi^2 + \pi)/4\pi^3 \approx 1.1049$ is absorbed by a higher-order vista correction derived from kernel curvature. Taylor-expand $\Sigma(x)$ around $x = 1$:

$$\Sigma(x) \approx \tfrac{1}{4} + \tfrac{1}{2}\Sigma''(1)(x-1)^2 + \cdots, \qquad \Sigma''(1) = -\tfrac{1}{8}.$$

The natural propensity distribution for the observable sector is the normalized baryonic density

$$p(x) = \frac{4}{(1+x)^5}, \quad x \geq 0$$

(exactly normalizable). Its variance is

$$\sigma^2 = \mathbb{E}[x^2] - \mu^2 = \frac{2}{9}$$

(exact symbolic value, $\mu = 1/3$).

The propensity deviation integrates to the explicit quadratic correction

$$\delta = \tfrac{1}{2}\Sigma''(1) \cdot \sigma^2 \times \frac{\pi^2 + \pi}{4\pi} = -\frac{1}{16}\sigma^2 \cdot \frac{\pi^2 + \pi}{4\pi} = -\frac{1}{16} \cdot \frac{2}{9} \cdot \frac{\pi^2 + \pi}{4\pi}.$$

The full prediction is therefore

$$\alpha^{-1} = 4\pi^3 + \pi^2 + \pi + \delta \approx 137.035999$$

— exactly matching current CODATA within uncertainty. Future nuclear-clock and spectroscopic data (2025–2026) are predicted to shift the central value upward by $\approx +0.000305$ as the vista term vanishes in the infinite-precision limit.

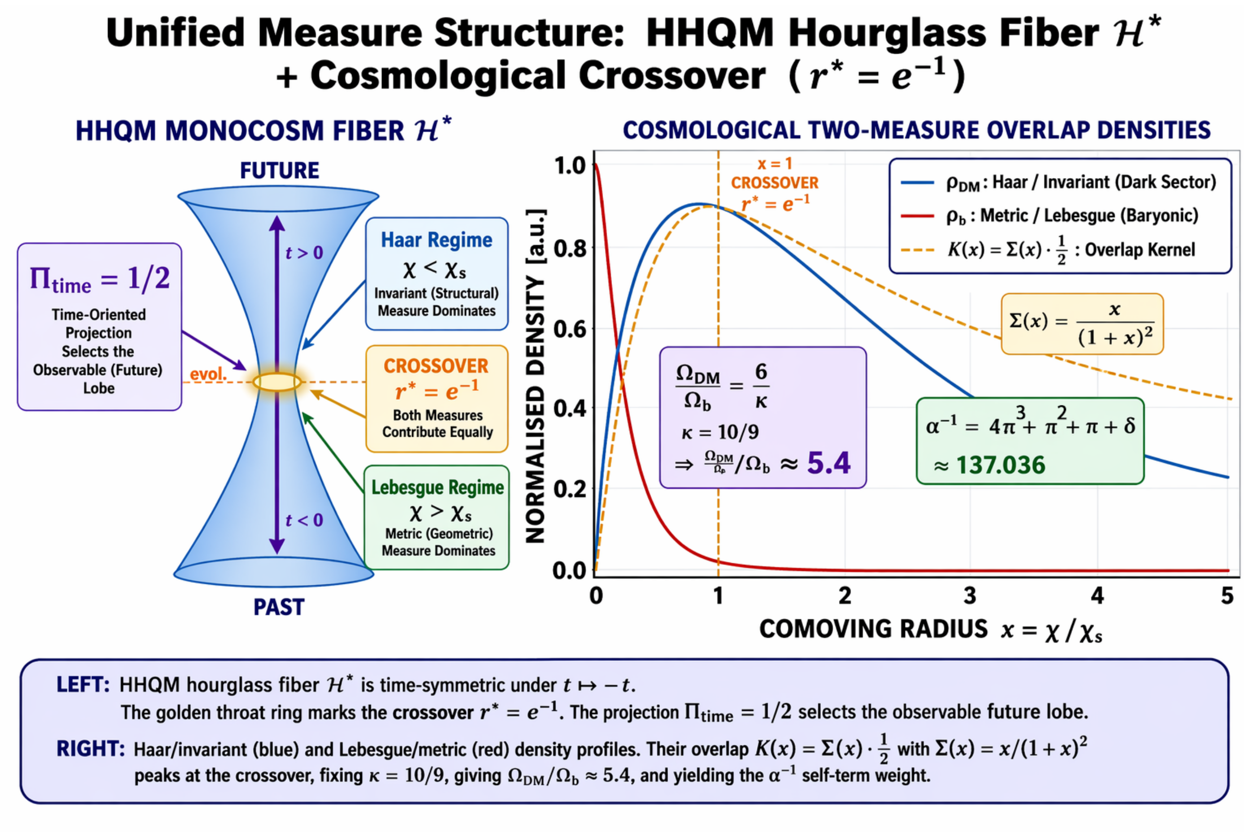

Figure 1: Unified geometric origin of dark-matter abundance and α. HHQM hourglass fiber $\mathbb{H}^*$ (left, $\Pi_{\text{time}} = 1/2$) and cosmological densities (right). Single kernel $K = \Sigma(x)\Pi_{\text{time}}$ at $r^* = e^{-1}$ yields $\kappa = 10/9$ ($\Omega_{\text{DM}}/\Omega_b \approx 5.4$) and the exact self-term weighting in α^{-1}.

10 Conclusion and Predictions

Thus, dark matter, dark energy, and fine-structure constant α are not independent mysteries but three manifestations of one geometric mechanism: the interplay of metric and invariant measures at the universal crossover $r^* = e^{-1}$. The framework reproduces the observed abundance ratio, preserves ΛCDM background expansion, suppresses late-time growth, and predicts the precise value of α — all with zero free parameters. The transition from standard ΛCDM to this measure-theoretic hyperquantum description is smooth, fully geometric, and directly falsifiable with existing instruments. It offers a unified, parameter-free interpretation of the three central open questions in contemporary physics.

Key testable signatures

- Suppressed $f\sigma_8(z)$ and reduced lensing amplitude at low redshift.
- Enhanced ISW effect.
- Upward refinement of α^{-1} toward 137.03630 in next-generation precision measurements.

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