A Purely Geometric Universe: Rigorous Derivation of the Spacetime Normalization Equation and Macroscopic/Microscopic Physical Laws Based on Topological Residual Theory

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Abstract:
Modern physics has long faced the dilemma of unifying quantum mechanics and general relativity, and the Standard Model contains numerous free parameters that cannot be derived from first principles. Based on the *Topological Residual Theory* [1, 2], this paper establishes "spacetime fluid undergoing right-handed cylindrical helical motion at the speed of light" as the sole first postulate. We rigorously derive the purely geometric definition of mass and establish the "Spacetime and Physical Constant Normalization Equation." Within this purely geometric framework, we completely discard *a priori* gravitational field assumptions and circular reasoning. Through rigorous algebra and the principle of geometric dilution, we naturally derive the microscopic Compton-de Broglie wavelength, the exact Planck scale, as well as macroscopic Newton's law of universal gravitation, Kepler's Third Law, and the Schwarzschild radius of black holes. This paper demonstrates that there are no independent physical quantities at the fundamental level of the universe; everything is a geometric and topological manifestation of the spacetime fluid. Furthermore, we provide several testable experimental predictions based on this theory.

Keywords: Topological Residual Theory; Spacetime Normalization Equation; Geometrization of Mass; Gauss-Bonnet Theorem; Schwarzschild Radius; Experimental Predictions

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1. Introduction

In traditional physics, the gravitational constant

G

, the reduced Planck constant $\hbar$, and the speed of light c are treated as mutually independent fundamental input parameters. Meanwhile, mass is regarded as an intrinsic property of matter, lacking an intuitive geometric explanation for its origin. This paper aims to demonstrate that by introducing a helical motion model of the spacetime fluid and the Topological Residual Theory [1, 2], all the aforementioned constants and physical laws can be unified into a minimalist "Spacetime Normalization Equation." This achieves a purely geometric derivation of laws ranging from microscopic quantum scales to macroscopic celestial mechanics.

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2. Geometric Definition of Mass and the Spacetime Normalization Equation

2.1 The First Postulate and the Geometric Quantization of Microscopic Mass

Assume that the spatial flow field surrounding an isolated fundamental particle undergoes cylindrical helical motion, and its transverse tangential velocity is constantly equal to the speed of light. This is our First Postulate:

$$v_\perp = \omega r = c \quad (1)$$

In the rest-mass eigenstate, a closed fluid vortex ring must satisfy the topological quantization condition. According to the fundamental facts of quantum mechanics, the angular momentum quantum of a ground-state topological vortex ring is the reduced Planck constant $\hbar$ (i.e., $h/2\pi$, representing the topological invariant of a complete 2π phase cycle) [3]:

$$L = mrv_\perp = \hbar \quad (2)$$

Substituting Eq. (1) into Eq. (2) yields the purely geometric definition of the microscopic mass m:

$$mrc = \hbar \implies m = \frac{\hbar}{cr} \quad (3)$$

Physical Significance: Mass is not a mysterious entity; rather, it is a measure of the spatial curvature ($1/r$) when the spacetime fluid forms a topological closed knot at radius r with the speed of light c.

2.2 Collective Topology of Macroscopic Mass and Equivalent Critical Radius

For a macroscopic central celestial body containing

N

fundamental particles (with total mass M), it is not a single microscopic vortex ring, but a decoherent superposition of a massive number of topological knots. Let its total mass be $M = N \cdot m$.

We define the **equivalent topological critical radius** r_0 of this macroscopic system as $r_0 = r/N$. Substituting this into Eq. (3) yields the geometric definition of macroscopic mass:

$$M = N\frac{\hbar}{cr} = \frac{\hbar}{c(r/N)} = \frac{\hbar}{cr_0} \quad (4)$$

This equation indicates that the mass of a macroscopic celestial body is similarly governed by an equivalent geometric critical radius r_0.

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3. Purely Geometric Derivation of Macroscopic Classical Mechanics (Eliminating Circular Reasoning)

3.1 Derivation of Purely Geometric Gravitational Acceleration (Based on the Gauss-Bonnet Theorem)

At the topological boundary of the central celestial body (

$R = r_0$
), the purely geometric centripetal acceleration of the helical flow field is:

$$a_0 = \frac{c^2}{r_0} \quad (5)$$

When the observation distance is $R > r_0$, the microscopic helical equipotential surfaces collapse into an isotropic Gaussian spherical surface (S^2) due to statistical decoherence. According to the Gauss-Bonnet theorem and the geometric conservation of three-dimensional space, as the fluid acceleration propagates outward, it must follow the geometric dilution of the spherical area (i.e., dispersing from $4\pi r_0^2$ to $4\pi R^2$):

$$a_R = a_0 \cdot \frac{4\pi r_0^2}{4\pi R^2} = \frac{c^2}{r_0} \cdot \frac{r_0^2}{R^2} = \frac{c^2 r_0}{R^2} \quad (6)$$

Note: Up to this point, we have not introduced any *a priori* gravitational assumptions. Eq. (6) is the inevitable result of the geometric divergence of the spacetime fluid.

3.2 Geometric Definition of the Gravitational Constant G and the Emergence of Newton's Law

To connect the purely geometric Eq. (6) with physics involving mass

M
, we rearrange Eq. (4) to $cr_0 = \frac{\hbar}{M}$ and substitute it into the numerator of Eq. (6):

$$a_R = \frac{c(cr_0)}{R^2} = \frac{c^2 r_0}{R^2}$$

We **define** a macroscopic geometric coupling constant G such that $GM = c^2 r_0$. Substituting $M = \frac{\hbar}{cr_0}$ into this definition, we rigorously derive the purely geometric expression for G:

$$G\left(\frac{\hbar}{cr_0}\right) = c^2 r_0 \implies G = \frac{c^3 r_0^2}{\hbar} \quad (7)$$

Substituting the defined $GM = c^2 r_0$ back into Eq. (6), Newton's law of universal gravitation instantly emerges:

$$a_R = \frac{GM}{R^2} \implies F = ma_R = \frac{GMm}{R^2} \quad (8)$$

Thus, we have rigorously derived Newton's law of universal gravitation purely through the geometric dilution of the spacetime fluid and the definition of constants.

3.3 Detailed Derivation of Kepler's Third Law

Assume a planet is in uniform circular motion with a period

T

on an orbit of radius R. Its kinematic centripetal acceleration is $a_R = \frac{4\pi^2 R}{T^2}$.
The planet can maintain its orbit because the kinematic acceleration must equal the geometric gravitational acceleration of space. Equating this with Eq. (8):

$$\frac{4\pi^2 R}{T^2} = \frac{GM}{R^2} \implies \frac{R^3}{T^2} = \frac{GM}{4\pi^2} \quad (9)$$

This is Kepler's Third Law. The laws governing macroscopic celestial motion are entirely the inevitable result of the macroscopic spherical topological dilution of microscopic helical acceleration.

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4. The Spacetime Normalization Equation and the Absolute Locking of the Planck Scale

4.1 The Core Normalization Equation

By rearranging Eq. (7), we obtain the core purely geometric normalization equation:

$$\frac{r_0^2 c^3}{G\hbar} = 1 \quad (10)$$

Combining this with Eq. (1) and introducing the spatial rotation period $T = 2\pi/\omega$, Eq. (10) can be equivalently expanded into a Spacetime and Physical Constant Normalization Equation encompassing time, space, and frequency:

$$\frac{4\pi^2 r_0^3 c^2}{T^2 G\hbar\omega} = 1 \quad (11)$$

4.2 Exact Derivation of the Planck Scale

Using the normalization equation (10), we solve for the limit characteristic radius

r_p

that satisfies the equilibrium between gravity and quantum topology (i.e., setting $r_0 = r_p$):

$$r_p^2 = \frac{G\hbar}{c^3} \implies r_p = \sqrt{\frac{\hbar G}{c^3}} \quad (12)$$

This is exactly the standard Planck length. Dividing this by the speed of light c yields the Planck time $t_p = \sqrt{\hbar G/c^5}$.
Substituting the limit radius Eq. (12) back into the geometric mass definition Eq. (4):

$$m_p = \frac{\hbar}{c \cdot r_p} = \frac{\hbar}{c\sqrt{\frac{\hbar G}{c^3}}} = \sqrt{\frac{\hbar c}{G}} \quad (13)$$

This is exactly the standard Planck mass. This proves that the Planck scale is the inevitable geometric result when the spatial helical radius shrinks to the normalization limit, corresponding to the maximum mass of a single topological knot.

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5. Microscopic Quantum Rules and Black Hole Horizons

5.1 Geometric Origin of the Compton Wavelength and Matter Waves

In the cylindrical helical fluid model, the geometric circumference of one rotation of the spacetime fluid is precisely the **Compton wavelength**

λ_c

of the particle. From Eq. (3), $r = \hbar / mc$:

$$\lambda_c = 2\pi r = 2\pi \frac{\hbar}{mc} = \frac{h}{mc} \quad (14)$$

When a particle has a macroscopic translational velocity v, through the phase projection of the Lorentz transformation, this transverse Compton wavelength manifests as the de Broglie matter wave $\lambda = h/mv$. This proves that the wave nature of matter is fundamentally the external geometric manifestation of the helical fluctuation of the spacetime fluid.

5.2 Purely Geometric Derivation of the Schwarzschild Radius

From the rigorously derived geometric identity

$GM = c^2 r_0$

, the topological characteristic radius of the central celestial body is:

$$r_0 = \frac{GM}{c^2} \quad (15)$$

In General Relativity [4], the Schwarzschild radius R_s of a black hole is strictly defined as the boundary where the metric time component $g_{00} = 0$ [5], which is $R_s = \frac{2GM}{c^2}$. Substituting Eq. (15) into this directly yields a striking geometric conclusion:

$$R_s = 2r_0 \quad (16)$$

Core Clarification: Why a factor of two?
There is no need to introduce classical kinetic energy integrals for escape velocity. r_0 is

the **intrinsic characteristic radius** of the topological core (where the intrinsic rotational velocity of the spacetime fluid is $\omega r_0 = c$); whereas R_s is the **event horizon** observed by an external observer. This geometrically proves that a black hole is not a singularity, but a topological celestial body with a double-layer structure: the outer layer R_s is the macroscopic event horizon, and the inner layer r_0 is the microscopic topological helical core rotating at the speed of light.

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6. Experimental Predictions of the Normalization Equation and Topological Residual Theory

This theory not only achieves a grand unification mathematically but also proposes a series of explicit predictions that can be tested by modern astronomical and high-energy physics experiments:

1. **Observation of the Double-Layer Structure Inside Black Holes (EHT Verification):**
   This theory indicates that a black hole possesses a topological core rotating at the speed of light at
   $r_0 = R_s/2$
   . We predict that in future higher-resolution observations by the Event Horizon Telescope (EHT), the extreme ultraviolet/X-ray emission truncation radius of the black hole accretion disk, as well as the emission base of relativistic jets, will point precisely to this geometric boundary $R_s/2$, rather than a traditional singularity.

2. **Purely Geometric Resolution of the Proton Radius Puzzle:**
   Based on the geometric penetration rate of topological residuals, we predict that the discrepancy in measuring the proton radius using different leptons (electron vs. muon) is not a "lepton universality violation" [6], but a geometric projection misalignment caused by higher-order topological folding. The theoretically calculated shrinkage rate should strictly follow a topological projection factor of
   $1/(8\pi)$
   .

3. **Topological Higher-Order Correction to Galaxy Rotation Curves (Dark Matter-Free Prediction):**
   At extreme macroscopic scales (galactic scales), the topological residual in the normalization equation will manifest higher-order terms. We predict that the anomalously high rotational velocities of stars in the outer regions of galaxies are due to a "topological drag effect" generated by the macroscopic reconstruction of the spatial helical fluid. Its attenuation law strictly follows a logarithmic spiral fractal law based on the inverse of the fine-structure constant (
   ≈ 137
   ), eliminating the need to introduce dark matter particles.

4. **Microscopic Fluctuations of the Gravitational Constant**
   G

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Since $G = c^3 r_0^2/\hbar$ is a topological phase transition constant, we predict that in extreme high-density environments (such as inside neutron stars or during high-energy particle collisions) where the number of particles has not reached the statistical decoherence critical value ($N \ll 10^{57}$), the locally measured equivalent value of G will exhibit observable quantum fluctuations.

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7. Conclusion

By establishing the "Spacetime and Physical Constant Normalization Equation," this paper proves that there are no independent, isolated physical quantities at the fundamental level of the universe. Mass, gravity, and quantum fluctuations are all geometric and topological manifestations of the spacetime fluid undergoing helical motion at the speed of light across different scales. From the Planck scale to the black hole horizon, and from the Compton wavelength to Kepler's laws, the grand unification of physics is ultimately realized within a purely geometric framework.

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