Resolution of the Hubble Tension from Octonionic Non-Associativity

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Abstract

We propose that the Hubble tension can be understood as a dynamical consequence of octonionic non-associativity. A physically relevant realization requires a genuinely nontrivial associator sector, which necessarily involves multiple octonionic fields. We therefore construct the minimal triplet model with three octonion-valued scalar fields and a covariant associator norm contribution $\|A(\Psi_1, \Psi_2, \Psi_3)\|^2$ in the action. From this action we derive the Einstein equations, the coupled octonionic matter equations, and the homogeneous FLRW reduction. In the resulting effective cosmology, the associator sector can be suppressed in the early universe while remaining nonzero at late times, thereby generating a redshift-dependent deformation of the expansion history. This produces a framework in which early-universe observables remain close to the standard background while late-time observables can infer a larger effective Hubble scale. We also formulate a directly testable phenomenological parametrization suitable for confrontation with Pantheon+ and DESI DR2. Current data do not yet establish a unique global best fit for the octonionic parameters, but they do show that a late-time, redshift-dependent departure from strict ΛCDM remains phenomenologically viable and is qualitatively aligned with the kind of deformation generated by the octonionic associator sector.

I. INTRODUCTION

The discrepancy between early- and late-time determinations of the Hubble constant suggests that the standard cosmological description may be incomplete. In the standard framework, the expansion history is governed by matter, radiation, and a cosmological constant, all embedded in an associative geometric background. The Hubble tension then appears as a mismatch between two observational windows that, within the same model, should infer the same present-day expansion rate. Planck 2018, interpreted within flat ΛCDM, gives

$$H_0 = (67.4 \pm 0.5)\,\mathrm{km\,s^{-1}\,Mpc^{-1}}, \tag{1}$$

while the SH0ES distance-ladder determination gives

$$H_0 = 73.04 \pm 1.04\,\mathrm{km\,s^{-1}\,Mpc^{-1}}, \tag{2}$$

and Pantheon+ with SH0ES calibration yields values near

$$H_0 \simeq 73.5\,\mathrm{km\,s^{-1}\,Mpc^{-1}}. \tag{3}$$

The purpose of the present work is to show that an octonionic framework naturally provides a mechanism for such a mismatch. The key idea is that non-associativity is not merely a mathematical curiosity but a dynamical ingredient. If the associator of the underlying algebra enters the action, then its effective energy density modifies the Einstein equations. Under cosmological symmetry reduction, this yields an additional contribution to the Friedmann equation.

A crucial structural point is that a physically nontrivial realization of the associator requires a multi-field sector. A single-field construction does not generate a dynamical contribution. The minimal consistent realization therefore involves a triplet of octonionic scalar fields coupled through a genuine associator norm. This framework provides a nontrivial covariant source for late-time cosmology and allows a direct phenomenological comparison with observational data.

II. ALGEBRAIC FOUNDATION

The octonions form an eight-dimensional normed division algebra

$$\mathbb{O} = \mathbb{R} \oplus \mathrm{Im}(\mathbb{O}), \tag{4}$$

with non-associative multiplication. The deviation from associativity is measured by the associator

$$A(a, b, c) = (ab)c - a(bc). \tag{5}$$

For associative algebras this vanishes identically, whereas for octonions it is generically nonzero.

We equip $\mathbb{O}$ with the real inner product

$$\langle X, Y \rangle = \mathrm{Re}(\bar{X}Y), \tag{6}$$

and the induced norm

$$\|X\|^2 = \langle X, X \rangle. \tag{7}$$

Let

$$\Psi_I(x) \in \mathbb{O}, \qquad I = 1, 2, 3, \tag{8}$$

be three octonion-valued scalar fields. The nontrivial associator sector is then defined by

$$U_A(\Psi_1, \Psi_2, \Psi_3) = \frac{\lambda}{2} \|A(\Psi_1, \Psi_2, \Psi_3)\|^2. \tag{1}$$

This quantity is real, nonnegative, and vanishes only when the triplet configuration dynamically collapses into an effectively associative subsector.

A key structural property is that a genuinely non-associative contribution requires at least three independent arguments. Consequently, the minimal nontrivial realization is intrinsically multi-field, and the triplet sector introduced above represents the simplest covariant implementation.

III. COVARIANT ACTION

We consider the minimal action

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} R + \sum_{I=1}^{3} \frac{\alpha_I}{2} g^{\mu\nu} \langle \nabla_\mu \Psi_I, \nabla_\nu \Psi_I \rangle - V(\Psi_1, \Psi_2, \Psi_3) - U_A(\Psi_1, \Psi_2, \Psi_3) \right]. \tag{2}$$

This form has three immediate advantages:

(i) it is manifestly covariant,

(ii) the associator sector is genuinely nontrivial,

(iii) in the homogeneous FLRW sector it can generate an effective late-time energy density.

For later use we write

$$\mathcal{L} = \frac{1}{16\pi G} R + \mathcal{L}_\Psi - \mathcal{V}, \tag{3}$$

with

$$\mathcal{L}_\Psi = \sum_{I=1}^{3} \frac{\alpha_I}{2} g^{\mu\nu} \langle \nabla_\mu \Psi_I, \nabla_\nu \Psi_I \rangle, \tag{4}$$

and

$$\mathcal{V} = V(\Psi_1, \Psi_2, \Psi_3) + U_A(\Psi_1, \Psi_2, \Psi_3). \tag{5}$$

IV. VARIATION WITH RESPECT TO THE METRIC

Using

$$\delta\sqrt{-g} = -\frac{1}{2}\sqrt{-g}\, g_{\mu\nu} \delta g^{\mu\nu}, \tag{6}$$

the variation of the Einstein-Hilbert term gives the standard result

$$\delta S_G = \frac{1}{16\pi G} \int d^4x \sqrt{-g}\, G_{\mu\nu}\, \delta g^{\mu\nu}. \tag{7}$$

For the kinetic sector one finds

$$\delta S_\Psi = \int d^4x \sqrt{-g} \sum_{I=1}^{3} \left[\frac{\alpha_I}{2} \langle \nabla_\mu \Psi_I, \nabla_\nu \Psi_I \rangle - \frac{1}{2} g_{\mu\nu} \frac{\alpha_I}{2} g^{\rho\sigma} \langle \nabla_\rho \Psi_I, \nabla_\sigma \Psi_I \rangle \right] \delta g^{\mu\nu}. \tag{8}$$

Since in the minimal realization V and U_A contain no explicit metric contractions beyond the volume factor, their metric variation arises only through $\sqrt{-g}$:

$$\delta S_{\mathcal{V}} = -\int d^4x \, \delta\sqrt{-g}\, \mathcal{V} = \frac{1}{2} \int d^4x \sqrt{-g}\, g_{\mu\nu} \mathcal{V}\, \delta g^{\mu\nu}. \tag{9}$$

Thus the stress-energy tensor is

$$T_{\mu\nu} = \sum_{I=1}^{3} \alpha_I \langle \nabla_\mu \Psi_I, \nabla_\nu \Psi_I \rangle - g_{\mu\nu} \left[\sum_{I=1}^{3} \frac{\alpha_I}{2} g^{\rho\sigma} \langle \nabla_\rho \Psi_I, \nabla_\sigma \Psi_I \rangle - \mathcal{V} \right]. \tag{10}$$

The Einstein equations therefore take the form

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}. \tag{11}$$

This is the fully varied metric field equation of the triplet theory.

V. VARIATION WITH RESPECT TO THE OCTONIONIC FIELDS

We now turn to the matter equations. Since the associator is trilinear, its variation is

$$\delta A(\Psi_1, \Psi_2, \Psi_3) = A(\delta\Psi_1, \Psi_2, \Psi_3) + A(\Psi_1, \delta\Psi_2, \Psi_3) + A(\Psi_1, \Psi_2, \delta\Psi_3). \tag{12}$$

Writing

$$A \equiv A(\Psi_1, \Psi_2, \Psi_3), \tag{13}$$

we obtain

$$\delta \|A\|^2 = 2\langle A, \delta A \rangle, \tag{14}$$

and hence

$$\delta U_A = \lambda \langle A, \delta A \rangle. \tag{15}$$

For the variation with respect to Ψ_1, this becomes

$$\delta_{\Psi_1} U_A = \lambda \langle A, A(\delta\Psi_1, \Psi_2, \Psi_3) \rangle. \tag{16}$$

We now define the adjoint associator operators D_1, D_2, D_3 by

$$\langle X, A(\delta\Psi_1, \Psi_2, \Psi_3) \rangle = \langle D_1(X; \Psi_2, \Psi_3), \delta\Psi_1 \rangle, \tag{17}$$

$$\langle X, A(\Psi_1, \delta\Psi_2, \Psi_3)\rangle = \langle D_2(X; \Psi_1, \Psi_3), \delta\Psi_2\rangle, \tag{18}$$

$$\langle X, A(\Psi_1, \Psi_2, \delta\Psi_3)\rangle = \langle D_3(X; \Psi_1, \Psi_2), \delta\Psi_3\rangle. \tag{19}$$

The octonionic field equations are then

$$\alpha_1 \Box \Psi_1 - \frac{\partial V}{\partial \bar{\Psi}_1} - \lambda D_1(A; \Psi_2, \Psi_3) = 0, \tag{20}$$

$$\alpha_2 \Box \Psi_2 - \frac{\partial V}{\partial \bar{\Psi}_2} - \lambda D_2(A; \Psi_1, \Psi_3) = 0, \tag{21}$$

$$\alpha_3 \Box \Psi_3 - \frac{\partial V}{\partial \bar{\Psi}_3} - \lambda D_3(A; \Psi_1, \Psi_2) = 0. \tag{22}$$

This is the master structure: gravity coupled to three octonionic scalar fields with a genuine non-associative backreaction term.

VI. OPERATOR FORM OF THE ASSOCIATOR SECTOR

The associator can be expressed through left- and right-multiplication operators,

$$L_a(x) = ax, \qquad R_a(x) = xa. \tag{23}$$

Then, schematically,

$$A(a, b, c) = L_{ab}c - L_a R_b c, \tag{24}$$

or, as an operator acting on the varying slot,

$$A(\delta a, b, c) = (\delta a\, b)c - \delta a(bc) = (R_b L_c - L_{bc})\, \delta a. \tag{25}$$

Hence the adjoint operator has the formal structure

$$D_1(X; b, c) = (R_b L_c - L_{bc})^\dagger X. \tag{26}$$

Analogous expressions hold for D_2 and D_3. In this sense the field equations are closed at the operator level.

VII. COSMOLOGICAL REDUCTION

We now impose homogeneity and isotropy and adopt the flat FLRW line element

$$ds^2 = -dt^2 + a(t)^2\, d\vec{x}^2. \tag{27}$$

For homogeneous fields

$$\Psi_I = \Psi_I(t), \tag{28}$$

the energy density and pressure of the octonionic sector become

$$\rho_A = \sum_{I=1}^{3} \frac{\alpha_I}{2} |\dot{\Psi}_I|^2 + V(\Psi_1, \Psi_2, \Psi_3) + U_A(\Psi_1, \Psi_2, \Psi_3), \tag{29}$$

$$p_A = \sum_{I=1}^{3} \frac{\alpha_I}{2} |\dot{\Psi}_I|^2 - V(\Psi_1, \Psi_2, \Psi_3) - U_A(\Psi_1, \Psi_2, \Psi_3). \tag{30}$$

The Friedmann equations then take the form

$$3H^2 = 8\pi G(\rho_m + \rho_r + \rho_A), \tag{31}$$

$$2\dot{H} + 3H^2 = -8\pi G(p_r + p_A). \tag{32}$$

If the late-time sector is potential dominated, then

$$w_A \equiv \frac{p_A}{\rho_A} \approx -1, \tag{33}$$

and the associator sector behaves effectively as dynamical dark energy.

VIII. EARLY-TIME SUPPRESSION AND LATE-TIME ACTIVATION

The central dynamical idea is that the early universe is driven toward highly symmetric and effectively associative sectors of the octonionic algebra. In such a regime, the triplet configuration is dynamically close to associative closure, so that

$$U_A(\Psi_1, \Psi_2, \Psi_3) \approx 0, \qquad \rho_A \approx 0. \tag{34}$$

Hence early-universe observables remain close to the standard matter-radiation background.

At late times, exact confinement to associative subsectors need no longer persist. Residual octonionic degrees of freedom can re-enter the effective dynamics, producing

$$\langle U_A \rangle > 0. \tag{35}$$

This generates a nonzero late-time energy component and thereby a redshift-dependent deformation of the expansion history.

The Hubble tension is then interpreted not as the existence of two physically distinct Hubble constants, but as the fact that early- and late-time observations probe different effective algebraic regimes of the same underlying cosmology.

IX. PHENOMENOLOGICAL BACKGROUND PARAMETRIZATION

For direct data analysis it is useful to introduce an effective background function

$$E(z)^2 \equiv \frac{H(z)^2}{H_0^2} = \Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda + \Omega_A f_A(z;\beta,\gamma), \tag{36}$$

with

$$f_A(z;\beta,\gamma) = (1+z)^\beta e^{-\gamma z}. \tag{37}$$

The interpretation is straightforward:

- β controls the algebraic scaling,
- γ enforces suppression at intermediate and high redshift,
- Ω_A is the present-day amplitude of the residual octonionic sector.

Flatness implies

$$\Omega_\Lambda = 1 - \Omega_r - \Omega_m - \Omega_A. \tag{38}$$

Thus

$$E(z)^2 = \Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda + \Omega_A(1+z)^\beta e^{-\gamma z}. \tag{39}$$

This form is particularly useful as a first test model: it is simple enough for immediate confrontation with Pantheon+ and DESI DR2, while remaining faithful to the octonionic triplet structure. A more microscopic derivation of Eq. (39) from the full triplet dynamics can then be developed separately.

X. CONSISTENCY ESTIMATES

To preserve the successful early-universe phenomenology, the additional sector must satisfy

$$\Omega_A f_A(z) \ll \Omega_m(1+z)^3 \qquad \text{for } z \gtrsim 1, \tag{40}$$

and even more strongly near recombination.

A simple estimate at $z = 2$ gives

$$\Omega_A\, 3^\beta e^{-2\gamma} \ll 27\,\Omega_m. \tag{41}$$

If one further demands that the additional sector be already substantially suppressed above the classical supernova regime around $z \sim 1$, a useful target condition is

$$3^{\beta} e^{-2\gamma} \lesssim 10^{-1}. \tag{42}$$

For β of order unity, this naturally drives γ to values also of order unity or larger.

A present-day residual sector at the percent level can already be sufficient to generate a Hubble shift of order 5–8%, provided the correction is concentrated toward low redshift, roughly $z \lesssim 0.5$. This is precisely the regime in which supernova and BAO data are sensitive to deviations from a pure cosmological constant.

XI. OBSERVATIONAL STRATEGY AND LIKELIHOOD FRAMEWORK

A practical first parameter vector is

$$\Theta = \{H_0, \Omega_m, M_B, \Omega_A, \beta, \gamma\}, \tag{43}$$

with flat geometry enforced by

$$\Omega_\Lambda = 1 - \Omega_r - \Omega_m - \Omega_A. \tag{44}$$

Reasonable initial parameter domains are

$$H_0 \in [65, 75], \quad \Omega_m \in [0.25, 0.35], \quad \Omega_A \in [0, 0.08], \quad \beta \in [0, 4], \quad \gamma \in [0, 5]. \tag{45}$$

A. Supernovae

The luminosity distance is

$$d_L(z) = \frac{c(1+z)}{H_0} \int_0^z \frac{dz'}{E(z')}, \tag{46}$$

and the theoretical distance modulus is

$$\mu_{\rm th}(z) = 5 \log_{10}\left(\frac{d_L(z)}{\rm Mpc}\right) + 25 + M_B. \tag{47}$$

The supernova likelihood is

$$\chi^2_{\rm SN} = \Delta\mu^T C_{\rm SN}^{-1} \Delta\mu, \qquad \Delta\mu_i = \mu_{{\rm obs},i} - \mu_{\rm th}(z_i). \tag{48}$$

B. BAO

For BAO one constructs

$$\chi^2_{\rm BAO} = \Delta D^T C^{-1}_{\rm BAO} \Delta D, \tag{49}$$

where ΔD contains the published observables, typically combinations such as

$$\frac{D_M(z)}{r_d}, \qquad \frac{D_H(z)}{r_d}, \tag{50}$$

or equivalent isotropic quantities.

C. Early-Time Information

As a first compressed implementation of early-time information, one may use Gaussian priors on

$$\omega_b = \Omega_b h^2, \qquad \omega_c = \Omega_c h^2, \tag{51}$$

or, at a more phenomenological level, on the Planck-compatible region of (H_0, Ω_m).

The total likelihood can then be written as

$$\chi^2_{\rm tot} = \chi^2_{\rm SN} + \chi^2_{\rm BAO} + \chi^2_{\rm early}. \tag{52}$$

XII. DATA-ORIENTED INTERPRETATION

The octonionic framework is structurally compatible with the observational fact that the late-time sector may admit a small additional dynamical contribution without significantly disturbing early-universe physics. In particular, a residual late-time component is not excluded *a priori* by the logic of current supernova and BAO analyses, and the relevant question is therefore quantitative: whether the posterior support for $(\Omega_A, \beta, \gamma)$ is improved relative to ΛCDM once the public covariance matrices are used in a full MCMC or nested-sampling analysis.

At this stage, what can be claimed cleanly is the following: the triplet field theory provides a mathematically consistent covariant source term and a directly implementable phenomenological fit framework based on public supernova and BAO products. What should not be claimed without explicit computation is an already established numerical global best fit for $(\Omega_A, \beta, \gamma)$.

XIII. STRUCTURAL DISTINCTION FROM STANDARD EXTENSIONS

This proposal differs conceptually from standard dark-energy parametrizations and early-dark-energy models.

In early-dark-energy scenarios, the additional component is introduced in order to affect the pre-recombination universe. In the present framework, the correction is instead a late-time remnant of a fundamentally non-associative algebraic structure.

In phenomenological $w(z)$ models, the deformation is introduced at the level of an effective fluid. Here the correction is tied to the norm of the octonionic associator in a genuine multi-field sector.

In modified gravity models, one alters the gravitational action directly, for example by replacing R with $f(R)$. Here the Einstein-Hilbert term remains unchanged at leading order; the novelty lies in the matter-pregeometry coupling generated by the non-associative sector.

XIV. CONCLUSION

We have reformulated the octonionic Hubble-tension scenario in a nontrivial covariant form. The decisive point is that a physically relevant associator contribution requires a genuinely multi-field sector. In the minimal triplet realization developed here, the associator norm

$$\|A(\Psi_1, \Psi_2, \Psi_3)\|^2 \tag{53}$$

provides a nontrivial covariant contribution to the stress-energy tensor and yields three coupled octonionic matter equations with non-associative backreaction operators.

Under homogeneous FLRW reduction, this sector contributes an effective late-time energy density and pressure, and in the potential-dominated regime it behaves as dynamical dark energy.

The resulting picture is structurally clear: the universe can pass through an early effectively associative regime with negligible associator energy and a late weakly non-associative regime with nonzero residual associator energy. Early- and late-time observations then probe different effective algebraic sectors of the same cosmology, which provides a natural mechanism for a Hubble-scale mismatch in inference.

For immediate phenomenological testing, the theory motivates the effective background form

$$E(z)^2 = \Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda + \Omega_A(1+z)^\beta e^{-\gamma z}, \tag{54}$$

which can be confronted directly with Pantheon+ and DESI DR2.

The strongest concise statement of the present result is therefore the following:

The originally proposed single-field associator potential $\|A(\Psi,\Psi,\Psi)\|^2$ vanishes identically by alternativity of the octonions. A nontrivial octonionic cosmology therefore requires at least a multi-field associator sector. In the minimal triplet realization developed here, the associator norm provides a genuine covariant contribution to the late-time energy budget and admits a direct phenomenological test against Pantheon+ and DESI DR2.

A full numerical assessment requires the official covariance matrices and a genuine MCMC or nested-sampling analysis. But at the formal level, the field structure and the associated data-driven fit framework are now in place.

XV. FIT RESULTS AND DATA COMPARISON

We now add a data-oriented interpretation of the phenomenological model in order to make explicit in what sense the octonionic scenario matches currently available cosmological observations.

A. Reference observational values

The relevant empirical situation can be summarized as follows.

Planck 2018, interpreted in base flat ΛCDM, gives

$$H_0 = (67.4 \pm 0.5)\,\mathrm{km\,s^{-1}\,Mpc^{-1}}, \qquad \Omega_m = 0.315 \pm 0.007. \tag{55}$$

Pantheon+ finds, from supernovae alone in flat ΛCDM,

$$\Omega_M = 0.334 \pm 0.018, \tag{56}$$

and, when combined with SH0ES calibration in flat w_0CDM,

$$H_0 = 73.5 \pm 1.1\,\mathrm{km\,s^{-1}\,Mpc^{-1}}. \tag{57}$$

The SH0ES distance-ladder analysis reports as baseline

$$H_0 = 73.04 \pm 1.04\,\mathrm{km\,s^{-1}\,Mpc^{-1}}, \tag{58}$$

and also quotes

$$H_0 = 73.30 \pm 1.04\,\mathrm{km\,s^{-1}\,Mpc^{-1}}. \tag{9}$$

DESI DR2 reports that the BAO measurements are individually well described by flat ΛCDM, but that the combination of DESI BAO with CMB prefers time-evolving dark energy over ΛCDM at 3.1σ, and that the preference rises to 2.8–4.2σ when supernovae are also included, with favored direction

$$w_0 > -1, \qquad w_a < 0. \tag{59}$$

These results imply an important point for the octonionic model: the data do not simply say "ΛCDM is exact." Rather, the current combined late-time datasets still allow, and in some combinations even favor, a redshift-dependent dark-energy deformation that is small at high redshift and more relevant at low redshift.

B. Why the octonionic background has the right qualitative shape

In the octonionic parametrization,

$$\rho_A(z) \propto f_A(z;\beta,\gamma) = (1+z)^{\beta} e^{-\gamma z}. \tag{60}$$

For $\gamma > 0$, the exponential factor suppresses the associator sector toward intermediate and high redshift. This is exactly the qualitative behavior required if one wants to preserve early-universe observables while permitting a low-redshift deformation of the expansion history.

Moreover, if one interprets ρ_A as an effective dark-energy component, then from

$$\frac{d\ln\rho_A}{dz} = \frac{\beta}{1+z} - \gamma \tag{61}$$

one obtains an effective equation of state

$$w_A(z) = -1 + \frac{1+z}{3}\frac{d\ln\rho_A}{dz} = -1 + \frac{\beta}{3} - \frac{\gamma}{3}(1+z). \tag{62}$$

Thus the octonionic sector naturally produces a time-dependent effective equation of state. In particular, for moderate positive β and γ, the model can mimic a low-redshift departure from strict Λ behavior while still being strongly damped at larger redshift. That is qualitatively consistent with the fact that DESI DR2 prefers an evolving dark-energy direction rather than a rigid cosmological constant in several combined analyses.

C. Benchmark octonionic fit region

Without claiming a fully optimized global best fit, one can identify a benchmark region that is simultaneously compatible with the structural constraints of the model and with the broad observational trends:

$$\Omega_m \approx 0.30\text{–}0.33, \qquad \Omega_A \approx 0.01\text{–}0.04, \qquad \beta \approx 0.5\text{–}2.0, \qquad \gamma \approx 1.5\text{–}3.5. \tag{63}$$

This region has the following properties.

First, it keeps the extra octonionic component subdominant above $z \sim 1$, as required by Planck compatibility and by the success of the standard early-universe background.

Second, it allows a residual positive energy density at $z \lesssim 0.5$, precisely where supernova and BAO measurements are most sensitive to late-time departures from ΛCDM.

Third, it can shift the locally inferred expansion scale upward without requiring a large early-time modification. This is the basic phenomenological requirement for any late-time explanation of the Hubble tension.

D. In what sense the model matches current data

The current observational status can be stated carefully in four parts.

a. 1. Match to early-time data. Because the octonionic term is exponentially suppressed for $\gamma > 0$, the model can be arranged so that

$$\Omega_A(1+z)^\beta e^{-\gamma z} \ll \Omega_m(1+z)^3 \tag{10}$$

already by $z \sim 1$ and certainly by recombination. Therefore the model can remain close to the Planck background solution at high redshift. This is a necessary condition, and the octonionic form satisfies it naturally for the benchmark range above. Planck's tight determination of H_0 and Ω_m therefore does not immediately exclude the model, provided the associator sector is truly late-time.

b. 2. Match to supernova data. Pantheon+ supernovae alone prefer $\Omega_M = 0.334 \pm 0.018$ in flat ΛCDM, which is still broad enough that a small low-z extra component can be hidden inside the expansion history unless the deformation becomes too large. Because the octonionic sector is constructed to be small and smooth over the Pantheon+ redshift lever arm, it is not in obvious conflict with the SNe dataset. More strongly: Pantheon+ combined with SH0ES continues to point toward $H_0 \sim 73$–74, which is exactly the direction that the octonionic late-time correction is intended to accommodate.

c. 3. Match to DESI DR2. DESI DR2 is the most important current stress test for any late-time model. DESI finds that flat ΛCDM still describes BAO distances well, but combined analyses with CMB and SNe prefer evolving dark energy over strict ΛCDM, with the preferred direction $w_0 > -1$, $w_a < 0$. The octonionic component is not written directly in CPL form, but it produces exactly the same type of phenomenology: a low-redshift modification that weakens with redshift. Therefore the octonionic model is qualitatively aligned with the deformation direction currently preferred by DESI DR2 combinations.

d. 4. Match to the Hubble tension itself. The model is built to explain why early-time probes infer an effectively lower H_0 while late-time probes infer a higher one. It does so not by postulating two different Hubble constants, but by introducing a redshift-dependent extra energy component that is negligible in the early universe and non-negligible at late times. This mechanism is directly aligned with the empirical structure of the tension: Planck prefers $H_0 \approx 67.4$, while SH0ES and Pantheon+SH0ES prefer $H_0 \approx 73$–73.5. The octonionic scenario was constructed precisely to interpolate between these inference regimes.

E. What the current data do not yet show

A crucial point of scientific honesty is that current public results do not yet provide an official published best-fit tuple

$$(\Omega_A, \beta, \gamma) \tag{11}$$

for the octonionic model itself.

So the correct claim is not:

"the octonionic model has already been numerically established as the best solution."

The correct claim is:

- the model is mathematically well posed at the covariant and FLRW levels;
- its phenomenological deformation has the right redshift structure to preserve early-universe physics and modify late-time inference;

- that deformation is qualitatively compatible with the current direction indicated by DESI DR2 combined analyses;
- the model remains viable and testable, but it still requires a dedicated MCMC or nested-sampling run on the public Pantheon+ and DESI DR2 covariance products before one can quote a statistically rigorous best fit, $\Delta\chi^2$, AIC, BIC, or Bayes factor.

F. Strongest data-based statement

The strongest data-based statement that can presently be defended is the following:

The octonionic triplet model is not ruled out by current background-expansion data and has the correct qualitative structure to address the Hubble tension: it is close to ΛCDM at early times, introduces a small late-time redshift-dependent deformation, and matches the current observational fact that combined DESI DR2 analyses prefer evolving dark energy over a strictly constant Λ in several dataset combinations. However, a statistically definitive claim of superiority over ΛCDM requires a dedicated full-likelihood fit of

$$(H_0, \Omega_m, M_B, \Omega_A, \beta, \gamma) \tag{12}$$

to Pantheon+, DESI DR2, and a chosen early-time prior set.

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