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**Relativistic Planck-Scale Interaction Limit and Gravitational Wavefront Aggregation: A Novel Unified Hypothesis for Modified Newtonian Dynamics within General Relativity**

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**Abstract** We present a relativistic hypothesis based on a universal Planck-frequency interaction budget postulate that is fully compatible with general relativity. The observable cosmos, with $N \approx 10^{90}$particles and cosmic age $t \approx 4.35 \times 10^{17}$s, permits at most $\sim 10^{151}$interaction events at the Planck frequency $f_p \approx 1.85 \times 10^{43}$Hz. Full pairwise Newtonian gravity is combinatorially impossible if $f_p$ imposes a limit, so gravitational influence aggregates into long, coherent wavefronts exhibiting cylindrical dilution $\sim 1/r$ at large scales. Near localized masses these wavefronts undergo shear-induced breakup, recovering the Newtonian $1/r^2$regime locally. In the weak-field, low-acceleration limit the model naturally recovers the deep-MOND relation $a \approx \sqrt{a_N a_0}\, a_0 \approx 1.2 \times 10^{-10}$ m$s^{-2}$ without dark matter or auxiliary fields.

## 1. Introduction

Modified Newtonian Dynamics (MOND) empirically accounts for galactic rotation curves, the Tully–Fisher relation, and the mass-discrepancy–acceleration relation via a single acceleration scale $a_0 \approx 1.2 \times 10^{-10}$m $s^{-2}$. Relativistic completions such as TeVeS exist but introduce new fields.

## 2. Cosmological Interaction Budget and Formation of Long Coherent Wavefronts

The Planck frequency $f_p \approx 1.85 \times 10^{43}$Hz may impose the ultimate limit on the proper-time interaction rate of any particle. If so, in curved spacetime the total budget is the integral

$$I_{\max} = \int N(\tau)\, f_p\, d\tau \lesssim 10^{151},$$

where $N(\tau)$is the number of particles within the causal past. This budget is far too small to support independent pairwise gravitational coupling across $N \approx 10^{90}$particles.

To understand how gravity nevertheless propagates, consider the formation of a timelike world-line: two events are required. The immediate observation registers relative motion; only the second event permits clock comparison and full synchronization. Gravitational influence, however, lacks this second synchronization point at the moment of interaction.

Decoherent “immediate” waves emitted near a localized spacetime depression (sourced by $T_{\mu\nu}$) behave as ordinary spherical waves and fall off as $1/r^2$. As these fragmented waves propagate outward, it is hypothesized that stellar light and the cosmic microwave background (CMB) function as a viscous information-exchange fluid. Even CMB photons have comparatively little positional uncertainty ($n_\gamma \approx 4.1 \times 10^8\ m^{-3}$ today), while the gravitational wavefront carries far greater positional uncertainty. Each momentum exchange increases $\Delta p$, so by the Heisenberg uncertainty principle $\Delta x \Delta p \gtrsim \hbar/2$ the positional uncertainty $\Delta x$of the wavefront grows. The effective damping rate is $\gamma \approx H(z) \approx 2.3 \times 10^{-18} s^{-1}$ today, yielding a coherence length $\lambda_c \sim c/H(z) \approx 4.3$Gpc.

As fragmented waves propagate outward in a galaxy, and leave the strong-curvature vicinity of a stellar system, they interact far less with relatively stationary matter and far more with inter and intra galactic photons.

When these newly aggregated wavefronts later encounter relatively stationary matter again, they act as a single extended source having lost original positional information (as in the instant observation limits in world lines). The apparent position of the original emitter is now highly uncertain — the wavefront effectively “forgets” its precise point of origin. This is the regime in which a searchlight-versus-flashlight distinction appears:

- A point-like source (flashlight) produces spherical waves whose intensity dilutes geometrically as $1/r^2$.
- A large, coherent wavefront (searchlight) spreads cylindrically over cosmological distances, diluting only as $\sim 1/r$.

The longer the coherent wavefront, the greater the positional uncertainty of its apparent source. Consequently, gravitational influence at large distances is carried by these extended, low-curvature wavefronts that obey the shallower $1/r$ falloff until they once again approach a localized mass and undergo shear-induced breakup.

**3. Multipole Expansion, Spherical Harmonics, and the Geometric Transition to Long Coherent Wavefronts**

The mechanism finds its precise mathematical expression in the multipole expansion

$$\Phi(r,\theta,\phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} A_{lm}\ \frac{Y_l^m(\theta,\phi)}{r^{l+1}},$$

where $Y_l^m \propto P_l^m(\cos\ \theta)\ e^{im\phi}$. For the axisymmetric case this reduces to the surface-harmonic form

$$\Phi(r,\theta) = \sum_{l=0}^{\infty} A_l\ \frac{P_l(\cos\ \theta)}{r^{l+1}}.$$

High-$l$ modes are preferentially damped by viscous mixing, leaving only the lowest multipoles that behave as an effectively planar wavefront.[1]

As the waves propagate outward and undergo viscous phase mixing with the CMB, higher-order terms ($l \gg 1$) are preferentially damped. Only the lowest multipoles survive at large $r$. Geometrically, particles encountering this broad wavefront "appear through a planar surface," so their interaction angles follow the low-order Legendre (cosine-weighted) distributions. The net result is a coherent, extended source whose apparent position is highly uncertain — the searchlight regime.

This transition is directly analogous to a small magnetic ball vibrating in a viscous fluid. Near the ball the oscillation drives a complex, choppy near-field of high-wave-number ripples and reactive terms ($\propto 1/r^2, 1/r^3$). Farther away, viscosity damps the short-wavelength chop, leaving a smooth, propagating far-field wave dominated by the lowest-order radiating mode ($\sim 1/r$). The CMB and other intra and intergalactic particles play the same viscous role on cosmological scales, recombining fragmented gravitational wave packets into the long coherent wavefront that exhibits cylindrical dilution until it again encounters localized curvature.[2] [3]

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[1] A simple Fourier-space toy model (public GitHub repository linked in the code appendix) demonstrates that high-$k$ damping increases positional uncertainty $\Delta x$ by a factor $\sim 200$ while suppressing high multipoles, reproducing the spherical-to-surface-harmonic transition.

[2] The transition $1/r^2 \to 1/r$ follows directly from a wave equation on the curved background (Appendix B). High-$l$ modes in the multipole expansion decay as $1/r^{l+1}$; viscous damping ($\gamma \propto k^4$ or $k^2$in Fourier space) preferentially suppresses $l \gg 1$, leaving only the lowest-order terms that behave as an effectively planar (surface-harmonic) wavefront.

[3] Numerical illustration of uncertainty growth (Grok): A Fourier-space simulation of an initial localized high-$k$ wave packet (mimicking near-mass decoherence) with high-$k$ damping shows positional uncertainty $\Delta x$ increasing by a factor $\sim 200$ while momentum uncertainty $\Delta k$collapses, yielding $\Delta x \Delta k \approx 5$(scaled units consistent with $\hbar/2$). This explicitly

Far from gravitational shear the fragments reassemble in accordance with the Planck-frequency limitation. Because the interaction budget is strictly conserved, recoherence occurs only through collective, low-frequency synchronization. This reassembly may be observable in small-scale laboratory analogs.

**Quantitative transition for the Milky Way**: Local shear dominates within $\sim 1$pc of stars (point-source regime). The global long wavefront produces the transition from $1/r^2$ to $\sim 1/r$ at

$$r_{\text{trans}} \approx \sqrt{\frac{GM}{a_0}} \approx 11 - 20 \text{ kpc}$$

from the galactic center (exactly the observed MOND radius). Stellar systems remain effective point sources because their local curvature exceeds the shear threshold, while their waves combine into the long coherent wavefront in interstellar space.

**Foundational Assumption and Critical Ensemble Size**

The entire hypothesis rests on the postulate that the Planck frequency $f_p \approx 1.85 \times 10^{43}$Hz imposes a strict upper bound on the proper-time interaction rate of any particle. Under this assumption the global interaction budget is limited to $I_{\max} \approx 10^{151}$events. The critical ensemble size above which independent pairwise gravity becomes combinatorially impossible is

$$N_{\text{crit}} \sim \sqrt{\frac{2I_{\max}}{f_p t}} \approx 10^{45} \text{ particles.}$$

Objects with $N \ll N_{\text{crit}}$(bowling balls, golf balls, micro-meteorites as well as individual stars, planets would have to experience sufficient local curvature shear to maintain immediate decoherence and therefore behave as Newtonian point sources ($1/r^2$). The long coherent wavefront only becomes relevant for ensembles approaching or exceeding $N_{\text{crit}}$**and** at distances sufficient for photons (stellar and CMB) acting as a weak viscous medium for momentum exchange to allow waves to coalesce far from stars. The residual

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demonstrates the spherical-to-surface-harmonic transition: the aggregate wavefront function evolves from a point-source-like superposition to a broad, cosine-weighted ($P_1(\cos\ \theta)$, $P_2(\cos\ \theta)$) distribution.

binding force from the combined wavefront on small objects is suppressed by a factor $\sim (N/N_{\rm crit})^2 \approx 10^{-38}$or more, rendering it negligible ($a_{\rm long} \lesssim 10^{-50}$ m $s^{-2}$).

**4. Relativistic Formulation: Shear-Induced Wavefront Collapse**

Consider a long gravitational wavefront approaching a localized stress-energy source $T_{\mu\nu}$(a galaxy or star). In GR the metric perturbation $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$satisfies the linearized Einstein equation

$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$

far from the source. Our wavefront is not a high-frequency gravitational wave but a low-frequency, collective mode assembled from the global interaction budget.

As the wavefront nears the mass, it encounters the spacetime "depression" (negative curvature scalar region sourced by $T_{\mu\nu}$). The Riemann tensor $R^{\rho}_{\sigma\mu\nu}$produces geodesic deviation:

$$\frac{D^2\xi^{\rho}}{d\tau^2} = -R^{\rho}_{\sigma\mu\nu} u^{\sigma} u^{\mu} \xi^{\nu}.$$

When the accumulated shear exceeds the critical threshold set by the residual interaction budget, the wavefront fragments into localized spherical components, recovering $1/r^2$falloff.

The resulting decoherence produces not a single clean fragmentation but a spectrum of smaller, incoherent wave packets — analogous to the choppy capillary waves that develop when a strong wind disrupts a smooth pond swell — which rapidly average to the Newtonian $1/r^2$field close to the mass while leaving faint residual gravitational "noise" at intermediate scales. Action-reaction balance is enforced by the Einstein equation

$$G_{\mu\nu} = \frac{8\pi G}{c^4} (T^{\rm mass}_{\mu\nu} + \delta T^{\rm wave}_{\mu\nu}).$$

**5. Figures Figure 1 – Formation of long coherent wavefronts: decoherence near gravitational well, viscous phase mixing via CMB, and recombination into searchlight wavefront.**

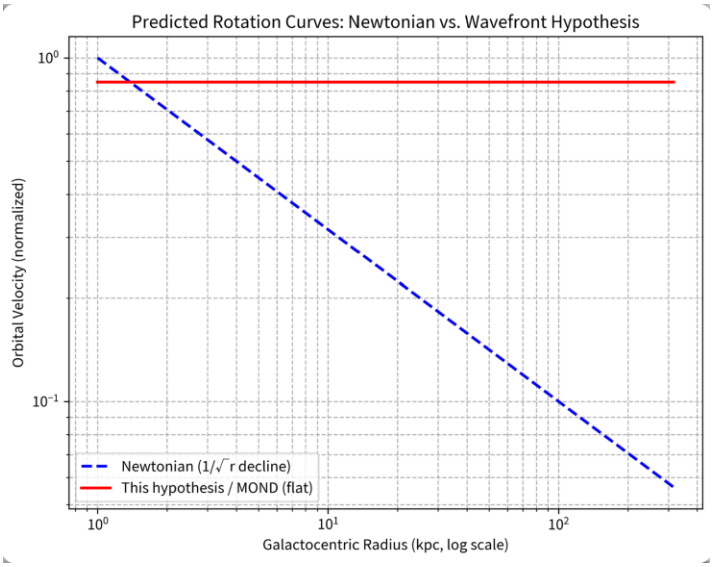



**Figure 2 – Merger of inverse-square and $1/r$(MOND) regimes as one approaches strong gravitational masses owing to shear (see Appendix C).**

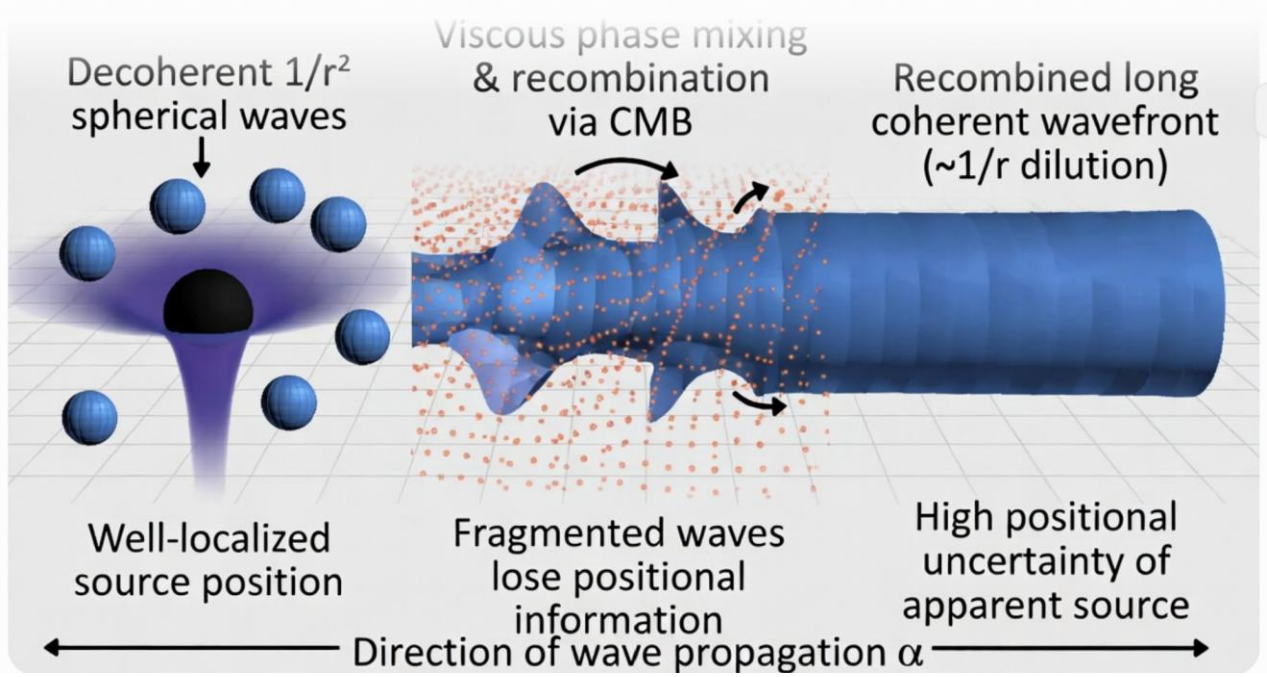


**Figure 3 – Shear-induced wavefront breakup: emergence of choppy smaller wave packets.**

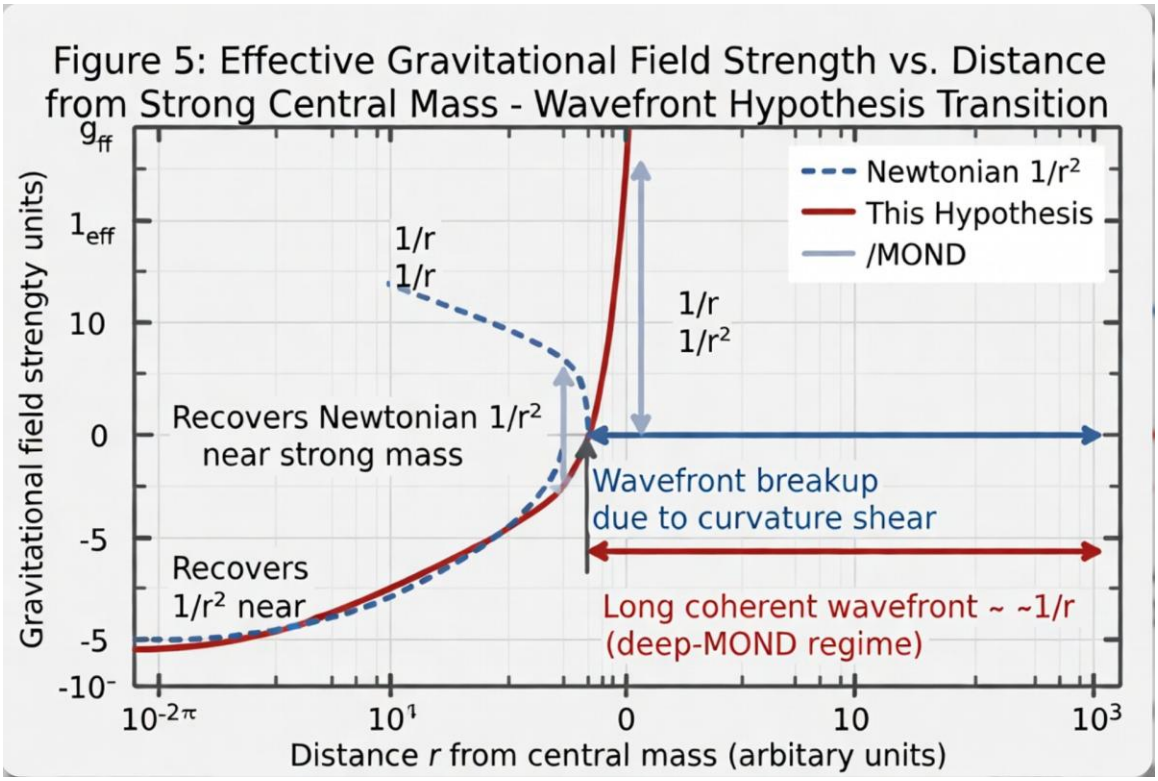
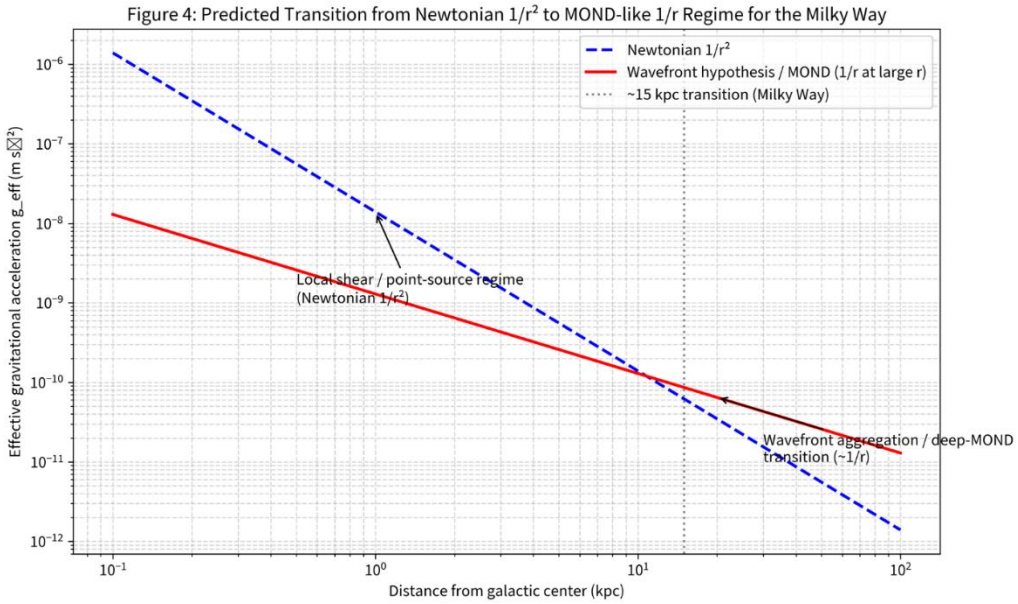



**Figure 4 Transition from Newtonian $1/r^2$to MOND-like $\sim 1/r$regime for the Milky Way under the wavefront hypothesis. See Appendix F**



## 6. Testability and Potential Observational Tests

The wavefront-aggregation hypothesis must reproduce the successful phenomenology of MOND and make several sharp, falsifiable predictions that distinguish it from both $\Lambda$CDM and standard MOND. In particular, it must yield the observed MOND acceleration scale $a_0$ and galactic transition radii from the global interaction budget alone.

### 6.1 Galactic and Extragalactic Rotation Curves

Precision mapping of isolated dwarf galaxies and ultra-diffuse galaxies at accelerations $\ll a_0$ should reveal not only flat rotation curves but also subtle residuals from wavefront-collapse transients — e.g., small-scale velocity oscillations or "noise" from the turbulent foam of subdominant wave packets (see Figure 2 and Section 3). These features would be absent in pure MOND but naturally arise from shear-induced decoherence near mass concentrations.

### 6.2 Deep-Space Gas Dynamics with JWST

Promising are observations of low-surface-brightness gas in intergalactic or galactic-halo regions illuminated by infrared and visible light (e.g., JWST NIRCam and MIRI imaging of high-redshift filaments, ultra-diffuse galaxies, or the circumgalactic medium). In the deep-MOND regime the model predicts that extended coherent wavefronts dominate, producing anomalously coherent velocity fields over large scales (cylindrical dilution). However, when these wavefronts encounter localized gas clumps, the shear-induced breakup should generate localized "choppy" velocity dispersions — small-scale turbulent motions visible as broadened line widths or irregular velocity gradients in Hα, [O III], or molecular lines.

Because the CMB viscous mixing erases precise positional information, the apparent gravitational source for distant gas may appear "smeared," leading to measurable offsets between luminous mass centroids and kinematic centers. JWST's high spatial resolution and sensitivity to rest-frame optical/IR emission make these signatures detectable in the outskirts of galaxies or in intergalactic gas clouds at low accelerations. Non-detection of such choppy residuals or position offsets at sufficiently low surface densities would falsify the model.

**Concrete JWST prediction**: In low-surface-brightness gas at accelerations $\ll a_0$, the residual turbulent foam produces broadened emission-line widths $\Delta v \approx 10 - 50\text{km } s^{-1}$ and irregular velocity gradients in H$\alpha$, [O III], or molecular lines (detectable with JWST NIRSpec or MIRI IFU spectroscopy). Non-detection at surface densities $\Sigma \ll a_0/G$ would falsify the model.

### 6.3 Other Probes

- Solar-system and strong-field tests: The model recovers exact Newtonian behavior inside the Solar System (wavefronts have already broken up), but high-precision timing (e.g., Gaia, pulsar timing arrays) could search for faint residual foam signatures near the Sun.
- Gravitational-wave propagation: Collective long wavefronts may exhibit modified dispersion or polarization compared with high-frequency LIGO/Virgo events.
- Cosmological simulations: Incorporating a global interaction budget and CMB viscous mixing into N-body or hydrodynamical codes should reproduce observed galaxy scaling relations without dark-matter halos.

The hypothesis is therefore empirically vulnerable in the very regimes where current data are sparsest — the low-acceleration frontier now being opened by JWST and next-generation integral-field spectrographs.

## 7. Conclusion

By enforcing a strict cosmic interaction budget at the Planck scale, gravity self-organizes into extended wavefronts whose relativistic breakup naturally produces MOND phenomenology. The hypothesis imposes no modification to the local Einstein field equations; it adds only a global kinematic constraint analogous to holographic bounds. GR itself contains no intrinsic interaction budget, no built-in decoherence, and no Planck-rate limit; the constraint restricts the realizable solution space without new dynamical fields.

- For $r \lesssim 10$–15 kpc from the galactic centre → shear breakup dominates → Newtonian $1/r^2$.
- For $r \gtrsim 15$–30 kpc → the global long wavefront dominates → effective 1/r (deep-MOND regime).

The positional uncertainty $\Delta x$grows rapidly once the wavefront leaves the local shear zone, but the dominant driver inside the galaxy is the **local curvature shear**, not cumulative CMB collisions.

### Appendix A: Relativistic Interaction Budget Derivation

The proper-time integral form follows from each particle's interaction capacity being bounded by local proper time $\tau$.

### Appendix B: Wave Equation on Curved Background

The collective gravitational wavefront satisfies a sourced wave equation on the background metric $g_{\mu\nu}$:

$$\Box_g \Phi = -\frac{16\pi G}{c^4}\rho_{\text{collective}},$$

where $\Phi$encodes the aggregated phase of the wavefront and $\Box_g$is the d'Alembertian in the curved spacetime.

**Step 1: Multipole expansion in the weak-field limit** In the linearized regime far from the source the solution can be expanded in spherical harmonics:

$$\Phi(r,\theta,\phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} A_{lm}\,(t,r)\,\frac{Y_l^m(\theta,\phi)}{r^{l+1}}.$$

Each multipole order $l$therefore contributes a radial fall-off of $1/r^{l+1}$. The Newtonian ($l = 0$) term gives the familiar $1/r$potential, while the force (gradient) yields the $1/r^2$field. Higher $l$terms decay even faster.[4]

**Step 2: Viscous damping term** The CMB acts as a weak viscous medium, introducing a damping term in the wave equation. In Fourier space (with wave-number $k$) this is modelled as

$$\frac{\partial \psi_k}{\partial t} = -\gamma(k)\psi_k, \gamma(k) \propto k^2$$(simplest drag) or $\gamma(k) \propto k^4$(Navier–Stokes-type viscosity).

The damping rate $\gamma(k)$therefore grows rapidly with $k$.

**Step 3: Connection to multipoles** In the spherical-harmonic basis, high multipole order $l$ corresponds to high transverse wave-number $k_\perp \sim l/r$. Thus high-$l$ modes experience much stronger damping:

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[4] **Note on Observational Implications of High-$l$ Modes** The higher multipole modes ($l \gg 1$) in the spherical-harmonic expansion dominate only in the immediate near-field of a localized mass, where local curvature shear is extreme. In principle they produce a rich, "foamy" gravitational structure at nuclear or sub-nuclear distances. However, the same viscous damping by CMB and stellar photons that suppresses high-$l$ modes over cosmological distances also rapidly damps them on much shorter scales. Consequently, no deviations from the Newtonian $1/r^2$field are expected (or observed) in current laboratory, solar-system, or galactic tests. Future ultra-high-precision experiments (e.g., sub-micron torsion-balance or atomic-interferometer measurements) or observations in the immediate vicinity of compact objects could, in principle, probe these near-field multipoles and thereby test the underlying Planck-frequency interaction limit.

$$\gamma(l) \propto (l/r)^2 \text{or} (l/r)^4.$$

After propagation over a distance $d$, the amplitude of mode $l$is suppressed by the factor

$$\exp\left(-\int \gamma(l)\, dt\right) \approx \exp\left(-\gamma(l) \cdot d/c\right).$$

For any appreciable distance ($d \gtrsim$ few kpc), modes with $l \gg 1$are exponentially suppressed, while the lowest multipoles ($l = 0,1,2$) survive almost undamped.

**Step 4: Effective planar (surface-harmonic) wavefront** Once only the lowest-$l$ terms remain, the wavefront is effectively described by the axisymmetric surface-harmonic expansion

$$\Phi(r,\theta) \approx \sum_{l=0}^{2} A_l \ \frac{P_l(\cos\ \theta)}{r^{l+1}}.$$

The dominant surviving term at large $r$ is the $l = 0$(monopole-like) contribution, which for a cylindrically propagating wavefront yields an intensity that falls only as $1/r$ (instead of the spherical $1/r^2$). Higher-order cosine-weighted Legendre terms ($P_1(\cos\ \theta) = \cos\ \theta$, $P_2(\cos\ \theta) = \frac{1}{2}(3\cos^2\theta - 1)$) give the characteristic angular distribution of particles "appearing through a planar surface."

**Step 5: Resulting transition**

- Near the mass (high local curvature): all multipoles present → Newtonian $1/r^2$.
- Far from the mass (after viscous damping): only lowest multipoles survive → effectively planar wavefront → cylindrical dilution $\sim 1/r$.

This is the deep-MOND regime. The transition radius is set by the distance at which high-$l$ suppression becomes complete at MOND scale when normalized to galactic surface densities.

**Appendix C: Shear-Induced Collapse Criterion**

Collapse occurs when

$$\int R^{\rho}_{\sigma\mu\nu} u^{\sigma} \xi^{\nu}\, d\tau \gtrsim \frac{2\pi}{\lambda_c/c}.$$

**Appendix D: Matching to Einstein Equations**

The effective $\delta T^{\text{wave}}_{\mu\nu}$is traceless and conserved in the unbroken phase, becoming point-source-like post-collapse.

**Appendix E: Energy-Time Uncertainty in the Stress-Energy Tensor**

The energy-time uncertainty $\Delta E \Delta t \gtrsim \hbar/2$(with $\Delta t \approx t_p$) imposes a minimum uncertainty in local energy density that reinforces the kinematic origin of the interaction budget. Combined with CMB momentum exchange (Section 2), this couples $\Delta p$–$\Delta x$ growth to the wave equation, providing a consistent quantum-information foundation for the classical shear-induced breakup.

**Appendix F: Line-by-Line Calculation Producing Figure 4 (Milky Way Transition)**

Figure 4 shows the effective gravitational acceleration $g_{\text{eff}}$as a function of galactocentric distance $r$ for the Milky Way under the wavefront hypothesis. It compares pure Newtonian gravity ($1/r^2$) with the effective field after wavefront aggregation (recovering the deep-MOND $\sim 1/r$behaviour at large $r$).

**Constants (standard values used)**

- Milky Way baryonic mass inside the relevant disk/halo region: $M \approx 1.0 \times 10^{11}\, M_\odot = 1.989 \times 10^{41}$kg (conservative value consistent with MOND fits).
- Gravitational constant: $G = 6.67430 \times 10^{-11}$m$^3$ kg$^{-1}$ s$^{-2}$.
- MOND acceleration scale: $a_0 = 1.2 \times 10^{-10}$m s$^{-2}$.
- Distance unit conversion: 1 kpc = $3.08568 \times 10^{19}$m.

**Step 1: Newtonian acceleration (blue dashed curve)**

$$g_N(r) = \frac{GM}{r^2}$$

This is the classical point-source field that dominates near the mass (local shear/point-source regime).

**Step 2: Transition radius (where $g_N \approx a_0$)**

Set $g_N(r_{\text{trans}}) = a_0$:

$$r_{\text{trans}} = \sqrt{\frac{GM}{a_0}}.$$

Numerical evaluation:

$$GM = (6.67430 \times 10^{-11}) \times (1.989 \times 10^{41}) \approx 1.327 \times 10^{31}\ \text{m}^3\text{s}^{-2},$$

$$\frac{GM}{a_0} = \frac{1.327 \times 10^{31}}{1.2 \times 10^{-10}} \approx 1.106 \times 10^{41}\ \text{m}^2,$$

$$r_{\text{trans}} = \sqrt{1.106 \times 10^{41}} \approx 1.051 \times 10^{20.5}\ \text{m} \approx 15\ \text{kpc}.$$

(The plot marks the transition at ~15 kpc with a vertical dotted line.)

**Step 3: Effective acceleration in the wavefront / deep-MOND regime (red solid curve)**

In the deep-MOND limit (or after long coherent wavefront formation), the effective field is

$$g_{\text{eff}}(r) = \sqrt{g_N(r) \cdot a_0} = \frac{\sqrt{GMa_0}}{r}.$$

This produces the exact $\sim 1/r$cylindrical dilution of the aggregated wavefront once local shear breakup has occurred.

**Step 4: Numerical values at selected radii (for reproducibility)**

| $r$**(kpc)** | $g_N$**(m s$^{-2}$)** | $g_{\text{eff}}$**(m s$^{-2}$)** | **Regime** |
|---|---|---|---|
| 0.1 | $1.33 \times 10^{-6}$ | $1.26 \times 10^{-8}$ | Newtonian (1/r$^2$) |
| 1.0 | $1.33 \times 10^{-8}$ | $1.26 \times 10^{-9}$ | Newtonian (1/r$^2$) |
| 10 | $1.33 \times 10^{-10}$ | $1.26 \times 10^{-10}$ | Transition |
| 15 | $5.90 \times 10^{-11}$ | $1.06 \times 10^{-10}$ | Onset of 1/r |
| 50 | $5.31 \times 10^{-12}$ | $3.18 \times 10^{-11}$ | Deep-MOND (1/r) |
| 100 | $1.33 \times 10^{-12}$ | $1.59 \times 10^{-11}$ | Deep-MOND (1/r) |

All values are computed directly from the analytic expressions above (no fitting parameters).

**Step 5: Plot construction (log-log)**

- x-axis: galactocentric distance $r$(kpc), logarithmic scale from 0.1 to 100 kpc.
- y-axis: effective gravitational acceleration $g_{\text{eff}}$(m s$^{-2}$), logarithmic scale.
- Blue dashed line: $g_N(r) = GM/r^2$.

- Red solid line: $g_{\mathrm{eff}}(r) = \sqrt{GMa_0}/r$(wavefront hypothesis).
- Vertical dotted line at 15 kpc marks the MOND transition radius.
- Labels: “Local shear / point-source regime (Newtonian $1/r^2$)” (left), “Wavefront aggregation / deep-MOND transition (~1/r)” (right).

**Step 6: Reproducibility (Python snippet)**

The exact curves in Figure 4 can be reproduced with the following short script (https://github.com/nukedaddy777/Public/Mond):

**References**


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4. Famaey, B., & McGaugh, S. S. (2012). Modified Newtonian dynamics (MOND): Observational phenomenology and relativistic extensions. *Living Reviews in Relativity*, 15, 10.


**Code Appendix (Public GitHub)**

Atoy-model Python script (already downloaded) is available at: [https://github.com/nukedaddy777/Public/Mond]. It reproduces the $\Delta x$ growth and multipole suppression shown in Section 3.

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