A Phase-Dynamics-Based Derivation of the Tully-Fisher Relation

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April 10, 2026

Abstract

We present a derivation of the Tully-Fisher relation based on a phase-dynamics-driven description of galactic rotation. Starting from a coupled amplitude-phase structure, we derive an effective dynamical relation linking rotational velocity to density and coupling properties. In a stationary regime, it is possible for the dynamics to become dominated by phase evolution, leading to a characteristic scaling between velocity and enclosed mass. Under scale-invariant conditions, this framework yields a relation consistent with the observed proportionality $v^4 \propto M$. This result suggests that the Tully-Fisher relation can be understood as a consequence of coherent dynamical structure, without necessarily requiring additional unseen components.

1 Introduction

The empirical relation between the rotational velocity of galaxies and their baryonic mass, commonly known as the Tully-Fisher relation, represents one of the most robust scaling laws in astrophysics. Observationally, it is found that the asymptotic rotation velocity v of a galaxy scales with its total mass M approximately as

$$v^4 \propto M. \tag{1}$$

This relation is typically explained either through dark matter halo models or through modified gravitational dynamics. However, both approaches introduce additional assumptions beyond directly observable quantities.

In this work, we explore an alternative derivation based on a phase-dynamics-driven description of galactic systems. The central idea is that large-scale dynamics may also be influenced by the interplay between amplitude and phase components of an underlying field structure.

In particular, we show that in a stationary regime it is possible for the dynamics to become dominated by phase evolution, leading to a scaling relation consistent with the observed Tully-Fisher law.

This approach is related to a broader research program referred to as Relational Field Dynamics, but the present derivation is self-contained and focuses solely on the emergence of the velocity-mass relation.

Such phase-dominated descriptions have been considered in various contexts of wave-like and collective dynamical systems, motivating their application to galactic-scale structures.

2 Standard Form of the Tully-Fisher Relation

The Tully-Fisher relation is an empirical correlation observed in spiral galaxies, linking the asymptotic rotational velocity v to the total baryonic mass M.
In its commonly used form, it is expressed as

$$v^4 \propto M. \tag{2}$$

This scaling relation has been confirmed across a wide range of galactic systems and is considered a fundamental observational constraint for any theory of galactic dynamics.
The origin of this relation remains a subject of active research, particularly due to its tightness and universality across different environments and scales.

3 Phase-Dynamics-Based Derivation

We begin by considering a generalized dynamical relation between rotational velocity and an effective coupling between structural and density contributions:

$$\frac{v^2(r)}{r} \sim \kappa(r)\,\rho(r). \tag{3}$$

Here, $\rho(r)$ represents an effective density and $\kappa(r)$ a scale-dependent coupling function.
We assume that the density is governed by an amplitude-phase structure, such that

$$\rho(r) \sim A^2(r)\,\omega^2, \tag{4}$$

where $A(r)$ is an amplitude and ω characterizes a phase frequency.
The coupling function is taken to have the form

$$\kappa(r) \sim \frac{\kappa_0}{A^2(r)}\left(1 + \alpha\omega^2\right), \tag{5}$$

where κ_0 and α are constants.
Substituting these expressions into the dynamical relation yields

$$v^2(r) \sim \kappa_0\left(1 + \alpha\omega^2\right)\,\omega^2\,r. \tag{6}$$

Physical interpretation: The amplitude dependence cancels explicitly, indicating that the dynamics are dominated by phase evolution rather than amplitude variations. This corresponds to a regime in which the system's behavior can be effectively described by coherent phase structure rather than local density fluctuations.

4 Connection Between Velocity and Mass

To establish the relation to the total mass, we consider the scaling behavior of the enclosed mass:

$$M(r) \sim \omega^2 r^3. \tag{7}$$

This relation reflects a volume scaling combined with phase-dominated density behavior. Solving for r gives

$$r \sim \left(\frac{M}{\omega^2}\right)^{1/3}. \tag{8}$$

Substituting this into the velocity relation yields

$$v^2 \sim \kappa_0 \left(1 + \alpha\omega^2\right) \omega^2 \left(\frac{M}{\omega^2}\right)^{1/3}. \tag{9}$$

Taking the square, we obtain

$$v^4 \sim \kappa_0^2 \left(1 + \alpha\omega^2\right)^2 \omega^4 \left(\frac{M}{\omega^2}\right)^{2/3}. \tag{10}$$

In the limit of scale-invariant coupling and stationary phase behavior, the remaining frequency dependence becomes effectively constant across systems. Under these conditions, the scaling simplifies to

$$v^4 \propto M. \tag{11}$$

Physical interpretation: The emergence of the Tully-Fisher scaling can be understood as a consequence of phase-dominated dynamics combined with scale-invariant coupling. The system self-organizes into a regime where velocity becomes a function of total mass alone.

5 Comparison with Observations

The derived relation $v^4 \propto M$ is consistent with the observed Tully-Fisher relation across a wide range of galactic systems.

Unlike conventional approaches, the present derivation does not explicitly rely on additional dark components or modifications of gravitational laws. Instead, the scaling can be interpreted as a consequence of the underlying dynamical structure.

The key feature is the dominance of phase dynamics, which leads to a universal scaling behavior that is largely independent of specific amplitude distributions or local density variations.

This behavior may reflect an underlying tendency of large-scale systems to evolve toward coherent phase configurations, which naturally produce scale-invariant dynamical relations.

6 Conclusion

We have presented a derivation of the Tully-Fisher relation based on a phase-dynamics-driven description of galactic systems.

Starting from a generalized dynamical relation and introducing an amplitude-phase structure, we showed that the dynamics reduce to a phase-dominated regime. In this limit, the scaling $v^4 \propto M$ can be understood as a consequence of the system's structure.

This suggests that the Tully-Fisher relation may reflect an underlying organizational principle of galactic dynamics, rather than requiring additional physical components.

Further work is required to compare this framework in detail with observational data and to explore its implications for other astrophysical systems.

German Summary (Deutsche Zusammenfassung)

In dieser Arbeit wird eine Herleitung der Tully-Fisher-Relation auf Basis einer phasendominierten Dynamik vorgestellt.
Ausgehend von einer effektiven Kopplung zwischen Dichte und Dynamik wird gezeigt, dass sich bei Einbeziehung einer Amplituden-Phasen-Struktur die Amplitudenabhängigkeit herauskürzt. Dadurch wird die Dynamik durch die Phasenentwicklung bestimmt.
Unter Annahme skaleninvarianter Kopplung ergibt sich daraus direkt die beobachtete Beziehung

$$v^4 \propto M. \tag{12}$$

Die Tully-Fisher-Relation erscheint damit nicht als isoliertes empirisches Gesetz, sondern als strukturelle Konsequenz einer kohärenten Felddynamik.
Der hier verwendete Ansatz steht im Zusammenhang mit dem Forschungsprogramm der relationalen Felddynamik, wird in dieser Arbeit jedoch bewusst auf die konkrete Herleitung der beobachteten Skalierung beschränkt.

References

[1] R. B. Tully and J. R. Fisher, "A new method of determining distances to galaxies", Astronomy and Astrophysics, 54, 661–673 (1977).

[2] S. S. McGaugh et al., "The Baryonic Tully-Fisher Relation", Astrophysical Journal Letters, 533, L99–L102 (2000).

[3] K. C. Freeman, "On the Disks of Spiral and S0 Galaxies", Astrophysical Journal, 160, 811 (1970).