Gauge Incompatibility Between Dark and Visible Sectors as a Consequence of Force Polarity Inversion

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"God does not play dice." — A. Einstein
He doesn't. We are just too far from the table to see where they land.

Abstract

Core Hypothesis. We propose that the invisibility of dark matter to all non-gravitational probes may result from structural incompatibility between gauge sectors, rather than weak coupling. We formalize a discrete transformation $\mathscr{T}$ — the *force polarity inversion* — acting on the gauge fiber bundle of an embedding group $\mathscr{G}$, producing a derived sector (our Standard Model) whose gauge representations are orthogonal to the substrate's. Gravity, being a property of the base manifold rather than the fiber bundle, is shown to be invariant under $\mathscr{T}$.

Scope and limitation. This paper is *kinematic*, not *dynamical*. We define the algebraic structure of $\mathscr{T}$ and prove two consequences: gauge incompatibility (Theorem 2.3) and gravitational invariance (Theorem 3.1). We do *not* construct the scalar field Φ or the potential $V(\Phi)$ that would implement $\mathscr{T}$ dynamically. Without this dynamical construction, the framework cannot predict when the inversion occurred, the dark-to-baryonic mass ratio, or the exact relic spectrum. These are open problems, not solved ones.

Conjectural predictions: (i) All dark matter direct detection experiments should continue to yield null results ($\sigma = 0$ exactly) — this follows from the kinematic structure alone; (ii) Dark matter should exhibit nonzero self-interaction ($\sigma/m \sim 0.3$–$1\ \text{cm}^2/\text{g}$) — requires specifying G_0; (iii) A first-order phase transition GW signature in LISA's band — requires the dynamical construction.

Appendices A–C present a minimal toy model ($SU(5)_L \times SU(5)_R \times \mathbb{Z}_2$) demonstrating viability: anomaly cancellation, BBN compatibility, and order-of-magnitude predictions.

Contents

1. Geometric Preliminaries

1.1 The Fiber Bundle Structure of Gauge Theory

The Standard Model of particle physics is formulated on a principal fiber bundle. We establish notation.

Definition 1.1 (Principal Bundle)**.** Let $\mathscr{P}(M,G)$ be a principal fiber bundle where M is a 4-dimensional Lorentzian manifold (spacetime) equipped with metric $g_{\mu\nu}$, and G is the structure group (gauge group).

The physics on this bundle is captured by three objects:
Base manifold (gravity). The metric $g_{\mu\nu}$ on M, governed by the Einstein–Hilbert action:

$$S_{\text{EH}}[g] = \frac{1}{16\pi G_N} \int_M d^4x \sqrt{-g}\, R \tag{1}$$

Fiber (gauge fields). A connection 1-form $A = A^a_\mu T_a\, dx^\mu$ on $\mathscr{P}$. The field strength and Yang–Mills action:

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu, \qquad S_{\text{YM}}[A,g] = -\frac{1}{4}\int_M d^4x \sqrt{-g}\, F^a_{\mu\nu} F^{a\mu\nu} \tag{2}$$

Matter fields. Sections ψ of associated vector bundles, in representations R of G:

$$S_{\text{matter}}[\psi, A, g] = \int_M d^4x \sqrt{-g}\, \bar{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi \tag{3}$$

where $D_\mu = \partial_\mu - i g_s A^a_\mu T^{(R)}_a$.

The total action is:

$$\boxed{S_{\text{total}} = S_{\text{EH}}[g] + S_{\text{YM}}[A,g] + S_{\text{matter}}[\psi,A,g]} \tag{4}$$

The critical observation is the **asymmetry of dependencies**:

$$S_{\text{EH}} = S_{\text{EH}}[g] \qquad \text{(depends only on base manifold)} \tag{5}$$
$$S_{\text{YM}} = S_{\text{YM}}[A,g] \qquad \text{(depends on fiber and base)} \tag{6}$$
$$S_{\text{matter}} = S_{\text{matter}}[\psi,A,g] \qquad \text{(depends on all three)} \tag{7}$$

Gravity depends only on the base manifold. Gauge interactions and matter depend on the fiber. This asymmetry is the geometric foundation of the entire framework.

2. The Inversion Operator

2.1 Two-Sector Structure

We postulate the existence of two gauge sectors sharing the same base manifold M.

Definition 2.1 (Two-Sector Bundle)**.** Let M carry two principal bundles:

$$\mathscr{P}_0(M,G_0) \quad \text{(substrate sector)}, \qquad \mathscr{P}_1(M,G_1) \quad \text{(derived/visible sector)} \tag{8}$$

with $G_0 = SU(3)' \times SU(2)' \times U(1)'$ and $G_1 = SU(3) \times SU(2) \times U(1)$.

The total action:

$$\boxed{S = S_{\text{EH}}[g] + S^{(0)}_{\text{YM}}[A_0,g] + S^{(0)}_{\text{matter}}[\psi_0,A_0,g] + S^{(1)}_{\text{YM}}[A_1,g] + S^{(1)}_{\text{matter}}[\psi_1,A_1,g]} \tag{9}$$

Both sectors source gravity:

$$G_{\mu\nu} = 8\pi G_N \left(T^{(0)}_{\mu\nu} + T^{(1)}_{\mu\nu}\right) \tag{10}$$

2.2 The Inversion Operator $\mathscr{T}$

Definition 2.2 (Force Polarity Inversion)**.** $\mathscr{T}$ is a discrete map $\mathscr{T} : \mathscr{P}_0(M,G_0) \to \mathscr{P}_1(M,G_1)$ defined by:

(i) **Acts on fiber:** $\mathscr{T}$ maps $A_0 \mapsto A_1$ and representations $R_0 \mapsto R_1$

(ii) **Preserves base:** $\mathscr{T}^* g_{\mu\nu} = g_{\mu\nu}$ (identity on M)

(iii) **Produces orthogonal representations:** $R_1 = \mathscr{T}(R_0)$ are structurally incompatible with R_0

Concrete realizations, with known technical challenges stated explicitly:

Option A: Chirality Flip. $\mathscr{T} : \psi_L \mapsto \psi'_R,\ \psi_R \mapsto \psi'_L$. Closest to mirror matter (Foot, Volkas [1]). Without dynamics forbidding kinetic mixing $\varepsilon F^{\mu\nu}F'_{\mu\nu}$, the distinction from standard mirror matter blurs. Novelty claim is weakest under this option.

Option B: Generalized Charge Conjugation. $\mathscr{T} : (q_c, q_w, q_Y) \mapsto (-q_c, -q_w, -q_Y)$. Raises anomaly cancellation issues: the SM is chiral, and inverting all quantum numbers may produce an inconsistent anomaly structure unless compensated by a chirality flip.

Option C: Orthogonal Embedding. $G_0, G_1 \subset \mathscr{G}$ with $G_0 \perp G_1$:

$$\mathrm{Tr}\left[T_a^{(0)} T_b^{(1)}\right] = 0 \qquad \forall\, T_a^{(0)} \in \mathrm{Lie}(G_0),\; T_b^{(1)} \in \mathrm{Lie}(G_1) \tag{11}$$

Strongest version. Candidates for $\mathscr{G}$: $SU(5) \times SU(5)$, $SO(10) \times SO(10)$, or subgroups of $E_8 \times E_8$. Known issues: constructing chiral spectra from orthogonal embeddings is nontrivial; E_8 compactifications generically produce unwanted massless states; absence of bifundamentals is not automatic.

2.3 From Orthogonality to Incompatibility

The central claim is that gauge incompatibility *follows* from the embedding structure, not assumed.

Proposition 2.3 (Commutativity from Direct Product)**.** *If $\mathscr{G}$ breaks to $G_0 \times G_1$, the generators commute:*

$$[T_a^{(0)}, T_b^{(1)}] = 0 \qquad \forall\, T_a^{(0)} \in Lie(G_0),\; T_b^{(1)} \in Lie(G_1) \tag{12}$$

Proof. In $G_0 \times G_1$, $\mathrm{Lie}(G_0 \times G_1) = \mathrm{Lie}(G_0) \oplus \mathrm{Lie}(G_1)$, a direct sum of ideals. Generators in different ideals commute by definition. □

Lemma 2.4 (Singlet Decomposition)**.** *If $[T_a^{(0)}, T_b^{(1)}] = 0$, any irreducible representation R_1 of G_1 is a singlet of G_0:*

$$T_a^{(0)} |\psi_1\rangle = 0 \qquad \forall\, T_a^{(0)} \in Lie(G_0),\; \psi_1 \in R_1 \tag{13}$$

Proof. Representations of $G_0 \times G_1$ decompose as $R = R_0 \otimes R_1$. A field defined as belonging to G_1 with no mechanism to assign G_0 quantum numbers transforms trivially under G_0. The singlet property is *derived* from the direct product structure. □

Theorem 2.5 (Gauge Incompatibility)**.** *If $\mathscr{G} \to G_0 \times G_1$, the gauge vertex between sectors vanishes:*

$$\mathscr{M}\,(\psi_0 + \psi_1 \to anything\ via\ A_0\ or\ A_1) = 0 \tag{14}$$

Proof. The vertex $g_s\,\bar{\psi}_1 \gamma^\mu T_a^{(R_0)} \psi_1 A_{0\mu}^a$ vanishes because ψ_1 is a singlet of G_0 by Lemma 2.4: $T_a^{(R_0)} \psi_1 = 0$. □

The logical chain: **direct product** $\to$ **commutativity (Prop. 2.3)** $\to$ **singlet (Lemma 2.4)** $\to$ **zero vertex (Thm. 2.5)**. The only input is the breaking $\mathscr{G} \to G_0 \times G_1$.

Caveat: The singlet decomposition assumes no bifundamental fields (charged under both G_0 and G_1). If bifundamentals exist, they constitute portal operators. Whether their absence is natural depends on $\mathscr{G}$ and its breaking potential — an open question (Section 2.4).

2.4 Status of $\mathscr{T}$: Declared Symmetry, Not Yet Dynamical Construction

An important limitation: $\mathscr{T}$ is *kinematic*, not *dynamical*. We establish:

- The algebraic structure of $\mathscr{T}$
- Gauge incompatibility (Theorem 2.5)
- Gravitational invariance (Theorem 3.1, below)

We do **not** establish:

- No Lagrangian generates $\mathscr{T}$
- No field Φ implements the transition
- No potential $V(\Phi)$ determines the dynamics

The closest structural analogue is electroweak symmetry breaking before the Higgs mechanism: $SU(2)_L \times U(1)_Y \to U(1)_{\text{em}}$ was a declared relation until the Higgs field provided the dynamical construction. By analogy, $\mathscr{T}$ requires:

(i) A scalar Φ transforming under $\mathscr{G}$

(ii) A potential $V(\Phi)$ with minima breaking $\mathscr{G} \to G_0 \times G_1$

(iii) A cosmological phase transition implementing the breaking

A toy model is presented in Appendix A. We proceed with the kinematic definition because Theorems 2.5 and 3.1 depend only on algebraic properties, not on the mechanism. The kinematic structure constrains; the dynamical construction will determine.

3. Gravitational Invariance

3.1 The Theorem

We postulate the following invariance principle:

The transformation $\mathscr{T}$ acts exclusively on internal gauge symmetries (the fiber bundle) and leaves spacetime geometry (the base manifold) invariant.

Theorem 3.1 (Gravitational Invariance)**.** *$\mathscr{T}$ leaves the Einstein–Hilbert action invariant: $\mathscr{T}[S_{EH}[g]] = S_{EH}[g]$.*

Proof. S_{EH} depends exclusively on $g_{\mu\nu}$ and its derivatives. By Definition 2.2, $\mathscr{T}^* g_{\mu\nu} = g_{\mu\nu}$. Therefore $\mathscr{T}^*\Gamma^\rho_{\mu\nu} = \Gamma^\rho_{\mu\nu}$, $\mathscr{T}^*R = R$, and $\mathscr{T}[S_{\text{EH}}] = S_{\text{EH}}$. □

3.2 Why Gravity Sees Both Sectors

Both sectors source gravity via the combined stress-energy tensor:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T^{\text{total}}_{\mu\nu}, \qquad T^{\text{total}}_{\mu\nu} = T^{(0)}_{\mu\nu} + T^{(1)}_{\mu\nu} \tag{15}$$

The metric responds to energy-momentum regardless of gauge origin.

Corollary 3.2. *The only observable signature of sector 0 from within sector 1 is gravitational.*

4. Distinction from Existing Models

4.1 Parametric vs. Structural Decoupling

Existing models achieve inter-sector decoupling through:

- **Parametric decoupling:** Cross-sector interactions governed by a small parameter (ε, λ_{HP}, y_ν). Suppressed, not forbidden. Detectable at sufficient sensitivity.
- **Structural decoupling (this work):** Forbidden by algebra (Thm. 2.5). No sensitivity increase can detect $\sigma = 0$.

4.2 Model-by-Model Comparison

Mirror matter [1, 2]: allows kinetic mixing $\mathscr{L}_{\text{mix}} = \frac{\varepsilon}{2} F^{\mu\nu} F'_{\mu\nu}$ with $\varepsilon \sim 10^{-9}$. Parametric. We predict $\varepsilon = 0$ exactly.

Hidden sectors [3, 11]: portal operators (Higgs, neutrino, kinetic mixing) allowed. Parametric. We predict all portals forbidden if no bifundamentals exist.

CPT cosmology [4]: sterile neutrinos with residual seesaw coupling. Partial. We predict zero non-gravitational coupling.

4.3 Summary of Distinguishing Predictions

Observable	This work	Mirror	Hidden sector	WIMP
Direct detection σ_{SI}	$= 0$ (exact)	> 0 ($\propto \varepsilon^2$)	> 0 (portal)	> 0 ($\propto g^4/m_\chi^2$)
Self-interaction σ/m	0.1–1 cm^2/g	similar	model-dep.	≈ 0
GW spectrum	broken power law	similar	none predicted	none predicted

5. Testable Predictions

5.1 Null Prediction for Direct Detection (kinematic)

$$\sigma_{\text{SI}}^{(0\leftrightarrow 1)} = 0, \qquad \sigma_{\text{SD}}^{(0\leftrightarrow 1)} = 0 \tag{16}$$

Falsifiable: any confirmed direct detection event rules out the framework. This prediction follows from Theorem 2.5 alone — no dynamical input needed.

5.2 Dark Matter Self-Interaction (requires G_0 specification)

The Born-regime transfer cross-section for Yukawa potential [9] (conversion: 1 GeV$^{-3} \approx 2.2 \times 10^{-4}$ cm^2/g):

$$\frac{\sigma_T}{m_{\text{DM}}} \approx \frac{8\pi \alpha_D^2 m_{\text{DM}}}{m_\phi^4} \tag{17}$$

For $\alpha_D = 0.01$, $m_{\text{DM}} = 10$ GeV, $m_\phi = 50$ MeV: $\sigma_T/m \approx 0.9$ cm^2/g. Within Bullet Cluster bounds [14]. Velocity-dependent: suppressed at cluster scales, enhanced at dwarf scales.

5.3 Gravitational Wave Signature (requires dynamical construction)

Following Caprini et al. [10]:

$$f_{\text{peak}} \approx 1.65 \times 10^{-5}\ \text{Hz} \times \left(\frac{T_*}{100\ \text{GeV}}\right) \left(\frac{\beta}{H_*}\right) \left(\frac{g_*}{100}\right)^{1/6} \tag{18}$$

Spectral shape: $\Omega_{\text{GW}} \propto f^3$ below peak, f^{-1} above — distinct from inflation (f^0) and SMBH binaries ($f^{-2/3}$).

6. Discussion and Open Problems

This paper is deliberately limited in scope. It establishes two kinematic results and does not claim dynamical completeness. The contribution is the proof that structural decoupling is a viable, distinguishable alternative to parametric decoupling.

The dynamical construction (highest priority). Appendix A presents a toy model based on $SU(5)_L \times SU(5)_R \times \mathbb{Z}_2$. Appendix B verifies consistency. Appendix C derives order-of-magnitude predictions. The definitive model requires specifying $V(\Phi)$ and deriving the transition thermodynamics.

The embedding group $\mathscr{G}$. Candidates include $SU(5) \times SU(5)$, $SO(10) \times SO(10)$, or subgroups of $E_8 \times E_8$. Each has known challenges from the GUT/string literature.

Quantum implications. A companion paper [15] develops broader implications for wave-particle duality, the measurement problem, and quantum non-locality within the two-sector framework.

A. Toy Model: $SU(5)_L \times SU(5)_R \times \mathbb{Z}_2$

This appendix is pedagogical, not definitive. It demonstrates viability.

A.1 Setup

Embedding group: $\mathscr{G} = SU(5)_L \times SU(5)_R \times \mathbb{Z}_2$. Scalar Σ in the bifundamental $(\mathbf{5}, \bar{\mathbf{5}})$, with potential:

$$V(\Sigma) = -\mu^2 \mathrm{Tr}(\Sigma^\dagger \Sigma) + \lambda_1 \left[\mathrm{Tr}(\Sigma^\dagger \Sigma)\right]^2 + \lambda_2 \mathrm{Tr}\left[(\Sigma^\dagger \Sigma)^2\right] + \delta\, \mathrm{Tr}(\Sigma^\dagger \Sigma) \cdot \left(|H_L|^2 - |H_R|^2\right) \quad (19)$$

The δ-term softly breaks $\mathbb{Z}_2$, creating asymmetry between sectors.

A.2 Breaking Pattern

$\langle \Sigma \rangle = v_\Sigma \cdot \mathbb{I}_{5\times 5}$ breaks the bifundamental coupling. Below v_Σ:

$$SU(5)_L \times SU(5)_R \xrightarrow{\langle \Sigma \rangle} SU(5)_L \times SU(5)_R \xrightarrow{\langle H_L \rangle, \langle H_R \rangle} G_0 \times G_1 \quad (20)$$

No light bifundamentals remain, enforcing Theorem 2.5.

A.3 Asymmetric Reheating

The δ-term produces $v_L \neq v_R$, hence different temperatures. With $\xi \equiv T_0/T_1$ [2]:

$$\frac{\Omega_{\mathrm{DM}}}{\Omega_b} \approx \frac{m_{\mathrm{DM}}}{m_p} \cdot \frac{\eta_0}{\eta_1} \cdot \xi^3 \quad (21)$$

For $\Omega_{\mathrm{DM}}/\Omega_b \approx 5.4$ with $m_{\mathrm{DM}} \approx m_p$, $\eta_0 \approx \eta_1$: $\xi \approx 1.75$. For $\xi \approx 0.5$: $m_{\mathrm{DM}} \approx 43\, m_p$, achievable if $\Lambda'_{\mathrm{QCD}} \gg \Lambda_{\mathrm{QCD}}$.

B. Consistency Checks

Anomaly cancellation: Each $SU(5)$ independently cancels via $\bar{\mathbf{5}} \oplus \mathbf{10}$.

Dark baryon stability: B' conserved at renormalizable level $\Rightarrow$ lightest dark baryon is stable DM candidate.

BBN: $\Delta N_{\mathrm{eff}} \approx 6.14 \times \xi^4$. For $\xi \lesssim 0.5$: $\Delta N_{\mathrm{eff}} \lesssim 0.38$, marginally compatible with Planck.

C. Quantitative Predictions

C.1 Self-Interaction

Born-regime Yukawa (conversion: 1 $\text{GeV}^{-3} \approx 2.2 \times 10^{-4}$ cm^2/g):

α_D	m_{DM} (GeV)	m_ϕ (MeV)	σ_T/m (cm^2/g)
0.01	10	50	0.9
0.01	50	100	0.3
0.01	10	30	6.8 (requires v-dep. suppression)

C.2 Gravitational Waves

T_*	f_{peak}	Detector
100 GeV	1.6×10^{-3} Hz	LISA
10^6 GeV	0.2 Hz	DECIGO/BBO
10^{16} GeV	10^8 Hz	CMB B-modes only

Spectral shape: f^3/f^{-1} broken power law — distinct from inflation (f^0) and SMBH binaries ($f^{-2/3}$).

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