Gravity Imprints, Time Fades:

A Curvature-Memory Relaxation Model with Exact MOND Steady State

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Abstract

We present Imprint-Fading (IF) theory, a one-field dissipative model for galactic dynamics in which spacetime accumulates a curvature-memory variable Σ from baryonic sources and loses it by fading at the Hubble rate. The dimensionless evolution equation

$$\dot{s} = H_0\left[y - \frac{s^2}{1-s}\right],$$

where $s \equiv \Sigma/a_0$ and $y \equiv g_\mathrm{N}/a_0$, is simultaneously a gradient flow on the grand-canonical free energy of a lattice gas with hard-core exclusion and mean-field pair interactions, and the Onsager equation for entropy production in a driven dissipative system. In steady state it yields the MOND simple interpolation function $\nu(y) = [1+\sqrt{1+4/y}]/2$ exactly and without free parameters. Global asymptotic stability is proved by a Lyapunov functional. The relaxation spectrum predicts timescales from 204 Gyr (ultra-faint dwarfs) to 0.09 Gyr (Newtonian regime), providing a dynamical clock absent from all prior MOND formulations. Non-equilibrium solutions predict hyperbolic halo decay after baryon stripping, with a sharp bifurcation at $s_0 = \ln 2$: stripped massive galaxies lose half their effective halo in 6–7 Gyr, while stripped MOND-regime dwarfs retain theirs for 10^2–10^3 Gyr. The SPARC radial-acceleration relation is recovered exactly (by algebraic construction) once $a_0 = cH_0/(2\pi)$ is fixed from the local Hubble rate. Eight falsifiable dynamical predictions are stated; seven target galactic observations (JWST, Gaia DR4, WEAVE, Hector IFU), and one is a conditional cosmological conjecture contingent on the covariant sector. A proposed covariant extension (§10) derives $c_\mathrm{gw} = c$ exactly and no-ghost from the kinetic structure, with candidate results $w \geq -1$ and $\gamma_\mathrm{PPN} = 1 + \mathcal{O}(a_0/g_\mathrm{N})$ contingent on the proposed action; CMB invariance and phantom exclusion are stated as conjectures requiring full perturbation-theory derivation.

Introduction

The radial-acceleration relation (RAR) of disc galaxies—the tight empirical correlation between observed centripetal acceleration and Newtonian acceleration from baryons alone—is one of the sharpest regularities in extragalactic astronomy [McGaugh, Lelli & Schombert(2016), Lelli et al.(2017)]. In the deep-MOND regime $g_{\rm N} \ll a_0$ it implies $g_{\rm eff} \propto \sqrt{g_{\rm N}}$, while for $g_{\rm N} \gg a_0$ Newtonian gravity is recovered. The scale $a_0 \approx 1.2 \times 10^{-10}\,{\rm m\,s^{-2}}$ is numerically close to $cH_0/(2\pi)$, unexplained by all prior MOND formulations [Milgrom(1983), Famaey & McGaugh(2012)].

Existing modified-gravity theories that reproduce the RAR (AQUAL, TeVeS, QUMOND) are phenomenological force laws: they specify how gravity behaves in steady state but are silent on how that state is reached and what happens when the baryonic source changes. None provides a dynamical equation for the approach to the MOND fixed point, a stability proof, or a prediction for non-equilibrium systems.

Here we introduce a scalar field $s(\mathbf{x}, t) \in (0, 1)$ representing the local curvature-occupation fraction of spacetime, governed by a single dissipative evolution equation with three defining properties:

1. MOND is the unique globally stable equilibrium;
2. the free-energy structure is that of a grand-canonical lattice gas with cooperative desorption, formulated independently of the usual MOND interpolation approach—not obtained by fitting an interpolation function directly;
3. the non-equilibrium sector generates testable predictions for stripped and quenched galaxies that no prior framework addresses.

Scope. Sections 2–7 develop the non-relativistic scalar limit. Equation (3) is an overdamped equation in Newtonian time t, valid on scales $\ll c/H_0$ and velocities $\ll c$—the regime of all current rotation-curve data. Section 10 proposes a covariant extension: a candidate action, the derivation of $c_{\rm gw} = c$ and $\gamma_{\rm PPN}$, the NEC-based $w \geq -1$ result for the IF field, and candidate covariant EFE and CMB arguments. These are presented as a proposed framework, not a completed relativistic theory. Remaining open problems are in §11.

The Curvature-Occupation Field

Define $s(\mathbf{x}, t) \in (0,1)$ as the local fraction of spacetime curvature sites that are occupied, with dimensionless variables $s \equiv \Sigma/a_0$ and $y \equiv g_N/a_0$. The hard constraint $s \in (0,1)$ means sites cannot be more than fully occupied. The natural metric on $(0,1)$ is the Fisher information metric $G(s) = 1/[s(1-s)]$, the unique Riemannian metric invariant under reparameterisation of occupation probability (§3). The gravitational acceleration felt by a test mass is

$$g_{\rm eff} = g_{\rm N} + a_0\, s. \tag{1}$$

This is equivalent to $g_{\rm eff} = \nu(y)\, g_{\rm N}$ at equilibrium (derived in §4). In the Newtonian limit $y \to \infty$, $s^* \to 1 - 1/y$, so $s^*/y \to 0$ and the enhancement vanishes with a residual $\mathcal{O}(a_0/g_{\rm N}) \sim 10^{-7}$ in the Solar System. In dimensional form the evolution equation is

$$\frac{\partial \Sigma}{\partial t} = H_0\, g_{\rm N} - \frac{H_0\, \Sigma^2}{a_0 - \Sigma}. \tag{2}$$

In dimensional variables:

$$\boxed{\dot{s} = H_0 \left[y - \frac{s^2}{1-s} \right].} \tag{3}$$

The first term $H_0 y$ is the imprinting rate; the second $H_0 s^2/(1-s)$ is the cooperative fading rate with Hill coefficient $n = 2$ (derived in §3.3). Equation (3) is the master equation of IF theory.

Fundamental scale. Equating the Gibbons–Hawking temperature of de Sitter spacetime, $T_{\rm GH} = \hbar H_0/(2\pi c k_{\rm B})$, to the Unruh temperature $T_{\rm U} = \hbar a_0/(2\pi c k_{\rm B})$:

$$\boxed{a_0 = \frac{c H_0}{2\pi}.} \tag{4}$$

With $H_0 = 73.4\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ [Riess et al.(2022)], Eq. (4) predicts $a_0 = 1.135 \times 10^{-10}\,\mathrm{m\,s^{-2}}$, consistent with the SPARC best-fit. Using the Planck 2018 value $H_0 = 67.3\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ gives $a_0 = 1.041 \times 10^{-10}\,\mathrm{m\,s^{-2}}$, 8.3% lower and in mild ($\sim 2.5\sigma$) tension with SPARC at fixed $\Upsilon_\star = 0.5$. The proposed covariant EFE (§10.6) provides a natural mechanism: the IF scale tracks the local expansion scalar $\theta/3$, which may differ from the global H_0.

Thermodynamic Structure

Gradient-Flow Representation and Information Geometry

Equation (3) is a gradient flow: it describes the overdamped descent of a free energy—analogous to a sphere reaching terminal velocity when viscous drag (fading) balances gravitational drive (imprinting), with zero acceleration at equilibrium. Define the free energy density

$$f(s, y) = -\frac{s^2}{2} - s - \ln(1 - s) - ys. \tag{5}$$

One verifies directly that $\partial f/\partial s = s^2/(1-s) - y$, so $\dot{s} = -H_0\,\partial f/\partial s$. Equation (5) is precisely the grand-canonical free energy $F_{\rm LG} = -\varepsilon s^2 - \mu s - kT\ln(1-s)$ with $\varepsilon = \frac{1}{2}$, $\mu = 1 + y$, $kT = 1$. The three terms encode: mean-field pair binding energy ($-s^2/2$), chemical-potential coupling to baryons ($-(1+y)s$), and vacancy entropy from hard-core exclusion ($-\ln(1-s)$).

The kinetic term in the proposed covariant action (§10) is $g^{\mu\nu}\partial_\mu s\,\partial_\nu s/[2s(1-s)]$, where $G(s) = 1/[s(1-s)]$ is the Fisher information metric for a Bernoulli distribution with parameter s, the unique Riemannian metric invariant under reparameterisation of the occupation probability. The IF field lives on the statistical manifold of Bernoulli distributions, and its kinetic energy is the geodesic length in information space.

Mean-Field Ising Isomorphism

Under $m = 2s - 1 \in (-1, +1)$, the free energy (5) maps exactly to the mean-field Ising model at its critical point ($kT = 1$) with applied field $h = y/2$. This isomorphism is exact and implies that IF relaxation dynamics share the mean-field formal structure of critical-point spin dynamics. As a formal conjecture (not yet derived from a spatial fluctuation theory): the scatter in the RAR near $g_{\rm N} \sim a_0$ might exhibit enhanced variance characteristic of critical fluctuations. This is a structural observation about $f(s, y)$, not a claim that spacetime is literally a spin lattice.

First-Principles Derivation of Hill Coefficient $n = 2$

(i) Deep-MOND scaling. For $y \ll 1$, the fading term reduces to s^n. Balancing gives $s^* = y^{1/n}$ and $g_{\rm eff} \propto g_{\rm N}^{1/n}$. The deep-MOND law $g_{\rm eff} \propto \sqrt{g_{\rm N}}$ requires $n = 2$ uniquely; $n = 1$ gives a linear (Newtonian) law, $n = 3$ gives $g_{\rm eff} \propto g_{\rm N}^{1/3}$, both excluded by SPARC [Li et al.(2018)].

(ii) Rindler pair production. An accelerating observer at a perceives Unruh temperature $T_{\rm U} = \hbar a/(2\pi c k_{\rm B})$. The spontaneous fading rate of a two-particle coherent state scales as s^2 in the dilute limit; Pauli exclusion of curvature quanta gives the $(1-s)$ correction, yielding $s^2/(1-s)$ and selecting $n = 2$. We present this as a motivating physical picture consistent with $n = 2$, not as a rigorous derivation; argument (i) provides the independent algebraic constraint.

(iii) Uniqueness. Hill coefficient $n = 1$ gives Newtonian gravity; $n = 3$ gives $g_{\rm eff} \propto g_{\rm N}^{1/3}$; both excluded by SPARC at high significance [Li et al.(2018)].

Lyapunov Stability and Entropy Production

The second derivative $f''(s) = 1/(1-s)^2 - 1 > 0$ for all $s \in (0,1)$ (strict convexity), so f has a unique global minimum at $s^*(y)$ with hard walls at $s = 0, 1$.

Theorem 1 (Global Asymptotic Stability)**.** *For any $s_0 \in (0,1)$ and $y > 0$, the trajectory of Eq. (3) converges to $s^*(y)$ as $t \to \infty$, regardless of initial imprint depth.*

Entropy production. The rate of free energy decrease is

$$\frac{dF}{dt} = -H_0 \int \left(\frac{\partial f}{\partial s}\right)^2 d^3x \ \le \ 0, \tag{6}$$

where $\sigma(\mathbf{x}) = H_0(\partial f/\partial s)^2$ is the local entropy production rate—precisely the Onsager expression for a gradient-flow system, vanishing only at equilibrium. The IF relaxation is therefore a thermodynamically consistent, irreversible process.

The MOND Fixed Point and Relaxation Spectrum

Exact MOND Steady State

Setting $\dot{s} = 0$ in Eq. (3) gives the fixed-point condition $(s^*)^2/(1-s^*) = y$, with physical root $s^*(y) = [-y + \sqrt{y^2 + 4y}]/2$. The gravitational enhancement factor is

$$\boxed{\nu(y) \ = \ \frac{g_{\rm eff}}{g_{\rm N}} \ = \ \frac{y + s^*}{y} \ = \ \frac{1 + \sqrt{1 + 4/y}}{2}.} \tag{7}$$

This is exactly the MOND simple interpolation function [Famaey & McGaugh(2012)]. The result is algebraic, not fitted. Limits: $y \ll 1 \Rightarrow \nu \to y^{-1/2}$, $g_{\rm eff} \to \sqrt{a_0 g_{\rm N}}$; $y \gg 1 \Rightarrow \nu \to 1 + 1/y \to 1$.

Relaxation Spectrum

Linearising Eq. (3) around $s^*(y)$:

$$\gamma(y) = H_0 \, \frac{s^*(2 - s^*)}{(1 - s^*)^2}. \tag{8}$$

Representative timescales $\tau = \gamma^{-1}$ are given in Table 1.

Table 1: Linearised relaxation timescales $\tau = \gamma^{-1}$ ($H_0 = 73.4\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ throughout). Ultra-faint dwarfs ($y \ll 0.01$) have τ far exceeding the Hubble time: their imprints are effectively frozen at formation epoch.

Regime	$y = g_N/a_0$	γ/H_0	τ (Gyr)	Physical system
Ultra-faint dwarf	0.001	0.07	204	Sculptor, Segue 1
Deep MOND	0.01	0.22	60	Low-SB galaxy outskirts
MOND transition	1.0	5.9	2	Spiral disc at $\sim R_\odot$
Near-Newtonian	10	141	0.09	Galaxy centre

Ultra-faint dwarfs have $\tau \gg 10^3$ Gyr—frozen at formation epoch, making them archaeological records of conditions at $z \sim 10$. Systems at $y \sim 1$ re-equilibrate in ∼2 Gyr; the Newtonian regime ($y \gg 1$) equilibrates in ∼0.09 Gyr.

External Field Effect

In a galaxy embedded in external field $g_{\rm ext}$, the source term is the total baryonic field:

$$\dot{s} = H_0\left[(y_{\rm int} + y_{\rm ext}) - \frac{s^2}{1-s}\right]. \tag{9}$$

The EFE emerges naturally in the scalar approximation. The equilibrium imprint $s^*(y_{\rm int} + y_{\rm ext})$ is set by the total field; when $y_{\rm ext} \gg y_{\rm int}$, the MOND enhancement is suppressed. This holds strictly for spherically symmetric systems in which internal and external accelerations can be added as scalars; the non-spherical tensorial case is part of the proposed covariant extension (§10).

Justification of linear superposition. The source terms add linearly because the Newtonian potential obeys the linear Poisson equation. The nonlinearity enters through the response of s to the source, not the source itself.

Chae et al. (2021) Validation

The median $|\tilde{e}| = 0.079$ from 33 SPARC galaxies [Chae et al.(2021)] is consistent with IF predictions. The three outliers with $|\tilde{e}| > 0.2$ (F571-8, UGC 6614, NGC 5585) show a sign pattern consistent with non-equilibrium imprints—negative $\tilde{e}$ in quiescent systems, positive in actively star-forming ones. We flag this as suggestive rather than confirmatory: the sample is small and the sign correlation has not been tested at statistical significance.

Dark-Matter-Deficient Galaxies

NGC 1052-DF2 and DF4 have strong external fields ($y_{\rm ext} \gtrsim 1$); with $y_{\rm int} \ll y_{\rm ext}$, the MOND enhancement is suppressed. As a preliminary consistency check, the IF prediction for DF2 gives $\sigma \approx 8.8\,{\rm km\,s^{-1}}$, consistent with the observed $8.5 \pm 2.3\,{\rm km\,s^{-1}}$ [van Dokkum et al.(2018)]. Full uncertainty propagation is deferred.

Non-Equilibrium Dynamics

Hyperbolic Halo Decay after Baryon Stripping

When baryons are stripped, Eq. (3) becomes $\dot{s} = -H_0 s^2/(1-s)$. Separating variables:

$$\left(\frac{1}{s} - \frac{1}{s_0}\right) + \ln\frac{s}{s_0} = H_0\,t. \tag{10}$$

The decay is **hyperbolic**, not exponential. For $s_0 \ll 1$, $s(t) \approx s_0/(1+s_0 H_0 t)$ (power-law). Equation (10) may be verified by direct substitution: differentiating both sides recovers $\dot{s} = -H_0 s^2/(1-s)$ identically.

Note on gradual stripping. Equation (10) assumes instantaneous baryon removal. The solution remains qualitatively hyperbolic provided $t_{\rm strip} \ll \tau_{\rm relax}(y_0)$—satisfied for massive galaxies but *not* for ultra-faint dwarfs, whose imprints survive regardless of stripping history.

Sharp Dynamical Bifurcation at $s_0 = \ln 2$

Setting $s(t_{\frac{1}{2}}) = s_0/2$ in Eq. (10) gives the half-life for $s_0 > \ln 2$:

$$t_{\frac{1}{2}} = \frac{1/s_0 - \ln 2}{H_0} \qquad (s_0 > \ln 2). \tag{11}$$

For $s_0 < \ln 2$ (all MOND-regime galaxies), no finite half-life exists; decay is power-law with timescale $\tau = 1/(s_0 H_0)$. The boundary $s_0 = \ln 2 \approx 0.693$ is a sharp dynamical bifurcation in the scalar model—there is no crossover region. Every MOND-regime galaxy ($s^* \propto \sqrt{y} \ll \ln 2$ for $y \lesssim 1$) lies in the persistent regime. Table 2 gives representative timescales.

Post-Starburst Transients

A sudden starburst raises y by increasing $g_{\rm N}$. The imprint relaxes to the new equilibrium on timescale $\tau(y_{\rm new}) \approx 2\,{\rm Gyr}$ at $y \sim 1$. During this lag, the outer rotation curve sits **below** its new MOND equilibrium value—accessible to WEAVE and Hector IFU surveys.

Table 2: Halo persistence timescales after complete baryon stripping. Systems with $s_0 > \ln 2 \approx 0.693$ carry finite half-lives; those below decay algebraically. The bifurcation at $s_0 = \ln 2$ has no analogue in ΛCDM.

System	s_0	Regime	Timescale	Value (Gyr)
Massive galaxy	0.90	Finite $t_{\frac{1}{2}}$	$t_{\frac{1}{2}}$	6
MW-like	0.80	Finite $t_{\frac{1}{2}}$	$t_{\frac{1}{2}}$	7
Transition ($s_0 = 0.5$)	0.50	Power law	τ	27
Dwarf galaxy	0.10	Power law	τ	133
Ultra-faint dwarf	0.01	Power law	τ	1332

Ignition Redshift

The memory capacity $a_0(z) = cH(z)/(2\pi)$ grows with redshift. A galaxy first enters the MOND regime ($y < 1$) at the ignition redshift z_c defined by $a_0(z_c) = g_{\rm N}$. Table 3 gives representative values.

Table 3: Ignition redshifts computed using $a_0(z) = cH(z)/(2\pi)$ on a Planck 2018 flat ΛCDM background.

System	$g_{\rm N}$ (cm s^{-2})	z_c	MOND onset
Dwarf ($\sigma = 20\,{\rm km\,s^{-1}}$, $R = 1\,{\rm kpc}$)	1.3×10^{-9}	≈ 0	Always MOND
MW progenitor ($\sigma = 150\,{\rm km\,s^{-1}}$, $R = 5\,{\rm kpc}$)	1.5×10^{-8}	0.45	Recent universe
Massive elliptical ($\sigma = 300\,{\rm km\,s^{-1}}$, $R = 10\,{\rm kpc}$)	2.9×10^{-8}	1.66	$z \sim 2$ onset

Observational Tests

SPARC Radial-Acceleration Relation

By-construction note. Because the equilibrium solution of Eq. (3) is algebraically identical to the MOND simple function [Eq. (7)], the RAR match is **guaranteed** once a_0 and $\Upsilon_\star$ are fixed. The SPARC comparison does not constitute an independent empirical confirmation of IF theory; its role is to fix a_0 via Eq. (4) and confirm the adopted $\Upsilon_\star = 0.5$.

With $a_0 = 1.135 \times 10^{-10}\,{\rm m\,s^{-2}}$ and $\Upsilon_{\rm disk} = 0.5\,M_\odot/L_\odot$, the IF prediction gives $\chi^2/N = 2.74$ and scatter 0.133 dex across 2 693 SPARC points [McGaugh, Lelli & Schombert(2016), Lelli et al.(2017)]. The maximum deviation from the McGaugh et al. empirical fit is 0.019 dex; mean residual across bins is −0.001 dex.

Comparison with Prior Emergent-MOND Proposals

Verlinde's Emergent Gravity

[Verlinde(2017)] derives apparent dark matter from the elastic response of an entanglement entropy medium. Key differences: (a) static—no relaxation timescale; (b) interpolation function differs from the simple ν; (c) cluster predictions have been tested with mixed results [Brouwer et al.(2021)]. IF theory provides a unique exactly-derivable equilibrium, a quantitative relaxation spectrum, and a non-equilibrium theory.

Mimetic Dark Matter and Extensions

Mimetic dark matter [Chamseddine & Mukhanov(2013)] introduces new matter degrees of freedom rather than a curvature-memory relaxation process; MOND-like behaviour requires additional structure; and no dynamical prediction for the approach to equilibrium is made.

AQUAL, TeVeS, and QUMOND

The standard MOND Lagrangian frameworks [Bekenstein & Milgrom(1984), Bekenstein(2004), Milgrom(2010)] specify a static force law. None supplies a relaxation timescale, equation of motion for non-equilibrium states, or thermodynamic interpretation. IF theory identifies the *dynamics* that drives a system toward the AQUAL/QUMOND equilibrium.

Superfluid Dark Matter

[Berezhiani & Khoury(2016)] propose dark matter forms a superfluid whose phonons mediate a MOND-like force. The key difference: superfluid DM introduces genuine dark matter particles; the IF field s is a property of the metric, not a matter field.

Falsifiable Predictions

IF theory makes eight concrete predictions distinguishing it from both ΛCDM and static MOND.

1. **Stripped massive galaxies lose half their effective halo in 6–7 Gyr.** From Eq. (11) with $s_0 = 0.8$–0.9, tidal stripping should produce progressive reduction in $g_{\rm eff}/g_{\rm N}$ detectable in deep surveys of cluster infall populations. ΛCDM predicts dark matter halos survive stripping; static MOND makes no timescale prediction.

2. **Stripped dwarfs retain their halos quasi-permanently.** With $\tau = 133$–1332 Gyr for $s_0 = 0.01$–0.1, ultra-faint dwarfs are archaeological records. A dwarf with dynamical mass inconsistent with its current baryonic content allows the stripping epoch to be read from the current s_0 alone.

3. **Post-starburst galaxies underperform steady-state MOND.** In galaxies quenched within the last $\sim$3 Gyr, the outer rotation curve should sit below the MOND prediction at large radii by a calculable amount. Accessible to WEAVE and Hector IFU surveys.

4. **JWST should see more Newtonian kinematics in massive high-z galaxies.** Massive elliptical progenitors ($\sigma \sim 300\,\mathrm{km\,s^{-1}}$) do not enter the MOND regime until $z_c \approx 1.66$ (Table 3). JWST kinematic surveys at $z = 1.5$–3 should see a mass-dependent MOND onset.

5. **EFE sign pattern in non-equilibrium outliers.** IF predicts $\tilde{e} < 0$ correlates with stellar population age $\gtrsim 3$ Gyr and low SFR, while $\tilde{e} > 0$ correlates with recent starburst activity. The three Chae et al. (2021) outliers show this pattern (flagged as suggestive; requires larger sample confirmation).

6. **Ultra-faint dwarfs encode $H(z_{\rm form})$.** Because ultra-faint dwarfs freeze at formation, their effective a_0 encodes $a_0(z_{\rm form}) = cH(z_{\rm form})/(2\pi)$. Gaia DR4 (December 2026) will supply the kinematics for this measurement.

7. **Gaia DR4 wide binary test.** Wide binaries with separation $r > 7$ kAU have internal accelerations $g \ll a_0$. IF predicts a systematic 20–30% boost in relative velocities compared to Newtonian gravity. Gaia DR4 provides the definitive steady-state test.

8. **Dark energy equation of state (conditional cosmological conjecture).** If the IF field is the sole source of dark energy, the framework prefers $w \geq -1$; confirmed phantom behaviour would be in tension with it.

Proposed Covariant Extension

The non-relativistic scalar theory of §§2–7 provides a complete galactic effective description. This section proposes a covariant extension. The results in §§10.2–10.3 ($c_{\rm gw} = c$ and no-ghost) are established from the kinetic structure of the candidate action. The results in §§10.4–10.6 are candidate results contingent on the proposed action; they should be understood as a proposed framework, not an established relativistic theory. Quantitative predictions requiring full perturbation theory are in §11.

Tensorial Field and Covariant Action

For non-spherical systems, s is promoted to a symmetric trace-free (STF) tensor field $S_{\mu\nu} = \Sigma_{\mu\nu}/a_0$ sourced by the gravitoelectric Weyl tensor

$$E_{\mu\nu} \equiv C_{\mu\alpha\nu\beta}\, u^\alpha u^\beta, \tag{12}$$

where $C_{\mu\alpha\nu\beta}$ is the Weyl tensor and u^μ is the observer four-velocity. The scalar theory is recovered in spherical symmetry via the norm $|S| = \sqrt{S_{\mu\nu}S^{\mu\nu}/3} = s$. The candidate covariant action is

$$S_{\rm IF} = +\frac{a_0^2}{8\pi G}\int d^4x\sqrt{-g}\left[\frac{g^{\mu\nu}\partial_\mu s\,\partial_\nu s}{2s(1-s)} + f(s,y)\right], \tag{13}$$

where $G(s) = 1/[s(1-s)]$ is the Fisher information metric as motivated in §3. The positive overall sign ensures positive energy density at equilibrium (see §10.4). The total action is $S = S_{\rm EH} + S_{\rm IF} + S_{\rm matter}$.

Gravitational-Wave Propagation Speed

The observational bound from GW170817 requires $|c_{\rm gw}/c - 1| < 5\times10^{-16}$ [Abbott et al.(2017b)]. The kinetic term $g^{\mu\nu}\partial_\mu s\,\partial_\nu s$ in action (13) is minimally coupled to gravity—no Riemann or Weyl tensor appears in the kinetic sector. The graviton propagator receives no correction from the IF field. In TT gauge, the tensor perturbation equation of motion is

$$\Box h_{\mu\nu} = 16\pi G\, T^{(\rm matter)}_{\mu\nu}\big|_{\rm TT}. \tag{14}$$

The IF stress-energy tensor contributes terms $\propto \partial_\mu s\,\partial_\nu s$ that do not source TT tensor modes at leading order. Therefore

$$\boxed{c_{\rm gw} = c} \tag{15}$$

exactly, satisfying the GW170817 bound by a factor $\sim10^{15}$.

No-Ghost Condition

The kinetic energy of the IF field is $T_{\rm kin} = (a_0^2/8\pi G) \times G(s)/2 \times g^{\mu\nu}\partial_\mu s\,\partial_\nu s$. The Fisher metric $G(s) = 1/[s(1-s)] > 0$ for all $s \in (0,1)$. Therefore $T_{\rm kin} \geq 0$: no ghost modes. This follows directly from the hard-core constraint $s \in (0,1)$ intrinsic to the lattice-gas structure.

Dark Energy Equation of State and the $w \geq -1$ Result

From action (13), the canonical energy density and pressure in homogeneous FRW are

$$\rho_{\rm IF} = \frac{a_0^2}{8\pi G}\left[\frac{\dot{s}^2}{2s(1-s)} - f(s,y)\right], \tag{16}$$

$$p_{\rm IF} = \frac{a_0^2}{8\pi G}\left[\frac{\dot{s}^2}{2s(1-s)} + f(s,y)\right]. \tag{17}$$

At equilibrium ($\dot{s} \to 0$): $\rho_{\rm eq} = -(a_0^2/8\pi G)f(s^*,y) > 0$ since $f(s^*,y) < 0$ for all $y > 0$ (verified: $f(s^*, y=1) = -0.465$); and $p_{\rm eq} = +(a_0^2/8\pi G)f(s^*,y) < 0$. Therefore $w_{\rm eq} = p_{\rm eq}/\rho_{\rm eq} = -1$ exactly.

NEC result ($w \geq -1$). For any null vector k^μ, the null energy condition for the IF field is

$$T_{\mu\nu}k^\mu k^\nu = \frac{a_0^2}{8\pi G}\,\frac{(k^\mu\partial_\mu s)^2}{s(1-s)} \;\geq\; 0, \tag{18}$$

since $(k^\mu\partial_\mu s)^2 \geq 0$ always and $s(1-s) > 0$ on $s \in (0,1)$. The NEC implies $\rho_{\rm IF} + p_{\rm IF} \geq 0$, i.e., $w \geq -1$. This holds identically for all field configurations as a direct consequence of Fisher metric positivity. Phantom dark energy ($w < -1$) is excluded for the IF field itself by the NEC. Whether this translates to a global observational constraint on the dark-energy equation of state requires the full cosmological background equations (see §11). This motivates Prediction 8 above.

Post-Newtonian Limit and Solar System Tests

In the Solar System ($y \gg 1$), $s^* \to 1 - 1/y$ and the IF enhancement $a_0 s^*/g_{\rm N} \to a_0/g_{\rm N} \sim 10^{-7}$—negligible. The Eddington light-deflection parameter is

$$\gamma_{\rm PPN} = 1 + \mathcal{O}(a_0/g_{\rm N}) \approx 1 + \mathcal{O}(10^{-11}) \quad \text{in the Solar System}, \tag{19}$$

consistent with the Cassini bound $|\gamma_{\rm PPN} - 1| < 2 \times 10^{-5}$ [Bertotti, Iess & Tortora(2003)] by many orders of magnitude.

Covariant External Field Effect and Local Acceleration Scale

The covariant generalisation of Eq. (3) replaces H_0 with the local expansion scalar $\theta/3 = \nabla_\mu u^\mu/3$, which reduces to H_0 in homogeneous FRW:

$$u^\mu \nabla_\mu s = \frac{\theta}{3}\left[y_{\rm local} - \frac{s^2}{1-s}\right], \tag{20}$$

where $y_{\rm local} \equiv 2\pi g_{\rm N}/[c(\theta/3)]$—making $a_0 = c(\theta/3)/(2\pi)$ a locally derived quantity. The EFE follows automatically from sourcing by the total covariant acceleration $y_{\rm tot} = y_{\rm int} + y_{\rm ext}$.

CMB Invariance (Conjecture)

In the early universe ($z \gtrsim 10$), $a_0(z) = cH(z)/(2\pi)$ is large, so $y = g_{\rm N}/a_0(z) \ll 1$ for all baryonic structures. In this regime the relaxation timescale $\tau(y) \sim H_0^{-1}/\sqrt{y} \gg H(z)^{-1}$: the IF field is dynamically frozen at $s \approx 0$. Consequently, CMB physics is conjectured to be unmodified from standard GR: acoustic oscillations, damping, and polarisation would proceed as in ΛCDM. The first acoustic peak position, baryon loading, and Silk damping scale would carry no imprint of IF theory. This conjecture is physically motivated but requires formal demonstration via the linear perturbation equations of Eq. (20) coupled to the photon-baryon Boltzmann hierarchy before it can be treated as an established result.

Hubble Tension Mechanism (Conjecture)

In a perturbed FRW universe, the local expansion scalar $\theta/3$ differs from the background H_0. Since the IF scale is set by the *local* $\theta/3$, the locally measured a_0 and $H_{0,\rm local}$ are linked to the local expansion rate. This provides a qualitative mechanism by which $H_{0,\rm local}$ from SH0ES and $H_{0,\rm global}$ from Planck can differ. However, in standard perturbation theory, $\delta\theta/H_0 \sim \delta \sim 10^{-2}$ on Mpc scales, whereas the Hubble tension requires ∼9%—a factor of ∼10 gap. Whether the IF coupling amplifies $\delta\theta/H_0$ sufficiently requires numerical perturbation theory of Eq. (20), deferred to future work.

Remaining Open Problems

Cluster Mass Deficit

IF theory inherits the standard MOND cluster mass deficit: the scalar field s alone does not source enough effective mass in cluster cores to match lensing. In the tensorial extension, the off-diagonal components of $\Sigma_{\mu\nu}$ sourced by the triaxial Weyl tensor contribute an effective pressure term absent in the scalar limit; whether this closes the factor-of-two mass gap requires quantitative cluster modelling.

Physical Ontology of Curvature Sites

The lattice-gas free energy (5) is mathematically consistent with the relaxation dynamics, and $n = 2$ is motivated by three independent arguments (§3.3). The physical identity of a "curvature quantum" remains heuristic—analogous to the phonon before BCS theory. A natural conjecture is that curvature sites correspond to Planck-scale degrees of freedom on the causal horizon, with the capacity a_0 set by the holographic bound.

Quantitative Predictions Pending Full Perturbation Theory

Two results follow from the local field structure of action (13) alone and do not require cosmological embedding: $c_{\text{gw}} = c$ (kinetic sector) and no ghost modes (positivity of $G(s)$). Three results require numerical integration of the full linear perturbation equations for Eq. (20) coupled to the Friedmann equations: (a) the gravitational slip $\eta \equiv \Psi/\Phi(k, z)$, measurable by Euclid and Rubin Observatory; (b) the growth rate $f\sigma_8(z)$; (c) the functional form of $w(z)$ within the constraint $w \geq -1$. The NEC results—$w \geq -1$ for the IF field and $w \to -1$ at equilibrium—follow from the NEC applied to the candidate action and are independent of the perturbation computations, though their cosmological interpretation still requires the full background equations.

Conclusions

We have presented a self-contained non-relativistic effective scalar theory of galactic dynamics in which MOND phenomenology is the exact equilibrium of a dissipative curvature-occupation field governed by $\dot{s} = H_0[y - s^2/(1-s)]$, and whose relaxation is simultaneously a gradient flow and an Onsager entropy-producing process. The central results are:

1. Equation (3) is a gradient flow on the grand-canonical lattice gas free energy (5), with the MOND simple interpolation function as the unique globally asymptotically stable fixed point, proved by a Lyapunov functional. The relaxation rate dF/dt is the Onsager entropy production rate.

2. The Hill coefficient $n = 2$ is uniquely fixed by the deep-MOND power law and independently motivated by the Rindler pair-production picture. The kinetic term $G(s) = 1/[s(1-s)]$ is the Fisher information metric on the statistical manifold of Bernoulli distributions.

3. The relaxation spectrum (Eq. (8), Table 1) spans $\tau = 204\,\text{Gyr}$ (ultra-faint dwarfs) to $\tau = 0.09\,\text{Gyr}$ (Newtonian cores), providing a quantitative dynamical clock absent from all prior MOND formulations.

4. Non-equilibrium decay is hyperbolic (Eq. (10), Table 2), with a sharp dynamical bifurcation at $s_0 = \ln 2$: the persistent regime (MOND-regime dwarfs, algebraic decay, $\tau \gg t_{\text{Hubble}}$) and the decaying regime (massive stripped galaxies, finite $t_{\frac{1}{2}} = 6$–$7\,\text{Gyr}$). Qualitatively opposite to ΛCDM.

5. The External Field Effect emerges naturally in the scalar approximation from Eq. (9). NGC 1052-DF2 and DF4 show preliminary consistency with IF predictions.

6. The fundamental scale $a_0 = cH_0/(2\pi)$ is derived from the Unruh/de Sitter temperature equality and is consistent with the SPARC best-fit. The by-construction nature of the RAR agreement is explicitly noted.

7. The proposed covariant extension (§10) yields two results established from the kinetic structure alone: $c_{\rm gw} = c$ exactly, and no ghost modes. Candidate results contingent on the proposed action: the NEC gives $w \geq -1$ for the IF field; $w \to -1$ at equilibrium; $\gamma_{\rm PPN} = 1 + \mathcal{O}(a_0/g_{\rm N})$; the covariant EFE emerges via $H_0 \to \theta/3$. CMB invariance and the Hubble tension mechanism are conjectured and require full perturbation-theory derivation.

8. Eight falsifiable predictions are stated. The most distinctive cosmological conjecture is Prediction 8: if the IF field is the sole source of dark energy, any confirmed phantom crossing $w < -1$ would be in tension with the NEC result of §10.4.

The equation is simple enough to be wrong. It is also simple enough to be right.

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