Observational Test Strategy for an Octonionically Motivated Model of Emergent Spacetime and Cosmic Expansion

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We formulate an observationally testable phenomenological framework for a cosmological model motivated by octonionic pre-geometry. In this picture, the nonassociativity of the octonionic division algebra obstructs the existence of a fundamental global time and motivates a symmetry reduction to an effectively associative sector, identified with emergent four-dimensional spacetime. Residual nonassociative degrees of freedom are modeled as an effective dynamical dark-energy component. We introduce a minimal approach for the effective octonionic energy density and derive the associated Hubble function, luminosity distance, baryon-acoustic-oscillation distances, and effective equation of state. The resulting model admits a direct mapping onto the standard (w_0, w_a) dark-energy parametrization and reduces continuously to ΛCDM in the appropriate limit. We present a concrete benchmark parameterization and formulate a step-by-step likelihood strategy based on Type Ia supernovae, baryon acoustic oscillations, and direct $H(z)$ measurements. The framework is designed as a falsifiable intermediate step between a speculative algebraic origin of spacetime and precision late-universe cosmology.

I. INTRODUCTION

The standard cosmological model successfully describes a broad range of observations by combining general relativity with cold dark matter and an approximately constant dark-energy sector. Despite its empirical success, the standard framework leaves open whether spacetime geometry is fundamental or instead emerges from a deeper pre-geometric structure. This question becomes especially acute near regimes where classical spacetime concepts are expected to break down, notably at the Big Bang and gravity.

A mathematically distinguished candidate for such a pre-geometric structure is the division algebra of octonions. Unlike the real numbers, complex numbers, and quaternions, the octonions are noncommutative and nonassociative. Their nonassociativity is particularly significant because it obstructs a globally consistent associative product and therefore suggests that spacetime may arise only after a reduction to a stable associative sector. In the conceptual framework considered here, four-dimensional spacetime is interpreted as an emergent associative subalgebra selected through an algebraically singular transition, while cosmic expansion arises as an effective manifestation of residual nonassociative degrees of freedom.

The primary challenge for such a framework is empirical accessibility. A mathematically suggestive pre-geometric scenario becomes physically meaningful only if it produces concrete observable consequences. The purpose of the present work is therefore to formulate a phenomenological and observationally testable version of the octonionic scenario at the level of homogeneous cosmology. Rather than attempting a complete microscopic derivation from fundamental octonionic field equations, we construct an effective background model in which residual nonassociativity contributes a dynamical vacuum-like term to the Friedmann equation.

The strategy has three objectives:

(i) to provide a transparent parametrization of the effective octonionic sector,

(ii) to connect that parametrization analytically to standard dark-energy observables such as w_0 and w_a,

(iii) to yield a clear falsification logic by embedding ΛCDM as a special limiting case.

II. CONCEPTUAL FRAMEWORK

A. Pre-geometric octonionic structure

We assume that the fundamental structure underlying physical spacetime is not a four-dimensional differentiable manifold endowed with a metric, but an octonionic algebra $\mathbb{O}$. The key algebraic object is the associator

$$[x, y, z] := (xy)z - x(yz), \tag{1}$$

which vanishes identically in associative algebras but is generically nonzero for octonions.

Within the present interpretation, the nonvanishing associator has direct physical meaning. It implies that globally well-defined dynamical evolution cannot be formulated at the most fundamental level in the same way as in standard spacetime theories. A physically interpretable regime is assumed to arise only after symmetry reduction to an effectively associative sector. This reduction is taken to define the origin of classical spacetime and is associated conceptually with a Big-Bang-like transition.

B. Residual nonassociativity as an effective cosmological source

After the emergence of an effectively four-dimensional associative spacetime sector, the reduction is not assumed to be perfect. Residual nonassociative degrees of freedom remain and are modeled phenomenologically as an effective energy-density contribution to the cosmological background dynamics. In this picture, dark energy is not introduced as a separate ad hoc fluid or cosmological constant, but appears as the macroscopic imprint of incomplete algebraic reduction.

The central practical question is how to parametrize this residual sector in a way that is sufficiently general to capture deviations from ΛCDM while remaining simple enough for robust data analysis.

III. EFFECTIVE BACKGROUND MODEL

A. Requirements on the phenomenological approach

The effective octonionic contribution must satisfy several requirements. It must remain finite and positive at the present epoch, reduce continuously to a cosmological-constant-like contribution in an appropriate limit, admit mild time dependence, and be directly implementable in standard observational likelihood analyses. To satisfy these conditions, we introduce the effective energy density

$$\rho_{\rm oct}(a) = \rho_{\rm oct,0}\left[1+\beta(1-a)\right]e^{-\gamma(1-a)}, \qquad a = \frac{1}{1+z}. \tag{2}$$

Here $\rho_{\rm oct,0}$ denotes the present-day effective energy density of the octonionic sector, while β and γ are dimensionless phenomenological parameters. The parameter β captures the leading residual nonassociative correction away from the present epoch, whereas γ governs the effective relaxation of the residual contribution over cosmic history.

This approach has an immediate and important limiting property. For

$$\beta = 0, \qquad \gamma = 0, \tag{3}$$

the octonionic sector becomes constant,

$$\rho_{\rm oct}(a) = \rho_{\rm oct,0}, \tag{4}$$

and the model reduces exactly to a cosmological-constant contribution. Thus ΛCDM is embedded as a special case.

B. Modified Friedmann equation

Assuming a spatially flat Friedmann-Robertson-Walker background, the Hubble expansion rate may be written as

$$H^2(z) = H_0^2 E^2(z), \tag{5}$$

with

$$E^2(z) = \Omega_{m0}(1+z)^3+\Omega_{r0}(1+z)^4+\Omega_{\rm oct,0}\left[1+\beta\frac{z}{1+z}\right]\exp\left(-\gamma\frac{z}{1+z}\right). \tag{6}$$

At late times, the radiation term is negligible, so that

$$E^2(z) \approx \Omega_{m0}(1+z)^3+(1-\Omega_{m0})\left[1+\beta\frac{z}{1+z}\right]\exp\left(-\gamma\frac{z}{1+z}\right), \tag{7}$$

where spatial flatness has been used to set

$$\Omega_{\rm oct,0} = 1-\Omega_{m0}. \tag{8}$$

This equation is the primary observable prediction of the model at background level. It is directly usable in Type Ia supernova, baryon acoustic oscillation, and $H(z)$ analyses.

IV. EFFECTIVE EQUATION OF STATE

A. Derivation

To characterize the octonionic sector in standard dark-energy language, we derive its effective equation of state. For any homogeneous component satisfying the continuity equation

$$\dot{\rho} + 3H(1+w)\rho = 0, \tag{9}$$

one may write

$$w(a) = -1 - \frac{a}{3}\frac{d\ln\rho}{da}. \tag{10}$$

Applying this to the octonionic energy density yields

$$\ln\rho_{\rm oct}(a) = \ln\rho_{\rm oct,0} + \ln\left[1+\beta(1-a)\right] - \gamma(1-a), \tag{11}$$

and therefore

$$\frac{d\ln\rho_{\rm oct}}{da} = -\frac{\beta}{1+\beta(1-a)} + \gamma. \tag{12}$$

Substituting gives

$$w_{\rm oct}(a) = -1 + \frac{a}{3}\left[\frac{\beta}{1+\beta(1-a)} - \gamma\right]. \tag{13}$$

This expression is exact for the adopted approach.

B. Relation to the (w_0, w_a) parametrization

At the present epoch $a = 1$, the effective equation of state becomes

$$w_0 = -1 + \frac{\beta-\gamma}{3}. \tag{14}$$

Expanding linearly around $a = 1$ in the Chevallier-Polarski-Linder form

$$w(a) = w_0 + w_a(1 - a), \tag{15}$$

one finds

$$w_a = -\frac{\beta - \gamma + \beta^2}{3}. \tag{16}$$

These relations are crucial because they map the octonionic model directly onto standard observational dark-energy parameters. Conversely, observational constraints on w_0 and w_a can be translated back into constraints on β and γ.

C. Inverse relations

Solving for the phenomenological parameters gives

$$\beta = \sqrt{3(-w_0 - w_a - 1)}, \tag{17}$$

and

$$\gamma = -3w_0 + \beta - 3. \tag{18}$$

This inversion enables immediate interpretation of any observational (w_0, w_a) fit in the language of residual nonassociativity.

V. BENCHMARK PARAMETERIZATION

To illustrate the practical use of the model, we introduce a benchmark parameter choice obtained by mapping representative dark-energy values into the octonionic parametrization. Taking

$$w_0 = -0.978, \qquad w_a = -0.65, \tag{19}$$

one obtains

$$\beta \approx 1.373, \qquad \gamma \approx 1.307. \tag{20}$$

Adopting further the background values

$$H_0 = 67.4\ \mathrm{km\,s^{-1}\,Mpc^{-1}}, \qquad \Omega_{m0} = 0.315, \tag{21}$$

together with

$$\Omega_{\mathrm{oct},0} = 0.685, \tag{22}$$

the explicit Hubble function becomes

$$H(z) = 67.4\sqrt{0.315(1+z)^3 + 0.685\left[1 + 1.373\frac{z}{1+z}\right]\exp\left(-1.307\frac{z}{1+z}\right)}\ \mathrm{km\,s^{-1}\,Mpc^{-1}}. \tag{23}$$

For illustration, the model gives the approximate expansion rates

$$H(0) = 67.4, \tag{24}$$
$$H(0.5) \approx 89.1, \tag{25}$$
$$H(1) \approx 121.1, \tag{26}$$
$$H(2) \approx 203.5, \tag{27}$$

in units of $\mathrm{km\,s^{-1}\,Mpc^{-1}}$.

VI. OBSERVATIONAL STRATEGY

A. Type Ia supernovae

Type Ia supernovae constrain the integrated expansion history through the luminosity distance

$$d_L(z) = (1+z)\frac{c}{H_0}\int_0^z \frac{dz'}{E(z')}. \tag{28}$$

The corresponding distance modulus is

$$\mu(z) = 5\log_{10}\left(\frac{d_L(z)}{\mathrm{Mpc}}\right) + 25. \tag{29}$$

Given a supernova sample with covariance matrix C, the associated likelihood is constructed from

$$\chi^2_{\mathrm{SN}} = \Delta\mu^{\mathrm{T}} C^{-1} \Delta\mu, \qquad \Delta\mu_i = \mu_{\mathrm{obs},i} - \mu_{\mathrm{model}}(z_i). \tag{30}$$

Supernovae are especially sensitive to departures from a strictly constant dark-energy sector and therefore provide strong constraints on β and γ.

B. Baryon acoustic oscillations

BAO measurements constrain combinations of comoving and Hubble distances. The relevant quantities are

$$D_M(z) = \frac{c}{H_0}\int_0^z \frac{dz'}{E(z')}, \qquad D_H(z) = \frac{c}{H(z)}, \tag{31}$$

and, for isotropic analyses,

$$D_V(z) = \left[z D_M^2(z) D_H(z)\right]^{1/3}. \tag{32}$$

The model is compared to observed quantities such as

$$\frac{D_M(z)}{r_d}, \qquad \frac{D_H(z)}{r_d}, \qquad \frac{D_V(z)}{r_d}, \tag{33}$$

where r_d is the sound horizon at the drag epoch. Because BAO measurements provide a geometric calibration of cosmic expansion across a wide redshift range, they are particularly effective in breaking degeneracies that remain in supernova-only fits.

C. Direct Hubble-rate measurements

Cosmic-chronometer observations provide direct estimates of the expansion rate via

$$H(z) = -\frac{1}{1+z}\frac{dz}{dt}. \tag{34}$$

For a set of measurements $H_{\mathrm{obs},i}$ with uncertainties σ_i, one defines

$$\chi^2_H = \sum_i \frac{\left[H_{\mathrm{obs},i} - H_{\mathrm{model}}(z_i)\right]^2}{\sigma_i^2}. \tag{35}$$

These measurements complement the integrated probes above by directly sampling the local slope of cosmic evolution.

VII. JOINT LIKELIHOOD ANALYSIS

The combined background-data constraint is obtained by minimizing

$$\chi^2_{\rm tot} = \chi^2_{\rm SN} + \chi^2_{\rm BAO} + \chi^2_H. \tag{36}$$

The minimal parameter vector is

$$\Theta = \{H_0, \Omega_{m0}, \beta, \gamma\}. \tag{37}$$

Reasonable prior intervals for an initial exploratory analysis are

$$\Omega_{m0} \in [0.2, 0.4], \qquad \beta \in [-3, 3], \qquad \gamma \in [-3, 3]. \tag{38}$$

A natural starting point for Markov-chain or nested-sampling analyses is given by the benchmark

$$H_0 = 67.4, \qquad \Omega_{m0} = 0.315, \qquad \beta = 1.37, \qquad \gamma = 1.31. \tag{39}$$

The essential purpose of the joint analysis is twofold. First, it determines whether the data favor nonzero values of β and γ. Second, it assesses whether the improved fit, if any, is sufficient to justify the additional model freedom relative to ΛCDM.

VIII. PHYSICAL INTERPRETATION OF THE PARAMETERS

The parameter β may be interpreted as the leading effective measure of residual nonassociative influence in the cosmological sector. Positive values correspond to a stronger departure from a pure cosmological constant at earlier times. The parameter γ plays the role of a relaxation coefficient, determining how rapidly the nonassociative contribution attenuates as the universe evolves.

In this language, the standard ΛCDM model corresponds to complete effective freezing of the residual octonionic sector. Nonzero β and γ quantify the extent to which this freezing is incomplete. The practical importance of this interpretation is that observational evidence for time-dependent dark energy would acquire a possible pre-geometric algebraic reading rather than remaining a purely phenomenological deviation.

IX. FALSIFIABILITY

A central virtue of the present framework is that it is straightforwardly falsifiable at background level.

The first falsification channel is statistical consistency with

$$\beta = 0, \qquad \gamma = 0 \tag{40}$$

within tight uncertainties. In that case, the octonionic sector becomes observationally indistinguishable from a cosmological constant and the model loses any nontrivial background-level predictive content.

The second falsification channel arises if nonzero best-fit values of β and γ are allowed but fail to improve information criteria such as AIC or BIC relative to ΛCDM. In that case, the octonionic reinterpretation would remain mathematically possible but would not gain empirical support.

The third falsification channel is theoretical consistency. If the fitted parameter region were to generate unphysical behavior, such as negative effective energy density over the relevant redshift range or severe late-time instability, the adopted approach would be ruled out in its present form even if statistically viable in a narrow domain.

Thus the model is not protected by vagueness: it either yields a statistically favored deviation from ΛCDM with physically acceptable evolution, or it does not.

X. LIMITATIONS AND NEXT STEPS

The present analysis is deliberately restricted to homogeneous background cosmology. This is both its strength and its limitation. It is a strength because the model becomes immediately testable using standard late-universe data. It is a limitation because a complete cosmological theory must also address perturbations, structure formation, early-universe dynamics, and possibly signatures in strong-gravity systems.

A natural next step is therefore to extend the framework to linear perturbations. At the phenomenological level, one may expect a modified growth equation of the form

$$\delta'' + 2H\delta' - 4\pi G_{\rm eff}\rho_m \delta = 0, \tag{41}$$

where $G_{\rm eff}$ may itself depend on the residual nonassociative sector. Such an extension would permit tests against redshift-space distortions and weak-lensing data.

Further extensions include studying whether the underlying algebraic symmetry reduction leaves traces in the cosmic microwave background, for example in the form of weak statistical anisotropies or nonstandard primordial signatures. One may also ask whether residual nonassociativity produces corrections in strong-field regimes, such as black-hole ringdown or horizon-scale imaging.

These developments require a deeper microscopic formulation. Nevertheless, the background model presented here already plays a crucial role: it determines whether the octonionic scenario has any empirical foothold at all.

XI. CONCLUSION

We have formulated a concrete observational test strategy for a cosmological model motivated by octonionic pre-geometry. In this framework, nonassociativity is interpreted as the fundamental algebraic obstruction that prevents the existence of global spacetime at the deepest level, while four-dimensional spacetime emerges as a stable associative sector after symmetry reduction. Residual nonassociative degrees of freedom are modeled phenomenologically as an effective dynamical dark-energy component.

The resulting background theory is defined by the approach

$$\rho_{\rm oct}(a) = \rho_{\rm oct,0}\big[1+\beta(1-a)\big]e^{-\gamma(1-a)}, \qquad (42)$$

which leads to the expansion law

$$E^2(z) = \Omega_{m0}(1+z)^3+(1-\Omega_{m0})\left(1+\beta\frac{z}{1+z}\right)\exp\left(-\gamma\frac{z}{1+z}\right). \qquad (43)$$

We derived the exact effective equation of state,

$$w_{\rm oct}(a) = -1+\frac{a}{3}\left[\frac{\beta}{1+\beta(1-a)}-\gamma\right], \qquad (44)$$

and established explicit relations between the phenomenological octonionic parameters and the standard dark-energy parameters w_0 and w_a. This provides a direct bridge from observational constraints to the underlying pre-geometric interpretation.

A benchmark parameterization,

$$\beta\approx 1.373, \qquad \gamma\approx 1.307, \qquad \Omega_{m0}=0.315, \qquad H_0 = 67.4\,{\rm km\,s^{-1}\,Mpc^{-1}}, \qquad (45)$$

yields an explicit Hubble function that can be fitted immediately to supernova, BAO, and direct $H(z)$ data. Because ΛCDM is recovered in the limit $\beta=\gamma=0$, the framework admits a clean falsification logic and a statistically fair comparison to the standard model.

The primary outcome of the present work is therefore not the proof of an octonionic cosmology, but the reduction of a speculative algebraic idea to a precise empirical question: does the observed expansion history favor a nontrivial residual pre-geometric sector over a strict cosmological constant? If the answer is negative, the model is strongly constrained. If the answer is positive, then late-universe cosmology may already contain measurable evidence for a deeper nonassociative origin of spacetime.

XII. OBSERVATIONAL INDICATIONS RELEVANT TO THE OCTONIONIC SCENARIO

In this section we summarize the current observational situation and assess to what extent existing cosmological data provide support for, or are compatible with, the octonionically motivated framework introduced above. The discussion is restricted to robust, publicly available late-universe datasets and their standard parameterizations.

A. Evidence for Dynamical Dark Energy

The most significant observational development relevant to the present framework arises from recent combined analyses of baryon acoustic oscillations (BAO), cosmic microwave background (CMB), and Type Ia supernova data. In particular, results from the Dark Energy Spectroscopic Instrument (DESI), in combination with Planck CMB data and supernova compilations such as Pantheon+, indicate a preference for a time-dependent dark-energy sector.

In the commonly used Chevallier–Polarski–Linder (CPL) parametrization,

$$w(a) = w_0 + w_a(1-a), \qquad (46)$$

representative combined constraints yield

$$w_0\approx -0.978, \qquad w_a\approx -0.65, \qquad (47)$$

with a statistical preference over ΛCDM at the level of approximately 2.8σ to 3.1σ depending on the dataset combination, and up to $\sim 4\sigma$ for certain supernova samples.

Within the present framework, the effective octonionic energy density leads to the relations

$$w_0 = -1+\frac{\beta-\gamma}{3}, \qquad w_a = -\frac{\beta-\gamma+\beta^2}{3}, \qquad (48)$$

which can be inverted to give

$$\beta=\sqrt{3(-w_0-w_a-1)}, \qquad \gamma = -3w_0+\beta-3. \qquad (49)$$

Inserting the above observational values yields the benchmark estimate

$$\beta\approx 1.37, \qquad \gamma\approx 1.31. \qquad (50)$$

This result corresponds to a clear deviation from the ΛCDM limit $(\beta,\gamma)=(0,0)$ and can therefore be interpreted as an empirical indication that a nontrivial effective component beyond a constant cosmological term may be present. In the octonionic interpretation, this component is associated with residual nonassociative degrees of freedom.

B. Consistency with Standard Background Parameters

At the same time, high-precision CMB observations from Planck remain in excellent agreement with the standard background cosmological parameters. In particular,

$$H_0 = 67.4\pm 0.5\ {\rm km\,s^{-1}\,Mpc^{-1}}, \qquad \Omega_{m0}=0.315\pm 0.007, \qquad (51)$$

provide a consistent baseline description of the expansion history.

The octonionic model constructed in this work is fully compatible with these values. In particular, the effective expansion law

$$E^2(z) = \Omega_{m0}(1+z)^3 + (1-\Omega_{m0})\left(1+\beta\frac{z}{1+z}\right)\exp\left(-\gamma\frac{z}{1+z}\right) \tag{52}$$

reduces continuously to the standard ΛCDM form in the limit $(\beta,\gamma)\to(0,0)$, ensuring that the model does not conflict with existing precision constraints.

C. Large-Scale Anomalies in the CMB

In addition to background expansion measurements, several large-scale anomalies in the CMB temperature field have been reported, including hemispherical power asymmetry and alignments of low multipoles. While these features are not statistically decisive and may be attributable to cosmic variance, they are qualitatively consistent with a mild violation of exact statistical isotropy.

Within the octonionic framework, such effects may arise naturally from residual nonassociative structure. In particular, the reduction from a fully nonassociative algebra to an effectively associative subalgebra need not preserve complete rotational symmetry, potentially leaving small directional imprints at the largest observable scales.

However, it is important to emphasize that current data do not provide statistically compelling evidence for such effects, and no unique octonion-specific signature has been established at the level of CMB observations.

D. Statistical Interpretation

From a statistical perspective, the current data indicate a preference for models with two additional degrees of freedom relative to ΛCDM, as reflected in improvements of the likelihood corresponding to $\Delta\chi^2 \sim 10$–20. This translates into a negative ΔAIC of order -6 to -17, depending on the dataset combination, indicating moderate to strong support for dynamical dark energy.

Since the octonionic parametrization (β,γ) is in one-to-one correspondence with the CPL parameters (w_0,w_a) at background level, it yields an identical maximum likelihood and therefore identical information criteria relative to ΛCDM. Consequently, current background data favor $\beta,\gamma\neq 0$, but do not distinguish between the octonionic interpretation and other dynamical dark-energy models.

E. Summary

The present observational situation can be summarized as follows. Current cosmological data provide a nontrivial indication for a dynamical dark-energy component, which is fully compatible with the effective octonionic model introduced here and corresponds to nonzero values of (β,γ). However, this evidence is not specific to the octonionic framework, since alternative parametrizations of dynamical dark energy provide equally good fits at the level of background expansion.

Therefore, while the data support the existence of a nontrivial effective component beyond a cosmological constant, they do not yet provide a unique empirical signature of an underlying nonassociative (octonionic) structure. Establishing such a signature requires extending the model beyond background cosmology, for example through predictions for structure formation, anisotropy, or strong-gravity observables.

Appendix A: Compact Formula Set for Implementation

The essential equations for numerical analysis are collected here.

Effective energy density:

$$\rho_{\rm oct}(a) = \rho_{\rm oct,0}[1+\beta(1-a)]e^{-\gamma(1-a)}. \tag{A1}$$

Dimensionless expansion rate:

$$E^2(z) = \Omega_{m0}(1+z)^3 + (1-\Omega_{m0})\left(1+\beta\frac{z}{1+z}\right)\exp\left(-\gamma\frac{z}{1+z}\right). \tag{A2}$$

Hubble function:

$$H(z) = H_0E(z). \tag{A3}$$

Effective equation of state:

$$w_{\rm oct}(a) = -1 + \frac{a}{3}\left[\frac{\beta}{1+\beta(1-a)} - \gamma\right]. \tag{A4}$$

Present value:

$$w_0 = -1 + \frac{\beta-\gamma}{3}. \tag{A5}$$

Linear evolution coefficient:

$$w_a = -\frac{\beta-\gamma+\beta^2}{3}. \tag{A6}$$

Inverse mapping:

$$\beta = \sqrt{3(-w_0-w_a-1)}, \qquad \gamma = -3w_0+\beta-3. \tag{A7}$$

Luminosity distance:

$$d_L(z) = (1+z)\frac{c}{H_0}\int_0^z \frac{dz'}{E(z')}. \tag{A8}$$

Distance modulus:

$$\mu(z) = 5\log_{10}\left(\frac{d_L(z)}{\rm Mpc}\right) + 25. \tag{A9}$$

BAO distances:

$$D_M(z) = \frac{c}{H_0}\int_0^z \frac{dz'}{E(z')}, \qquad D_H(z) = \frac{c}{H(z)}, \qquad D_V(z) = \left[z D_M^2(z) D_H(z)\right]^{1/3}. \tag{A10}$$

Total likelihood:

$$\chi^2_{\rm tot} = \chi^2_{\rm SN} + \chi^2_{\rm BAO} + \chi^2_H. \tag{A11}$$

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