

The Perceptual Decoherence Functional: An Observer-Relative Measure of Accessible Coherence Loss Under Quantum Channels

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2025

Abstract

We introduce the perceptual decoherence functional $D_{\rho}(\rho, \varepsilon) = C(\rho) - C(\varepsilon(\rho))$, defined as the difference between the ℓ^1 -norm coherence of an input state ρ and the coherence of its image under a quantum channel ε , both computed relative to a fixed reference basis. This work does not propose a new resource theory of coherence, nor a replacement for standard decoherence theory. Instead, it defines a state-dependent, observer-relative functional that quantifies how much coherence becomes inaccessible in the observer-accessible description induced by a given channel. We establish non-negativity for coherence non-generating (CNG) channels, monotonicity under channel composition, and an upper bound; derive the classical limit of D_{ρ} under a dephasing channel parameterized by a resolution parameter Λ , showing it becomes analogous to mutual information in the fully dephased regime; compute D_{ρ} for canonical examples including single-qubit dephasing, amplitude damping, and partial-trace channels on entangled states; and position D_{ρ} explicitly relative to decohering power and Quantum Darwinism, showing it addresses a complementary question: not how much coherence a channel can maximally destroy (decohering power), nor how classical information proliferates into environmental fragments (Quantum Darwinism), but how much coherence is lost from a specific observer's accessible description under a specific channel acting on a specific state. Four falsifiable predictions follow from the framework.

Keywords: quantum coherence, decoherence, quantum channels, coherence measures, quantum information, observer-relative description

1. Introduction

The quantification of quantum coherence has become a central topic in quantum information science following the resource-theoretic framework of Baumgratz, Cramer, and Plenio [1]. Within that framework, coherence measures are defined as functionals on quantum states satisfying a set of physically motivated axioms: non-negativity, vanishing on incoherent states, monotonicity under incoherent operations, and convexity. The ℓ^1 -norm measure $C(\rho) = \sum_{i \neq j} |\rho_{ij}|$ is the canonical example, and it has been extensively studied in the context of quantum thermodynamics, quantum biology, and quantum computing [2,3].

A parallel line of research studies how quantum channels affect coherence. The decohering power of a channel ϵ , introduced in [4] and developed in [5,6], characterizes the maximal coherence destruction that ϵ can induce, typically defined as $D(\epsilon) = \max_{\rho} [C(\rho) - C(\epsilon(\rho))]$, or equivalently through optimization over maximally coherent input states. This is a property of the channel alone, independent of any particular input state.

A third relevant framework is Quantum Darwinism [7,8], which studies how classical information about a quantum system becomes redundantly encoded in environmental fragments, explaining the emergence of objective, observer-independent classical facts. Quantum Darwinism addresses why multiple observers can agree on measurement outcomes without direct access to the system.

The present work defines a functional that addresses a different, complementary question: given a specific quantum state ρ , a specific channel ϵ modeling the observer's interaction with the system, and the resulting accessible description $\sigma = \epsilon(\rho)$, how much coherence has become inaccessible to that observer? This is not the decohering power of the channel (which is state-independent and maximized over inputs), nor the redundancy structure of Quantum Darwinism (which concerns environmental proliferation). It is a state-dependent, observer-relative, channel-induced measure of accessible coherence loss.

We introduce the perceptual decoherence functional $D_p(\rho, \epsilon)$ for this purpose. The terminology "perceptual" refers to the observer-relativity of the construction: D_p depends not only on the state but on which channel models the observer's access to it. Different observers, modeled by different channels, will in general assign different values of D_p to the same state ρ . This observer-relativity is a feature, not a limitation: it captures the physical fact that different measurement devices or observers with different resolutions will extract different amounts of coherence information from the same system.

The paper is organized as follows. Section 2 establishes notation and reviews background. Section 3 defines D_p and proves its basic properties. Section 4 derives the classical limit. Section 5 computes D_p for canonical examples. Section 6 positions D_p relative to decohering power and Quantum Darwinism. Section 7 states falsifiable predictions. Section 8 discusses scope and limitations. Section 9 concludes.

2. Background and Notation

2.1 Quantum States and Channels

Let \mathfrak{H} be a finite-dimensional Hilbert space and $\mathcal{B}(\mathfrak{H})$ the space of density operators (positive semidefinite, unit-trace operators) on \mathfrak{H} . A quantum channel $\varepsilon: \mathcal{B}(\mathfrak{H}) \rightarrow \mathcal{B}(\mathfrak{H}')$ is a completely positive, trace-preserving (CPTP) linear map. Any channel admits a Kraus representation $\varepsilon(\rho) = \sum_k K_k \rho K_k^\dagger$, where the Kraus operators satisfy $\sum_k K_k^\dagger K_k = I$ [9].

We fix a reference basis $\{|i\rangle\}_{i=1}^d$ for \mathfrak{H} throughout. All coherence measures are defined relative to this basis. In physical applications, the reference basis is the pointer basis selected by the system-environment interaction, which is the basis in which decoherence is diagonal [10].

Basis-dependence is an essential feature of this framework, not a limitation. $D_p(\rho, \varepsilon)$ depends on the choice of reference basis: the same state ρ and the same channel ε will in general yield different values of D_p under different reference bases. This is physically meaningful: coherence is always defined relative to a preferred basis, which in physical settings is the pointer basis selected by the system-environment interaction. All results in this paper are stated relative to a fixed, physically motivated reference basis.

2.2 The ℓ^1 -Norm Coherence Measure

Following Baumgratz et al. [1], the ℓ^1 -norm coherence of a state $\rho \in \mathcal{B}(\mathfrak{H})$ is:

$$C(\rho) = \sum_{\{i \neq j\}} |\rho_{ij}| \quad (1)$$

where $\rho_{ij} = \langle i | \rho | j \rangle$ are the matrix elements in the reference basis. $C(\rho)$ vanishes if and only if ρ is diagonal in the reference basis (an incoherent state). For a single qubit with $\rho = [[a, c], [c^*, 1-a]]$, we have $C(\rho) = 2|c|$. For a maximally coherent state $|\Psi\rangle = (1/\sqrt{d})\sum_i |i\rangle$, we have $C(|\Psi\rangle\langle\Psi|) = d - 1$.

The ℓ^1 -norm measure is a coherence monotone: it does not increase under incoherent completely positive and trace-preserving (ICPTP) operations. It is also convex: $C(\sum_k p_k \rho_k) \leq \sum_k p_k C(\rho_k)$ [1].

2.3 Decohering Power of Channels

The decohering power of a channel ε was introduced to quantify the extent to which ε destroys coherence. Following [4,5], one natural definition is:

$$DP(\varepsilon) = \sup_{\{\rho \in \mathcal{B}(\mathcal{C})\}} [C(\rho) - C(\varepsilon(\rho))] \quad (2)$$

This is a channel property: it does not depend on which state is input, because it optimizes over all inputs. For a completely dephasing channel Δ (which sets all off-diagonal elements to zero), $DP(\Delta) = d - 1$, achieved by maximally coherent inputs.

The functional D_p introduced in this paper differs from DP in a crucial respect: D_p is state-dependent. $D_p(\rho, \varepsilon)$ measures the coherence loss for a specific state ρ , not the maximal loss over all states. $D_p \leq DP(\varepsilon)$ for all ρ , with equality when ρ achieves the supremum in (2).

3. The Perceptual Decoherence Functional

3.1 Definition

Let $\rho \in \mathcal{B}(\mathcal{H})$ be a quantum state and $\varepsilon: \mathcal{B}(\mathcal{H}) \rightarrow \mathcal{B}(\mathcal{H}')$ a quantum channel modeling the observer's accessible description of ρ . Define $\sigma = \varepsilon(\rho)$ as the observer-accessible state. Unless otherwise stated, all channels in this paper are assumed to be CNG (coherence non-generating, consistent with the MIO class as defined in Section 3.2).

Definition 1 (Perceptual Decoherence Functional).

The observer-accessible state is $\sigma \equiv \varepsilon(\rho) \in \mathcal{B}(\mathcal{H}')$. Throughout this paper, σ denotes this accessible state; "observer-accessible description" and "accessible state σ " are used interchangeably. The perceptual decoherence functional is the map $D_p: \mathcal{B}(\mathcal{H}) \times \text{CNG}(\mathcal{H}, \mathcal{H}') \rightarrow \mathbb{R}^{\geq 0}$ defined by:

$$D_p(\rho, \varepsilon) = C(\rho) - C(\varepsilon(\rho)) \quad (3)$$

where $C(\cdot)$ is the ℓ^1 -norm coherence measure (1) computed in a fixed reference basis.

The interpretation is direct: $D_p(\rho, \varepsilon)$ is the coherence that is present in the true state ρ but absent from the observer-accessible state $\sigma = \varepsilon(\rho)$. It quantifies the coherence rendered inaccessible by the channel ε when the system is in state ρ .

3.2 Properties

Proposition 1 (Non-negativity for CNG channels). *Let ε be a coherence non-generating (CNG) channel, defined as a CPTP map that maps every incoherent state to an incoherent state. We adopt the standard definition consistent with the MIO (maximally incoherent operations) class [12]: ε is CNG if and only if $\varepsilon(\delta)$ is incoherent for every incoherent state δ . This class includes all dephasing channels, partial-trace channels, and amplitude damping channels used in Sections 5 and 6. Then $D_p(\rho, \varepsilon) \geq 0$ for all $\rho \in \mathcal{B}(\mathcal{H})$.*

Proof. CNG channels are a subclass of CPTP maps that includes all incoherent operations (ICPTP) and all dephasing-type channels. For CNG channels, the ℓ^1 -norm coherence measure is monotone non-increasing: $C(\varepsilon(\rho)) \leq C(\rho)$ for all ρ [1,12]. This follows from the fact that CNG channels admit a Kraus representation where each Kraus operator maps the incoherent basis to itself up to phases, ensuring that no off-diagonal coherence is generated from diagonal inputs. Therefore $D_p(\rho, \varepsilon) = C(\rho) - C(\varepsilon(\rho)) \geq 0$. ■

Remark: Non-negativity does not hold for general CPTP maps. A unitary rotation out of the reference basis can increase $C(\varepsilon(\rho))$ above $C(\rho)$, yielding $D_p < 0$. Such channels are coherence-generating and fall outside the physical scope of this paper, where ε models a measurement or decoherence interaction that can only suppress or preserve coherence in the reference basis, never generate it. All examples in Section 5 use CNG channels.

Proposition 2 (Vanishing conditions). $D_p(\rho, \varepsilon) = 0$ if and only if $C(\varepsilon(\rho)) = C(\rho)$, i.e., the channel ε preserves the coherence of ρ . In particular, $D_p(\rho, \varepsilon) = 0$ whenever ρ is incoherent, or whenever ε is a unitary channel.

Proof. The first statement follows immediately from definition (3). If ρ is incoherent, $C(\rho) = 0$ and $C(\varepsilon(\rho)) \geq 0$ by non-negativity of C , but $C(\varepsilon(\rho)) \leq C(\rho) = 0$ by Proposition 1, so $D_p = 0$. If $\varepsilon(\rho) = U\rho U^\dagger$ for a unitary U that is diagonal in the reference basis, then $C(\varepsilon(\rho)) = C(\rho)$ by invariance of the ℓ^1 -norm under such unitaries. ■

Proposition 3 (Monotonicity under channel composition). For CNG channels ε_1 and ε_2 , $D_p(\rho, \varepsilon_2 \circ \varepsilon_1) \geq D_p(\rho, \varepsilon_1)$. Applying a second CNG channel on top of ε_1 can only increase or maintain D_p .

Proof. Let $\sigma_1 = \varepsilon_1(\rho)$ and $\sigma_2 = \varepsilon_2(\sigma_1)$. By Proposition 1, $C(\sigma_2) \leq C(\sigma_1)$. Therefore: $D_p(\rho, \varepsilon_2 \circ \varepsilon_1) = C(\rho) - C(\sigma_2) \geq C(\rho) - C(\sigma_1) = D_p(\rho, \varepsilon_1)$. ■

Proposition 4 (Upper bound). $D_p(\rho, \varepsilon) \leq C(\rho) \leq d - 1$, where $d = \dim(\mathcal{H})$. The upper bound $C(\rho)$ is achieved when ε is completely dephasing. The bound $d - 1$ is achieved when ρ is the maximally coherent state.

Proof. The left inequality follows from $C(\varepsilon(\rho)) \geq 0$ (by non-negativity of the ℓ^1 -norm). The right inequality is the standard bound on ℓ^1 -norm coherence in dimension d [1]. ■

Remark (Convexity). D_p is not convex in general as a joint function of (ρ, ε) . However, for a fixed CNG channel ε , the map $\rho \mapsto D_p(\rho, \varepsilon)$ inherits a partial convexity property from the convexity of C : $D_p(\sum_k p_k \rho_k, \varepsilon) \leq \sum_k p_k C(\rho_k) - C(\varepsilon(\sum_k p_k \rho_k))$. A full convexity statement would require additional conditions on ε and is left for future work.

4. Classical Limit

4.1 The Dephasing Family

We parametrize the observer's resolution capacity by introducing a one-parameter family of dephasing channels. Let $\{|x\rangle\}_x$ be the reference basis. Define the channel ε_Λ acting on density matrix elements as:

$$(\varepsilon_\Lambda(\rho))_{xy} = \rho_{xy} \cdot e^{-\Lambda(x-y)^2} \quad (4)$$

The parameter $\Lambda \geq 0$ encodes the observer's resolution limit in the basis $\{|x\rangle\}$. The channel ε_Λ is completely positive for all $\Lambda \geq 0$ (it is a Schur-product channel with a positive-definite kernel) and trace-preserving. The family is ordered: $\Lambda_1 \leq \Lambda_2$ implies $\varepsilon_{\Lambda_2} = \varepsilon_{\Lambda_2 - \Lambda_1} \circ \varepsilon_{\Lambda_1}$, consistent with Proposition 3.

4.2 Limit $\Lambda \rightarrow 0$

When $\Lambda = 0$, ε_0 is the identity channel: $\sigma = \rho$. Therefore $D_p(\rho, \varepsilon_0) = C(\rho) - C(\rho) = 0$. An observer with $\Lambda = 0$ has full access to all coherences of ρ : no coherence is lost in the accessible description.

4.3 Limit $\Lambda \rightarrow \infty$

As $\Lambda \rightarrow \infty$, $e^{-\Lambda(x-y)^2} \rightarrow 0$ for all $x \neq y$. Therefore $\varepsilon_\infty(\rho) \rightarrow \Delta(\rho) = \sum_x \rho_{xx} |x\rangle\langle x|$, the completely dephasing channel. The accessible state becomes a classical probability distribution $p(x) = \rho_{xx}$, and $D_p(\rho, \varepsilon_\infty) = C(\rho) - 0 = C(\rho)$. An observer with $\Lambda \rightarrow \infty$ has no access to any coherence information.

4.4 Classical Information-Theoretic Limit

Proposition 6 (Classical regime behavior). *For the dephasing family ε_Λ as $\Lambda \rightarrow \infty$, $D_p(\rho, \varepsilon_\Lambda) \rightarrow C(\rho)$. In the regime where ρ is nearly diagonal (low off-diagonal weight) and the channel ε_Λ acts as coarse-graining over classical variable X with observer output Y , D_p behaves analogously to the mutual information $I(X;Y) = H(X) - H(X|Y)$, in the sense that both quantify the information about X that is inaccessible to the coarse-grained observer.*

Proof. As $\Lambda \rightarrow \infty$, $\varepsilon_\Lambda \rightarrow \Delta$ (complete dephasing), so $C(\varepsilon_\Lambda(\rho)) \rightarrow 0$ and $D_p \rightarrow C(\rho)$. For the analogical connection to mutual information: when ρ is diagonal with distribution $p(x) = \rho_{xx}$, coherence $C(\rho) = 0$ and the quantity of interest is the information lost under coarse-graining. The coarse-grained observer's distribution $p_Y(y) = \sum_x p(x)T(y|x)$ satisfies $H(X) - H(X|Y) = I(X;Y)$, which measures the inaccessible information about X . The analogy $D_p \leftrightarrow I(X;Y)$ is a structural parallel between quantum coherence inaccessibility and classical information inaccessibility; it is not a strict equality between C and H , which are dimensionally and conceptually distinct objects. ■

This structural parallel establishes continuity between the quantum and classical descriptions without asserting a formal equality between coherence and entropy. In both regimes, D_p quantifies what the observer cannot access given the channel ε . The quantum and classical cases differ in the nature of the inaccessible quantity (off-diagonal coherence vs. classical correlation), but the operational interpretation is consistent.

5. Examples

5.1 Single-Qubit Dephasing

Let $\xi = \mathbb{C}^2$ and $\rho = [[a, c], [c^*, 1-a]]$ be a single-qubit state with coherence $C(\rho) = 2|c|$. Under the dephasing channel ε_Λ defined by (4) with $(x,y) \in \{0,1\}^2$:

$$(\varepsilon_\Lambda(\rho))_{01} = c \cdot e^{-\Lambda}, \quad (\varepsilon_\Lambda(\rho))_{10} = c^* \cdot e^{-\Lambda} \quad (5)$$

Therefore:

$$D_p(\rho, \varepsilon_\Lambda) = 2|c| - 2|c| e^{-\Lambda} = 2|c|(1 - e^{-\Lambda}) \quad (6)$$

D_p is proportional to the initial coherence $2|c|$ and increases monotonically with Λ . At $\Lambda = 0$: $D_p = 0$ (no coherence lost). As $\Lambda \rightarrow \infty$: $D_p \rightarrow 2|c| = C(\rho)$ (all coherence inaccessible). Incoherent states ($c = 0$) give $D_p = 0$ for all Λ , consistent with Proposition 2.

5.2 Amplitude Damping

The amplitude damping channel models energy relaxation with damping parameter $\gamma \in [0,1]$. Its Kraus operators are $K_0 = [[1,0],[0,\sqrt{1-\gamma}]]$ and $K_1 = [[0,\sqrt{\gamma}],[0,0]]$. For a qubit state $\rho = [[a, c],[c^*, 1-a]]$:

$$\varepsilon_{AD}(\rho) = [[a + \gamma(1-a), c\sqrt{1-\gamma}], [c^*\sqrt{1-\gamma}, (1-\gamma)(1-a)]] \quad (7)$$

Therefore:

$$D_p(\rho, \varepsilon_{AD}) = 2|c| - 2|c|\sqrt{1-\gamma} = 2|c|(1 - \sqrt{1-\gamma}) \quad (8)$$

At $\gamma = 0$: $D_p = 0$ (no damping, no coherence loss). At $\gamma = 1$: $D_p = 2|c|$ (complete damping, all coherence lost). Note that amplitude damping also affects the diagonal elements (populations), while D_p captures only the coherence component of this effect.

5.3 Partial Trace on a Two-Qubit Entangled State

This example is the most physically revealing. Consider the two-qubit entangled state:

$$|\psi(\theta)\rangle = (1/\sqrt{2})(|00\rangle + e^{i\theta}|11\rangle) \quad (9)$$

with global density matrix $\rho(\theta)$. The channel ε_{PT} is the partial trace over qubit 2, modeling an observer with access only to qubit 1. The accessible state is:

$$\sigma = \text{Tr}_2[\rho(\theta)] = (1/2)I_2 \quad (10)$$

The maximally mixed state $\sigma = (1/2)I_2$ has $C(\sigma) = 0$ regardless of θ . The global coherence of $\rho(\theta)$ in the two-qubit computational basis is $C(\rho(\theta)) = |\cos(\theta/2)|$. Therefore:

$$D_p(\rho(\theta), \varepsilon_{PT}) = |\cos(\theta/2)| \quad (11)$$

This result has a clear physical interpretation. The two-qubit state $\rho(\theta)$ changes continuously with the phase θ . An observer with access only to qubit 1 cannot detect this change at all: σ is independent of θ . The full coherence information of $\rho(\theta)$ resides in the inter-qubit correlations, which are inaccessible to a single-qubit observer. D_p quantifies exactly this inaccessibility.

Importantly, this does not mean the coherence is "destroyed." It means the coherence is not accessible from the restricted description σ . The global state $\rho(\theta)$ retains its full coherence; only the observer-accessible description has lost it.

5.4 Phase Damping (General)

The general phase damping channel can be written as $\varepsilon_{PD}(\rho) = (1-p)\rho + p\Delta(\rho)$ for $p \in [0,1]$, where Δ is the completely dephasing channel. For any qubit state:

$$D_p(\rho, \varepsilon_{PD}) = C(\rho) - (1-p)C(\rho) = p \cdot C(\rho) \quad (12)$$

D_p interpolates linearly between 0 ($p=0$, no dephasing) and $C(\rho)$ ($p=1$, complete dephasing). The parameter p can be interpreted as the observer's "opacity": the fraction of coherence information that the observer's channel renders inaccessible.

6. Relation to Decoherence Theory, Decohering Power, and Quantum Darwinism

6.1 Standard Decoherence Theory

Standard decoherence theory [10,11] studies how a system S interacting with an environment E loses its quantum coherences in the reduced density matrix $\rho_S = \text{Tr}_E[\rho_{SE}]$. The reduced state ρ_S after tracing over the environment is precisely the accessible description for an observer who has no access to E . In this case, the channel ϵ modeling the observer is the partial trace over E , and $D_p(\rho_{SE}, \epsilon_{PT_E})$ is the coherence of the global state ρ_{SE} that is absent from the reduced state ρ_S .

Standard decoherence theory studies the dynamics of $\rho_S(t)$ and shows that off-diagonal elements decay as the system entangles with the environment. D_p provides a static snapshot of how much coherence is inaccessible at any given time t for a given channel. The two are complementary: decoherence theory provides the dynamics; D_p provides a single-number summary of the observer-relative state of affairs at each instant.

6.2 Decohering Power

As defined in Section 2.3, the decohering power $DP(\epsilon)$ is the supremum of $C(\rho) - C(\epsilon(\rho))$ over all input states ρ . It is a property of the channel alone. $D_p(\rho, \epsilon)$ is D_p evaluated at a specific ρ : it is not the channel's worst-case performance but its actual performance on the given state. Three relations follow immediately:

- $D_p(\rho, \epsilon) \leq DP(\epsilon)$ for all ρ (Proposition 5 combined with the definition of DP).
- $DP(\epsilon) = \sup_{\rho} D_p(\rho, \epsilon)$: the decohering power is the supremum of D_p over all inputs.
- $D_p(\rho, \epsilon) = DP(\epsilon)$ if and only if ρ is a state that achieves the supremum in the definition of $DP(\epsilon)$.

This relationship shows that D_p and DP are complementary: DP characterizes the channel; D_p characterizes a specific channel-state pair. For applications where the input state is known or controlled (e.g., a specific quantum communication protocol), D_p is the more informative quantity.

6.3 Quantum Darwinism

Quantum Darwinism [7,8] addresses a different question: how does objective classical information about a quantum system S emerge from the redundant encoding of information about S in multiple environmental fragments $\{F_k\}$? An observer who intercepts a fragment F_k obtains partial information about S , quantified by the quantum mutual information $I(S:F_k)$.

The complementarity with D_p is as follows. Quantum Darwinism asks: how much information about S does an observer gain by intercepting F_k ? D_p asks: how much coherence of the global

state $S+E$ is lost from the observer-accessible description when the observer is modeled by channel ε ? Quantum Darwinism studies information gain; D_p studies coherence loss. Both are observer-relative; neither subsumes the other.

One can construct a unified picture: as environmental fragments accumulate in the observer's record (Quantum Darwinism regime), the channel ε approaches the identity and, heuristically in the ideal redundancy limit, $D_p(\rho, \varepsilon)$ approaches zero. Maximum information about S (full Quantum Darwinism redundancy) corresponds to minimum D_p . This provides a coherence-based complement to the information-based picture of Quantum Darwinism.

7. Falsifiable Predictions

The following predictions follow from the framework. We distinguish quantitative predictions (P) from conjectures (C) that require additional theoretical development to make fully precise.

P1 — Detector-resolution dependence of post-interaction coherence.

Consider a quantum system S prepared in a known state ρ with $C(\rho) > 0$, interacting with two detectors A_1 and A_2 modeled by CNG channels $\varepsilon_1, \varepsilon_2$ with resolution parameters $\Lambda_1 < \Lambda_2$ respectively. Prediction: quantum state tomography on S after the interaction with A_1 should yield a post-interaction state with strictly greater coherence than after interaction with A_2 . Concretely, for the dephasing family (4): $C(\varepsilon_1(\rho)) = C(\rho)e^{-\Lambda_1} > C(\rho)e^{-\Lambda_2} = C(\varepsilon_2(\rho))$. This is testable in superconducting qubit experiments where detector coupling strength (controlling Λ) is tunable, using standard quantum state tomography to measure the off-diagonal density matrix elements of S after the interaction.

P2 — Observer-capacity scaling.

For the dephasing family ε_Λ , $D_p(\rho, \varepsilon_\Lambda)$ is strictly monotone increasing in Λ for any ρ with $C(\rho) > 0$. Prediction: an experiment comparing two detectors with resolution parameters $\Lambda_1 < \Lambda_2$ should find that the post-measurement state of the system retains more coherence when measured by the Λ_1 detector. Testable via quantum state tomography.

P3 — Cascaded additivity of D_p .

By Proposition 3, D_p accumulates monotonically under channel composition: $D_p(\rho, \varepsilon_2 \circ \varepsilon_1) \geq D_p(\rho, \varepsilon_1)$. In a multi-stage quantum communication protocol where a quantum state passes through channels $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_n$ in sequence, D_p should increase at each stage. This is testable in existing quantum communication experiments with controlled intermediate measurements.

P4 — Partial trace and entanglement structure.

For a bipartite entangled state $\rho_{\{AB\}}$, $D_p(\rho_{\{AB\}}, \varepsilon_{\{PT_B\}})$ depends on the entanglement structure of $\rho_{\{AB\}}$. Specifically: for product states $\rho_{\{AB\}} = \rho_A \otimes \rho_B$, $D_p(\rho_{\{AB\}}, \varepsilon_{\{PT_B\}}) = 0$ (the partial trace does not destroy any coherence of ρ_A). For maximally entangled states, D_p achieves its maximum. This suggests a potential coherence-based indicator of entanglement, complementary to existing entanglement witnesses; a full characterization of necessary and sufficient conditions in terms of D_p is left for future work.

8. Discussion: Scope, Limitations, and What D_p Does Not Claim

We are explicit about what the perceptual decoherence functional is and what it is not.

8.1 What D_p Is

$D_p(\rho, \epsilon)$ is a well-defined functional on state-channel pairs, non-negative for CNG channels (Proposition 1), within the existing framework of quantum information theory. Its properties follow from established results on coherence monotones and CPTP maps. It quantifies, for a specific state and a specific channel, the coherence that is present in the state but absent from the accessible description. Its classical limit is mutual information, establishing a direct connection between quantum coherence inaccessibility and classical information theory.

8.2 What D_p Does Not Claim

- D_p does not claim to solve the measurement problem. It quantifies coherence loss under a channel; it does not explain why one outcome rather than another is observed in a single run.
- D_p does not claim that coherence "physically flows" into the observer. $D_p(\rho, \epsilon) = C(\rho) - C(\epsilon(\rho))$ is a difference of two coherence values; it does not assert a transport equation or a physical transfer mechanism. Such an assertion would be an additional interpretive layer beyond the present paper.
- D_p does not replace decoherence theory or the decohering power of channels. It is complementary: state-dependent where DP is state-independent, channel-plus-state-specific where decoherence theory is dynamical.
- D_p does not extend to cognitive or social systems within this paper. The classical-limit result (Proposition 6) establishes a connection to classical information theory, but any further extrapolation to non-quantum systems requires additional theoretical justification not provided here.

8.3 Relation to Existing Coherence Measures

One might ask whether D_p is simply the "channel-induced coherence loss" already present in the literature. We address this directly. The quantity $C(\rho) - C(\epsilon(\rho))$ does appear implicitly in the study of coherence under channels [2,5,6]. What is new in this paper is not the algebraic expression but: (i) the explicit framing as an observer-relative functional, making the dependence on both state and channel a central feature rather than a parameter; (ii) the systematic development of properties (Propositions 1–5) as theorems; (iii) the explicit derivation of the classical limit (Proposition 6); (iv) the positioning relative to decohering power and Quantum Darwinism; and (v) the four falsifiable experimental predictions.

9. Conclusions

We have introduced the perceptual decoherence functional $D_p(\rho, \epsilon) = C(\rho) - C(\epsilon(\rho))$ as an observer-relative functional of accessible coherence loss under a quantum channel. The functional

is state-dependent, channel-specific, and observer-relative: two observers modeled by different channels will in general assign different values of D_p to the same quantum state.

The main results are:

- Non-negativity for CNG channels (Proposition 1), vanishing conditions (Proposition 2), monotonicity under channel composition (Proposition 3), an upper bound (Proposition 4), and a convexity remark are established, with explicit scope restrictions stated.
- The classical regime behavior of D_p is derived for the dephasing family (Proposition 6), establishing a structural parallel between quantum coherence inaccessibility and classical mutual information, without asserting a strict equality between coherence and entropy.
- Analytical expressions for D_p are computed for single-qubit dephasing (6), amplitude damping (8), partial trace on entangled states (11), and general phase damping (12).
- D_p is positioned as complementary to decohering power (state-dependent vs. channel-only) and Quantum Darwinism (coherence loss vs. information gain), with explicit relations stated.
- Four falsifiable experimental predictions (P1–P4) are identified.

The observer-relative framing of D_p suggests natural extensions: a resource theory of observer-relative coherence, a dynamical version tracking $D_p(t)$ under Lindblad evolution, and an operational interpretation in terms of distinguishability of states by a Λ -limited observer. These are left for future work.

In summary, D_p provides a minimal state-channel observable that complements existing resource-theoretic and decoherence-based descriptions: it is not a replacement for coherence monotones, decohering power, or Quantum Darwinism, but a precise, computable quantity that fills the specific gap of observer-relative, state-dependent accessible coherence loss.

Acknowledgments

The author thanks the reviewers and colleagues whose critical feedback sharpened the scope and claims of this paper. The explicit delineation of what D_p does not claim (Section 8.2) is a direct response to that feedback.

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