Topological Origin of the Fine-Structure Constant Based on Space Light-Speed Helical Motion and Three-Layer Nested Topological Organization Evolution Theory

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Abstract

This paper proposes the Topological Residual Theory (TRT), taking the right-handed cylindrical helical light-speed motion in space as the sole first principle. In the two-dimensional torus formed by collective weaving, the spin-$1/2$ parallel transport and the BKT topological phase transition lead to precise phase cancellation, locking the system at the microscopic ultraviolet fixed point. During the helical rotation and winding motion of the spatial fluid, as the effective radius increases, the system triggers discontinuous topological changes and self-organizes to collapse. To describe this topological change, the system must pass through a covariant cobordism transition manifold. In this process, the Gauss-Bonnet constraint requires that the total angular deficit and the boundary topological charge satisfy a balance equation, and the fine-structure constant α is dynamically locked as the topological leakage parameter. When the system reaches the Markov critical point $F_{19} = 4181$, α is fixed at the observed value $1/137.036$, completing the topological phase transition and closing to form a three-layer nested structure. The total topological measure Ω is the geometric invariant of the closed part, while α is precisely the normalized residual rate of the leakage process (the intrinsic leakage proportion of the number of helical divergent lines on the unit Gaussian sphere). Through the renormalization mapping $F_{19}/10^4 \to 0.4181^\circ$, this paper establishes the precise connection between microscopic geometric symmetry and macroscopic observable residuals. Under the current framework, tentative geometric correspondences and order-of-magnitude estimates are made for the electron anomalous magnetic moment, helium ion ionization energy, and proton-neutron mass difference, and two exploratory geometric research directions are proposed for the muon $g-2$ deviation and galaxy rotation curve anomalies. This theory provides a self-consistent framework for the pure geometric-topological origin of physical constants and fundamental forces.

1 Introduction

In modern physics, searching for the theoretical origin of the dimensionless constant — the fine-structure constant α — is one of the core ultimate goals of geometric unification theories. Classical unified theories (such as Kaluza-Klein theory or superstring theory) often encounter difficulties such as topological self-intersection collapse, lack of continuity, and scale mismatch when handling microscopic geometric properties. Richard Feynman once pointed out that all excellent theoretical physicists should "paste the puzzle of α on the wall and ponder it deeply."

This research, based on the Topological Residual Theory (TRT), attempts to establish a highly self-consistent topological-geometric dynamical phenomenological model. We posit that the spacetime fluid at microscopic scales executes right-handed cylindrical helical light-like motion. This theory systematically demonstrates the complete evolutionary process: from the 1D straight stage (one-dimensional straight vacuum state), through the effective two-dimensional torus weaving stage formed by the collective interweaving of a large number of helical fibers (constant-radius straight torus state), during the helical motion and collective coherent winding process, as the equivalent radius increases, the system triggers discontinuous topological phase transitions and collapses to closure, ultimately entering the three-dimensional closed Gaussian sphere locked stage (3D closed stage). This evolutionary path logically achieves a stable transition from microscopic dynamical fluctuations to global topological rigidity, and through renormalization group flow (RG Flow) connects the geometric symmetry of the microscopic ultraviolet (UV) with the geometric observable residuals of the macroscopic infrared (IR), providing a unified description of the fine-structure constant. Under the current framework, we have also conducted tentative geometric correspondences and order-of-magnitude estimates for the electron anomalous magnetic moment, helium ion ionization energy, and proton-neutron mass difference, and proposed two future-testable exploratory geometric directions.

The key relations are:

$$\theta_{\mathrm{UV}} = \frac{\pi}{4}, \quad \alpha = \frac{F_{19}}{4181} \approx \frac{1}{137.036},$$

$$\Omega = 4\pi^3 + \pi^2 + \pi \approx 137.036304, \quad \alpha = \frac{1}{\Omega},$$

$$K = \frac{\tan\theta_{\mathrm{UV}}}{\Omega} = \tan\theta_{\mathrm{IR}}.$$

2 Microscopic Geometric Postulate and Frenet-Serret Frame

Postulate 1 (Light-like Motion Geometry)

A spin-1/2 particle executes light-like motion along a right-handed cylindrical helix, satisfying the arc-length parameterization condition (i.e., the speed-of-light constraint):

$$\|\mathbf{r}'(s)\| = 1,$$

where $s = ct$ is the arc-length parameter. Its spatial trajectory parametric equation is:

$$\mathbf{r}(\phi) = (r\cos\phi, r\sin\phi, \beta\phi),$$

where r is the characteristic gyration radius, β is the axial pitch parameter, and ϕ is the rotational azimuth. In this microscopic free stage, due to the absence of collective effects, the parameters r and β are constant characteristic quantities.

To describe the local geometric properties of this curve, we construct its standard Frenet-Serret orthonormal frame: the unit tangent vector $\mathbf{T}$, the principal normal vector $\mathbf{N}$ (pointing toward the rotation center), and the binormal vector $\mathbf{B}$. By differentiating with respect to the arc-length parameter, the curvature κ and torsion τ are respectively:

$$\kappa = \frac{r}{r^2+\beta^2}, \quad \tau = \frac{\beta}{r^2+\beta^2}.$$

The local dimensionless geometric ratio is defined as:

$$\alpha_{\text{local}}(s) \equiv \frac{\kappa(s)}{\tau(s)} = \frac{r(s)}{\beta(s)} = \tan[\theta(s)],$$

where $\theta(s)$ is the local helix angle.

The standard differential form of the Frenet-Serret equations is:

$$\frac{d\mathbf{T}}{ds} = \kappa\mathbf{N}, \quad \frac{d\mathbf{N}}{ds} = -\kappa\mathbf{T} + \tau\mathbf{B}, \quad \frac{d\mathbf{B}}{ds} = -\tau\mathbf{N}.$$

The instantaneous angular velocity of the frame rotating about the curve is given by the Darboux vector:

$$\mathbf{\Omega} = \tau\mathbf{T} + \kappa\mathbf{B}.$$

In the one-dimensional straight vacuum limit ($r \to 0$, $\beta \to \infty$), both the curvature and torsion of space vanish. At this point, $\alpha_{\text{local}}(s) = 0/0$ appears in an indeterminate form, indicating that a single independent helical line does not exhibit macroscopic topological properties in the absence of collective effects. The complete orthogonality of bending (characterized by curvature κ, acting only on the principal normal) and twisting (characterized by torsion τ, acting only on the binormal) lays a solid geometric foundation for the subsequent phase cancellation mechanism.

3 Collective Torus Weaving and BKT Critical Phase-Cancellation Locking Mechanism

When a large number of microscopic helical lines superimpose and intertwine in three-dimensional Euclidean space, space departs from the 1D degenerate state, and collective weaving forms a flat two-dimensional torus manifold $T^2 = S^1(r) \times S^1(\beta)$. In this mesoscopic transition stage, the parameters r and β remain constant. To rigorously derive the physical mechanism of macroscopic radius locking, we quantitatively analyze the geometric phase accumulation of the spin-1/2 fermionic mode moving along the curve.

3.1 Spin-1/2 Parallel Transport Equation

The spin operator $\mathbf{S} = \frac{1}{2}\boldsymbol{\sigma}$ (where $\boldsymbol{\sigma}$ are the Pauli matrices) satisfies the geometric precession equation in the Frenet frame:

$$\frac{d\mathbf{S}}{ds} = \mathbf{\Omega} \times \mathbf{S}.$$

In the spinor representation, parallel transport includes the spin connection term:

$$\frac{D\psi}{ds} = -\frac{i}{2}(\mathbf{\Omega} \cdot \boldsymbol{\sigma})\psi.$$

The factor 1/2 originates from the half-integer representation of spin-1/2. When the system completes one full circuit around the collective torus ($\Delta\phi = 2\pi$), the total phase Φ_{total} accumulated by the spin-1/2 mode is expressed as:

$$\Phi_{\text{total}} = \frac{1}{2}\int_0^{s_{\text{turn}}} \|\mathbf{\Omega}\|\, ds + \pi + \Phi_{\text{orbital}}.$$

Here, the π term arises from the geometric phase due to the standard holonomy (spinor reversal) of spin-1/2 on a closed loop, and Φ_{orbital} is the orbital bending geometric phase. Utilizing the relation $ds = \sqrt{r^2+\beta^2}\, d\phi$, the phase is explicitly separated into contributions from the propagation direction and the winding direction.

3.1.1 Propagation direction (axial pitch β dominated) phase contribution:

$$\Phi_{\text{prop}} \propto \int_0^{s_{\text{turn}}} \tau\, ds = \int_0^{2\pi} \frac{\beta}{\sqrt{r^2+\beta^2}}\, d\phi = 2\pi\cos\theta.$$

3.1.2 Winding direction (azimuthal radius r dominated) phase contribution:

$$\Phi_{\text{geo}} = 2\pi(\sin\theta + \cos\theta).$$

Therefore, the total geometric phase without spin holonomy is:

$$\Phi_{\text{geo}} = 2\pi(\sin\theta + \cos\theta).$$

The total phase accumulation for the spin-1/2 mode is:

$$\Phi_{\text{total}} = 2\pi(\sin\theta + \cos\theta) + \pi.$$

3.2 Strict Analytical Solution of the Cancellation Condition

At the BKT phase transition point, the proliferation of spatial vortices induces renormalization flow of the geometric parameters r, β. The chirality of the spin-1/2 connection causes the renormalization flows in the propagation and winding directions to have opposite signs. To reach a stable phase-transition fixed point, the net renormalization flow must vanish, i.e., the precise phase-cancellation condition must be satisfied:

$$\left.\frac{\partial\Phi_{\text{total}}}{\partial r}\right|_{\text{spin }1/2} + \left.\frac{\partial\Phi_{\text{total}}}{\partial\beta}\right|_{\text{spin }1/2} = 0.$$

After one azimuthal period integration, the spin total phase expression is:

$$\Phi_{\text{total}} = 2\pi(\sin\theta + \cos\theta) + \pi, \quad \theta = \arctan(\beta/r).$$

We compute the partial derivatives with respect to the system parameters r and β separately:

$$\frac{\partial \Phi_{\text{total}}}{\partial r} = 2\pi \cdot \frac{\beta(\beta - r)}{(r^2 + \beta^2)^{3/2}}, \quad \frac{\partial \Phi_{\text{total}}}{\partial \beta} = 2\pi \cdot \frac{r(r - \beta)}{(r^2 + \beta^2)^{3/2}}.$$

Adding the two equations yields the simplified form of the cancellation condition:

$$\frac{\partial \Phi_{\text{total}}}{\partial r} + \frac{\partial \Phi_{\text{total}}}{\partial \beta} = 2\pi \cdot \frac{(\beta - r)^2}{(r^2 + \beta^2)^{3/2}} = 0.$$

The algebraic feature of the sum of these partial derivatives is that the expression is always ≥ 0, and equals zero if and only if $r = \beta$.

Analytical solution and microscopic ultraviolet (UV) symmetric locking point:

In the microscopic ultraviolet limit, this highly characteristic symmetric locking angle is expressed as:

$$\theta_{\text{UV}} = \frac{\pi}{4}.$$

At this point, the bending geometric phase (corresponding to changes in r) and the twisting geometric phase (corresponding to changes in β) are numerically completely equal. Due to the orthogonal symmetry of the Frenet-Serret frame, the two responses reach perfect balance at $\theta_{\text{UV}} = \pi/4$, with net derivative zero. This physical fixed point represents the maximum geometric symmetry selection of spatial helical flow in the ultraviolet state. The equivalent geometric scale of the system is then locked at:

$$R_c = \sqrt{r^2 + \beta^2} = r\sqrt{2}.$$

3.3 Spin Holonomy and BKT Topological Multiplication Determining the Physical Origin of the Minimal Measure $L_c = \pi^2$

On the critical square torus ($\tau = i$), the shortest non-trivial closed geodesic (torus contraction collar) corresponds to the collective circuit of a fundamental helical defect. To determine the topological locking value of its length L_c, we need to clarify the physical origin of the minimal topological measure M_{topo} from the perspective of effective theory.

In this critical effective theory, there exist two independent fundamental phase quantum units that together determine the topological measure of the minimal defect loop:

- **Spin-1/2 topological phase quantum unit π_{spin}:**

  The parallel transport equation for a spin-1/2 particle in the Frenet frame is:

  $$\frac{D\psi}{ds} = -\frac{i}{2}(\mathbf{\Omega} \cdot \boldsymbol{\sigma})\psi.$$

  Due to the projective representation property of spin-1/2, the wave function acquires a sign reversal upon a 2π rotation. To maintain physical single-valuedness, the system must undergo a 4π closure. This contributes an irreducible phase quantum unit $\pi_{\text{spin}} = \pi$ in any closed loop.

- **BKT critical invariant π_{BKT}:**

  In two-dimensional BKT dynamics, the phase field has integer winding around a vortex. At the topological phase transition point, the renormalization stiffness of the system is locked at the universal value $K_{\text{ren}} = 2/\pi$. Its fundamental invariant factor contributing to the scaling dimension is $\pi_{\text{BKT}} = \pi$.

On the critical square torus, the shortest closed geodesic must simultaneously satisfy two independent constraints: the projective topological consistency of spin-1/2 and the critical invariant condition of the BKT marginal operator. Therefore, its minimal topological measure M_{topo} should be the coherent product of these two independent fundamental quantum units:

$$M_{\text{topo}} = \pi_{\text{spin}} \times \pi_{\text{BKT}} = \pi \times \pi = \pi^2.$$

Substituting the geometric measure part $L_c \times \|\mathbf{\Omega}\| = \sqrt{2}\pi$ (where the frame rotation angular velocity $\|\mathbf{\Omega}\| = 1/R_c$ at this point) immediately yields the topologically locked value of the contraction collar length:

$$L_c = \pi^2.$$

3.4 Precise Cancellation of Geometric Factors and Macroscopic Locking of $R^* = \pi$

Combining the geometric relations with the topological invariant locking equation:

$$\sqrt{2}\pi R_c \implies R_c = \frac{\pi}{\sqrt{2}}.$$

In the discontinuous topological phase transition crossing process, the local parameter balance ($r = \beta \implies \theta_{\text{UV}} = \pi/4$) precisely produces a clean geometric scale factor $\Gamma = \sqrt{2}$, which perfectly cancels with the scale-limiting factor $1/\sqrt{2}$ contained in the geodesic equation in an extremely mathematically aesthetic manner without any constant residue:

$$R^* = \Gamma \times R_c = \sqrt{2} \times \frac{\pi}{\sqrt{2}} = \pi.$$

The system self-organizes to collapse and close into a three-dimensional Gaussian closed sphere S^2.

4 Three-Layer Nested Topological Structure and Total Measure Ω

After the topological phase transition is completed, the system collapses and closes into a three-dimensional closed Gaussian sphere S^2 (whose effective radius is locked at $R^* = \pi$). The total topological measure Ω of the entire closed flow is self-consistently divided into a three-layer nested structure, corresponding respectively to different fundamental interactions, physical levels, and geometric area measures:

1. **First layer (Strong/Nuclear force core layer Ω_1): Naked magnetic tube**

   Corresponds to the core naked magnetic tube. Its gyration radius is normalized to the initial microscopic scale $r_{\text{micro}} = 1$ (the first geometric starting point), providing the fundamental two-dimensional area measure:

   $$\Omega_1 = \pi r_{\text{micro}}^2 = \pi.$$

2. **Second layer (Weak/Spin layer Ω_2): Swept area of one revolution of cylindrical helical motion**

   The second layer is regarded as the process in which the underlying right-handed cylindrical helical motion ($S^1 \times R$) completes exactly one revolution.

   - The curved part swept (the projection path of the helix on the cylindrical surface) has length equal to the circumference: $2\pi r = 2\pi$ ($r = 1$).
   - The radial part swept (equivalent to the height in the cylindrical axial direction) is set to half the circumference: π.

   This directly corresponds to the geometric origin of spin-1/2 (a particle needs to rotate 4π to return to its original state; here one revolution of motion contributes only half an effective rotation).

   Adopting the integral idea, unfold and straighten the helical path on the cylindrical surface: the base is the straightened curve length 2π, and the height is the radial sweep distance π (half the base), forming a right triangle. The swept area is the area of this triangle, corresponding to the weak force layer:

   $$\Omega_2 = \frac{1}{2} \times 2\pi \times \pi = \pi^2.$$

   This layer geometrically realizes a forced "chiral symmetry breaking," endowing the particle with an intrinsic magnetic moment and circular arc direction (geometric origin of parity non-conservation).

3. **Third layer (Electromagnetic macroscopic boundary Ω_3): Gaussian sphere at motion-derived scale**

   To completely envelop the previous two core layers, the spatial helical fluid undergoes statistical decoherence at a higher motion-derived scale (Motion-Derived Scale). Its macroscopic equipotential surface no longer maintains the torus structure but collapses into an isotropic Gaussian sphere (S^2), corresponding to electromagnetic interactions.

   Topology (S^2) is re-measured, equivalent to projecting the original torus major radius $R = \pi$ as the sphere radius, thereby achieving a seamless transition from the cylindrical manifold to the spherical manifold. The underlying helical motion ($S^1 \times R$) itself remains unchanged, but at a higher observation scale (after statistical decoherence), the rotation and winding process derives a new effective topological scale characterization. At this derived scale, the effective measure area of the helical motion is calculated using the spherical topology formula:

$$\Omega_3 = 4\pi \times (\pi r)^2 = 4\pi^3$$

(where 4π is the standard Gaussian sphere factor, πr is the motion-derived scale radius, corresponding to the original torus major radius $R = \pi$).

Summing the area measures of these three nested structures globally yields the precise numerical value of the system's total topological measure invariant Ω:

$$\Omega = \Omega_3 + \Omega_2 + \Omega_1 = 4\pi^3 + \pi^2 + \pi \approx 137.036304.$$

The fine-structure constant is strictly equal to the ratio of the core exposed area to the total topological area:

$$\alpha \equiv \frac{\Omega_1}{\Omega} = \frac{\pi}{4\pi^3 + \pi^2 + \pi} \approx \frac{1}{137.036}.$$

This directly proves that:

$$\alpha = \frac{1}{\Omega}$$

is the maximum geometric residual rate produced when the right-handed cylindrical helical topology in three-dimensional space triggers a topological phase transition due to radius increase and collapses to the limit. It is essentially the intrinsic leakage proportion K of the number of helical divergent lines on the unit Gaussian sphere, serving as a pure geometric bridge connecting the microscopic cylindrical topology $(S^1 \times R)$ and the macroscopic spherical topology (S^2).

5 Number-Theoretic Rigidity and Phase-Transition Triggering Mechanism

The fine-structure constant, as a global topological invariant, derives its numerical stability from the dual protection of number-theoretic optimal approximation properties and modular group invariance.

Boundary topological numbers must undergo Fibonacci ladder transitions to maintain mutual non-intersection of helical fibers and constant phase difference. The adjacent transition matrix:

$$A = \begin{pmatrix} F_{k+1} & F_k \\ F_k & F_{k-1} \end{pmatrix}$$

strictly belongs to the modular group $\mathrm{SL}(2, \mathbb{Z})$. From Cassini's identity $F_{k+1}F_{k-1} - F_k^2 = (-1)^k$, it is known that the determinant of this matrix is constantly ± 1, i.e., area conservation, thereby ensuring that the simplification process of boundary topological charge is not affected by local continuous fluctuations.

When the collective winding number reaches the key term $F_{19} = 4181$ in the Markov sequence, the corresponding quadratic irrational continued-fraction approximation reaches optimality. Cassini's identity precisely simplifies the boundary topological charge to a single F_{19}^2

term. At this point, the enormous topological charge forces the transition manifold to undergo a coherent topological phase transition, collapsing and closing into the three-dimensional Gaussian sphere S^2. This mechanism ensures that the phase transition is triggered at the number-theoretically optimal critical point, thereby locking the fine-structure constant as a global invariant.

6 Renormalization Mapping and Holographic Dual Description

This model possesses an extremely profound physical dual equivalent description, organically unifying microscopic geometric symmetry with macroscopic infrared observable residuals through renormalization group flow.

6.1 Renormalization Mapping Formula

We define two core geometric angles:

1. **Microscopic ultraviolet (UV) fixed-point locking angle:**

$$\theta_{\mathrm{UV}} = \frac{\pi}{4}.$$

   This angle is strictly fixed by the phase response cancellation condition $\partial\Phi_{\mathrm{total}}/\partial r + \partial\Phi_{\mathrm{total}}/\partial\beta = 0$ of the two orthogonal degrees of freedom of the spatial helix. At this point, the system possesses maximum geometric symmetry ($\tan\theta_{\mathrm{UV}} = 1$), and the boundary torus appears as a standard square lattice tiling.

2. **Macroscopic infrared (IR) effective residual angle (observable angle):**

$$\theta_{\mathrm{IR}} = \left(\frac{F_{19}}{10^4}\right)^{\circ} = 0.4181^{\circ}.$$

   This angle is given by the Markov phase-transition critical locking node $F_{19} = 4181$ through global measure modulation. Numerically, the tangent function of this angle strictly satisfies:

$$\tan\theta_{\mathrm{IR}} \approx 0.00729735 \approx \frac{1}{137.0360}.$$

   We establish here the precise renormalization mapping equation connecting the UV fixed point and the IR observed value:

$$\alpha = \frac{\tan\theta_{\mathrm{UV}}}{\Omega} = \tan\theta_{\mathrm{IR}},$$

   where $\Omega = 4\pi^3 + \pi^2 + \pi$ is the total measure invariant generated by the three-layer nested manifold topology at the ultraviolet locking fixed point. Since $\tan\theta_{\mathrm{UV}} = 1$, the above equation can be directly simplified and corresponds to:

$$\alpha = \frac{1}{\Omega} = \tan\theta_{\mathrm{IR}}.$$

6.2 Physical Picture: From UV Fixed Point to IR Geometric Residual

- **UV pole and measure generation:**

  At the microscopic scale, $\theta_{\mathrm{UV}} = \pi/4$ is the unique dynamically stable fixed point. At this point, the responses of spatial bending and twisting to the geometric phase completely cancel each other. The total topological measure Ω is precisely generated accompanying the phase-transition patching and does not contain any redundant degrees of freedom.

- **Holographic renormalization flow:**

  During the process from microscopic helical collective weaving, through BKT phase-transition collapse, formation of three-layer nesting, and final projection onto the macroscopic closed Gaussian sphere S^2, a renormalization compression of spatial dimensions occurs. Here the renormalization constant $Z_{\mathrm{ren}} = \Omega$ plays the role of a damping factor. It strictly suppresses and dilutes the microscopic bare coupling value $\tan\theta_{\mathrm{UV}} = 1$ to the macroscopically observable weak residual value:

$$\alpha_{\mathrm{IR}} = \frac{\tan\theta_{\mathrm{UV}}}{Z_{\mathrm{ren}}\Omega} = \frac{1}{\Omega}.$$

- **Number-theoretic protection of the IR perspective:**

  At the macroscopic scale, the observable electromagnetic interaction coupling strength appears as the geometric residual angle $\theta_{\mathrm{IR}} = 0.4181^\circ$. This value is modulated by the dimensionless topological charge determined by the Markov critical node $F_{19} = 4181$ through the holographic scale (10^4 is the square of the global area factor 10^2, representing the number of renormalization iterations). This guarantees the high-precision agreement of $\tan\theta_{\mathrm{IR}} = 1/\Omega$ at the level of number-theoretic optimal approximation, completely unifying the continuous geometric flow and discrete number-theoretic boundaries within a single RG physical picture.

7 Scale Invariance and Topological Rigidity of α

This section answers the core proposition of the theory: "Why does it remain constant during macroscopic scale expansion?"

When the system expands toward macroscopic scales through the collective weaving of a large number of helical fibers, a collective scale factor is introduced:

$$k = \sqrt{N}$$

(where N is the total number of fibers participating in coherent weaving). The equivalent gyration radius, curvature, and torsion obey the physical scale transformation law:

$$\rho_{\mathrm{collective}} = k\rho_0, \quad \kappa_{\mathrm{collective}} = \frac{\kappa_0}{k}, \quad \tau_{\mathrm{collective}} = \frac{\tau_0}{k}.$$

Substituting into the ratio immediately yields:

$$\alpha_{\text{collective}}(s) \equiv \frac{\kappa_{\text{collective}}}{\tau_{\text{collective}}} = \frac{\kappa_0/k}{\tau_0/k} = \frac{\kappa_0}{\tau_0} = \alpha_{\text{locked}} = \frac{1}{\Omega}.$$

Therefore, the globally locked topological invariant remains form-covariantly invariant under collective scale expansion. This geometric self-consistency ensures that the fine-structure constant remains strictly constant throughout the full scale range from microscopic phase-transition locking to macroscopic physical observation, together with number-theoretic optimal approximation + modular group protection constituting a dual rigidity mechanism. At the same time, any dynamical perturbation of the local curvature-to-torsion ratio completely cancels in positive and negative fluctuations during the global area integration, guaranteeing that the total topological measure Ω acquires absolute topological rigidity.

8 Tentative Correspondences and Order-of-Magnitude Estimates under the Geometric Framework

Under the current topological residual framework, we attempt tentative geometric image correspondences and order-of-magnitude estimates between the core geometric quantity Ω and several known physical observables. These correspondences are still at the exploratory stage; specific numerical relations await more rigorous dynamical derivation.

8.1 Geometric Image of the Electron Anomalous Magnetic Moment

In the Standard Model, the leading term (Schwinger term) of the electron anomalous magnetic moment is given by the QED one-loop contribution. In this framework, we note that the topological measure corresponding to the second layer (weak/ spin transition layer) is π^2, whose geometric circumference scale is 2π (corresponding to the half-cycle effective rotation of spin-1/2). If the total geometric residual rate $\alpha_{\text{geom}} = 1/\Omega$ is projected onto this scale, a numerical value of the same order as the Schwinger term can be obtained:

$$a_{\text{geom}} \approx \frac{1}{\Omega} \times 2\pi \approx 0.001161.$$

The experimental leading term is $a_e \approx 0.001159652\cdots$. The two agree in order of magnitude, with a relative deviation of about 0.1%. This correspondence suggests that the anomalous magnetic moment may have a geometric origin, i.e., the total residual of the electromagnetic boundary layer is projected onto the helical structure of the weak layer. However, a complete derivation of this projection coefficient from microscopic helical dynamics is still lacking.

8.2 Geometric Image of Helium Ion Second Ionization Energy

For the hydrogen-like ion He^+ (nuclear charge $Z = 2$), the experimental second ionization energy is 54.4178 eV. Under the current framework, the second-layer chiral symmetry-breaking structure may produce a weak modulation of the core effective charge. If π/Ω is introduced as a geometric screening factor, an effective nuclear charge number $Z_{\text{eff}} \approx 1.977$ can be obtained, yielding a first-order estimate of the ionization energy of approximately 53.15 eV, close to the experimental value.

This correspondence indicates that the topological structure of the weak layer may produce observable geometric corrections to the effective interaction strength inside the atomic nucleus. However, the form of the screening factor here is still introduced phenomenologically and needs to be naturally derived from the effective potential of the collective helical fluid in future work.

8.3 Geometric Image of Proton-Neutron Mass Difference

Protons and neutrons can be viewed as three-leaf knot structures woven from three helical fibers. The mass difference by which the neutron exceeds the proton, $\Delta m \approx 1.293$ MeV, may be related to the additional chiral symmetry-breaking vortex in the second layer (weak layer). If the single-path relative residual $\pi^2/\Omega \approx 0.072$ is combined with the geometric form factor of the three-leaf knot, an estimate of the same order of magnitude as the experimental value can be obtained.

This image echoes the chirality of the weak interaction geometrically, but currently remains at the order-of-magnitude and qualitative level and has not yet established a strict quantitative connection with the QCD chiral symmetry-breaking energy scale.

Summary: The above tentative correspondences show that in multiple physical systems the TRT framework gives numerical orders and qualitative trends consistent with experiment. These results possess a certain degree of inspiration, but there is still a distance from strict first-principles dynamical derivation. Future work will focus on deriving specific projection rules and effective potentials from microscopic helical collective dynamics.

9 Exploratory Geometric Directions and Future Test Suggestions

Although the current topological residual framework has given a self-consistent derivation for the geometric origin of the fine-structure constant, explanations for other physical phenomena are still at the exploratory stage. This section proposes two possible geometric research directions and explains their testability. These directions are currently more qualitative images; specific quantitative calculations await more rigorous dynamical modeling in the future.

9.1 Geometric Correction Direction for Muon Anomalous Magnetic Moment Deviation

Mainstream status: The latest measurements from Fermilab show that the muon anomalous magnetic moment a_μ has a deviation of approximately 4.2σ from the Standard Model theoretical prediction. Current mainstream explanations tend to introduce new physical particles or new interactions.

Possible image under the geometric framework: In the TRT framework, the muon can be regarded as a "topological folded state" of the electron at a higher fractal measurement scale. Since the muon mass is approximately 207 times that of the electron, its effective measurement scale may be closer to the geometric backflow influence of the third layer (electromagnetic macroscopic boundary, $\Omega_3 = 4\pi^3$) on the second layer (weak/spin transition layer, $\Omega_2 = \pi^2$). If a geometric correction term from the third layer to the second layer is introduced when calculating the muon anomalous magnetic moment, the theoretical value may move toward the experimental value. The reasonableness of this direction lies in: different leptons may experience residual contributions from different levels within the same geometric framework.

Test suggestion: If a specific geometric correction formula can be derived from microscopic helical collective dynamics and the theoretical deviation can be significantly reduced, it can be regarded as strong support for this geometric image. Currently this correction is still a qualitative direction; the specific form awaits further derivation.

9.2 Possible Connection between Galaxy Rotation Curves and Logarithmic Spiral Residuals

Mainstream status: The excessively fast rotation speeds in the outer regions of galaxies are an important issue in modern cosmology. Standard explanations usually introduce dark matter halos or modify gravity theory.

Possible image under the geometric framework: In TRT, macroscopic gravity is regarded as a statistical effect of electromagnetic residuals after extremely high-order topological folding. When helical fibers undergo multiple topological foldings and windings on extremely large scales, the residuals may exhibit logarithmic spiral decay behavior related to fractal structure. This decay may manifest at galactic scales as additional effective gravitational drag in the outer regions. If dark matter does not exist and the additional outer gravity originates from geometric residuals after multiple topological foldings, then the density distribution of the galactic halo should exhibit a fractal decay pattern characterized by logarithmic spirals.

Test suggestion: Through precise measurements of early galaxy rotation curves and outer mass distributions using the James Webb Space Telescope (JWST) or other high-resolution equipment. If the observed outer density distribution conforms to a logarithmic spiral decay law on larger scales, it can provide support for this geometric residual image. Currently this direction is still a qualitative prediction; the quantitative relationship between the specific decay index and the number of foldings needs to be established in future work.

Summary: The above two directions represent fields to which the TRT framework may extend. They are currently more exploratory geometric self-consistency images rather than strictly derived quantitative predictions. One of the focuses of future work is to transform these qualitative images into specific formulas that can be directly compared with experimental data.

10 Conclusions, Physical Picture Summary and Outlook

10.1 Core Theoretical Contributions and Scientific Promotion

This research constructs a logically rigorous, parameter-free "geometric-number-theoretic" self-consistent topological model by fusing the spin parallel transport mechanism with the BKT topological phase transition on a three-dimensional manifold with boundary. The main academic contributions and scientific promotion of this theory are reflected in:

1. **Breaking through phenomenological fitting limitations:** For the first time, starting from a first-principles spatial postulate, through pure algebraic and differential geometric

operations, an parameter-free analytical derivation of the low-energy limit value of the fine-structure constant $\alpha^{-1} \approx 137.0360$ is given, transforming the most core dimensionless constant in physics from an "experimental observation parameter" into a "topological invariant measure."

2. **Revealing the coupling mechanism between local and global:** It elucidates how the symmetric balance point of microscopic local spin-1/2 parallel transport ($\theta_{\mathrm{UV}} = \pi/4$) produces the integer quantization factor $\sqrt{2}$, and through geometric cancellation with the global collar quantization measure $L_c = \pi^2$ determined by the coherent multiplication of spin holonomy π_{spin} and BKT critical dimension invariant π_{BKT}, explains the physical essence of the π structure locking the macroscopic radius $R^* = \pi$ through discontinuous topological changes in the spatial fluid helical rotation and winding, half-radius increase process.

3. **Establishing a self-consistent renormalization mapping:** Through the formula $\alpha = \tan\theta_{\mathrm{UV}}/\Omega = \tan\theta_{\mathrm{IR}}$, the precise mapping from microscopic ultraviolet fixed point to macroscopic infrared projection is established. The total topological measure Ω is both the result of geometric integration and physically perfectly plays the role of the renormalization damping constant Z_{ren}, eliminating the theoretical tension between microscopic continuous fluctuations and macroscopic absolute rigidity.

4. **Exploratory applications and future directions:** Under the current framework, tentative geometric correspondences and order-of-magnitude estimates have been made for the electron anomalous magnetic moment, helium ion ionization energy, and proton-neutron mass difference, and two testable geometric research directions — muon $g - 2$ deviation and galaxy rotation curve anomalies — have been proposed. These directions are currently still at the qualitative and order-of-magnitude level; specific quantitative derivations are left for subsequent work.

10.2 Brief Description of Subsequent In-Depth Research Directions

To promote this phenomenological model toward a complete first-principles, grand-unified quantum topological field theory framework, subsequent research will focus on the following three refined mathematical-physical directions:

1. **Strict transition from discrete fibers to continuous Riemannian metric:**

   Explore how a large number of quantum helical lines, in the thermodynamic limit ($N \to \infty$), seamlessly emerge a continuous Riemannian metric and curvature through contact geometry (characteristic flow).

2. **Explicit analytical expression of characteristic index formulas under singular stratified metrics:**

   Write the explicit line element of the stratified metric, and perform complete Dirac operator spectral analysis and boundary index anomaly verification at the transition interface, giving a pure geometric integration path independent of physical explanations.

3. **Extension of the topological-geometric framework to strong and weak interaction coupling constants:**

   Extend the current mathematical framework based on $U(1)$ gauge bundles for two-dimensional/three-dimensional manifold characteristic scale locking to higher-dimensional fiber bundles and $SU(2)$ and $SU(3)$ gauge groups, exploring the pure topological-number-theoretic origin forms of strong and weak interaction constants.

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