A Sieve of Sundaram for Twin prime

By

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Abstract. A new Twin prime sieve based on a modified sieve of Sundaram is introduced. It sieves through the set of natural numbers n such that n is not representable in either of the forms 2ij + i + j or2ij + i + j -1 for positive integers i, j.

Theorem

Let n be a positive integer. Then the two numbers $2n+1$ and $2n+3$ are both prime if and only if n cannot be written in either of the forms $n=2ij+i+j$ or $n = 2ij+i+j-1$ for any positive integers i and j.

Proof

We prove the contrapositive for each number separately and then combine the conditions.

1. Characterization of compositeness of $2n+1$

Assume first that $2n+1$ is composite and greater than 3. Then it can be factored as a product of two odd integers each at least 3. Write

$2n+1=(2i+1)(2j+1)$

with $i,j\in Z+$ (positive integers). Expanding the right-hand side gives

$2n+1=4ij+2i+2j+1.$

Subtracting 1 and dividing by 2 yields

$n=2ij+i+j.$

Conversely, if $n=2ij+i+j$ for some positive integers i,j then substituting back shows

$$2n+1=(2i+1)(2j+1),$$

which is a product of two integers greater than 1; hence $2n+1$ is composite (unless one factor equals 1, but that would require $i=0$ or $j=0$, which are not allowed). Therefore $2n+1$ is prime $\iff n\neq 2ij+i+j$ for all positive integers i, j.

2. Characterization of compositeness of $2n+3$

Now suppose $2n+3$ is composite and greater than 3. Again write it as a product of two odd integers:

$$2n+3=(2i+1)(2j+1),\ i,j\in Z+.$$

Expanding:

$$2n+3= 4ij+2i+2j+1.$$

Subtract 1:

$$2n+2=4ij+2i+2j.$$

Divide by 2:

$$n+1 = 2ij+i+j,$$

and hence

$$n = 2ij+i+j-1.$$

Conversely, if $n=2ij+i+j-1$ for some positive i, j then

$$2n+3=2(2ij+i+j-1)+3=4ij+2i+2j+1=(2i+1)(2j+1),$$

so $2n+3$ is composite. Thus

$$2n+3 \text{ is prime } \iff n\neq 2ij+i+j-1 \text{ for all positive integers } i, j.$$

3. Combining the two conditions

The statement "2n+1 and 2n+3 are both prime" is true exactly when **both** individual primality conditions hold. Therefore n must avoid both representing forms:

Conclusion

We have shown

2n+1, 2n+3 composite ⟺ n=2ij+i+j or n=2ij+i+j-1.

Taking negations gives

2n+1 and 2n+3 are prime ⟺ n is not representable in those forms.

Reference

[1] Ahmed Diab, Development of sieve of Eratosthenes and sieve of Sundaram's proof, arVix: 2102.06653v2 [math.NT] 3 May 2021.