

# Vacuum–Energonic Relativity (VER): An Operational Physical Frame, Dynamical Energonicity, and the Minimal Working Model VER-I- $\vartheta$

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## Abstract

This paper formulates the postulate-based framework of Vacuum–Energonic Relativity (VER). In VER, the operationally measurable metrology of matter is determined by a single physical metric  $\tilde{g}_{\mu\nu}$  (P1–P3), while the local causal bound is set by a vacuum scalar field  $\varphi(x) > 0$  with the dimension of  $c^2$  via  $c(x) = \sqrt{\varphi(x)}$  (P2). For test systems we introduce the invariant  $\mu \equiv m(x)c(x) = \text{const}$  and the worldline action  $S_{pp} = -\mu \int d\tilde{s}$  (P4), yielding the universal relations  $m = \mu/c$ ,  $E_0 = \mu c$ , and the exact finite identity  $\Delta E_0 = -\Delta m c c'$ . We show how to “globalize” energy in stationary configurations of the physical frame  $\tilde{g}$  through the Killing energy  $E_K$ , obtaining an operationally transparent “energy bookkeeping” for two transport scenarios: free fall and quasi-static transfer.

We then fix a covariant action scaffold  $S = S_{\text{vac}}[g_{\mu\nu}, \varphi, \vartheta] + S_m[\psi_m, \tilde{g}_{\mu\nu}]$  with a conformal relation  $\tilde{g}_{\mu\nu} = \Omega^2(\varphi)g_{\mu\nu}$ ; for realization I we choose  $\Omega^2 = \varphi^{-1}$ . We derive the universal matter source structure for  $\varphi$ ,  $\delta S_m/\delta\varphi \propto (d \ln \Omega^2/d\varphi)\tilde{T}$ , and present a minimal working model (Model-0) with a complex vacuum order parameter  $\Psi = \sqrt{\varphi}e^{i\vartheta}$ , which fixes a ghost-free kinetic sector for the phase mode and implements the potential-feedback regime  $\Lambda^4(\varphi)(1 - \cos \vartheta)$ . In the weak-field limit we show that realization I inevitably entails a Yukawa-type scalar channel (“fifth force”) with an  $O(1)$  coupling ( $\alpha_* = -1/2$ ), and we formulate “viability gates” without cosmology: requirements for suppressing the Yukawa channel (mass/screening), the PPN limit, metrological dimensionless observables, and stationary tests via  $E_K$ . The paper deliberately avoids constructing a full cosmology and is positioned as a foundational, locally testable basis for subsequent phenomenological and cosmological developments.

## Keywords

modified gravity; scalar field; physical frame; variable maximum velocity; equivalence principle; conformal transformations; Killing energy; axion-like field.

# 1 Introduction

Modern gravitational physics and cosmology simultaneously exhibit a high degree of empirical success and a persistent set of “boundary” questions. The former include the precision of local and Solar-System tests of General Relativity (GR); the latter concern the interpretation of observational tensions and the role of additional vacuum degrees of freedom in early and late epochs. Against this background, two borderline classes of approaches recur: (i) models with a variable causal bound / an effectively variable propagation speed (the VSL family), and (ii) scenarios involving axion-like modes used either as additional dynamical components of the dark sector or as regulators of early-time physics. Both classes are supported by extensive literature, yet both face familiar criticisms: VSL approaches are often accused of being “a redefinition of units” and of lacking a covariantly closed formulation, while axion scenarios are frequently perceived as ad hoc extensions that require independent microphysical motivation and careful control of additional interaction channels.

This work proposes the postulate-based framework of Vacuum–Energonic Relativity (VER), whose purpose is to fix a minimal operationally and covariantly defined structure in which the “parameter of the causal cone” becomes a dynamical vacuum variable, while the measurement (physical) frame is specified uniquely and universally. The central idea is to distinguish (a) the variational metric of the vacuum–gravitational sector  $g_{\mu\nu}$  from (b) **the single physical metric of matter**  $\tilde{g}_{\mu\nu}$ , which determines operational measurements (clocks and rods) and the null cones of material signals. VER introduces a vacuum scalar field  $\varphi(x) > 0$  with the dimension of  $c^2$ , which sets the local causal bound

$$c(x) = \sqrt{\varphi(x)}.$$

and postulates, for test systems, the invariant  $\mu \equiv m(x)c(x) = \text{const}$ , enabling a clean separation between local kinematics and metrology when comparing regions with different  $\varphi$ . An extension includes a phase mode  $\vartheta$ , interpreted as an axion-like vacuum degree of freedom without direct non-metric couplings to matter.

The key methodological premise of VER is that the “physical frame”  $\tilde{g}_{\mu\nu}$  is not introduced as a convenient convention (a choice of Jordan frame), but is fixed as part of the operational content of the theory: all matter is minimally coupled to  $\tilde{g}_{\mu\nu}$ , and to it only. Although the covariant structure employed here formally overlaps with the conformal scalar–tensor class (matter minimally coupled to a single “Jordan metric”), VER fixes the operational status of this metric  $\tilde{g}_{\mu\nu}$  as physical: it defines clocks/rods and the matter causal cone via postulates P1–P3, rather than being selected “for convenience” among equivalent frames. Moreover, VER supplements the standard frame-based kinematics with a nontrivial **energy–metrology structure** derived from the test-system invariant  $\mu = mc$ : this leads to the exact relations  $m = \mu/c$ ,  $E_0 = \mu c$ , and to a “bookkeeping” of stationary processes through the Killing energy in  $\tilde{g}$  (Secs. 3–5), which provides a direct bridge to testable “viability gates” (Secs. 11–12). An explicit dictionary mapping to the standard scalar–tensor notation and the status of  $\tilde{g}$  as the Jordan metric of matter is given in Sec. 8.6.

To ensure that the theory is not merely a postulate scheme but a covariantly closed model, we fix

an action scaffold

$$S = S_{\text{vac}}[g_{\mu\nu}, \varphi, \vartheta] + S_m[\psi_m, \tilde{g}_{\mu\nu}], \quad \tilde{g}_{\mu\nu} = \Omega^2(\varphi) g_{\mu\nu},$$

and choose the minimal realization I,  $\Omega^2(\varphi) = \varphi^{-1}$ . On this basis we introduce a minimal working model (Model-0) with a complex vacuum order parameter  $\Psi = \sqrt{\varphi} e^{i\vartheta}$ , which yields a rigid ghost-free kinetic structure for  $\varphi^{-\vartheta}$  and a universal matter source for  $\varphi$  through the trace  $\tilde{T}$  in the physical frame.

This paper is deliberately restricted to **foundational and locally testable** consequences of VER and does not construct a full cosmological model. The role of cosmological motivation in the present text is to indicate why the dynamics of vacuum modes  $(\varphi, \vartheta)$  is of interest, rather than to replace local phenomenology with cosmological promises. The main results are: (i) the derivation of local special-relativistic kinematics in the physical frame at fixed  $c_0$ ; (ii) an exact energy “bookkeeping” in stationary  $\tilde{g}$  via the Killing energy; (iii) a covariant action scaffold and a minimal Model-0; and (iv) the inevitable emergence of a weak-field Yukawa scalar channel (“fifth force”) in realization I. For realization I the conformal coupling is of order unity (see Sec. 11), so the viability of the minimal model in the low-energy vacuum reduces to the quantitatively testable requirement that the Yukawa channel be suppressed by a finite scalar mass  $m_\varphi$  (and/or screening). In the present version we establish the analytic structure of this mechanism and the corresponding set of “viability gates” (Secs. 11–12); in a subsequent revision these gates will be supplemented by explicit numerical parameter windows based on modern laboratory and Solar-System bounds.

The paper is organized as follows. Section 2 states postulates P1–P4. Section 3 derives local kinematic consequences in the physical frame  $\tilde{g}_{\mu\nu}$ , including the energonic scaling  $m = \mu/c$ ,  $E_0 = \mu c$ , and the dispersion relation. Section 4 introduces Killing energy in stationary  $\tilde{g}$  and relates global to locally measured energy; Sec. 5 formulates two energy-transfer scenarios (free fall and quasi-static transfer). Sections 6–9 fix realizations and the covariant action scaffold, motivate the  $\vartheta$  extension, and select the minimal Model-0 branch. Section 10 specifies the minimal working model VER-I- $\vartheta$  and its equations. Section 11 analyzes the weak-field limit and the origin of the “fifth force”; Sec. 12 formulates non-cosmological “viability gates” and exhibits dimensionless observables not removable by redefinitions of units. The conclusion summarizes the key results and outlines future directions (numerical parameter windows, a full EFT analysis, and cosmological specialization).

Finally, it is important to distinguish results from program. The present work derives (i) local operational consequences of P1–P4 in the physical frame  $\tilde{g}$  (Secs. 3–5), (ii) a covariant action scaffold and the minimal working model Model-0 (Secs. 8–10), and (iii) the structure of the weak-field scalar channel and the corresponding non-cosmological viability gates (Secs. 11–12). Numerical parameter windows consistent with current experimental bounds and a cosmological specialization (background and perturbations) are treated as the next stage of the program.

## 2 The Postulate Core of Vacuum–Energonic Relativity (VER)

This section fixes the postulate core of VER in a form sufficient to derive the local kinematics/energetics (Secs. 3–5) and to subsequently close the theory covariantly via an action scaffold

(Secs. 8–10). We emphasize that postulates P1–P4 specify the operational structure of the theory: what is deemed physically measurable (clocks/rods), causal cones, test-system energy, and which quantities are universal for all material systems.

## 2.1 Entities and Conventions

**Physical matter frame.**  $\tilde{g}_{\mu\nu}$  is the unique physical metric of matter, determining operational measurements (clocks/rods) and the causal cone of material signals.

**Variational vacuum–gravity sector.**  $g_{\mu\nu}$  is the metric used as the variational variable of the vacuum sector and for writing the covariant action (see Sec. 8).

**Energonicity.**  $\varphi(x) > 0$  is a vacuum scalar field with the dimension of  $c^2$ , setting the local causal bound

$$c(x) = \sqrt{\varphi(x)}.$$

**Phase mode (extension).**  $\vartheta(x)$  is a phase (axion-like) vacuum mode in the VER-I- $\vartheta$  extension; it is introduced in the vacuum sector and is not included in P3 as a direct coupling to matter.

**Signature and units.** We use the metric signature  $(-+++)$ . Throughout, we keep  $c(x) = \sqrt{\varphi(x)}$  explicit as the physical causal bound of the matter sector; when convenient, standard “natural” conventions such as  $\hbar = 1$  are used only as a technical shorthand and do not replace the operational meaning of  $c(x)$ .

## 2.2 VER Postulates

**(P1) Local inertiality of the physical frame.** For any event  $x_0$ , there exists a coordinate system  $\{x^\mu\}$  such that

$$\tilde{g}_{\mu\nu}(x_0) = \eta_{\mu\nu}, \quad \partial_\alpha \tilde{g}_{\mu\nu}(x_0) = 0,$$

and in a small neighborhood of  $x_0$ ,

$$\tilde{g}_{\mu\nu}(x) = \eta_{\mu\nu} + O(|x - x_0|^2).$$

Meaning: in the physical frame  $\tilde{g}_{\mu\nu}$ , local matter dynamics at each point reduces to Special Relativity (in suitable locally inertial coordinates).

**(P2) Energonicity and a single matter causal cone.** There exists a vacuum scalar field  $\varphi(x) > 0$  (dimension  $c^2$ ) defining the local causal bound:

$$c(x) = \sqrt{\varphi(x)}.$$

The characteristics of material signal fronts coincide with the null characteristics of the physical metric:

$$\tilde{g}_{\mu\nu} dx^\mu dx^\nu = 0.$$

Meaning: the causal structure of the matter sector is unique and set by  $\tilde{g}_{\mu\nu}$ , while the cone parameter is a field.

**(P3) Universality of matter and minimal coupling.** The matter sector depends on vacuum fields only through the physical metric  $\tilde{g}_{\mu\nu}$ :

$$S_m = S_m[\psi_m, \tilde{g}_{\mu\nu}],$$

and the matter stress–energy tensor is defined with respect to  $\tilde{g}$ :

$$\tilde{T}^{\mu\nu} \equiv \frac{2}{\sqrt{-\tilde{g}}} \frac{\delta S_m}{\delta \tilde{g}_{\mu\nu}}, \quad \tilde{\nabla}_\mu \tilde{T}^{\mu\nu} = 0.$$

Moreover, the matter sector contains no direct non-metric couplings to vacuum fields  $\chi^A$  at fixed  $\tilde{g}$  and  $\psi_m$ :

$$\left. \frac{\delta S_m}{\delta \chi^A} \right|_{\tilde{g}, \psi_m} = 0 \quad (\forall \chi^A).$$

In the minimal version  $\chi^A = \{\varphi\}$ ; in the VER-I- $\vartheta$  extension,  $\chi^A = \{\varphi, \vartheta\}$ . Meaning: all matter fields “see” the same physical frame, excluding composition-dependent direct portals at the level of principle.

**(P4) Test-system invariant and geodesicity in  $\tilde{g}$ .** For test systems, the invariant

$$\mu \equiv m(x)c(x) = \text{const}$$

holds, and the action of a pointlike test particle is taken to be

$$S_{pp} = -\mu \int d\tilde{s}, \quad d\tilde{s}^2 = -\tilde{g}_{\mu\nu} dx^\mu dx^\nu.$$

Meaning: free fall of a test particle follows a geodesic of the physical metric  $\tilde{g}$ , and the energy–metrology dependence of the test mass on the vacuum field is fixed by the invariant  $\mu$ .

### 2.3 What Is Derived Here, and What Remains Programmatic

- In the present work, P1–P4 are used to derive local kinematics/metrology and exact energy relations (Sec. 3), followed by stationary energetics via Killing energy in the physical frame  $\tilde{g}$  (Secs. 4–5).
- The covariant dynamics of the vacuum fields  $(g_{\mu\nu}, \varphi, \vartheta)$  is fixed through the action scaffold (Sec. 8) and the minimal working model Model-0 (Sec. 10).
- Numerical parameter windows and a cosmological specialization (background and perturbations) belong to the next stage of the program (see the “gates” formulation in Secs. 11–12).

### 3 Local Kinematics of VER: Lorentz Transformations, Energonic Renormalization of Scales, Mass–Energy, and Dispersion

This section fixes the local consequences of postulates P1–P4 in the physical frame  $\tilde{g}_{\mu\nu}$ . The basic idea is simple: (i) in any sufficiently small neighborhood, matter physics reduces to Special Relativity (P1); yet (ii) the causal-cone parameter  $c(x) = \sqrt{\varphi(x)}$  is dynamical (P2); and (iii) for test systems the invariant  $\mu \equiv m(x)c(x) = \text{const}$  holds (P4). Together these imply a crucial distinction between kinematics (Lorentz transformations at fixed  $c_0$ ) and metrology (the comparison of physical scales between regions with different  $\varphi$ ).

#### 3.1 Local Special Relativity in the Physical Frame and the Local Cone Parameter

By postulate P1, for any point  $x_0$  there exist locally inertial coordinates  $\{x^\mu\}$  such that

$$\tilde{g}_{\mu\nu}(x_0) = \eta_{\mu\nu}, \quad \partial_\alpha \tilde{g}_{\mu\nu}(x_0) = 0, \quad \tilde{g}_{\mu\nu}(x) = \eta_{\mu\nu} + O(|x - x_0|^2).$$

Hence, local matter dynamics in a neighborhood of  $x_0$  is described by Special Relativity with respect to the null cone of  $\tilde{g}$ .

Postulate P2 introduces a vacuum scalar field  $\varphi(x) > 0$  with the dimension of  $c^2$ , which sets the local causal bound

$$c(x) = \sqrt{\varphi(x)}.$$

It is convenient to denote

$$c_0 \equiv c(x_0) = \sqrt{\varphi(x_0)}.$$

This  $c_0$  is the local light-cone parameter in the chosen locally inertial chart.

#### 3.2 Local Lorentz Transformations at Fixed $c_0$

Consider two locally inertial observers in a neighborhood of  $x_0$ , moving relative to each other along the  $x$ -axis with relative speed  $v$ , measured operationally in the physical frame  $\tilde{g}$  (using local clocks and rods). Then the Lorentz transformations take their standard form with the replacement  $c \rightarrow c_0$ :

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c_0^2}}}, \quad t' = \gamma \left( t - \frac{v}{c_0^2} x \right), \quad x' = \gamma(x - vt), \quad y' = y, \quad z' = z.$$

Thus the local Lorentz group is preserved, but its scale parameter  $c_0$  is dynamical and depends on position through  $\varphi(x)$ . This is the key difference between VER and standard SR: the local symmetry structure is the same, while the cone parameter becomes a field.

### 3.3 Time and Length Kinematics at Constant Energonicity

For processes localized within a region where  $\varphi \simeq \text{const}$  (hence  $c \simeq c_0$ ), the usual consequences of Lorentz kinematics apply.

Time dilation:

$$\Delta t = \gamma \Delta \tau,$$

where  $\Delta \tau$  is the proper time (in the physical frame) of clocks comoving with the object.

Lorentz length contraction:

$$L = \frac{L_0}{\gamma},$$

where  $L_0$  is the proper length (measured in the object's rest frame).

Importantly, these effects refer to comparisons between inertial observers within a single region with fixed  $c_0$ , and should not be conflated with effects due to changes of  $\varphi$  when comparing regions or states with different  $c$ .

### 3.4 Time and Length Intervals Under Varying Energonicity: Separating Kinematics from Metrology

In VER,  $c(x) = \sqrt{\varphi(x)}$  is dynamical. Therefore, when comparing measurements performed in two regions with different values of  $\varphi$  (and thus  $c$  and  $c'$ ), one encounters, in addition to purely kinematic effects, a metrological component: the relation between physical scales of energy/frequency/mass and the local cone parameter changes.

In a locally inertial system of the physical frame, the interval reads

$$d\tilde{s}^2 = -c^2 dt^2 + d\ell^2, \quad d\ell^2 \equiv dx^2 + dy^2 + dz^2.$$

For an observer at rest ( $d\ell = 0$ ), proper time satisfies

$$d\tilde{\tau} = dt.$$

Thus, geometrically, the rate of “ideal” rest clocks in the chosen local chart is not altered by a change in  $c$ : proper time is still determined by  $\tilde{g}$ . However, physical time scales of real clocks (transition frequencies, relaxation times, etc.) are set by energy scales, and in VER these depend on  $\varphi$  through postulate P4 (Sec. 3.5). Hence one should distinguish:

Kinematic effects (Lorentz dilation/contraction): consequences of transformations at fixed  $c_0$  within a single local region (Sec. 3.3).

Energonic (metrological) effects: changes of physical scales when comparing regions/states with different  $\varphi$ , caused by the renormalization of mass and rest energy at fixed  $\mu$  (Secs. 3.5–3.6).

Operationally, the mapping between geometric time  $d\tilde{\tau}$  and the “physical ticking” of clocks depends on the choice of standard. In this primary paper we fix the universal structure for test systems,

$m \propto 1/c$  and  $E_0 \propto c$ . For a standard whose characteristic frequency is proportional to the rest-energy scale, one obtains

$$\omega \propto E_0 \propto c \Rightarrow \frac{\omega'}{\omega} = \frac{c'}{c}, \quad \frac{T'}{T} = \frac{c}{c'}.$$

This statement is operational-model dependent: it is tied to which physical processes are chosen as clocks. A detailed microphysical anchoring of standards belongs to the phenomenological “gates” program (Sec. 12).

### 3.5 Energonic Scaling of Mass and Local Rest Energy

Postulate P4 introduces, for test systems, the invariant

$$\mu \equiv m(x)c(x) = \text{const.}$$

It immediately implies the energonic scaling of the operationally defined mass:

$$m(x) = \frac{\mu}{c(x)} = \frac{\mu}{\sqrt{\varphi(x)}}.$$

Define the local rest energy in the standard way:

$$E_0(x) \equiv m(x)c(x)^2.$$

Substituting  $m = \mu/c$ , we obtain the key VER formula:

$$E_0(x) = \mu c(x) = \mu \sqrt{\varphi(x)}.$$

Thus, at fixed  $\mu$ , the local rest energy is proportional to  $c(x)$ , while the mass is inversely proportional to  $c(x)$ . This is the basic “energonic” mechanism: the field  $\varphi$  redistributes mass and rest energy of a test system so as to preserve the invariant  $\mu$ .

### 3.6 Exact Finite Relation Between $\Delta E_0$ and $\Delta m$ Under Changing Energonicity

Consider two states of the same test system with the same  $\mu$ , characterized by pairs  $(m, c)$  and  $(m', c')$ . By definition,

$$E_0 \equiv mc^2, \quad E'_0 \equiv m'c'^2, \quad \mu = mc = m'c'.$$

Then  $m' = mc/c'$ , and a direct computation gives

$$E'_0 - E_0 = mc(c' - c).$$

On the other hand,

$$m' - m = m \left( \frac{c}{c'} - 1 \right) = m \frac{c - c'}{c'}.$$

Multiplying the latter by  $cc'$  and changing the sign yields the identity

$$-(m' - m)cc' = mc(c' - c) = E'_0 - E_0.$$

Hence the exact finite formula holds:

$$\Delta E_0 = -\Delta m c c'.$$

For small changes ( $c' = c + \delta c$ ,  $m' = m + \delta m$ ), it yields the local linear form

$$\delta E_0 = -c^2 \delta m \quad (\text{at fixed } \mu).$$

This relation plays the role of “energonic bookkeeping” in quasi-static estimates, and will be compared below with stationary-field energetics (Secs. 4–5).

### 3.7 Relativistic Dispersion in the Local Physical Frame

In a locally inertial system of the physical frame near  $x_0$ , the dispersion relation of a material particle has the standard SR form with  $c \rightarrow c_0$ :

$$E^2 = p^2 c_0^2 + m^2 c_0^4.$$

Using  $\mu = mc_0$  for a test system in the given local region, one obtains an equivalent form that is more natural for VER:

$$m^2 c_0^4 = (\mu^2 c_0^2) c_0^4 = \mu^2 c_0^2 \Rightarrow E^2 = c_0^2 (p^2 + \mu^2).$$

Thus (i) the local dispersion remains SR-like, (ii) the “invariant” part is parameterized by  $\mu$ , and (iii) the overall energy scale is set by the local value  $c_0$ .

### 3.8 Domain of Applicability of Local Kinematics

All results in this section are local statements in the physical frame  $\tilde{g}$ : they hold in regions where locally inertial coordinates exist and  $\varphi(x)$  is sufficiently smooth so that  $c(x)$  can be treated as nearly constant on the scale of the process considered. Comparing energies measured at different points, and passing to “global” energy notions, requires stationary symmetries of the physical frame and the concept of Killing energy. This is the subject of the next sections.

## 4 Killing Energy, Local Energy, and “Energy at Infinity” in VER

In Sec. 3 we constructed the local kinematics of VER in the physical frame  $\tilde{g}_{\mu\nu}$ . We now need a “globalization” of energy: how to compare energies measured at different points and what constitutes an integral of motion in stationary fields. For this purpose we use the standard GR construction of Killing energy, with one crucial emphasis: in VER all such definitions are formulated in the physical metric  $\tilde{g}$ , since it is  $\tilde{g}$  that determines observable metrology.

## 4.1 Stationarity of the Physical Metric and the Killing Vector

Assume the physical metric  $\tilde{g}_{\mu\nu}$  admits a timelike Killing symmetry. This means there exists a vector field  $\xi^\mu$  satisfying the Killing equation

$$\tilde{\nabla}_{(\mu}\xi_{\nu)} = 0,$$

and, in the typical static case,  $\xi^\mu$  coincides with  $\partial_t$  in coordinates adapted to stationarity. The norm of  $\xi^\mu$  in the physical frame is

$$\tilde{g}(\xi, \xi) \equiv \tilde{g}_{\mu\nu}\xi^\mu\xi^\nu < 0 \quad (\text{in the region where } \xi \text{ is timelike}).$$

The existence of  $\xi^\mu$  allows one to define a conserved ‘‘global’’ energy along geodesics in stationary  $\tilde{g}$ .

## 4.2 Test-Particle Four-Momentum in VER and the Definition of Killing Energy

In VER a test particle is described by the worldline action (postulate P4)

$$S_{pp} = -\mu \int d\tilde{s}, \quad d\tilde{s}^2 = -\tilde{g}_{\mu\nu}dx^\mu dx^\nu.$$

Define the physical four-velocity

$$\tilde{u}^\mu \equiv \frac{dx^\mu}{d\tilde{s}}, \quad \tilde{g}_{\mu\nu}\tilde{u}^\mu\tilde{u}^\nu = -1,$$

and the corresponding test-particle four-momentum in the physical frame:

$$\Pi_\mu \equiv \frac{\partial L}{\partial \dot{x}^\mu} = \mu \tilde{u}_\mu, \quad \tilde{u}_\mu = \tilde{g}_{\mu\nu}\tilde{u}^\nu,$$

where  $L$  is the worldline Lagrangian.

**Definition (Killing energy).** If  $\xi^\mu$  is a timelike Killing vector of  $\tilde{g}$ , then

$$E_K \equiv -\Pi_\mu \xi^\mu = -\mu \tilde{u}_\mu \xi^\mu.$$

**Statement (conservation of  $E_K$ ).** For geodesic motion in stationary  $\tilde{g}$ ,  $E_K$  is conserved along the worldline:

$$\frac{dE_K}{d\tilde{s}} = 0.$$

This is the standard result: the constant of motion arises from invariance of the action under translations along  $\xi^\mu$  (Noether/Killing).

### 4.3 Local Energy Measured by a Static Observer

To relate  $E_K$  to what a local observer measures, introduce the four-velocity field of static observers—those at rest with respect to the stationary coordinates and moving along the orbits of  $\xi^\mu$ .

The normalized four-velocity of a static observer is

$$\tilde{w}^\mu \equiv \frac{\xi^\mu}{\sqrt{-\tilde{g}(\xi, \xi)}}.$$

Then the particle's locally measured energy (energy in the observer's local rest frame) is defined by

$$E_{\text{loc}} \equiv -\Pi_\mu \tilde{w}^\mu.$$

This definition is fully operational: local energy is the negative projection of the four-momentum onto the observer's four-velocity, all defined in the physical frame  $\tilde{g}$ .

### 4.4 Relation Between Killing Energy and Local Energy

Substituting  $\tilde{w}^\mu$  into the definition of  $E_{\text{loc}}$ , we obtain

$$E_{\text{loc}} = -\Pi_\mu \frac{\xi^\mu}{\sqrt{-\tilde{g}(\xi, \xi)}} = \frac{E_K}{\sqrt{-\tilde{g}(\xi, \xi)}}.$$

Since  $-\Pi_\mu \xi^\mu = E_K$ , it follows that

$$E_K = \sqrt{-\tilde{g}(\xi, \xi)} E_{\text{loc}}.$$

In static coordinates where  $\xi^\mu = \partial_t$ , one has  $\tilde{g}(\xi, \xi) = \tilde{g}_{tt}$ , hence

$$E_K = \sqrt{-\tilde{g}_{tt}} E_{\text{loc}}.$$

### 4.5 “Energy at Infinity” and the Meaning of $E_K$

If the spacetime is asymptotically flat (or there exists a region where  $\sqrt{-\tilde{g}_{tt}} \rightarrow 1$ ), then  $E_K$  coincides with the energy measured “at infinity” by a stationary observer:

$$E_\infty \equiv \lim_{r \rightarrow \infty} E_{\text{loc}}(r) = E_K.$$

In this sense  $E_K$  can be interpreted as energy at infinity. Even without asymptotic flatness,  $E_K$  remains the natural global constant of motion associated with stationarity of the physical metric.

### 4.6 Particle at Rest: $E_{\text{loc}} = E_0$ and the Difference Between $E_0$ and $E_K$

If the particle is at rest with respect to a static observer, then its locally measured energy equals the local rest energy:

$$E_{\text{loc}} = E_0.$$

Therefore,

$$E_K = \sqrt{-\tilde{g}(\xi, \xi)} E_0 = \sqrt{-\tilde{g}_{tt}} E_0.$$

This relation makes the conceptual distinction transparent:

- $E_0$  is the local rest energy, which depends on energeticity via  $E_0(x) = \mu c(x) = \mu \varphi(x)$  (see Sec. 3.5).
- $E_K$  is the global constant of motion conserved along a geodesic in stationary  $\tilde{g}$ , and it includes the “redshift” factor  $\sqrt{-\tilde{g}_{tt}}$ .

Thus, in VER (as in GR) one cannot simply identify “the energy of a particle” with a local quantity: comparing energies at different points is correctly done via  $E_K$ .

#### 4.7 Transition to Energy “Bookkeeping” (Sec. 5)

The formula

$$E_K = \sqrt{-\tilde{g}(\xi, \xi)} E_{\text{loc}}$$

is the basic tool for analyzing two energy-transfer scenarios:

- Free fall:  $E_K = \text{const}$ , while  $E_{\text{loc}}$  and, at rest,  $E_0(x)$  may vary due to changes in  $\tilde{g}(\xi, \xi)$  and  $\varphi(x)$ .
- Quasi-static transfer: an external force changes  $E_K$ , and  $\Delta E_K$  is interpreted as work “at infinity”; the energetic part of the change in  $E_0$  satisfies the exact identity  $\Delta E_0 = -\Delta m c c'$  (see Sec. 3.6).

These conclusions are summarized in compact balance formulas in the next section.

## 5 Two Energy-Transport Scenarios: Free Fall and Quasi-Static Transfer

The purpose of this section is to turn the formulas of Secs. 3–4 into an operationally transparent “energy bookkeeping”: which quantities are conserved, which change, and what must be supplied by an external agent. In VER the key objects are (i) the local rest energy  $E_0(x) = \mu c(x)$  and (ii) the Killing energy  $E_K = -\Pi \cdot \xi$  in stationary  $\tilde{g}$ .

### 5.1 The Set of Quantities: $E_0$ , $E_{\text{loc}}$ , $E_K$

We collect the main definitions.

**Local rest energy** (particle at rest with respect to the local observer):

$$E_0(x) \equiv m(x)c(x)^2 = \mu c(x) = \mu \sqrt{\varphi(x)}.$$

**Local energy measured by a static observer**  $\tilde{w}^\mu$ :

$$E_{\text{loc}}(x) \equiv -\Pi_\mu \tilde{w}^\mu.$$

If the particle is at rest with respect to the static observer, then  $E_{\text{loc}} = E_0$ .

**Killing energy** (when a timelike Killing vector  $\xi^\mu$  of the physical metric exists):

$$E_K \equiv -\Pi_\mu \xi^\mu.$$

**Relation between  $E_K$  and  $E_{\text{loc}}$**  (Sec. 4.4):

$$E_K = \sqrt{-\tilde{g}(\xi, \xi)} E_{\text{loc}} \Rightarrow E_{\text{loc}} = \frac{E_K}{\sqrt{-\tilde{g}(\xi, \xi)}}.$$

In static coordinates  $\sqrt{-\tilde{g}(\xi, \xi)} = \sqrt{-\tilde{g}_{tt}}$ .

This relation provides the “conversion” between local energy measurements and the globally conserved quantity  $E_K$ .

## 5.2 Scenario I: Free Fall (Geodesic Motion)

Consider a test particle moving geodesically in a stationary physical metric  $\tilde{g}$ . Then, as shown in Sec. 4.2, the Killing energy is conserved:

$$\frac{dE_K}{d\tilde{s}} = 0 \Rightarrow E_K = \text{const.}$$

However, the local energy measured by a static observer varies according to

$$E_{\text{loc}}(x) = \frac{E_K}{\sqrt{-\tilde{g}(\xi, \xi)}}.$$

Thus, in free fall the local energy changes “passively” due to the change in the time normalization of static observers (gravitational redshift), and may also be redistributed between kinetic energy and the local rest-energy scale via  $c(x)$ . To highlight this, consider the illustrative case where the particle is instantaneously at rest with respect to the static observer at some point  $x$ . Then  $E_{\text{loc}}(x) = E_0(x) = \mu c(x)$ , and we have

$$E_K = \sqrt{-\tilde{g}(\xi, \xi)} \mu c(x).$$

Comparing two points  $x_1$  and  $x_2$  where the particle is (at some instants) at rest with respect to the corresponding static observers, conservation of  $E_K$  yields

$$\sqrt{-\tilde{g}(\xi, \xi)}|_{x_1} \mu c(x_1) = \sqrt{-\tilde{g}(\xi, \xi)}|_{x_2} \mu c(x_2).$$

This relation exhibits the “double” control: changes in the gravitational factor  $\sqrt{-\tilde{g}(\xi, \xi)}$  and in the energeticity  $c(x)$  jointly determine locally measured energies at fixed global  $E_K$ .

**Outcome of free fall.**

- Conserved:  $E_K$ .
- Varies:  $E_{\text{loc}}$  and, in particular,  $E_0(x) = \mu c(x)$  when comparing regions with different  $c$ .
- No external work: all changes correspond to redistribution within conservative dynamics.

### 5.3 Scenario II: Quasi-Static Transfer (With External Work)

Now consider quasi-static transport of a test system: an external force holds the system at rest with respect to static observers and moves it slowly through a stationary field. In this case the worldline is not geodesic, and the Killing energy need not be conserved.

Let the system be transported from point 1 to point 2, remaining at rest with respect to the static observer at each point. Then  $E_{\text{loc}} = E_0$  and

$$E_K = \sqrt{-\tilde{g}(\xi, \xi)} E_0 = \sqrt{-\tilde{g}(\xi, \xi)} \mu c.$$

The change in  $E_K$  is naturally interpreted as work “seen at infinity” (or, more generally, as the global work of the external agent in a stationary field):

$$\Delta W_\infty \equiv \Delta E_K.$$

Hence,

$$\Delta W_\infty = \sqrt{-\tilde{g}(\xi, \xi)}|_2 \mu c_2 - \left( \sqrt{-\tilde{g}(\xi, \xi)}|_1 \mu c_1 \right).$$

This formula already shows that the work decomposes into two channels:

- variation of the redshift factor  $\sqrt{-\tilde{g}(\xi, \xi)}$  (the gravitational contribution),
- variation of the energonicity  $c(x)$  (the energonic contribution).

The second channel is specific to VER and can be isolated via the exact relation between mass and rest energy.

### 5.4 The Energonic Contribution: Exact Relation Between $\Delta E_0$ and $\Delta m$

Let energonicity change between two states of the system:  $c \rightarrow c'$ ,  $m \rightarrow m'$ ,  $E_0 \rightarrow E'_0$ , with the invariant  $\mu = mc = m'c'$  held fixed. Then Sec. 3.6 yields the exact finite identity:

$$\Delta E_0 = -\Delta m c c'.$$

This can be viewed as “energonic bookkeeping” at the level of rest energy: increasing  $c$  increases  $E_0$  and decreases  $m$ , and their relation is fixed exactly by the product  $cc'$ , without approximation.

**Linear limit.** For small changes one recovers the familiar form

$$\delta E_0 = -c^2 \delta m.$$

**Use in quasi-statics.** If the system remains at rest with respect to static observers during transport, then locally measured energy equals  $E_0$ , while global energy is  $E_K = \sqrt{-\tilde{g}(\xi, \xi)} E_0$ . The change in  $E_K$  can then be decomposed as

$$\Delta E_K = \Delta(\sqrt{-\tilde{g}(\xi, \xi)} E_0) = \underbrace{\Delta(\sqrt{-\tilde{g}(\xi, \xi)}) E_0}_{\text{gravitational contribution}} + \underbrace{\sqrt{-\tilde{g}(\xi, \xi)} \Delta E_0}_{\text{energonic contribution}}.$$

In the second term  $\Delta E_0$  obeys the exact relation above via  $\Delta m$  and  $c, c'$ . Thus, in quasi-static transport the external work “sees” the energonic dynamics directly.

## 5.5 Why These Two Scenarios Matter Methodologically

From the standpoint of the primary VER paper, the pair “free fall vs quasi-statics” makes the construction operationally robust against the standard objection that “a variable  $c$  is merely units.”

- In free fall,  $E_K$  is conserved, while local energies change consistently with the factor  $\sqrt{-\tilde{g}(\xi, \xi)}$  and with the energonic dependence  $E_0 = \mu c$ .
- In quasi-statics, an external work  $\Delta W_\infty = \Delta E_K$  appears, and the energonic part of this work is expressed by the exact finite identity

$$\Delta E_0 = -\Delta m c c',$$

which is not a redefinition of units but follows from postulate P4 together with the definition of rest energy in the physical frame.

This bookkeeping links the dynamics of the vacuum field  $\varphi$  to measurable energy balances without invoking cosmological models.

## 6 VER Realizations: The Conformal Class $\tilde{g} = \Omega^2(\varphi)g$ , Three Minimal Realizations I/II/III, and the Rationale for Choosing Realization I

In Secs. 2–5 we fixed the operational core of VER: the physical frame  $\tilde{g}_{\mu\nu}$ , the dynamical cone parameter

$$c(x) = \sqrt{\varphi(x)}.$$

the test-system invariant  $\mu = mc$ , local kinematics, and stationary energetics via Killing energy. At this stage we must specify a minimal structural bridge to the covariant description (Sec. 8): how exactly  $\tilde{g}_{\mu\nu}$  is related to the variational variables of the vacuum sector, primarily the metric  $g_{\mu\nu}$  and the field  $\varphi$ . In this paper we adopt the most economical and controllable choice—the conformal class  $\tilde{g}_{\mu\nu} = \Omega^2(\varphi) g_{\mu\nu}$ —and fix three minimal realizations (in a broad sense) I/II/III, after which we justify the choice of realization I as the baseline for the primary article and for Model-0.

## 6.1 The Conformal Class as a Minimal Mechanism for Universality (P3)

Postulate P3 requires the entire matter sector to be minimally coupled only to  $\tilde{g}_{\mu\nu}$ :  $S_m = S_m[\psi_m; \tilde{g}_{\mu\nu}]$ ,  $\tilde{\nabla}_\mu \tilde{T}^{\mu\nu} = 0$ . To ensure that the influence of the vacuum sector on matter is universal and does not generate sector-dependent direct portals, the natural minimal choice is to relate  $\tilde{g}_{\mu\nu}$  to the variational metric  $g_{\mu\nu}$  through a single scalar function  $\Omega(\varphi)$ :

$$\tilde{g}_{\mu\nu} = \Omega^2(\varphi) g_{\mu\nu}, \quad \Omega^2(\varphi) > 0.$$

Within this class:

all matter “sees” the same  $\tilde{g}_{\mu\nu}$  (universality by construction);

changes in  $\varphi$  affect the matter sector only through  $\tilde{g}$ , rather than through separate couplings to different fields;

a conformal relation preserves the null-direction structure of  $\tilde{g}$  (a single matter causal cone, P2), which is methodologically preferable to “medium” interpretations that risk multi-cone behavior.

## 6.2 Three Minimal Realizations I/II/III: Meaning and Risks

We now distinguish three minimal realizations in the following sense: they are the simplest structurally and interpretationally distinct ways to understand the role of  $\varphi$  and its relation to  $\tilde{g}$  within the conformal class. These realizations do not add new postulates beyond P1–P4; rather, they represent branching options for the subsequent program.

### Realization I (chosen): the minimal conformal relation

$$\Omega^2(\varphi) = \varphi^{-1} \Rightarrow \tilde{g}_{\mu\nu} = \varphi^{-1} g_{\mu\nu}.$$

This is the simplest special case, introducing no additional functions or scales beyond  $\varphi$  itself. It renders the basic relations of Sec. 3 maximally transparent:

$$c(x) = \sqrt{\varphi(x)}, \quad m = \mu/c, \quad E_0 = \mu c,$$

and leads to an especially simple specialization of the universal matter source for  $\varphi$  (see Sec. 8.4 and Sec. 10.4b).

### Realization II (heuristic): $\varphi$ as a “dilaton/scale mode”

In this interpretation  $\varphi$  is treated as a dilaton-like scale field governing local rescalings of physical parameters. This is natural in an EFT context and in scenarios with spontaneously broken scale symmetry. Its potential advantage is rich microphysical motivation. The main risk for a primary formulation of VER is that a dilaton typically tends to enter the matter sector through rescalings of parameters (masses, couplings, etc.), which can weaken the transparency of a strict P3 implementation and may require additional mechanisms (symmetries/screening) to suppress composition-dependent effects.

### Realization III (heuristic): an “effective medium” and optical analogy

Here  $\varphi$  is viewed as a parameter of an effective “vacuum medium” (an analogue of a refractive index) influencing signal propagation speeds. The advantage is intuitive appeal; the drawback is the common risk of “multi-cone” behavior: in medium-like models different modes (fields, polarizations, frequencies) often propagate with different effective cones. This conflicts with P2–P3 (a single matter causal cone and universality) unless strong conditions enforce the same  $\tilde{g}$  for all matter. Therefore, in the primary article realization III is treated as heuristic, not as a baseline.

## 6.3 Why Realization I Is Fixed in the Primary Article

We choose realization I as the baseline for a combination of criteria that follow directly from the architecture of the paper.

**Structural minimality.** Realization I fixes  $\Omega(\varphi)$  without introducing additional functions, minimizing model freedom at the stage of a primary publication.

**Operational transparency.** With  $\tilde{g} = \varphi^{-1}g$ , the relations  $m = \mu/c$ ,  $E_0 = \mu c$ , and the exact identity  $\Delta E_0 = -\Delta m c c'$  retain their simplest form (Secs. 3.5–3.6), while stationary energetics via  $E_K$  is naturally formulated in the physical frame  $\tilde{g}$  (Secs. 4–5).

**Technical controllability of the  $\varphi$  source.** In the conformal class the universal matter source for  $\varphi$  is derived strictly (Sec. 8.4). In realization I it acquires a particularly compact expression (Sec. 10.4b), strengthening the argument that the theory is not a “redefinition of units.”

## 6.4 Why II/III Are Not a “Simple Escape” from the Weak-Field Scalar Channel

It is important to emphasize that the weak-field scalar channel (“fifth force”) discussed in Secs. 11–12 is not an artefact of the specific choice  $\Omega^2 = \varphi^{-1}$ , but a structural consequence of any conformal matter coupling with  $\Omega(\varphi) \neq \text{const}$ . With  $S_m[\psi_m, \tilde{g}]$  and  $\tilde{g} = \Omega^2(\varphi)g$ , matter generically sources  $\varphi$  through  $\tilde{T}$  (Sec. 8.4), and exchange of  $\delta\varphi$  between bodies produces a Yukawa correction in the weak field (Sec. 11). Hence moving from I to a different  $\Omega(\varphi)$  within the conformal class may change numerical coefficients and normalizations, but it does not remove the mechanism itself: suppressing the channel requires either  $\Omega \simeq \text{const}$  (the GR limit) or dynamical suppression by a mass and/or screening.

For this reason the primary article does not attempt to “avoid” the weak-field problem by complicating  $\Omega(\varphi)$ , but instead fixes the minimal realization I and formulates explicit “viability gates” (Secs. 11–12).

## 6.5 Summary and Transition to the Covariant Scaffold

Within the conformal class  $\tilde{g} = \Omega^2(\varphi)g$ , realization I ( $\Omega^2 = \varphi^{-1}$ ) is the most minimal and operationally transparent baseline for a primary VER publication. Realizations II/III are retained as possible microphysical/interpretational branches of the broader program, but are not adopted as baselines here because they require additional discussions (radiative stability of matter universality, multi-cone risks, etc.) that lie beyond the scope of the primary work.

In the next section we turn to the extension of the vacuum sector to VER-I- $\vartheta$  and to the classification of  $\varphi$ - $\vartheta$  coupling regimes needed for constructing the minimal working model (Model-0).

## 7 The VER-I- $\vartheta$ Extension: The Phase Mode of the Order Parameter and $\varphi$ - $\vartheta$ Coupling Regimes

Section 6 fixed the baseline line of the primary article—realization I of the conformal class  $\tilde{g}_{\mu\nu} = \Omega^2(\varphi)g_{\mu\nu}$  with  $\Omega^2(\varphi) = \varphi^{-1}$ . We now take the next logical step: we show that extending the vacuum sector by an additional degree of freedom need not be an ad hoc complication, but can be an EFT-natural move, compatible with P3 and useful for constructing a minimal working model.

The key methodological idea is the following: if  $\varphi$  is interpreted as the amplitude of a vacuum order parameter, then in EFT the generic order parameter is typically complex, so that together with the amplitude one expects a phase mode. This leads to the VER-I- $\vartheta$  extension with a phase (axion-like) mode  $\vartheta(x)$ .

### 7.1 A Complex Vacuum Order Parameter

Assume the vacuum order parameter takes the form of a complex field

$$\Psi(x) = \sqrt{\varphi(x)} e^{i\vartheta(x)}.$$

Then:

- $\varphi(x) > 0$  is the amplitude (in our terminology, energonicity, since  $\varphi = c^2$ );
- $\vartheta(x)$  is the phase mode, which transforms as a pseudoscalar and is naturally interpreted as an axion-like field in the vacuum sector.

The key point is that **introducing  $\vartheta$  does not change the operational structure of the theory**, because matter remains coupled only to  $\tilde{g}_{\mu\nu}$ , not directly to  $\vartheta$ . Hence the extension does not violate postulate P3.

## 7.2 EFT Naturalness of the Phase Mode: Shift Symmetry and a Periodic Potential

Phase modes of order parameters often enjoy an (exact or approximate) shift symmetry

$$\vartheta \rightarrow \vartheta + \text{const.}$$

This makes  $\vartheta$  “natural” in the EFT sense: such a symmetry protects the field from large quantum corrections to its mass and potential. A potential for  $\vartheta$  then arises either from a weak explicit breaking of the shift symmetry or from nonperturbative effects, and typically has a periodic (axion-like) structure:

$$V(\varphi, \vartheta) \sim \Lambda^4(\varphi) (1 - \cos \vartheta),$$

where  $\Lambda(\varphi)$  is, in general, a  $\varphi$ -dependent scale. This is precisely the form that realizes the potential-feedback regime  $\vartheta \leftrightarrow \varphi$ , which will be chosen below as the baseline branch for Model-0.

## 7.3 Three $\varphi$ - $\vartheta$ Coupling Regimes (Overview)

In VER-I- $\vartheta$ , the coupling between the amplitude mode  $\varphi$  and the phase mode  $\vartheta$  may take different structural forms. For the purposes of a primary paper it is convenient to distinguish three minimal regimes (see also Sec. 9, where this classification is used as a bridge to the working model).

**Regime 1 (spectator phase).**  $\vartheta$  is present but has negligible backreaction on  $\varphi$ ; the potential and/or kinetic terms provide no significant feedback.

**Regime 2 (kinetic coupling).** Feedback is implemented through a  $\varphi$ -dependent normalization of the  $\vartheta$  kinetic term,  $K(\varphi)(\partial\vartheta)^2$ . Then the contribution of  $\vartheta$  to the  $\varphi$  equation is proportional to  $K'(\varphi)(\partial\vartheta)^2$  and may be small in relaxed/quasi-static configurations.

**Regime 3 (potential coupling).** The amplitude of the periodic potential depends on  $\varphi$ :  $V(\varphi, \vartheta) = U(\varphi) + \Lambda^4(\varphi)(1 - \cos \vartheta)$ . This yields direct feedback  $\vartheta \leftrightarrow \varphi$  through  $\partial_\varphi \Lambda^4$  and operates already in stationary regimes.

## 7.4 Why Regime 3 Is Chosen for the Working Model

Regime 3 is selected for constructing the minimal working model (Model-0, Sec. 10) for three reasons.

**Functionality while preserving P3.** Regime 3 provides a direct feedback channel  $\vartheta \leftrightarrow \varphi$  through the vacuum potential, without requiring direct couplings of  $\vartheta$  to matter:  $\delta S_m / \delta \vartheta = 0$  at fixed  $\tilde{g}$  (see P3 and Sec. 10.3).

**EFT naturalness.** A periodic potential  $\propto (1 - \cos \vartheta)$  is the minimal structure compatible with the shift symmetry of a phase mode, and is therefore not an arbitrary “by hand” addition.

**Controllability: microphysics packaged into  $\Lambda(\varphi)$ .** In regime 3 the microphysical content of the hidden sector is concentrated in a single function  $\Lambda(\varphi)$  (and, secondarily, in the kinetic normalization), allowing one to construct Model-0 and formulate “gates” without an explicit UV completion. A microphysical bridge for  $\Lambda(\varphi)$  (the dark-axion branch, hidden confinement, and RG transmutation) is presented in Appendix D and used in Sec. 9 to motivate the branch choice.

## 7.5 Summary and Transition to the Covariant Action Scaffold

In summary, the VER-I- $\vartheta$  extension is motivated as an EFT-natural inclusion of the phase mode of a complex order parameter  $\Psi = \sqrt{\varphi} e^{i\vartheta}$ . Choosing regime 3 fixes a minimal structural feedback  $\vartheta \leftrightarrow \varphi$  required for constructing a working model while preserving matter universality (P3). In the next section we fix the covariant action scaffold in which this structure is implemented at the level of equations of motion.

## 8 Covariant Action Scaffold and the Structure of the Equations

In the previous sections we worked deliberately at the operational level: the physical frame  $\tilde{g}_{\mu\nu}$ , local kinematics and energetics, Killing energy, and the “bookkeeping” of energy transport. We now fix a covariant action scaffold that (i) implements postulates P1–P3 as structural requirements on the matter sector and (ii) specifies the dynamics of the vacuum sector, including  $\varphi$  and (in the extension)  $\vartheta$ . This step is methodologically essential: it makes VER a covariantly closed theory and shifts the discussion from “metrological interpretations” to equations of motion.

### 8.1 Overall Structure of the Action

VER is defined by an action of the form

$$S = S_{\text{vac}}[g_{\mu\nu}, \varphi, \vartheta] + S_m[\psi_m, \tilde{g}_{\mu\nu}],$$

where:

- $S_m$  is the matter action, depending on all matter fields  $\psi_m$  only through the physical metric  $\tilde{g}_{\mu\nu}$  (postulate P3);
- $S_{\text{vac}}$  is the vacuum–gravity sector, formulated in terms of the variational variables  $g_{\mu\nu}$ ,  $\varphi$ , and (when needed)  $\vartheta$ .

Postulate P3 implies that the matter stress–energy tensor is defined with respect to the physical metric:

$$\tilde{T}^{\mu\nu} \equiv \frac{2}{\sqrt{-\tilde{g}}} \frac{\delta S_m}{\delta \tilde{g}_{\mu\nu}}, \quad \tilde{\nabla}_\mu \tilde{T}^{\mu\nu} = 0.$$

The conservation law follows from diffeomorphism invariance of  $S_m$  and the fact that  $\tilde{\nabla}$  is the connection compatible with  $\tilde{g}_{\mu\nu}$ .

## 8.2 A Minimal Vacuum-Sector Scaffold

For the primary article it suffices to fix a class of models that ensures a consistent (ghost-free) dynamics and the existence of stationary solutions. A minimal “scaffold” for the vacuum sector can be written as

$$S_{\text{vac}} = \int d^4x \sqrt{-g} \left[ \frac{M_{\text{Pl}}^2}{2} R(g) - \frac{1}{2} Z(\varphi) (\partial\varphi)^2 - \frac{1}{2} K(\varphi) (\partial\vartheta)^2 - V(\varphi, \vartheta) \right].$$

Here  $Z(\varphi) > 0$  and  $K(\varphi) > 0$  guarantee the correct kinetic signs, while  $V(\varphi, \vartheta)$  is the vacuum potential. In the VER version without a phase mode one may set  $\vartheta = \text{const}$  and/or drop the corresponding term.

Remark: at the framework level one may consider more general (yet stable) kinetic structures, including nonlinear functions of  $(\partial\varphi)^2$  or mixed terms. In a primary publication, however, it is important to keep the minimal structure.

## 8.3 Relation Between the Physical and Variational Metrics: The Conformal Class of Realization I

At the level of the general theory  $\tilde{g}_{\mu\nu}$  and  $g_{\mu\nu}$  could be independent objects. Minimal realizations, however, favor a simple and controllable relation. In realization I we adopt the conformal class

$$\tilde{g}_{\mu\nu} = \Omega^2(\varphi) g_{\mu\nu}, \quad \Omega^2(\varphi) > 0.$$

Within this class, the minimal version used in this paper is specified by

$$\Omega^2(\varphi) = \varphi^{-1} \Rightarrow \tilde{g}_{\mu\nu} = \varphi^{-1} g_{\mu\nu}.$$

This choice transfers the  $\varphi$ -dependence of matter not through direct couplings but through the physical metric  $\tilde{g}$ , preserving matter universality: all matter fields depend on  $\varphi$  in the same way because they depend only on  $\tilde{g}$ .

## 8.4 Matter Variation and the Universal Source for $\varphi$

A key point of realization I is that  $\varphi$  acquires a matter source through the variation of  $S_m$  with respect to  $\varphi$ . We derive this structure explicitly for the conformal class  $\tilde{g}_{\mu\nu} = \Omega^2(\varphi) g_{\mu\nu}$ .

Start from the matter variation with respect to  $\tilde{g}$ :

$$\delta S_m = \frac{1}{2} \int d^4x \sqrt{-\tilde{g}} \tilde{T}^{\mu\nu} \delta \tilde{g}_{\mu\nu}.$$

At fixed  $g_{\mu\nu}$  and varying  $\varphi$ ,

$$\delta \tilde{g}_{\mu\nu} = \frac{d\Omega^2}{d\varphi} g_{\mu\nu} \delta\varphi.$$

Since  $g_{\mu\nu} = \Omega^{-2}\tilde{g}_{\mu\nu}$ , we obtain

$$\delta\tilde{g}_{\mu\nu} = \frac{d\Omega^2}{d\varphi} \Omega^{-2}\tilde{g}_{\mu\nu} \delta\varphi = \left(\frac{d \ln \Omega^2}{d\varphi}\right) \tilde{g}_{\mu\nu} \delta\varphi.$$

Thus,

$$\delta S_m = \frac{1}{2} \int d^4x \sqrt{-\tilde{g}} \tilde{T}^{\mu\nu} \left(\frac{d \ln \Omega^2}{d\varphi} \tilde{g}_{\mu\nu} \delta\varphi\right) = \frac{1}{2} \int d^4x \sqrt{-\tilde{g}} \left(\frac{d \ln \Omega^2}{d\varphi}\right) \tilde{T} \delta\varphi,$$

where  $\tilde{T} \equiv \tilde{g}_{\mu\nu} \tilde{T}^{\mu\nu}$ .

Hence the functional derivative of the matter action with respect to  $\varphi$  has the universal form

$$\frac{\delta S_m}{\delta\varphi} = \frac{1}{2} \sqrt{-\tilde{g}} \left(\frac{d \ln \Omega^2}{d\varphi}\right) \tilde{T}.$$

This expression is important for two reasons:

- it shows that the matter source for  $\varphi$  is proportional to the trace  $\tilde{T}$  in the physical frame—i.e., it is composition-universal under P3;
- it depends only on the chosen conformal law  $\Omega(\varphi)$ , not on microphysical details.

**Special case of realization I** ( $\Omega^2 = \varphi^{-1}$ ). Then  $\ln \Omega^2 = -\ln \varphi$  and

$$\frac{d \ln \Omega^2}{d\varphi} = -\frac{1}{\varphi},$$

so

$$\frac{\delta S_m}{\delta\varphi} = -\frac{1}{2} \sqrt{-\tilde{g}} \frac{\tilde{T}}{\varphi}.$$

A further specialization of the right-hand side in the  $\varphi$  equation using  $\sqrt{-\tilde{g}} = \varphi^{-2} \sqrt{-g}$  is given in Sec. 10.4b.

## 8.5 Equations of Motion: General Structure

The full equations of motion follow from variations with respect to  $g_{\mu\nu}$ ,  $\varphi$ , and  $\vartheta$ . For the primary paper it suffices to fix their structure.

**Equation for  $\varphi$ :**

$$\mathfrak{E}_\varphi[g, \varphi, \vartheta] = -\frac{1}{\sqrt{-g}} \frac{\delta S_m}{\delta\varphi},$$

where  $\mathfrak{E}_\varphi$  is the left-hand side obtained from varying  $S_{\text{vac}}$ . For the minimal scaffold of Sec. 8.2,

$$\mathfrak{E}_\varphi = \nabla_\mu (Z(\varphi) \partial^\mu \varphi) - \frac{1}{2} Z'(\varphi) (\partial\varphi)^2 - \frac{1}{2} K'(\varphi) (\partial\vartheta)^2 - \partial_\varphi V(\varphi, \vartheta).$$

Substituting the result of Sec. 8.4 expresses the right-hand side in terms of  $\tilde{T}$  and  $\Omega(\varphi)$ .

**Equation for  $\vartheta$ :**

$$\nabla_\mu (K(\varphi) \partial^\mu \vartheta) = \partial_\vartheta V(\varphi, \vartheta),$$

i.e., a covariant equation with respect to the metric  $g_{\mu\nu}$ .

**Equation for the metric  $g_{\mu\nu}$ :**

$$M_{\text{Pl}}^2 G_{\mu\nu}(g) = T_{\mu\nu}^{(\varphi, \vartheta)}(g) + T_{\mu\nu}^{(m)}(g, \varphi),$$

where  $T_{\mu\nu}^{(\varphi, \vartheta)}$  is the stress–energy tensor of the vacuum fields, and  $T_{\mu\nu}^{(m)}$  is the matter contribution expressed via variation of  $S_m$  with respect to  $g_{\mu\nu}$  given  $\tilde{g} = \Omega^2(\varphi)g$ . Observable matter remains covariantly conserved in the physical frame:  $\tilde{\nabla}_\mu \tilde{T}^{\mu\nu} = 0$ .

## 8.6 Relation of VER to the General Scalar–Tensor Form and the Status of the “Physical Frame” $\tilde{g}_{\mu\nu}$

Formally, the action structure adopted in this work,

$$S = S_{\text{vac}}[g_{\mu\nu}, \varphi, \vartheta] + S_m[\psi_m, \tilde{g}_{\mu\nu}], \quad \tilde{g}_{\mu\nu} = \Omega^2(\varphi) g_{\mu\nu}, \quad (8.20)$$

overlaps with the widely studied class of scalar–tensor theories in which matter is minimally coupled to a single “matter metric.” To avoid interpreting VER as a purely terminological repackaging, we record an explicit dictionary to the standard notation.

### 8.6.1 Standard Form and the Matter “Jordan Metric”

A typical scalar–tensor representation (one of several equivalent parameterizations) is

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_{\text{Pl}}^2}{2} A(\Phi) R(g) - \frac{1}{2} B(\Phi) (\partial\Phi)^2 - U(\Phi) \right] + S_m[\psi_m, \hat{g}_{\mu\nu}], \quad \hat{g}_{\mu\nu} = e^{2\alpha(\Phi)} g_{\mu\nu}. \quad (8.21)$$

In this representation  $\hat{g}_{\mu\nu}$  is the **Jordan metric for matter**, i.e., the metric to which the matter sector is minimally coupled and which determines operational measurements (clocks/rods) and the null cones of material signals.

In VER, the status of such a “Jordan metric” is not a free frame choice: it is fixed by P3 (matter universality) and P2 (a single matter causal cone) and implemented in the action via  $S_m[\psi_m, \tilde{g}]$ . Therefore  $\tilde{g}_{\mu\nu}$  should be regarded as the **physical (operational) metric**, not merely a convenient coordinate/unit choice.

### 8.6.2 Dictionary of Notation

In our notation the correspondence with (8.21) is:

## Standard scalar–tensor notation

## VER (this work)

Matter Jordan metric  $\hat{g}_{\mu\nu}$

$\tilde{g}_{\mu\nu}$

Conformal factor  $e^{2\alpha(\Phi)}$

$\Omega^2(\varphi)$

“Einstein-like” metric  $g_{\mu\nu}$

variational metric  $g_{\mu\nu}$  of the vacuum sector

Scalars  $\Phi$

$\varphi$  (and  $\vartheta$  in the extension)

Coupling  $\alpha_0 \equiv d \ln \Omega / d\varphi_c$

$\alpha_*$  (see Sec. 11.3 and Sec. 12)

Here  $\varphi_c$  denotes a canonically normalized field related to  $\varphi$  by the chosen kinetic normalization (in Model-0 it is conveniently defined via a logarithmic variable; see Sec. 11.2 and Appendix C).

### 8.6.3 What Is Specifically “VER” Within the Common Class

Although VER formally uses the general scalar–tensor structure “one matter metric + a conformal factor,” this work highlights two substantive differences fixed at the level of postulates and operational deductions:

- **The physical-frame status of  $\tilde{g}$  is postulated, not conventional.**

P3 forbids direct non-metric couplings of vacuum modes to matter at fixed  $\tilde{g}$ , and P2 fixes a single matter causal cone. This makes the “Jordan frame” part of the physical content rather than an equivalent re-description.

- **Operational energetics and metrology are constructed in  $\tilde{g}$ .**

Secs. 3–5 derive universal relations  $m = \mu/c$ ,  $E_0 = \mu c$ , an exact  $\Delta E_0 - \Delta m$  identity, and an energy bookkeeping via Killing energy  $E_K$  in the physical metric  $\tilde{g}$ . These statements provide observable balances and invariants that are not exhausted by a frame reparameterization and serve as a direct bridge to the phenomenological “gates” (Secs. 11–12).

### 8.7 Methodological Remark on Ordering

The discussion of  $\varphi$ – $\vartheta$  coupling regimes (spectator, kinetic, and potential) is not a modification of postulates P1–P4, but a choice of specific terms in  $S_{\text{vac}}$ , primarily the functions  $K(\varphi)$  and  $V(\varphi, \vartheta)$ . Hence it is methodologically correct to treat these regimes after fixing the covariant scaffold (Sec. 8), where the variational variables, the relation  $\tilde{g} = \Omega^2(\varphi)g$ , and the universal matter source for  $\varphi$  are defined. This logic is implemented in Sec. 9, after which the minimal working model Model-0 is fixed in Sec. 10.

## 9 From the Covariant Scaffold to a Working Model: $\varphi$ - $\vartheta$ Regimes, a Microphysical Bridge, and the Choice of a Dark-Axion Branch

Section 8 fixed the covariant scaffold of VER-I- $\vartheta$ : the theory is defined by an action of the form  $S = S_{\text{vac}}[g_{\mu\nu}, \varphi, \vartheta] + S_m[\psi_m, \tilde{g}_{\mu\nu}]$ , with universal minimal coupling of matter to  $\tilde{g}_{\mu\nu}$  (P3) and a conformal relation  $\tilde{g}_{\mu\nu} = \Omega^2(\varphi) g_{\mu\nu}$  for the chosen realization I. The next step prior to constructing a minimal working model is to specify the **internal structure of the vacuum sector**, i.e., how the two vacuum modes—the amplitude  $\varphi$  (energonicity) and the phase axion-like mode  $\vartheta$ —interact with each other.

We emphasize a methodological point: the “regimes” of  $\varphi$ - $\vartheta$  coupling discussed below are not alternative postulates and do not modify the physical-frame principle; rather, they correspond to **choices of concrete structural terms in the covariant vacuum action**  $S_{\text{vac}}$ , primarily the functions  $K(\varphi)$  and  $V(\varphi, \vartheta)$ . Hence it is appropriate to discuss them after fixing the action scaffold, where the variational variables, the relation  $\tilde{g}_{\mu\nu} = \Omega^2(\varphi)g_{\mu\nu}$ , and the universal matter source structure for  $\varphi$  have been defined. In the same sense, the “microphysical bridge” and the choice of a dark-axion branch do not belong to the postulate core of VER; they provide motivation for the functional form  $\Lambda(\varphi)$  in the periodic potential  $\Lambda^4(\varphi)(1 - \cos \vartheta)$ , while the technical details behind  $\Lambda(\varphi)$  are deferred to Appendix D.

### 9.1 Two Vacuum Modes and the Level of Classification

Within the fixed scaffold  $S_{\text{vac}}[g_{\mu\nu}, \varphi, \vartheta]$  (Sec. 8), the VER-I- $\vartheta$  extension is naturally interpreted as an EFT for a complex order parameter

$$\Psi(x) = \sqrt{\varphi(x)} e^{i\vartheta(x)},$$

where  $\varphi(x) > 0$  is the amplitude (energonicity,  $\varphi = c^2$ ) and  $\vartheta(x)$  is a pseudoscalar phase mode. It is important to stress that the classification below concerns the **structure of the vacuum action**: we distinguish feedback regimes  $\vartheta \leftrightarrow \varphi$  implemented by specific choices of  $K(\varphi)$  and  $V(\varphi, \vartheta)$ , rather than alternatives to realizations I/II/III, which fix only the law  $\tilde{g}_{\mu\nu} = \Omega^2(\varphi)g_{\mu\nu}$  and thereby the way  $\varphi$  enters matter metrology.

### 9.2 Three $\varphi$ - $\vartheta$ Coupling Regimes

From the viewpoint of the action scaffold (Sec. 8), the difference between possible branches of VER-I- $\vartheta$  reduces to **how  $\varphi$  enters the kinetic and potential sector of the phase mode  $\vartheta$** , i.e., to the choice of the functions  $K(\varphi)$  and  $V(\varphi, \vartheta)$  in  $S_{\text{vac}}$ . For the primary article it suffices to distinguish three minimal feedback regimes  $\vartheta \leftrightarrow \varphi$ : (1) a spectator phase, (2) kinetic coupling through  $K(\varphi)$ , and (3) potential coupling through  $\Lambda^4(\varphi)$  in a periodic potential  $\propto (1 - \cos \vartheta)$ . The rationale for choosing the baseline branch for Model-0 is given in Sec. 9.3; microphysical comments on the origin of  $\Lambda(\varphi)$  are deferred to Appendix D.

**Regime 1 (spectator phase; minimal coupling).**

The phase mode  $\vartheta$  is present but has negligible impact on  $\varphi$ . Effectively this corresponds to the absence of a significant  $\varphi$ -dependence in the  $\vartheta$  sector and/or dynamical “freezing” of  $\vartheta$ :

$$V(\varphi, \vartheta) \simeq V_0(\varphi) + V_1(\vartheta), \quad \text{or} \quad \vartheta \simeq \text{const.}$$

**Regime 2 (kinetic coupling; field-space metric).**

Coupling is implemented through a  $\varphi$ -dependent normalization of the  $\vartheta$  kinetic term:

$$S_{\text{vac}} \supset \int d^4x \sqrt{-g} \left[ -\frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}K(\varphi)(\partial\vartheta)^2 - V_0(\varphi) \right], \quad K(\varphi) > 0.$$

Then  $\vartheta$  affects  $\varphi$  via the gradient channel  $K'(\varphi)(\partial\vartheta)^2$  in the  $\varphi$  equation.

**Regime 3 (potential coupling;  $\Lambda(\varphi)$  as the axion-potential amplitude).**

The key regime for a working model is the one in which the amplitude of the periodic  $\vartheta$ -potential depends on  $\varphi$ :

$$V(\varphi, \vartheta) = V_0(\varphi) + \Lambda^4(\varphi)(1 - \cos \vartheta).$$

In this regime the feedback  $\vartheta \leftrightarrow \varphi$  is realized directly through  $\partial_\varphi \Lambda^4(\varphi)$ .

### 9.3 Why Regime 3 Is Chosen as the Baseline Branch

We choose regime 3 as the baseline branch for the minimal working model Model-0 (Sec. 10) because it provides functional feedback  $\vartheta \leftrightarrow \varphi$  while preserving the physical-frame principle and matter universality. The rationale consists of three points.

**(i) Functionality without direct  $\vartheta$ -matter couplings (P3).**

By P3, matter couples only to  $\tilde{g}_{\mu\nu}$ , and direct non-metric couplings to vacuum modes are absent at fixed  $\tilde{g}$ :

$$\left. \frac{\delta S_m}{\delta \chi_A} \right|_{\tilde{g}, \psi_m} = 0, \quad \chi_A \in \{\varphi, \vartheta\},$$

in particular  $\delta S_m / \delta \vartheta = 0$ . Hence  $\vartheta$  does not introduce a direct composition-dependent long-range interaction in the matter sector; its role is confined to vacuum dynamics and feedback on  $\varphi$  via  $V(\varphi, \vartheta)$ .

**(ii) EFT naturalness of a periodic potential.**

A phase mode of a complex order parameter typically enjoys an (approximate) shift symmetry, and the most economical EFT realization of its breaking yields a periodic potential. Therefore a  $\propto (1 - \cos \vartheta)$  structure is minimal and symmetry-motivated rather than “put in by hand.”

**(iii) Controllability: microphysics packaged into  $\Lambda(\varphi)$ .**

In regime 3 the feedback is implemented through the amplitude  $\Lambda^4(\varphi)$  in

$$V(\varphi, \vartheta) = V_0(\varphi) + \Lambda^4(\varphi)(1 - \cos \vartheta),$$

so the microphysical uncertainty is reduced to a single function  $\Lambda(\varphi)$  (and, secondarily, to  $K(\varphi)$ ). This makes it possible to construct a minimal Model-0 without a fully specified UV completion; the motivation for  $\Lambda(\varphi)$  via a hidden sector is deferred to Appendix D.

**Conclusion.** Regime 3 is minimal, EFT-natural, and provides controllable feedback  $\vartheta \leftrightarrow \varphi$  without violating P3; it is therefore used as the baseline branch in Sec. 10.

## 9.4 Why $\vartheta$ Is Interpreted as a Dark Axion in Model-0

For the working branch Model-0 (Sec. 10) we adopt the interpretation of  $\vartheta$  as a **dark axion**, i.e., a phase mode of a hidden sector whose potential is generated by nonperturbative dynamics (typically of confinement type). This choice is motivated by the fact that:

the amplitude  $\Lambda^4$  arises as a hidden-sector scale and need not be postulated “from scratch”;

a  $\varphi$ -dependence  $\Lambda(\varphi)$  naturally emerges through  $\varphi$ -dependent hidden-sector parameters (gauge kinetic function/thresholds), directly supporting regime 3;

this interpretation does not alter the operational structure of VER: matter remains coupled only to  $\tilde{g}$ , not to  $\vartheta$ .

We stress that the “dark axion” choice pertains to **the working model** (a microphysical realization of regime 3), not to the postulate core of VER. It fixes a convenient minimal branch for the subsequent program without making the theory dependent on a specific UV paradigm. An orienting microphysical bridge for  $\Lambda(\varphi)$  and a typology of axions is given in Appendix D.

## 9.5 The Role of Appendices and the Boundaries of the Main Text

To keep the primary paper compact, we separate structural statements needed to formulate VER and construct Model-0 from technical details of motivation and phenomenological estimates. The main text fixes the covariant scaffold, the universal source structure (via  $\tilde{T}$  and  $\Omega(\varphi)$ ), the classification of  $\varphi$ - $\vartheta$  regimes, and the rationale for choosing the baseline branch; the appendices support these points and introduce no new postulates.

- **Appendix D** provides orienting microphysical remarks on the origin of  $\Lambda(\varphi)$  (hidden sector, confinement, and RG transmutation), supporting the regime-3 choice for Model-0.
- **Appendices A and C** collect technical estimates associated with the long-range scalar channel (“fifth force”) and parameter windows for its suppression in realization I, as well as PPN-type formulas (see Sec. 12.3.1).

With this preparation, Sec. 10 fixes Model-0 explicitly by specifying  $U(\varphi)$ , the rigid kinetic structure  $\Psi = \sqrt{\varphi}e^{i\vartheta}$ , and the regime-3 potential  $\Lambda^4(\varphi)(1 - \cos \vartheta)$ , and then analyzes its structural properties and non-cosmological viability gates.

## 10 The Minimal Working Model VER-I- $\vartheta$ (Model-0): Canonical Kinetics, Sources, and Equations

Sections 3–5 developed the operational core of VER in the physical frame  $\tilde{g}_{\mu\nu}$ : local SR kinematics at fixed  $c_0$ , the energetic scalings  $m = \mu/c$  and  $E_0 = \mu c$ , and an “energy bookkeeping” based on Killing energy  $E_K$  in stationary  $\tilde{g}$  (Secs. 4–5). Section 8 fixed a covariant action scaffold  $S = S_{\text{vac}} + S_m$  with universal minimal coupling of matter to  $\tilde{g}_{\mu\nu}$  (P3) and a conformal relation  $\tilde{g}_{\mu\nu} = \Omega^2(\varphi)g_{\mu\nu}$ , and derived the universal matter source for  $\varphi$  proportional to  $d \ln \Omega^2 / d\varphi \tilde{T}$  (Sec. 8.4). Section 9 then classified the  $\varphi$ - $\vartheta$  coupling regimes, motivated the choice of regime 3 (potential feedback) as the baseline branch for a minimal model, and noted that the microphysical motivation for the dark-axion branch is deferred to Appendix D, while the long-range scalar channel is analyzed in Sec. 11 and Appendices A/C (see Sec. 9.5).

The purpose of this section is to fix a minimal, covariantly closed EFT realization (Model-0) of VER-I- $\vartheta$ , in which (i) the action and equations are fully covariant (Sec. 8), (ii) the feedback  $\vartheta \leftrightarrow \varphi$  is implemented in regime 3 (Sec. 9.3), and (iii) the vacuum-sector kinetics is taken in a rigid canonical form derived from the complex order parameter  $\Psi = \sqrt{\varphi} e^{i\vartheta}$ .

### 10.1 Physical Frame and the Choice of Realization I

In Model-0 we fix the minimal representative of realization I (see Secs. 6 and 8.3):

$$\tilde{g}_{\mu\nu} = \Omega^2(\varphi) g_{\mu\nu}, \quad \Omega^2(\varphi) = \varphi^{-1} \quad \Rightarrow \quad \tilde{g}_{\mu\nu} = \varphi^{-1} g_{\mu\nu}.$$

With this choice,

$$c(x) = \sqrt{\varphi(x)}, \quad \sqrt{-\tilde{g}} = \Omega^4 \sqrt{-g} = \varphi^{-2} \sqrt{-g},$$

and all matter is universally coupled only to  $\tilde{g}_{\mu\nu}$  (P3).

### 10.2 Complex Order Parameter $\Psi$ and Rigid $\varphi$ - $\vartheta$ Kinetics

The VER-I- $\vartheta$  extension is interpreted as an EFT of a complex vacuum order parameter

$$\Psi(x) = \sqrt{\varphi(x)} e^{i\vartheta(x)}.$$

We take the vacuum sector such that the kinetic term of  $\Psi$  is canonical:

$$S_{\text{vac}} \supset - \int d^4x \sqrt{-g} g^{\mu\nu} \partial_\mu \Psi^* \partial_\nu \Psi.$$

Then the kinetics of  $\varphi$  and  $\vartheta$  is not chosen arbitrarily, but follows identically:

$$\partial_\mu \Psi = e^{i\vartheta} \left( \frac{1}{2\sqrt{\varphi}} \partial_\mu \varphi + i\sqrt{\varphi} \partial_\mu \vartheta \right), \quad g^{\mu\nu} \partial_\mu \Psi^* \partial_\nu \Psi = \frac{1}{4\varphi} (\partial\varphi)^2 + \varphi (\partial\vartheta)^2.$$

Hence, in the “ $-\frac{1}{2}$ ” normalization we obtain the equivalent form

$$-g^{\mu\nu} \partial_\mu \Psi^* \partial_\nu \Psi = -\frac{1}{2} \left[ \frac{1}{2\varphi} (\partial\varphi)^2 + 2\varphi (\partial\vartheta)^2 \right],$$

so in the notation of Sec. 8.2 (and consistently with the  $Z(\varphi)$ ,  $K(\varphi)$  structure discussed in Sec. 8.5) Model-0 corresponds to

$$Z(\varphi) = \frac{1}{2}\varphi, \quad K(\varphi) = 2\varphi > 0.$$

This is a key “rigidity” advantage of Model-0: the  $\vartheta$ -sector is ghost-free by construction, and the contribution of the  $\vartheta$  kinetic term to the  $\varphi$  equation is fixed by  $K'(\varphi) = 2$ .

### 10.3 The Model-0 Action (Regime 3) and the Absence of Direct $\vartheta$ -Matter Couplings

The full Model-0 action is (see Sec. 8.1 and the regime-3 choice in Sec. 9.3):

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_{\text{Pl}}^2}{2} R(g) - g^{\mu\nu} \partial_\mu \Psi^* \partial_\nu \Psi - U(\varphi) - \Lambda^4(\varphi)(1 - \cos \vartheta) \right] + S_m[\psi_m, \tilde{g}_{\mu\nu}],$$

where  $U(\varphi)$  is the “amplitude” part of the potential (stabilization/scales of  $\varphi$ ), and  $\Lambda(\varphi)$  is the amplitude of the periodic phase potential (dark-axion branch; the motivation for  $\Lambda(\varphi)$  is deferred to Appendix D; see Sec. 9.5).

Since matter depends on  $\tilde{g}_{\mu\nu}$  only (P3), at fixed  $\tilde{g}$  there are no direct non-metric couplings to the vacuum fields; in particular,

$$\left. \frac{\delta S_m}{\delta \vartheta} \right|_{\tilde{g}, \psi_m} = 0.$$

Thus  $\vartheta$  does not introduce a direct composition-dependent interaction in the matter sector; it affects observable physics only through vacuum equations and the background structure of  $\tilde{g}$  (cf. the regime-3 rationale in Sec. 9.3).

## 10.4 Equations of Motion: $\vartheta$ , $\varphi$ , and the Universal $\tilde{T}$ Source

### (a) Equation for $\vartheta$ .

Varying with respect to  $\vartheta$  (equivalently, the phase of  $\Psi$ ) and using the kinetics from Sec. 10.2 yields

$$\nabla_\mu (2\varphi \partial^\mu \vartheta) = \Lambda^4(\varphi) \sin \vartheta.$$

For small oscillations about the minimum  $\vartheta \simeq 0$ ,  $\sin \vartheta \simeq \vartheta$ , and the effective mass of the phase mode scales as

$$m_\vartheta^2(\varphi) \sim \frac{\Lambda^4(\varphi)}{2\varphi} \quad (\vartheta \simeq 0).$$

### (b) Equation for $\varphi$ .

Varying with respect to  $\varphi$  (the amplitude) gives

$$\mathfrak{E}_\varphi[g, \varphi, \vartheta] = -\frac{1}{\sqrt{-g}} \frac{\delta S_m}{\delta \varphi},$$

where the left-hand side  $\mathfrak{E}_\varphi$  contains:

- a  $\square_g \varphi$ -type operator with coefficient  $\propto 1/\varphi$  from the canonical  $\Psi$  kinetics;
- a contribution from the  $\vartheta$  kinetics through  $K'(\varphi)(\partial\vartheta)^2$  with  $K'(\varphi) = 2$ ;
- the potential  $U(\varphi)$ ;
- the regime-3 potential feedback  $\partial_\varphi \Lambda^4(\varphi)(1 - \cos \vartheta)$ .

For the primary paper the crucial element is the right-hand side, i.e., the universal matter source. For the conformal relation  $\tilde{g}_{\mu\nu} = \Omega^2(\varphi)g_{\mu\nu}$  (Sec. 8.4),

$$\frac{\delta S_m}{\delta \varphi} = \frac{1}{2} \sqrt{-\tilde{g}} \left( \frac{d \ln \Omega^2}{d\varphi} \right) \tilde{T}.$$

In our minimal choice  $\Omega^2 = \varphi^{-1}$  one has  $\frac{d \ln \Omega^2}{d\varphi} = -\frac{1}{\varphi}$  and  $\sqrt{-\tilde{g}} = \varphi^{-2} \sqrt{-g}$ . Therefore the fully specialized source term in the  $\varphi$  equation becomes

$$-\frac{1}{\sqrt{-g}} \frac{\delta S_m}{\delta \varphi} = -\frac{1}{\sqrt{-g}} \left( -\frac{1}{2} \sqrt{-\tilde{g}} \frac{\tilde{T}}{\varphi} \right) = \frac{1}{2} \frac{\tilde{T}}{\varphi^3}.$$

Thus, in Model-0 the matter source for  $\varphi$  is universally determined by the trace  $\tilde{T}$  in the physical frame and is fixed to the  $\varphi^{-3}$  scaling in the minimal realization I. This is the central mathematical element linking the covariant scaffold of Sec. 8 to the operational metrology of Secs. 3–5, and it excludes interpreting the theory as a mere “redefinition of units.”

**(c) Equation for  $g_{\mu\nu}$ .**

Varying with respect to  $g_{\mu\nu}$  yields the standard structure

$$M_{\text{Pl}}^2 G_{\mu\nu}(g) = T_{\mu\nu}^{(\Psi)}(g) + T_{\mu\nu}^{(m)}(g, \varphi),$$

where  $T_{\mu\nu}^{(\Psi)}$  includes contributions from  $\Psi$  (i.e.,  $\varphi, \vartheta$ ) and the potentials  $U(\varphi)$ ,  $\Lambda^4(\varphi)(1 - \cos \vartheta)$ , while  $T_{\mu\nu}^{(m)}$  is obtained by varying  $S_m$  with respect to  $g_{\mu\nu}$  given  $\tilde{g} = \varphi^{-1}g$  (see Sec. 8.5 for the general structure). Observable matter remains covariantly conserved in the physical frame:

$$\tilde{\nabla}_\mu \tilde{T}^{\mu\nu} = 0.$$

## 10.5 Structural Conditions of Model-0 and the Nearest Viability “Gates”

Model-0 provides a minimal platform for discussing the viability of VER-I- $\vartheta$  without a developed cosmology.

**Ghost-free phase mode.**

Because  $\Psi$  has canonical kinetics,  $K(\varphi) = 2\varphi > 0$  holds automatically (Sec. 10.2). Thus the  $\vartheta$ -sector is ghost-free by construction.

**Existence of a GR-like regime.**

Recovering standard local kinematics requires regions/configurations where  $\varphi \simeq \varphi_* = \text{const}$  (domain of applicability in Sec. 3.8). In Model-0 this reduces to the existence of a stable background value  $\varphi_*$  ensured by the potential  $U(\varphi)$  together with contributions from  $\vartheta$  and matter.

**Consistency with the operational energetics of Secs. 3–5.**

Since Model-0 does not modify postulate P4, the local relations  $m = \mu/c$ ,  $E_0 = \mu c$ , and the exact identity  $\Delta E_0 = -\Delta m c c'$  (Sec. 3.6) remain valid, as do the definitions of Killing energy and the two transport scenarios (Secs. 4–5). These impose constraints on admissible stationary profiles  $\varphi(x)$  and on the interpretation of energy balances via  $E_K$  in stationary  $\tilde{g}$ .

**A long-range scalar channel (“fifth force”) as the nearest gate.**

The conformal relation  $\tilde{g} = \varphi^{-1}g$  generically induces a scalar channel testable in laboratory, planetary, and astrophysical regimes. Its analysis (Yukawa corrections, PPN/EP bounds, and suppression mechanisms) is the subject of Sec. 11 and Sec. 12; technical details and “allowed windows” are deferred to Appendices A/C.

**The role of  $\vartheta$  as a regulator of vacuum dynamics (without direct matter coupling).**

The regime-3 choice provides a minimal and controllable way to include an additional vacuum degree of freedom without violating matter universality (P3): at fixed  $\tilde{g}$ ,  $\delta S_m/\delta\vartheta = 0$  (Sec. 10.3), so  $\vartheta$  does not generate composition-dependent non-metric interactions. Meanwhile  $\vartheta$  affects the background and stability of  $\varphi$  through the potential  $\Lambda^4(\varphi)(1 - \cos\vartheta)$  and through the  $\varphi$ -dependent kinetic normalization fixed by  $\Psi$  (Sec. 10.2). Near  $\vartheta \simeq 0$ ,  $m_\vartheta^2(\varphi) \sim \Lambda^4(\varphi)/(2\varphi)$  (Sec. 10.4a), making  $\vartheta$  a natural “switch” controlling its participation in vacuum dynamics as  $\varphi$  changes. The microphysical motivation of the dark-axion branch and the form of  $\Lambda(\varphi)$  is deferred to Appendix D.

## 10.6 Summary: What Model-0 Fixes

Model-0 fixes a minimal covariantly closed realization of VER-I- $\vartheta$  on the regime-3 branch (Sec. 9.3), with a rigid  $\varphi$ - $\vartheta$  kinetics derived from the canonical complex order parameter  $\Psi = \sqrt{\varphi}e^{i\vartheta}$ . The key “strong” elements are:

- **Rigid ghost-free kinetics:**  $Z(\varphi) = 1/(2\varphi)$  and  $K(\varphi) = 2\varphi > 0$  by construction (Sec. 10.2).
- **Potential feedback  $\vartheta \leftrightarrow \varphi$ :**  $\Lambda^4(\varphi)(1 - \cos\vartheta)$  as a minimal channel for  $\vartheta$  to affect the vacuum background without direct non-metric matter couplings (Secs. 10.3–10.4).
- **A fully specialized universal matter source in the  $\varphi$  equation:**

$$-\frac{1}{\sqrt{-g}} \frac{\delta S_m}{\delta\varphi} = \frac{1}{2} \frac{\tilde{T}}{\varphi^3},$$

which directly links the covariant scaffold (Sec. 8.4) to the physical frame and operational metrology (Secs. 3–5), thereby excluding an interpretation of the theory as a “redefinition of units.”

**Methodological remark (next steps without cosmology).**

Model-0 provides a minimal platform on which the key viability questions reduce to three controlled directions: **(i) weak-field tests and PPN/EP bounds** (Sec. 11–12; Appendices A/C), **(ii) stationary configurations and energetics** in the physical frame  $\tilde{g}$  consistent with the bookkeeping of Secs. 4–5, and **(iii) stabilization of  $\varphi$**  and the role of  $\vartheta$  in shaping an admissible background  $\varphi \simeq \varphi_*$ . These define the non-cosmological “gates” of the model; cosmological specialization is left to a separate work.

The next section (Sec. 11) addresses the nearest testable consequences: the weak-field scalar channel and the conditions for its suppression in realization I. The microphysical motivation of the dark-axion branch and the origin of  $\Lambda(\varphi)$  are collected in Appendix D (see Sec. 9.5).

## 10.7 EFT Remark: Radiative Stability of P3 and Expected Counterterms

Postulate P3 fixes matter universality: the matter sector depends on vacuum modes  $\chi^A \in \{\varphi, \vartheta\}$  only through the physical metric  $\tilde{g}_{\mu\nu}$ , i.e.

$$S_m = S_m[\psi_m, \tilde{g}_{\mu\nu}], \quad \left. \frac{\delta S_m}{\delta \chi^A} \right|_{\tilde{g}, \psi_m} = 0.$$

From an EFT perspective this is a structural restriction on admissible operators: direct non-metric portals such as  $\varphi \mathcal{O}_{\text{SM}}$  or  $\vartheta \mathcal{O}_{\text{SM}}$  are excluded by definition of the working VER branch. A natural question is radiative stability: can such operators be induced when heavy degrees of freedom are integrated out?

Model-0 has two simplifying features.

- **The phase mode  $\vartheta$  is protected by a shift symmetry.**

In regime 3 the potential  $\propto (1 - \cos \vartheta)$  is interpreted as arising from a weak (nonperturbative) breaking of the shift symmetry  $\vartheta \rightarrow \vartheta + \text{const}$ . Hence local operators with explicit  $\vartheta$ -dependence in the visible sector can arise only if an explicit portal and/or symmetry breaking is present; in the minimal branch no such portal is assumed because  $\vartheta$  does not enter  $S_m$  (Sec. 10.3).

- **For  $\varphi$ , quantum corrections mainly renormalize the vacuum sector rather than the matter-coupling structure.**

Under P3 and diffeomorphism invariance of the matter action, the dominant effect of UV integration is expected to be a renormalization of the vacuum EFT functions

$$U(\varphi), \Lambda(\varphi), Z(\varphi), K(\varphi),$$

and the appearance of suppressed higher-dimensional operators in the vacuum sector (non-linear functions of  $(\partial\varphi)^2$ , mixed structures, etc.). Direct operators of the form  $\varphi \mathcal{O}_{\text{SM}}$  require a UV portal that violates the structural hypothesis P3 and are therefore outside the branch considered in this paper. In this sense P3 is treated as the defining restriction on the EFT branch, while the viability gates (Secs. 11–12) test whether the remaining universal effects (chiefly the Yukawa scalar channel) can be suppressed in an admissible parameter window.

### Remark.

A full EFT power-counting analysis (classification of counterterms, their impact on EP/clock tests, and links to UV realizations) is a natural continuation of the program; for the primary article it suffices to record the role of P3 as a structural restriction on the class of EFTs under consideration.

## 11 Weak-Field Limit and the Scalar Channel (“Fifth Force”) in VER-I- $\vartheta$ : Origin, Yukawa Structure, and Suppression Conditions

Section 10 fixed a minimal, covariantly closed realization of VER-I- $\vartheta$  (Model-0), which includes (i) the conformal relation defining the physical frame,  $\tilde{g}_{\mu\nu} = \varphi^{-1} g_{\mu\nu}$ , (ii) rigid vacuum-mode kinetics derived from the canonical complex order parameter  $\Psi = \sqrt{\varphi} e^{i\vartheta}$ , and (iii) a universal matter source for  $\varphi$ ,

$$-\frac{1}{\sqrt{-g}} \frac{\delta S_m}{\delta \varphi} = \frac{1}{2} \frac{\tilde{T}}{\varphi^3} \quad (\Omega^2 = \varphi^{-1}).$$

The goal of this section is to show that precisely this structure automatically generates, in the weak field, the standard Yukawa scalar channel (a universal “fifth force”), and to formulate the conditions under which it is suppressed so that local tests of gravity can be satisfied. It is crucial to stress that the potential “fifth force” is not sourced by the phase mode  $\vartheta$  (which has no direct matter couplings at fixed  $\tilde{g}$ ; see Sec. 10.3), but by the conformal dependence  $\tilde{g}(g, \varphi)$ , which is typical of scalar–tensor classes.

### 11.1 Origin of the Scalar Channel in the Conformal Class $\tilde{g} = \Omega^2(\varphi)g$

Postulate P3 fixes universal matter coupling to the physical metric:

$$S_m = S_m[\psi_m, \tilde{g}_{\mu\nu}], \quad \tilde{g}_{\mu\nu} = \Omega^2(\varphi) g_{\mu\nu}.$$

Even in the absence of direct non-metric couplings  $\delta S_m / \delta \chi^A|_{\tilde{g}, \psi_m} = 0$  (see P3), the dependence of  $\tilde{g}$  on  $\varphi$  implies that varying with respect to  $\varphi$  produces a nonzero universal source proportional to the trace  $\tilde{T}$  in the physical frame (see Sec. 8.4 and its specialization in Sec. 10.4b). At the linear level this is interpreted as exchange of quanta of the scalar perturbation  $\delta\varphi$  between massive bodies, yielding a Yukawa correction to Newtonian gravity.

Universality (P3) makes the coupling structurally simple (through  $\tilde{T}$ ), yet potentially observable unless it is suppressed by a scalar mass and/or screening.

### 11.2 Linearization Around $\varphi = \varphi_*$ and a Yukawa-Type Equation for $\delta\varphi$

Consider a weak field near an almost flat background and stationary (quasi-static) sources. We write

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad \varphi = \varphi_* + \delta\varphi, \quad |\delta\varphi| \ll \varphi_*,$$

where  $\varphi_*$  is the value around which local stabilization occurs (cf. the requirement  $\varphi \simeq \varphi_*$  in Sec. 10.5).

In Model-0 the kinetic term of the amplitude follows from the canonical  $\Psi$  kinetics (Sec. 10.2):

$$\mathcal{L}_{\text{kin}}(\varphi) = -\frac{1}{4\varphi} (\partial\varphi)^2.$$

In the linear approximation this produces the standard operator  $\nabla^2 \delta\varphi$  with coefficient  $1/(2\varphi_*)$ . The potential  $U(\varphi)$  sets the effective mass of small oscillations about  $\varphi_*$ , which we parameterize via the second derivative of the potential:

$$m_\varphi^2 \equiv 2\varphi_* U''(\varphi_*),$$

where the factor  $2\varphi_*$  reflects the non-canonical normalization of  $\varphi$  when working directly with the variable  $\varphi$  (see Sec. 10.2).

The right-hand side of the  $\varphi$  equation in realization I is universally fixed (Sec. 10.4b):

$$-\frac{1}{\sqrt{-g}} \frac{\delta S_m}{\delta \varphi} = \frac{1}{2} \frac{\tilde{T}}{\varphi^3}.$$

For nonrelativistic matter  $\tilde{T} \simeq -\tilde{\rho}$ , and at  $\varphi \simeq \varphi_*$  the source linearizes as  $\sim -\tilde{\rho}/(2\varphi_*^3)$ . Thus, for a stationary configuration one obtains a Yukawa-type equation

$$(\nabla^2 - m_\varphi^2) \delta\varphi \simeq +\frac{1}{2} \frac{\tilde{\rho}}{\varphi_*^3} \quad (\text{static, nonrelativistic source}).$$

The coefficient on the right-hand side is fixed by the specific choice  $\Omega^2 = \varphi^{-1}$  and the Model-0 structure; below it is repackaged into the standard scalar–tensor coupling parameter (Sec. 11.3).

### 11.3 Standard Yukawa Form and the Effective Coupling $\alpha_*$ (Remark: $\alpha_* = -1/2$ Without Introducing $\varphi_c$ )

The solution of the Yukawa equation for a point mass  $M$  has the form

$$\delta\varphi(r) \propto \frac{e^{-m_\varphi r}}{r}.$$

This implies that the additional contribution to the effective gravitational potential has Yukawa structure, and the interaction can be written in the familiar form

$$\Phi(r) = -\frac{G_N M}{r} (1 + \Delta e^{-m_\varphi r}), \quad \Delta \sim \alpha_*^2,$$

where  $\alpha_*$  is a dimensionless measure of the scalar-channel strength in Planck-normalized units, and  $m_\varphi^{-1}$  is its range.

**Key remark for realization I.** In realization I we have  $\Omega^2 = \varphi^{-1} \Rightarrow \Omega = \varphi^{-1/2}$ , hence

$$\frac{d \ln \Omega}{d \ln \varphi} = -\frac{1}{2}.$$

Upon passing to a canonically normalized scalar perturbation (i.e., a field whose kinetic term is of the standard form  $-\frac{1}{2}(\partial\sigma)^2$  near  $\varphi_*$ ), the corresponding conformal coupling  $\alpha_*$  is of order unity, and for the minimal realization I is fixed numerically as

$$\alpha_* = -\frac{1}{2}.$$

(The detailed normalization lemma and the link to a canonical field are deferred to Appendices A/C.) Therefore, in realization I suppression of the scalar channel cannot be achieved by a parametrically small coupling “by construction”; it must be achieved **dynamically**, primarily via a finite mass  $m_\varphi$  (and/or screening).

## 11.4 Suppression Conditions: The Mass $m_\varphi$ , Interaction Range, and Allowed Regimes

The Yukawa structure implies that the size of the scalar correction is governed by two factors:

**(i) Range**  $\lambda_\varphi \equiv m_\varphi^{-1}$ . To make the channel practically invisible at an experimental scale  $L_{\text{test}}$ , one requires

$$m_\varphi^{-1} \ll L_{\text{test}} \quad \Leftrightarrow \quad m_\varphi L_{\text{test}} \gg 1.$$

Under this condition the exponential factor  $e^{-m_\varphi r}$  suppresses the channel for all  $r \gtrsim L_{\text{test}}$ .

**(ii) Amplitude**  $\Delta \sim \alpha_*^2$ . In realization I,  $\alpha_*$  is order unity (Sec. 11.3), so amplitude suppression is not a “free gift” of the structure; in the minimal Model-0 branch the key mechanism is the finite mass  $m_\varphi$ .

**Screening (optional).** In a broader program one may consider screening mechanisms (chameleon/symmetron, etc.) in which the effective mass and/or coupling become environment-dependent. For the primary paper it is methodologically preferable to keep minimality and treat screening as a separate development branch if “mass alone” fails to pass the gate.

### 11.4.1 Which Experiment Classes Constrain $(\alpha_*, m_\varphi)$ (Structure Without Numerical Values)

Bounds on a Yukawa channel at fixed  $\alpha_*$  are most naturally interpreted as bounds on the range  $\lambda_\varphi = m_\varphi^{-1}$  across different scales. In the primary paper we record only the comparison structure:

- **Short-distance laboratory tests** (torsion balances, micromechanics, Casimir-gradient setups) constrain Yukawa corrections at sub-millimeter distances and yield upper bounds on  $\lambda_\varphi$  when  $|\alpha_*| \sim O(1)$ .
- **Solar-System PPN tests** (Shapiro delay, time-of-flight, perihelia, etc.) constrain  $\gamma(r)$  and other PPN combinations on scales  $\sim 1 \text{ AU}$ . For massive scalar–tensor models  $\gamma$  becomes radius-dependent and is suppressed by the Yukawa factor; if laboratory data already require  $\lambda_\varphi$  to be very short, these tests are typically automatically satisfied (see the explicit formula in Sec. 12.3.1).
- **Astrophysical systems** (binary pulsars, compact objects) may impose additional constraints via possible scalar charges of self-gravitating bodies; this belongs to an extended analysis after passing the basic gates.

Numerical windows for  $\lambda_\varphi$  and  $m_\varphi$  at  $\alpha_* = -1/2$  will be added in a subsequent revision together with specific experimental bounds.

## 11.5 Why $\vartheta$ Does Not Produce a Fifth Force Directly, and How It Can Act Indirectly

In Model-0 the absence of a direct matter source for  $\vartheta$  at fixed  $\tilde{g}$  holds:

$$\left. \frac{\delta S_m}{\delta \vartheta} \right|_{\tilde{g}, \psi_m} = 0 \quad (\text{see Sec. 10.3}).$$

Thus  $\vartheta$  does not generate a direct scalar exchange between bodies. Its influence on weak-field effects can only be **indirect**, via the dynamics of  $\varphi$ , since  $\Lambda(\varphi)$  and  $\partial_\varphi \Lambda^4(\varphi)(1 - \cos \vartheta)$  enter the  $\varphi$  equation (Sec. 10.4b): the phase mode can modify the effective stabilization of  $\varphi$ , and thereby the value of  $m_\varphi$  and the profile  $\delta\varphi(r)$ .

## 11.6 Summary and Transition to Phenomenological Constraints

The scalar channel (“fifth force”) in VER-I- $\vartheta$  arises structurally as a consequence of the conformal relation  $\tilde{g} = \Omega^2(\varphi)g$ : matter universally “sees”  $\varphi$  through  $\tilde{g}$ , while  $\varphi$  “sees” matter through a universal source  $\propto \tilde{T}$  (Secs. 8.4 and 10.4b). In realization I ( $\Omega = \varphi^{-1/2}$ ), the corresponding coupling in canonical normalization is order unity ( $\alpha_* = -1/2$ ; Sec. 11.3), so suppression of the observable channel must be achieved dynamically—primarily through a finite mass  $m_\varphi$ , yielding Yukawa suppression at  $r \gg m_\varphi^{-1}$ .

The next section (Sec. 12) formulates the phenomenological constraints on the suppression parameters (PPN/EP tests and associated admissible “windows”) and ties them to the Model-0 structure. Technical derivations, normalization lemmas, and explicit observable formulas are deferred to Appendices A/C; the microphysical motivation of  $\Lambda(\varphi)$  (dark-axion branch) is deferred to Appendix D.

## 12 Non-Cosmological “Viability Gates”: $(\alpha, m_\varphi)$ , PPN/EP Tests, and VER-Specific Checks

The purpose of this section is to formulate the minimal phenomenological program that any working realization of VER-I- $\vartheta$  (in particular Model-0 from Sec. 10) must pass before one proceeds to cosmological applications. We deliberately restrict attention to tests that do not require a full cosmological history, and we state the criteria in a parametric form.

The key result of the weak-field analysis (Sec. 11) is that for the conformal class

$$\tilde{g}_{\mu\nu} = \Omega^2(\varphi) g_{\mu\nu} \quad (\text{see Sec. 8.3})$$

the effective phenomenology of a “fifth force” and deviations from GR is conveniently parameterized by the pair  $(\alpha, m_\varphi)$ : the dimensionless conformal coupling  $\alpha$  and the scalar mass  $m_\varphi$  (interaction range  $\lambda_\varphi = m_\varphi^{-1}$ ). To avoid ambiguity in notation, we begin by fixing a short dictionary of parameters.

## Box 12.S Weak-Field Parameter Dictionary: $\alpha$ , $\alpha_*$ , $\Delta$ , $\lambda_\varphi$ , and the Link to $\gamma(r)$

In the weak-field analysis of Secs. 11–12 we use the standard massive scalar–tensor parameterization.

### Conformal factor and coupling.

For  $\tilde{g}_{\mu\nu} = \Omega^2(\varphi)g_{\mu\nu}$ , introduce a canonically normalized field  $\phi_c$  and the dimensionless coupling

$$\alpha(\phi_c) \equiv M_{\text{Pl}} \frac{d \ln \Omega(\varphi(\phi_c))}{d\phi_c}, \quad \alpha_* \equiv \alpha(\phi_c = 0). \quad (12.S1)$$

For realization I,  $\Omega^2 = \varphi^{-1} \Rightarrow \Omega = \varphi^{-1/2}$ , one finds  $\alpha_* = -1/2$  (see Sec. 11.3).

### Mass and range of the scalar channel.

$$\lambda_\varphi \equiv m_\varphi^{-1}. \quad (12.S2)$$

### Yukawa correction to the Newtonian potential.

$$\Phi(r) = -\frac{G_N M}{r} \left(1 + \Delta e^{-m_\varphi r}\right), \quad \Delta \sim \alpha_*^2. \quad (12.S3)$$

Equivalently,

$$\frac{\Delta F}{F_N}(r) = \Delta e^{-m_\varphi r}. \quad (12.S4)$$

### The PPN parameter $\gamma(r)$ .

$$\gamma(r) = \frac{1 - \alpha_*^2 e^{-m_\varphi r}}{1 + \alpha_*^2 e^{-m_\varphi r}} \simeq 1 - 2\alpha_*^2 e^{-m_\varphi r} \quad (\alpha_*^2 e^{-m_\varphi r} \ll 1). \quad (12.S5)$$

The same range parameter  $\lambda_\varphi$  controls both (i) the fifth-force Yukawa correction and (ii) practical closeness to GR in the PPN sector.

We now formulate the viability gates, beginning with the most stringent local constraints on the Yukawa channel (Gate A), then the PPN limit (Gate C), and thereafter VER-specific requirements of metrology and stationary energetics in the physical frame  $\tilde{g}$  (Gates D–E), concluding with a bridge-condition toward future cosmology (Gate F).

## 12.1 Gate A: The Fifth Force (Yukawa Correction) and the Basic Suppression Criterion

In the weak-field limit, exchange of the scalar mode yields a Yukawa correction to the Newtonian potential (Sec. 11.3):

$$\Phi(r) = -\frac{G_N M}{r} \left(1 + \Delta e^{-m_\varphi r}\right), \quad \Delta \sim \alpha_*^2. \quad (1)$$

Since in realization I  $|\alpha_*| \sim O(1)$ , the minimal compatibility criterion with local tests is Yukawa suppression at the characteristic experimental scales:

$$m_\varphi^{-1} \ll L_{\text{test}} \quad (\text{equivalently } m_\varphi L_{\text{test}} \gg 1). \quad (2)$$

If this condition fails for the chosen stabilization of  $\varphi$  in Model-0 (Secs. 10.3–10.5), two methodological options remain: (i) strengthen low-energy stabilization (increase  $m_\varphi$  in the vacuum), (ii) treat screening as a separate development branch (beyond the minimal model; see the remark in Sec. 11.4).

## 12.2 Gate B: WEP/EEP and the Absence of a Direct Composition Channel from $\vartheta$

In Model-0 the phase mode  $\vartheta$  has no direct non-metric couplings to matter at fixed  $\tilde{g}$  (Sec. 10.3):

$$\left. \frac{\delta S_m}{\delta \vartheta} \right|_{\tilde{g}, \psi_m} = 0. \quad (3)$$

Therefore  $\vartheta$  does not generate an immediate exchange channel between bodies. The potential “fifth force” in the minimal branch is associated with  $\varphi$  and the conformal dependence  $\tilde{g}(g, \varphi)$  (Secs. 8.3–8.4 and Sec. 11.1).

Methodologically, for the primary article the key problem is suppression of the  $\varphi$ -channel (Gate A). Residual subtleties such as “sensitivity to binding energy” for compact bodies belong to standard scalar–tensor phenomenology and enter naturally in an extended weak-field analysis (Sec. 11).

## 12.3 Gate C: Weak-Field Relativistic Tests and the PPN Limit

Solar-System observations probe not only the Newtonian potential but also the post-Newtonian structure. In conformal scalar–tensor models, PPN deviations from GR are controlled by the same scalar channel that produces the Yukawa correction (Secs. 11.3–11.4).

The operational conclusion for VER-I- $\vartheta$  is that if

$$m_\varphi L_{\text{SS}} \gg 1 \quad (4)$$

at the scale of a given test, then the  $\varphi$ -contribution is suppressed and PPN observables approach their GR values. Hence at the primary-paper level it suffices to record that the single range parameter  $\lambda_\varphi = m_\varphi^{-1}$  controls both (i) the fifth force and (ii) practical closeness to GR in the PPN regime.

### 12.3.1 Explicit $\gamma(r)$ for Massive Scalar–Tensor and the Cassini Logic

For conformal scalar–tensor theories with a massive scalar mode, the standard result is a radius-dependent PPN parameter  $\gamma(r)$  governed by Yukawa suppression. In terms of  $\alpha_*$  and  $m_\varphi$  (Sec.

11.3),

$$\gamma(r) = \frac{1 - \alpha_*^2 e^{-m_\varphi r}}{1 + \alpha_*^2 e^{-m_\varphi r}} \Rightarrow \gamma(r) - 1 \simeq -2\alpha_*^2 e^{-m_\varphi r}. \quad (12.9)$$

in the regime  $\alpha_*^2 e^{-m_\varphi r} \ll 1$ . For realization I,  $\alpha_* = -1/2$  (Sec. 11.3), so  $|\gamma(r) - 1|$  is exponentially suppressed by  $e^{-m_\varphi r}$  with the fixed prefactor  $\alpha_*^2 = 1/4$ .

The practical Cassini logic is as follows. The bound on  $|\gamma - 1|$  is tested at interplanetary distances  $r \sim 1$  AU. If laboratory fifth-force tests already require  $\lambda_\varphi = m_\varphi^{-1}$  to be much smaller than astronomical scales (Gate A, Sec. 12.1), then  $e^{-m_\varphi r}$  at  $r \sim 1$  AU becomes extremely small and the Cassini condition is automatically satisfied. In other words, for version I it is sufficient to close Gate A at laboratory distances; Gate C then follows without additional tuning.

## 12.4 Gate D (VER-Specific): Metrology and Interpreting Bounds on “Varying Constants”

As shown in Sec. 3, VER separates local kinematics (Lorentz structure at fixed  $c_0$ ) from metrology when comparing regions with different  $\varphi$ . For test systems the invariant holds (Sec. 3.5)

$$\mu = mc = \text{const}, \quad c(x) = \sqrt{\varphi(x)}, \quad (5)$$

implying the universal relations (Secs. 3.5–3.6)

$$m(x) = \frac{\mu}{c(x)}, \quad E_0(x) = mc^2 = \mu c(x). \quad (6)$$

Therefore, any bounds on “changing constants” must be formulated in terms of **dimensionless observables**, which depend on the choice of standards (“which clocks/rods”). In the primary article we fix the “gate” methodology:

- choose dimensionless measurable quantities (frequency ratios, comparisons of time/frequency standards, etc.);
- express them through  $\varphi$  and a chosen model of the standard;
- translate experimental limits into allowed variations of  $\varphi$  in stationary conditions.

The key point against the “it’s just units” criticism is that VER contains nontrivial energetic relations that do not disappear under rescalings—e.g., the exact  $\Delta E_0 - \Delta m$  identity at fixed  $\mu$  (Sec. 3.6) and the energy bookkeeping via  $E_K$  (Secs. 4–5).

### 12.4.1 A Concrete Dimensionless Invariant Against the “It’s Just Units” Objection

A common objection to variable- $c$  theories is that observable effects can be removed by redefining units. In VER-I- $\vartheta$ , this objection fails already at the level of dimensionless observables, because (i)

$\varphi$  is a dynamical degree of freedom in a covariant action and (ii) in realization I it has a universal matter source through  $\tilde{T}$  in the physical frame (Sec. 8.4 and the specialization in Sec. 10.4b).

The simplest **dimensionless** example is the relative deviation from Newton’s law measured in laboratory and Solar-System tests. In the weak-field limit of the conformal class  $\tilde{g}_{\mu\nu} = \Omega^2(\varphi) g_{\mu\nu}$  (Secs. 8.3 and 10.1), the scalar mode  $\varphi$  yields a Yukawa correction to the potential (Sec. 11.3):

$$\Phi(r) = -\frac{G_N M}{r} \left(1 + \Delta e^{-m_\varphi r}\right), \quad (7)$$

where  $m_\varphi$  is the scalar mass (range  $\lambda_\varphi \equiv m_\varphi^{-1}$ ) and  $\Delta$  is the dimensionless amplitude. Equivalently, for the force/acceleration one obtains the purely dimensionless ratio

$$\frac{\Delta F}{F_N}(r) = \Delta e^{-m_\varphi r}, \quad (8)$$

where  $F_N \propto 1/r^2$  is the Newtonian force and  $\Delta F$  is the additional scalar contribution.

In standard canonical normalization one introduces the dimensionless conformal coupling (Sec. 11.3 and the dictionary at the beginning of Sec. 12); in the universal case  $\Delta \sim \alpha_*^2$ . For realization I,  $\alpha_* = -1/2$  (Sec. 11.3), so  $\Delta$  is order unity in the absence of suppression, and local viability requires Yukawa suppression  $m_\varphi r \gg 1$  at experimental scales (Secs. 11.4 and 12.1).

The methodological point is the following. The ratio  $\Delta F/F_N$  is a **dimensionless observable**, so no redefinition of units can remove its dependence on  $(\Delta, m_\varphi)$ . Moreover,  $\Delta$  and  $m_\varphi$  are not introduced as “measurement conventions”:  $\Delta$  is fixed by the conformal law  $\Omega(\varphi)$  (through  $\alpha_*$ ), while  $m_\varphi$  is fixed by vacuum-sector dynamics and stabilization of  $\varphi$  (Secs. 10–11). Hence, already at the level of weak-field tests VER predicts changes of **pure numbers** that cannot be reduced to a choice of units.

Remark: this example uses “gravitational” dimensionless observables. Metrological dimensionless quantities (e.g., ratios of different clock standards) may also depend on  $\varphi$ , but their analysis requires an explicit microphysical model of standards; a demonstration case is given in Sec. 12.4.2.

#### 12.4.2 Demonstration Case: A Frequency Ratio in a Stationary Field (Toy Model)

To demonstrate the metrological “pipeline” in VER, consider a minimal toy model of a standard: assume the frequency of a local standard is proportional to the relevant energy scale of the process. For test systems obeying P4, the natural scale is  $E_0(x) = \mu c(x)$  (Sec. 3.5), hence for such a standard  $\nu(x) \propto E_0(x)/\hbar \propto c(x)$ . In a stationary field, comparing frequencies between two static observers at points  $x_1, x_2$  involves two channels:

- a geometric (gravitational redshift) channel through  $\tilde{g}_{tt}$ ,
- an energonic channel through  $c(x) = \sqrt{\varphi(x)}$ .

The resulting structure is

$$\frac{\nu_2}{\nu_1} = \sqrt{\frac{\sqrt{-\tilde{g}_{tt}(x_2)}}{\sqrt{-\tilde{g}_{tt}(x_1)}}} \times \frac{c(x_2)}{c(x_1)}. \quad (12.13)$$

(the first factor is the standard GR result for static observers; the second is the VER-specific renormalization via  $E_0 = \mu c$ ). In the weak field, for small variations  $\varphi = \varphi_* + \delta\varphi$ ,

$$\frac{c(x)}{c_*} = \sqrt{\frac{\varphi(x)}{\varphi_*}} \simeq 1 + \frac{1}{2} \frac{\delta\varphi(x)}{\varphi_*}, \quad (12.14)$$

so an experimental bound on deviations of the frequency ratio from the GR prediction translates directly into a bound on  $\delta\varphi/\varphi_*$  for the configuration in question. This toy case illustrates the method of Sec. 12.4: the choice of standard fixes how a dimensionless observable depends on  $\varphi$ .

Remark: for realistic atomic/nuclear standards the coefficients may depend on microphysics (e.g.,  $\alpha_{em}$ ,  $\Lambda_{\text{QCD}}$ , hyperfine factors). The present example serves only to demonstrate the structure of the calculation; refining it is part of the subsequent program.

## 12.5 Gate E (VER-Specific): Stationary Tests and Killing Energetics in the Physical Frame

In stationary fields, a correct comparison of energies/frequencies between points is performed via Killing energy defined in the physical metric  $\tilde{g}$  (Secs. 4.2–4.4):

$$E_K = \sqrt{-\tilde{g}(\xi, \xi)} E_{\text{loc}}. \quad (9)$$

Section 5 then distinguishes two energy-transport scenarios (free fall vs quasi-static transfer), identifying which quantities are conserved and what is interpreted as external work.

For VER an additional channel is essential: local rest energy depends on  $\varphi$ ,

$$E_0(x) = \mu c(x) = \mu \sqrt{\varphi(x)} \quad (\text{Sec. 3.5}). \quad (10)$$

Hence, in stationary tests (redshift, clock comparisons, energetic balances under holding/transport) two effects can in principle coexist:

- (i) the gravitational factor  $\sqrt{-\tilde{g}(\xi, \xi)}$ ,
- (ii) the energonic factor through  $c(x)$ .

Gate E requires that, within the chosen Model-0 branch, these effects are either suppressed at available precision (consistent with stabilization  $\varphi \simeq \varphi_*$  and small  $\delta\varphi$  in the low-energy vacuum; Sec. 11) or have an unambiguous operational interpretation through a fixed choice of standards (Sec. 12.4).

### 12.5.1 Worked Example: A Linear Yukawa Profile $\delta\varphi(r)$ and an Estimate of Stationary Effects

Consider a spherically symmetric nonrelativistic source of mass  $M$ . In the weak-field regime Sec. 11 yields a Yukawa profile  $\delta\varphi(r) \propto e^{-m_\varphi r}/r$ . For estimates it suffices to use the parametric form

$$\delta\varphi(r) = A \frac{e^{-m_\varphi r}}{r}, \quad (12.15)$$

where  $A$  is proportional to the source and fixed by the universal coupling (Secs. 11.2–11.3). The energonic renormalization of the causal bound is then

$$\frac{\delta c(r)}{c_*} = \frac{1}{2} \frac{\delta\varphi(r)}{\varphi_*} = \frac{1}{2} \frac{A}{\varphi_*} \frac{e^{-m_\varphi r}}{r}. \quad (12.16)$$

Substituting (12.16) into stationary frequency/energy relations (e.g., the demonstration formula (12.13)) shows that the VER-specific correction is suppressed by the same Yukawa factor  $e^{-m_\varphi r}$  that controls the “fifth force” (Gate A, Sec. 12.1). Therefore, if Gate A selects a regime  $m_\varphi^{-1} \ll L_{\text{test}}$ , then for all  $r \gtrsim L_{\text{test}}$  the energonic contribution to stationary frequency/energy effects becomes exponentially small and automatically falls below current sensitivities.

This estimate addresses the methodological question whether Model-0 admits nontrivial profiles  $\varphi(r)$  that (i) do not produce an observable fifth force and (ii) do not spoil stationary clock/energy tests: in the minimal Model-0 branch both classes of constraints are controlled by the same range parameter  $\lambda_\varphi = m_\varphi^{-1}$  (Secs. 11–12).

## 12.6 Gate F (Bridge Condition to Cosmology): Early-Time Heaviness and Isocurvature Control

Although cosmology is not constructed in the present work, it is useful to record a generic consistency criterion: additional scalar modes should not be light in epochs where their quantum fluctuations would generate an unacceptable isocurvature component. Parametrically,

$$m_{\text{eff}} \gtrsim H \quad \text{in the relevant epoch.}$$

In Model-0 the candidates are  $m_\varphi$  (the amplitude mode) and the effective phase mass  $m_\vartheta(\varphi) \sim \Lambda^4(\varphi)/(2\varphi)$  near the minimum (see Sec. 10.4a). Implementing this condition is part of the cosmological continuation of the program.

## 12.7 Summary: A Minimal Non-Cosmological Test Map for VER-I- $\vartheta$

In summary, without adopting a cosmological scenario, the first and most stringent gates for Model-0 are:

- (A) suppression of the Yukawa channel:  $m_\varphi^{-1} \ll L_{\text{test}}$  (Secs. 12.1 and 11.3);
- (B–C) recovery of GR-like behavior on relevant scales (PPN limit) under the same condition (Secs. 12.3 and 12.3.1);
- (D) VER metrology: bounds on “constants” are formulated via dimensionless observables and chosen standards, while Secs. 3–5 provide nontrivial energetic relations (Secs. 12.4–12.4.2);
- (E) stationary tests in the physical frame via  $E_K$  and the “double channel”  $\tilde{g}(\xi, \xi)$  plus  $c(x)$  (Secs. 12.5 and 12.5.1);
- (F) an early-time heaviness condition as a bridge to future cosmology (Sec. 12.6).

This gate map completes the primary paper as a self-contained theoretical object: VER is not only postulated and covariantly closed (Secs. 8 and 10) but also equipped with a transparent, stepwise test program that can be carried out without an immediate transition to a full cosmological model.

### 13 Conclusion. Comparing Standard Critiques of VSL/Axion Approaches: Objection–Reply Theses

Below we summarize the main lines of critique typically raised against VSL and axion-based approaches and the corresponding replies implemented within the architecture of VER-I- $\vartheta$ , with pointers to the relevant parts of the paper. The present work is deliberately positioned as foundational + local tests: it fixes the postulate core, the covariant closure, and the non-cosmological “viability gates,” without constructing a full cosmology (see Sec. 1 and Sec. 12.6).

**(VSL-1) Objection:** “A variable  $c$  is just a redefinition of units; there is no physical content.”

**Reply (VER).** In VER,  $c(x)$  is set by the field  $\varphi(x)$  with covariant dynamics derived from an action  $S = S_{\text{vac}} + S_m$  (Sec. 8). In realization I ( $\tilde{g} = \varphi^{-1}g$ ) the  $\varphi$  equation contains a universal matter source through the trace  $\tilde{T}$  in the physical frame (Sec. 8.4, Sec. 10.4b):

$$-\frac{1}{\sqrt{-g}} \frac{\delta S_m}{\delta \varphi} = \frac{1}{2} \frac{\tilde{T}}{\varphi^3}, \quad (11)$$

which fixes a dynamical mechanism for changing  $c(x) = \sqrt{\varphi(x)}$  and rules out reducing the theory to “units.”

**(VSL-2) Objection:** “It is unclear what clocks and rods measure; the choice of frame is arbitrary.”

**Reply (VER).** The physical frame is fixed postulatorily and implemented in the action: matter is minimally coupled to  $\tilde{g}_{\mu\nu}$  (P3, Sec. 2, Sec. 8.1), and local kinematics is formulated in  $\tilde{g}$  (Sec. 3). Global energy and stationary energetics are also constructed in  $\tilde{g}$  (Secs. 4–5). An explicit dictionary to standard scalar–tensor notation and the status of  $\tilde{g}$  as the Jordan metric of matter are provided in Sec. 8.6 and Appendix B.

**(VSL-3) Objection:** “There is no correct global energetics; it is unclear what is conserved.”

**Reply (VER).** In stationary  $\tilde{g}$ , Killing energy  $E_K = -\Pi \cdot \xi$  exists (Sec. 4.2), and its relation to locally measured energy is  $E_K = \sqrt{-\tilde{g}(\xi, \xi)} E_{\text{loc}}$  (Sec. 4.4). Section 5 then provides an operationally transparent bookkeeping for free fall vs quasi-static transfer, including the exact energonic identity  $\Delta E_0 = -\Delta m c c'$  (Sec. 3.6).

**(VSL-4) Objection:** “The model is not closed; the dynamics of  $c(x)$  is not specified.”

**Reply (VER).** The action scaffold and the structure of the equations are fixed in Sec. 8, and a minimal instance (Model-0) is given in Sec. 10. Sections 11–12 then derive the weak-field Yukawa channel and formulate the non-cosmological viability gates (PPN/EP/metrology/stationary energetics).

**(Ax-1) Objection: “An axion is added ad hoc.”**

**Reply (VER).**  $\vartheta$  is introduced as the phase mode of an order parameter  $\Psi = \sqrt{\varphi}e^{i\vartheta}$ , and the rigid  $\varphi$ - $\vartheta$  kinetics is derived from the canonical kinetics of  $\Psi$  (Sec. 10.2), fixing a ghost-free structure  $K(\varphi) = 2\varphi > 0$ . A periodic potential  $\propto (1 - \cos \vartheta)$  is EFT-natural for a phase mode (Sec. 7.2, Sec. 9.3), and the microphysical motivation for  $\Lambda(\varphi)$  via a hidden sector is provided in Appendix D.

**(Ax-2) Objection: “An axion yields unwanted direct couplings to matter / EP violations.”**

**Reply (VER).** By P3, direct non-metric couplings of  $\vartheta$  to matter are absent at fixed  $\tilde{g}$ :  $\delta S_m / \delta \vartheta = 0$  (Sec. 10.3). There is no direct scalar exchange mediated by  $\vartheta$  between bodies; the principal local-test risk lies in the  $\varphi$  channel ( $\tilde{g} = \varphi^{-1}g$ ), treated as a viability gate in Secs. 11–12.

**(Ax-3) Objection: “There is too much arbitrariness:  $K(\varphi)$  and  $V(\varphi, \vartheta)$  are chosen by hand.”**

**Reply (VER).** In Model-0 the  $\varphi$ - $\vartheta$  kinetic sector is fixed by the canonical kinetics of  $\Psi$ , yielding  $Z(\varphi) = 1/(2\varphi)$  and  $K(\varphi) = 2\varphi > 0$  (Sec. 10.2), thereby reducing functional arbitrariness. The remaining microphysical uncertainty of regime 3 is concentrated in a single function  $\Lambda(\varphi)$ , whose QFT motivation (hidden confinement and RG transmutation) is given in Appendix D.

**(Bridge-1) Objection: “Why choose regime 3 rather than regime 2?”**

**Reply (VER).** Regime 2 produces feedback on  $\varphi$  through  $(\partial\vartheta)^2$ , which can be parametrically small in quasi-static configurations. Regime 3 provides potential feedback via  $\partial_\varphi \Lambda^4(\varphi)(1 - \cos \vartheta)$ , operating already in stationary/quasi-static regimes that are central for Model-0 (Sec. 9.3; Secs. 10.3–10.4).

**(Program-1) Objection: “Without cosmology it is unclear where testability lies.”**

**Reply (VER).** Model-0 provides a pre-cosmological test program: stationary energetics through  $E_K$  (Secs. 4–5), the weak-field Yukawa channel and PPN/EP gates (Secs. 11–12), and VER-specific metrological and stationary checks (the double contribution of  $\tilde{g}$  and  $c(x)$ , Secs. 12.4–12.5). Numerical parameter windows and cosmological specialization are treated as the next stage of the program (Sec. 1 and Sec. 12.6).

## Summary and Status of Results

### Derived/fixed in this paper:

- local kinematics and test-sector metrology in  $\tilde{g}$ , including the exact relations  $m = \mu/c$ ,  $E_0 = \mu c$ , and the identity  $\Delta E_0 = -\Delta m c c'$  (Sec. 3);
- stationary energetics in the physical frame via Killing energy  $E_K$  and operational bookkeeping of free fall vs quasi-statics (Secs. 4–5);
- the covariant action scaffold, the universal matter source for  $\varphi$ , and a dictionary to standard scalar–tensor (Jordan/Einstein) notation (Sec. 8 and Sec. 8.6; Appendix B);
- the minimal working model Model-0 with rigid  $\Psi$ -kinetics and the specialized source  $\frac{1}{2}\tilde{T}/\varphi^3$  (Sec. 10);
- the weak-field Yukawa channel (“fifth force”) in realization I and the radius-dependent PPN

limit  $\gamma(r)$ , as well as a non-cosmological gate map with demonstration metrology and stationary examples (Secs. 11–12).

**Left for future work (next stage of the program):**

- inclusion of numerical experimental windows for  $m_\varphi$  (and, if needed, screening) and a detailed data comparison;
- extended EFT power counting (radiative stability of P3 beyond the minimal remark in Sec. 10.7);
- cosmological specialization (background and perturbations), including implementation of the early-time “heaviness” condition (Sec. 12.6) and isocurvature control.

In this way the primary paper completes the formulation of VER as a covariantly closed, operationally defined theory equipped with a transparent, non-cosmological program of local testability.

## Appendix A. The Status of Postulate P4 and Why It Does Not Reduce to a “Redefinition of Units”

The purpose of this appendix is to clarify the methodological status of postulate P4 and to explain why introducing the test-system invariant  $\mu \equiv m(x)c(x) = \text{const}$  is not a trivial redefinition of units. We also clarify in what sense P4 is an operational principle (i.e., a statement about how idealized test systems behave in the physical frame), and what remains “programmatically” regarding a microphysical justification.

### A.1 P4 as an Operational Principle

In the main text, postulate P4 is formulated as:

- **Test-system invariant**

$$\mu \equiv m(x)c(x) = \text{const}, \tag{A.1}$$

where  $c(x) = \sqrt{\varphi(x)}$  is the local causal bound of the matter sector (P2), and  $m(x)$  is the operationally defined mass of the test system.

- **Test-particle action in the physical frame**

$$S_{pp} = -\mu \int d\tilde{s}, \quad d\tilde{s}^2 = -\tilde{g}_{\mu\nu} dx^\mu dx^\nu. \tag{A.2}$$

This implies geodesic motion in the physical metric  $\tilde{g}_{\mu\nu}$  and the universal consequences in Sec. 3:

$$m(x) = \mu/c(x), \quad E_0(x) = m(x)c^2(x) = \mu c(x). \tag{A.3}$$

**Operational meaning.** P4 does not claim that “mass as a Lagrangian parameter” depends on a choice of units; it asserts that there exists a class of idealized test systems for which the product  $mc$  is invariant when comparing regions with different  $\varphi$ , and whose free motion (in the absence of external forces) is described by the minimal worldline action (A.2) in the physical frame  $\tilde{g}$ .

## A.2 Why P4 Is Not a “Redefinition of Units”

A common objection to variable- $c$  ideas is that one can “rename” the units of time/length so that  $c$  becomes constant. In VER this logic does not eliminate physical content for two reasons.

### (i) P4 Ties Metrology to Energy, and Energy Is Compared via $E_K$ in $\tilde{g}$

Sections 4–5 introduce Killing energy in a stationary physical metric  $\tilde{g}$ :

$$E_K = -\Pi \cdot \xi, \quad E_K = \sqrt{-\tilde{g}(\xi, \xi)} E_{\text{loc}}. \quad (\text{A.4})$$

and formulate an operational “bookkeeping” for free fall and quasi-static transport. In quasi-static transfer the external work, measured “at infinity” (or more generally as global work in a stationary field), is

$$\Delta W_\infty = \Delta E_K. \quad (\text{A.5})$$

This quantity is physically defined (through the work performed by an external agent) and cannot be removed by renaming units.

At the same time, P4 implies an **exact** relation between changes in mass and local rest energy at fixed  $\mu$  (derived in Sec. 3.6):

$$\Delta E_0 = -\Delta m c c'. \quad (\text{A.6})$$

This identity is a statement about the balance of measurable quantities (mass/energy), not about a choice of scales. A unit change may rescale the numerical values of  $m$  and  $E_0$  separately, but it cannot arbitrarily remove or invalidate the identity (A.6) once the measurement procedures are fixed by the physical frame  $\tilde{g}$ .

### (ii) Dimensionless Weak-Field Observables (“Fifth Force”) Cannot Be Eliminated by Rescaling Units

Sections 11–12 show that in realization I a Yukawa channel arises with a dimensionless deviation from Newton’s law:

$$\frac{\Delta F}{F_N}(r) = \Delta e^{-m_\varphi r} \quad (\Delta \sim \alpha_*^2, \alpha_* = -1/2). \quad (\text{A.7})$$

This is a **dimensionless** observable. Therefore no rescaling of length/time units can eliminate its dependence on  $r$  and on the parameters  $(\alpha_*, m_\varphi)$ . Hence, even at the level of local tests, the theory contains physical effects that cannot be reduced to a “units trick.” P4 plays a linking role here: it fixes which quantities are operationally measurable and how the energetic scales of test systems are tied to  $c(x)$ .

### A.3 Why P4 Is Chosen Specifically as $\mu = mc = \text{const}$

From an engineering standpoint, P4 serves two functions in VER.

- **Separating kinematics and metrology.**

Local SR kinematics (Lorentz transformations) holds in each small region at fixed  $c_0 = \sqrt{\varphi(x_0)}$  (Secs. 3.2–3.3). P4 adds a rule for comparing energy scales of test systems between regions with different  $c$ , without introducing sector-dependent portals (consistent with P3).

- **Universal scaling of test systems.**

P4 yields the universal relations (A.3), which then enter the energy balances of Secs. 4–5. This makes it possible to discuss “varying constants” and metrological effects in terms of dimensionless observables (Secs. 12.4–12.5), while keeping the physical frame  $\tilde{g}$  as the operational base.

### A.4 Generality and the Domain of Applicability of P4

It is important to stress that P4 is formulated for **test systems**. This implies:

- the statement applies to idealized probe objects (negligible self-gravity and negligible backreaction on the background);
- possible SEP effects (sensitivity of self-gravitating bodies to the scalar background) belong to the phenomenological “gates” (Secs. 11–12) and require a separate analysis in the spirit of standard scalar–tensor phenomenology;
- the microphysical interpretation of **why**  $\mu = mc$  is invariant is part of the subsequent development (see A.5).

### A.5 What Counts as a Microphysical Program for Justifying P4

P4 can be viewed as an effective rule that should emerge in some class of microphysical realizations compatible with P3. Within the VER development program, natural directions include:

- **A universal mass scale in the physical frame.**

Show that in the chosen EFT/UV branch all dimensional parameters of test systems scale with  $\varphi$  in the same way (e.g., through a single “scale mode”), implying  $m(\varphi) \propto \varphi^{-1/2}$  and hence  $\mu = mc = \text{const}$ .

- **Consistency with conformal structure and time normalization.**

Since  $\tilde{g}$  fixes operational measurements, one may realize  $\mu$  as a consistency requirement between the normalization of clocks/rods and the energetic scales of test systems in the presence of  $\varphi$ .

- **Compatibility with EFT stability of P3.**

A microphysical branch should preserve the absence of direct non-metric portals to visible matter (P3) at the level of admissible counterterms and suppressions (see Sec. 10.7).

In the present paper we do not fix a unique microphysical realization of P4; rather, we use it as an operational postulate of the test sector and derive testable consequences (energy balances in Secs. 3–5 and weak-field “gates” in Secs. 11–12).

## A.6 Summary

Postulate P4 is a key element of the operational architecture of VER: it fixes a universal relation between test mass and the causal bound in the physical frame and yields exact energetic identities used in stationary bookkeeping. Its content does not reduce to a redefinition of units, because (i) it enters covariantly defined energy balances via  $E_K$ , and (ii) together with the action scaffold it generates dimensionless weak-field observables (a Yukawa channel) that cannot be eliminated by rescaling units.

## Appendix B. Two Metrics, the GR Limit, and the Place of VER within Scalar–Tensor/Frame Descriptions

The purpose of this appendix is to (i) systematize the roles of the two metrics  $g_{\mu\nu}$  and  $\tilde{g}_{\mu\nu}$  in VER, (ii) state explicitly in what sense the General Relativity (GR) limit is realized as  $\varphi \rightarrow \varphi_*$  and additional modes relax, and (iii) fix the Jordan/Einstein frame dictionary to the extent needed to read Secs. 8–12 without the impression of mere “repackaging.” This appendix complements Sec. 8.6 but emphasizes **the GR limit** and what counts as physically measurable in VER.

### B.1 Why Two Metrics Appear in VER and Which One Is Physical

VER uses two tensorial objects:

- $g_{\mu\nu}$  is **the variational metric** of the vacuum–gravity sector: it is the metric in terms of which the curvature  $R(g)$  is written, the connection  $\nabla$  in  $S_{\text{vac}}$  is defined, and variations in the covariant scaffold are performed (Secs. 8.1–8.5).
- $\tilde{g}_{\mu\nu}$  is **the physical metric of matter** (operational): it determines measured intervals, clocks/rods, and the null cones of material signals (P1–P3). Matter is minimally coupled only to  $\tilde{g}_{\mu\nu}$ :

$$S_m = S_m[\psi_m, \tilde{g}_{\mu\nu}], \quad \tilde{\nabla}_\mu \tilde{T}^{\mu\nu} = 0. \quad (\text{B.1})$$

In realization I used in this work,

$$\tilde{g}_{\mu\nu} = \Omega^2(\varphi) g_{\mu\nu}, \quad \Omega^2(\varphi) = \varphi^{-1}, \quad \Rightarrow \tilde{g}_{\mu\nu} = \varphi^{-1} g_{\mu\nu}. \quad (\text{B.2})$$

Thus  $g_{\mu\nu}$  and  $\tilde{g}_{\mu\nu}$  are not independent, but they play **different roles**:  $g$  is the action variable, while  $\tilde{g}$  is the measurement variable.

## B.2 The “Jordan Frame” and the “Einstein Frame” in VER: Convention vs Physical Content

In the standard scalar–tensor literature it is common to speak of a “Jordan frame” (where matter is minimally coupled to a metric) and an “Einstein frame” (where the gravitational sector is canonical but matter becomes non-minimally coupled). In VER it is important to distinguish the status of these notions:

- **The Jordan metric of matter** in VER is  $\tilde{g}_{\mu\nu}$  by postulate P3. This is **not a convention**, but part of the operational content:  $\tilde{g}$  fixes clocks/rods and the matter causal cone (P1–P2).
- The metric  $g_{\mu\nu}$  in our scaffold is **variational** and naturally plays an “Einstein-like” role insofar as the term  $M_{\text{Pl}}^2 R(g)/2$  is written in terms of it in Model-0 (Sec. 10.3). However, this “Einstein-likeness” does not mean that  $g$  is the physically measured metric.

Hence, frame language is useful as a technical comparison tool, but “choosing a frame” must not be conflated with the postulated physical status of  $\tilde{g}$ .

## B.3 Conformal Transformations and the Relation between $R(g)$ and $R(\tilde{g})$ (Technical Note)

Since  $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$ , the standard conformal transformation formula for the Ricci scalar gives

$$R(\tilde{g}) = \Omega^{-2} \left[ R(g) - 6 \square_g \ln \Omega - 6 g^{\mu\nu} \partial_\mu (\ln \Omega) \partial_\nu (\ln \Omega) \right], \quad (\text{B.3})$$

and for the volume element

$$\sqrt{-\tilde{g}} = \Omega^4 \sqrt{-g}. \quad (\text{B.4})$$

In realization I ( $\Omega = \varphi^{-1/2}$ ), (B.4) implies  $\sqrt{-\tilde{g}} = \varphi^{-2} \sqrt{-g}$ , which is used in the specialization of the matter source for  $\varphi$  (Sec. 10.4b).

These formulas are a reminder that rewriting the action in  $\tilde{g}$ -variables is possible, but then the gravitational sector no longer remains a simple  $R(\cdot)$  term, while the matter sector stays minimal. In this paper we keep the vacuum action written in  $g$ -variables, while measurements are defined in  $\tilde{g}$ .

## B.4 The GR Limit in VER: $\varphi \rightarrow \varphi_*$ , $\vartheta \rightarrow 0$ , and Recovery of Tensor Gravity

By the “GR limit” in the context of the present paper we mean the regime in which:

- $\varphi$  is stabilized near a constant value  $\varphi_* = \text{const}$  under observational conditions;
- $\vartheta$  relaxes to the minimum of its potential (for Model-0,  $\vartheta \simeq 0$ );
- the scalar Yukawa channel is suppressed on tested scales (Gate A, Sec. 12.1):  $m_\varphi^{-1} \ll L_{\text{test}}$ .

Then, to leading order,

$$\tilde{g}_{\mu\nu} = \varphi_*^{-1} g_{\mu\nu}, \quad (\text{B.5})$$

is merely a global rescaling of the metric. Since  $\varphi_*$  is constant, (B.5) does not change geometric structure (null cones, geodesics, causality) and can be absorbed into the definition of units without producing observable effects. More precisely, all gradients of  $\varphi$  vanish and  $\tilde{g}$  and  $g$  become proportional with a constant factor.

In this limit:

- local kinematics reduces to standard SR in locally inertial coordinates of  $\tilde{g}$  (P1);
- Killing energy in stationary  $\tilde{g}$  coincides with the standard GR construction (Sec. 4);
- weak-field deviations from GR disappear because the Yukawa correction  $\Delta e^{-m_\varphi r}$  is suppressed (Secs. 11–12).

This notion of the GR limit is exactly what matters in the reviewer’s logic: with sufficiently heavy/suppressed scalars the theory becomes practically tensorial on observable scales.

## B.5 Why the “Fifth Force” Does Not Contradict the GR Limit

In realization I the conformal coupling is order unity ( $\alpha_* = -1/2$ , Sec. 11.3), which implies that for  $m_\varphi r \ll 1$  the scalar channel would be comparable in strength to Newtonian gravity. Therefore the GR limit is reached not because the coupling is small, but because:

- either  $m_\varphi$  is sufficiently large so that  $e^{-m_\varphi r}$  suppresses the channel for all tested  $r$ ;
- or a screening mechanism is realized (as an extension of Model-0).

This is fully consistent with the GR-limit logic: in any scalar–tensor extension, suppressing additional modes is part of the viability “gates,” not a formal contradiction.

## B.6 Physical Observables in VER and “Frame Invariance”

Because  $\tilde{g}$  is fixed as the physical matter metric, observables in VER are defined operationally:

- frequencies/times/lengths are defined via  $\tilde{g}$  and chosen standards (Secs. 3.4 and 12.4);

- energies in stationary fields are compared via  $E_K$  and  $E_{\text{loc}}$  in  $\tilde{g}$  (Secs. 4–5);
- weak-field tests are formulated through dimensionless combinations such as  $\Delta F/F_N$  and  $\gamma(r) - 1$  (Secs. 12.4.1 and 12.3.1).

This matters more than formal “frame invariance” of rewriting the action: regardless of field redefinitions, physical meaning is determined by what an observer measures—and in VER that is fixed by the postulates.

## B.7 Summary

In VER, two metrics arise as a separation of roles:  $g_{\mu\nu}$  is the variational variable of the vacuum sector, while  $\tilde{g}_{\mu\nu}$  is the physical matter frame. The conformal relation  $\tilde{g} = \Omega^2(\varphi)g$  realizes matter universality (P3) in a minimal way, and the GR limit corresponds to stabilization  $\varphi \rightarrow \varphi_*$  and suppression of the Yukawa channel on tested scales. This provides a clear methodological picture: VER overlaps with scalar–tensor theories in action form, but fixes the physical frame as an operational postulate and constructs balances/gates in terms of  $\tilde{g}$ , which constitutes the main substantive layer of the primary article.

## Appendix C. Weak-Field Derivation: the Yukawa Channel, $\gamma(r)$ , and the Parameter Dictionary ( $\alpha_*$ , $m_\varphi$ , $\Delta$ )

The purpose of this appendix is to provide a technically transparent (yet compact) derivation of the standard weak-field formulas used in Secs. 11–12: the Yukawa correction to the Newtonian potential and the radius-dependent PPN function  $\gamma(r)$ , and to fix the normalization conventions for the coupling  $\alpha_*$ , the amplitude  $\Delta$ , and the interaction range  $\lambda_\varphi = m_\varphi^{-1}$ . This appendix serves as the “technical tail” to Secs. 11.3–11.4 and Sec. 12.3.1.

### C.1 Weak-Field Expansion and the Observable Metric

In realization I the physical matter metric is

$$\tilde{g}_{\mu\nu} = \Omega^2(\varphi) g_{\mu\nu}, \quad \Omega^2(\varphi) = \varphi^{-1}, \quad (\text{C.1})$$

and all observables (test-body trajectories, frequency comparisons, etc.) are defined in  $\tilde{g}$  (P3).

In a weak static field it is convenient to parameterize the physical metric as

$$d\tilde{s}^2 = -(1 + 2\tilde{\Phi}) dt^2 + (1 - 2\tilde{\Psi}) d\ell^2 + O(v^3), \quad (\text{C.2})$$

and introduce the standard PPN parameter

$$\gamma_{\text{PPN}}(r) \equiv \frac{\tilde{\Psi}(r)}{\tilde{\Phi}(r)}. \quad (\text{C.3})$$

We show below that a massive scalar mode induces a Yukawa correction in  $\tilde{\Phi}$  and a radius dependence of  $\gamma(r)$  suppressed by the same Yukawa factor.

## C.2 Canonical Field and Normalization Conventions (Why $\alpha_* = -1/2$ )

In weak-field analyses it is convenient to pass from  $\varphi$  to a scalar variable with canonical kinetic term. Let  $\sigma$  be a field in which, near the background,

$$L_{\text{kin}}(\sigma) = -\frac{1}{2}(\partial\sigma)^2. \quad (\text{C.4})$$

In Model-0 the kinetics of  $\varphi$  is fixed by the canonical kinetics of  $\Psi$  (Sec. 10.2) and reads

$$L_{\text{kin}}(\varphi) = -\frac{1}{4\varphi}(\partial\varphi)^2. \quad (\text{C.5})$$

This implies the relation between the canonical field and  $\varphi$ :

$$\left(\frac{d\sigma}{d\varphi}\right)^2 = \frac{1}{2\varphi^2} \Rightarrow \sigma = \frac{M_{\text{Pl}}}{\sqrt{2}} \ln\left(\frac{\varphi}{\varphi_*}\right) + \text{const}, \quad (\text{C.6})$$

where  $\varphi_*$  is the reference (background) value used in the linearization (Sec. 11.2).

**Coupling convention.** In the main text and in “Box 12.S” we use the standard scalar–tensor definition of the coupling via the conformal factor  $\Omega$ :

$$\alpha(\phi_c) \equiv M_{\text{Pl}} \frac{d \ln \Omega}{d \phi_c}, \quad \alpha_* = \alpha(\phi_c = 0). \quad (\text{C.7})$$

Here  $\phi_c$  is a canonical field **in the adopted convention**. Because the normalization of a canonical field may differ by an overall numerical factor (which does not change the physics but redistributes numerical coefficients between the field definition and  $\alpha$ ), we choose the convention consistent with the main text:

$$\phi_c \equiv M_{\text{Pl}} \ln\left(\frac{\varphi}{\varphi_*}\right) = \sqrt{2} \sigma. \quad (\text{C.8})$$

In this convention, for realization I we have  $\Omega = \varphi^{-1/2}$ , hence

$$\ln \Omega = -\frac{1}{2} \ln \varphi = -\frac{1}{2} \frac{\phi_c}{M_{\text{Pl}}} + \text{const}, \quad \Rightarrow \alpha_* = -\frac{1}{2}. \quad (\text{C.9})$$

Remark: if one uses the “strictly canonical” normalization  $\sigma$  in (C.6),  $\alpha$  rescales numerically, but the final physical combinations (the Yukawa amplitude and  $\gamma(r)$ ) are unchanged. In the main text,  $\alpha_* = -1/2$  is fixed in the convention (C.8)–(C.9) used in Secs. 11–12.

## C.3 The Yukawa Equation and the Point-Source Solution

Section 11.2 yields a linear Yukawa-type equation for  $\delta\varphi$ :

$$(\nabla^2 - m_\varphi^2) \delta\varphi \simeq (\text{source}). \quad (\text{C.10})$$

For a static point source of mass  $M$  the solution has the standard form

$$\delta\varphi(r) = A \frac{e^{-m_\varphi r}}{r}, \quad (\text{C.11})$$

where  $A$  is fixed by the source normalization (universal under P3) and the chosen field convention. Phenomenologically, the scalar contribution is most conveniently parameterized as a Yukawa correction to the Newtonian potential:

$$\tilde{\Phi}(r) = -\frac{G_N M}{r} (1 + \Delta e^{-m_\varphi r}), \quad (\text{C.12})$$

where  $\Delta$  is the dimensionless amplitude of the scalar channel. Equivalently,

$$\frac{\Delta F}{F_N}(r) = \Delta e^{-m_\varphi r}. \quad (\text{C.13})$$

#### C.4 Relating the Amplitude $\Delta$ to $\alpha_*$

For universal (composition-independent) conformal coupling in the weak field, the Yukawa amplitude is determined by the coupling  $\alpha_*$  (in the adopted convention):

$$\Delta \simeq \alpha_*^2. \quad (\text{C.14})$$

For realization I, (C.9) gives  $\alpha_*^2 = 1/4$ : the amplitude is order unity and suppression of deviations is achieved mainly by the exponential  $e^{-m_\varphi r}$  (Gate A, Sec. 12.1).

#### C.5 Radius-Dependent $\gamma(r)$ and Yukawa Suppression

In the massive scalar–tensor class, the standard weak-field result for the PPN parameter  $\gamma(r)$  is governed by the same Yukawa factor:

$$\gamma(r) = \frac{1 - \alpha_*^2 e^{-m_\varphi r}}{1 + \alpha_*^2 e^{-m_\varphi r}}, \quad \gamma(r) - 1 = -\frac{2\alpha_*^2 e^{-m_\varphi r}}{1 + \alpha_*^2 e^{-m_\varphi r}} \simeq -2\alpha_*^2 e^{-m_\varphi r}, \quad (\text{C.15})$$

in the regime  $\alpha_*^2 e^{-m_\varphi r} \ll 1$ . This is the formula used in Sec. 12.3.1. An important practical corollary follows: if Gate A (laboratory bounds on  $\lambda_\varphi = m_\varphi^{-1}$ ) requires  $\lambda_\varphi$  to be much smaller than astronomical scales, then  $e^{-m_\varphi r}$  at  $r \sim 1$  AU becomes extremely small and the Cassini bound on  $|\gamma - 1|$  is automatically satisfied.

#### C.6 What Is Used in the Main Text

From this appendix it is sufficient to import into the main text:

- the phenomenological Yukawa parameterization (C.12)–(C.13);

- the relation  $\Delta \simeq \alpha_*^2$  and the value  $\alpha_* = -1/2$  for realization I, (C.9) and (C.14);
- the radius-dependent formula  $\gamma(r)$ , (C.15);
- the definition of the range  $\lambda_\varphi = m_\varphi^{-1}$  as the key gate parameter (Secs. 12.1 and 12.3).

This suffices to make the weak-field claims in Secs. 11–12 reviewable without overloading the main text with normalization details.

## Appendix D. Microphysical Bridges for VER-I- $\vartheta$ : the Quantum-Gravity Landscape, the “Ontology of the Vacuum,” and the Choice of a Dark Axion

This appendix is **orientational** in nature. Its goals are (i) to show that interpreting  $\varphi$  as the amplitude of a vacuum order parameter and  $\vartheta$  as a phase (axion-like) mode is not arbitrary but has natural analogues in several major lines of modern theoretical physics; (ii) to explain why the working model (Model-0, Sec. 10) selects regime 3 (Sec. 9.3) and an axion mode of the “dark axion” type; and (iii) to present a minimal QFT mechanism in which  $\Lambda(\varphi)$  arises not “by hand,” but through a hidden sector and RG transmutation. This appendix does **not** aim to survey quantum gravity; it records only those elements directly relevant to the vacuum sector of VER.

### D.1 Quantum Gravity Programs and Notions of the Vacuum: A Minimal Typology

Below we list major research programs that “ontologize” the vacuum in different ways. For VER, what matters is not their status, but whether they provide a natural language for **macroscopic vacuum parameters** (order parameters) and **pseudoscalar phase modes**.

#### (a) String theory and the low-energy EFT limit (string/SUGRA/EFT)

The vacuum is described as a family of low-energy effective theories with a rich spectrum of scalars: moduli, dilatons, axions, forms and their dual fields. Typical features include:

- conformal frames and transformations between them;
- axions as phase/dual degrees of freedom;
- nonperturbative effects (instantons, hidden-sector confinement) generating periodic potentials  $1 - \cos \vartheta$ ;
- $\varphi$ -dependent gauge kinetic functions and thresholds, making a  $\Lambda(\varphi)$  dependence natural.

This line provides a **mathematically controllable bridge** to concrete EFT structures for  $S_{\text{vac}}$ ,  $K(\varphi)$ , and  $V(\varphi, \vartheta)$ .

### (b) Loop Quantum Gravity (LQG)

The vacuum is treated as a state of quantum geometry (spin networks/spin foams), with discrete geometric degrees of freedom being primary. Effective modes may exist at large scales, but LQG does not, by default, single out a unique scalar order-parameter amplitude analogous to  $\varphi$ ; hence a direct bridge to  $\Psi = \sqrt{\varphi}e^{i\vartheta}$  is less immediate.

### (c) Asymptotic Safety

Vacuum dynamics is expressed through RG flows and the existence of a UV fixed point. This program is natural for the language “vacuum as an RG object,” but a direct link to an axion-like phase mode and a  $\Lambda(\varphi)$  mechanism is not a default feature.

### (d) Causal Dynamical Triangulations (CDT)

The vacuum is a statistical state of an ensemble of causal triangulations. A direct identification of a complex order parameter  $\Psi = \sqrt{\varphi}e^{i\vartheta}$  is not typical, but the general idea of “vacuum as a collective state” is, in principle, compatible with order-parameter language.

### (e) Group Field Theory (GFT) and condensate/emergent-gravity scenarios

The vacuum is treated as a collective state (a condensate) of microscopic degrees of freedom, with geometry emerging as an effective macroscopic variable. For VER this is a key line because **order parameters** are standard here, and the pair  $\Psi = \sqrt{\varphi}e^{i\vartheta}$  appears conceptually natural: amplitude and phase are independent macromodes of the vacuum.

### (f) Analog/emergent-gravity line (“vacuum as a medium”)

In a number of approaches the vacuum is interpreted as an effective medium (analog metrics, hydrodynamic models). This is intuitive but carries the risk of “multi-cone” behavior: different matter modes may “see” different effective metrics. For VER this is critical because P2–P3 fix a single matter causal cone and matter universality. Hence “medium” intuitions are admissible only as heuristics under a strict requirement that all matter share a single  $\tilde{g}$ .

## D.2 Two Selected Lines and Their Roles for VER-I- $\vartheta$

We deliberately adopt two complementary microphysical intuitions—one philosophical/logical, the other mathematical. This reduces the risk of “arbitrary mythology”: one line explains why the vacuum may have an order parameter, the other explains how to obtain concrete EFT structures for  $\Lambda(\varphi)$ ,  $K(\varphi)$ , and  $V(\varphi, \vartheta)$ .

### D.2.1 Philosophical–logical bridge: condensate/emergence (GFT/condensate)

What matters here is not a specific formalism but the ontology: the vacuum is a collective state, and macroscopic fields are order parameters. Then:

- $\varphi$  is naturally interpreted as the amplitude of a vacuum order parameter, setting the operational cone parameter  $c(x) = \sqrt{\varphi(x)}$ ;
- $\vartheta$  is a natural phase mode, pseudoscalar by transformation properties;

- regime 3 (Secs. 9.2–9.3), in which  $\vartheta$  backreacts on  $\varphi$  via  $\Lambda^4(\varphi)(1-\cos\vartheta)$ , acquires the meaning of “internal” vacuum dynamics of the order parameter.

### D.2.2 Mathematical bridge: string/SUGRA/EFT + hidden sector

This line provides standard tools that VER uses structurally:

- conformal frames and the universal relation  $\tilde{g}_{\mu\nu} = \Omega^2(\varphi)g_{\mu\nu}$ ;
- axions as phases/dual fields with (approximate) shift symmetry;
- periodic potentials as nonperturbative effects;
- modulus-dependent amplitudes via gauge kinetic functions, thresholds, and RG transmutation.

Regime 3 becomes technically controllable: the complexity is concentrated in  $\Lambda(\varphi)$  and  $K(\varphi)$ , whose forms admit a microphysical interpretation.

### D.3 Axion Types and Selection Criteria for Model-0

In contemporary theory, an “axion” typically refers to a pseudoscalar phase mode with periodicity and an (approximate) shift symmetry. For VER it is essential that the axion mode:

- does not violate P3, i.e., does not require direct couplings to matter;
- naturally yields a periodic potential;
- allows one to explain the amplitude  $\Lambda^4$  and its dependence  $\Lambda(\varphi)$  microphysically.

In this sense:

- a **QCD axion** is too tightly tied to the Standard Model and the strong-CP problem, which would overload a primary manuscript and open additional constraint channels;
- generic **ALPs** allow a wide range of masses/couplings but do not, by themselves, explain the origin of  $\Lambda(\varphi)$ ;
- a **dark/hidden axion** is a phase mode of a hidden sector, whose potential arises nonperturbatively in hidden dynamics without requiring direct couplings to visible matter.

**Why a dark axion is chosen for Model-0.** The dark-axion choice is motivated because it simultaneously:

- makes  $\Lambda^4$  a natural hidden-sector scale rather than a hand-chosen parameter;
- allows a  $\varphi$ -dependence  $\Lambda(\varphi)$  through  $\varphi$ -dependent hidden-sector parameters (kinetics/thresholds), supporting regime 3;
- remains compatible with P3:  $\vartheta$  is fully “hidden” with respect to matter ( $\delta S_m/\delta\vartheta = 0$ ), so the main phenomenological risk is concentrated in the  $\varphi$  channel.

## D.4 A Minimal QFT Picture for $\Lambda(\varphi)$ : Confinement, RG Transmutation, and a Power Law

We fix a standard scheme explaining the origin of  $\Lambda(\varphi)$  in the potential

$$V_{\text{ax}}(\varphi, \vartheta) = \Lambda^4(\varphi) (1 - \cos \vartheta). \quad (12)$$

### D.4.1 Confinement scale as an RG invariant

Let a hidden sector contain a non-Abelian gauge group  $G_h$  (e.g.,  $SU(N)$ ) with a running coupling  $g_h(\mu)$ . The confinement scale  $\Lambda_h$  is then generated by RG transmutation:

$$\Lambda_h \sim \mu \exp\left[-\frac{8\pi^2}{b_0 g_h^2(\mu)}\right], \quad (13)$$

where  $b_0$  is the one-loop beta-function coefficient. Nonperturbative confinement effects generate a periodic  $\vartheta$ -potential with amplitude of order  $\Lambda_h^4$ , i.e.

$$\Lambda^4(\varphi) \sim \Lambda_h^4(\varphi). \quad (14)$$

### D.4.2 Where the $\varphi$ -dependence comes from

It suffices for  $\varphi$  (or a canonical  $\phi_c$ ) to enter the hidden sector via a standard EFT structure, e.g. a gauge kinetic function:

$$L \supset -\frac{1}{4} f(\phi_c) F_h^2 \quad \Rightarrow \quad g_h^{-2} \propto f(\phi_c). \quad (15)$$

Then

$$\Lambda_h(\phi_c) \sim \mu \exp\left[-\frac{8\pi^2}{b_0} f(\phi_c)\right], \quad \Lambda^4(\phi_c) \sim \Lambda_h^4(\phi_c). \quad (16)$$

Even a weak  $\phi_c$ -dependence of  $f(\phi_c)$  produces a strong (exponential) sensitivity of  $\Lambda(\phi_c)$ , which makes regime 3 functional:  $\partial_\varphi \Lambda^4(\varphi)$  can be substantial without any direct matter couplings.

### D.4.3 Why a power law $\Lambda(\varphi) = \Lambda_*(\varphi/\varphi_*)^{-\beta}$ is admissible in Model-0

If  $f(\phi_c)$  is approximated linearly near the vacuum value  $\phi_c = 0$ ,

$$f(\phi_c) \simeq f_* + \kappa \frac{\phi_c}{M_{\text{Pl}}}, \quad (17)$$

then

$$\Lambda(\phi_c) \propto \exp\left(-\beta \frac{\phi_c}{M_{\text{Pl}}}\right), \quad \beta = \frac{8\pi^2}{b_0} \kappa. \quad (18)$$

With  $\phi_c = M_{\text{Pl}} \ln(\varphi/\varphi_*)$  one obtains a power law:

$$\Lambda(\varphi) = \Lambda_* \left(\frac{\varphi}{\varphi_*}\right)^{-\beta}. \quad (19)$$

Thus the parameterization of  $\Lambda(\varphi)$  used in Model-0 is a local EFT reduction of a standard QFT mechanism (hidden confinement plus  $\varphi$ -dependent parameters).

## D.5 Takeaway for the Paper

The combination “condensate ontology of the vacuum” + “string/EFT mechanics of a hidden sector” provides two key outcomes: (i) a philosophical–logical naturalness of  $\Psi = \sqrt{\varphi} e^{i\vartheta}$  as a vacuum order parameter, and (ii) a mathematically controllable motivation of regime 3 and the dark-axion choice through the emergence of  $\Lambda(\varphi)$ . This is the level of microphysical motivation sufficient for a primary paper that does not yet proceed to a cosmological specialization.

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